

# Chapter 10

## Motion Planning

### Part 3

#### 10.3 Real Time Global Motion Planning



# Outline

- 10.3 Real Time Global Motion Planning
  - 10.3.1 Introduction
  - 10.3.2 Depth Limited Approaches
  - 10.3.3 Anytime Approaches
  - 10.3.4 Plan Repair Approach: D\* Algorithm
  - 10.3.5 Hierarchical Planning
  - Summary

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# 10.3.1 Introduction

- Unknown and dynamic environments can be **treated similarly** because a dynamic environment is partially unknown.
- Unknown Environments
  - Limited perception limits what you can know.
  - Often, the only way to learn more is to move.
  - You may eventually learn that the path you are on is wrong.
- Dynamic Environments
  - Limited prediction fidelity limits what you can know.
  - Often, the only way to learn more is to wait.

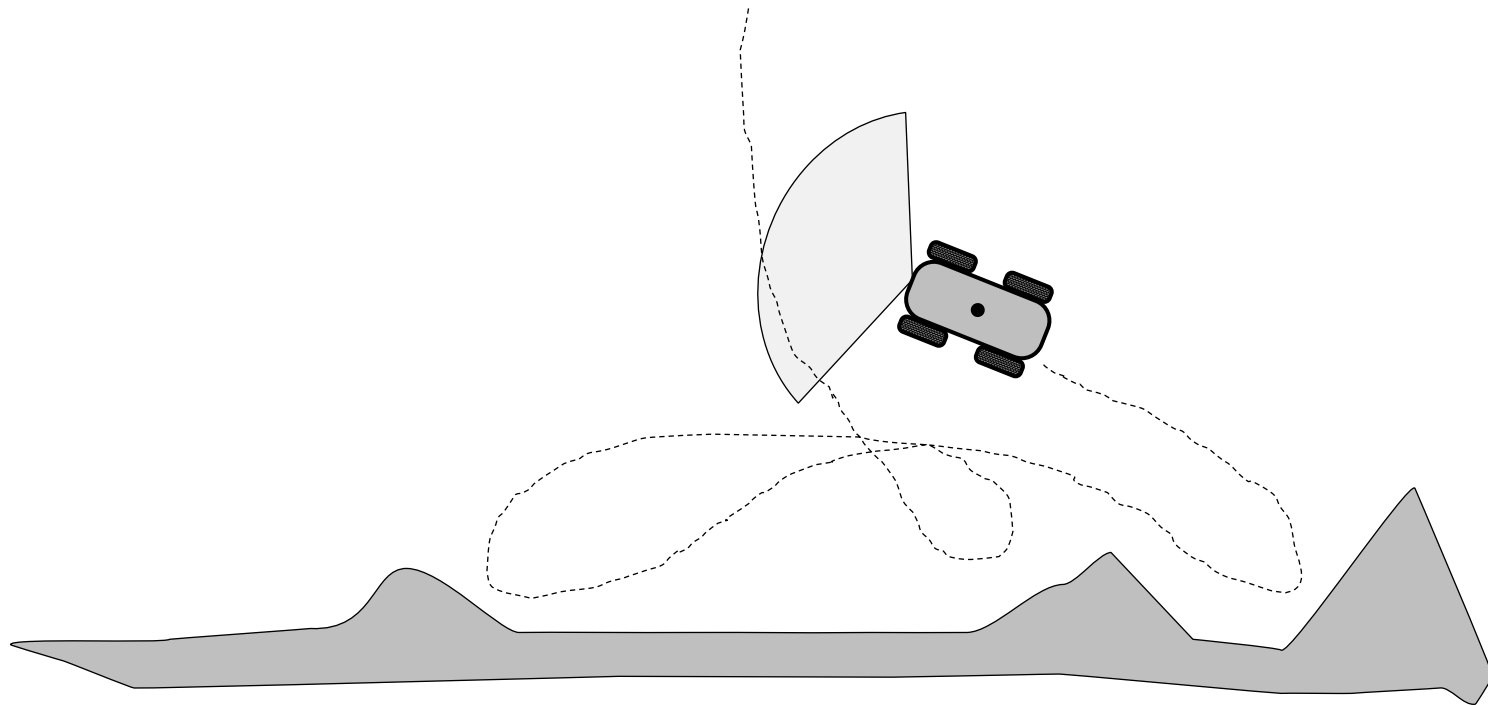
# 10.3.1 Introduction

(Thinking vs Doing)

- Often, it is possible to trade off the cost of execution and planning.
  - More planning time makes better use of available information.
  - More motion gathers more information.
- Sometimes its better to stop and think, other times not.

# 10.3.1.1 Unknown Environments

## (Changing Strategy)



- It is not unusual for a robot to continue to change its mind as it learns new information.

# 10.3.1 Introduction

(Four Techniques)

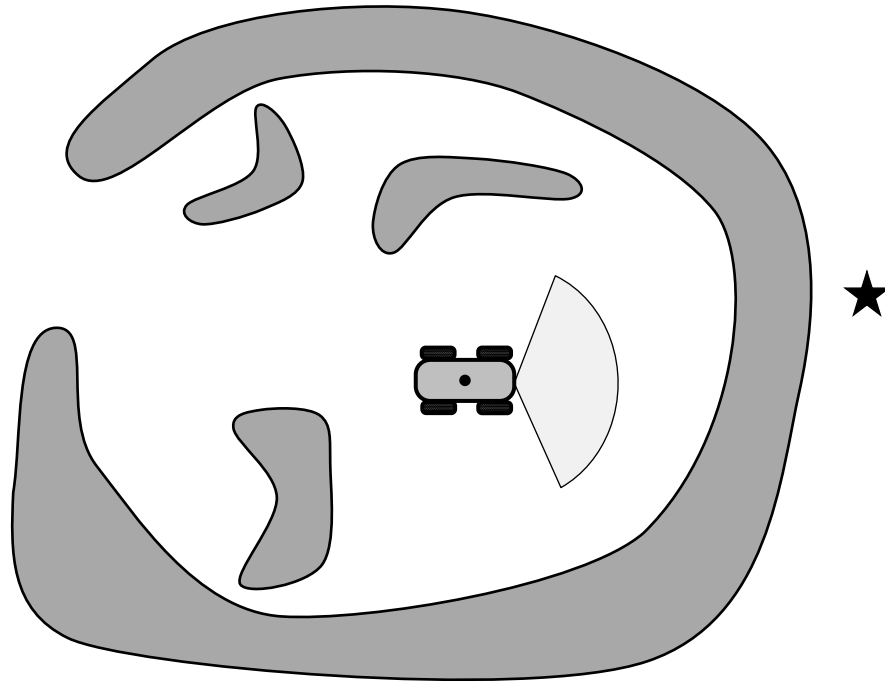
- Four techniques are available to deal with the real time / limited computation issues:
  1. Limited Horizon
    - Don't predict too far
  2. Anytime Approaches
    - Always have an answer available
  3. Plan Repair
    - Reuse elements of last plan.
  4. Hierarchical Planning
    - Ignore detail when possible

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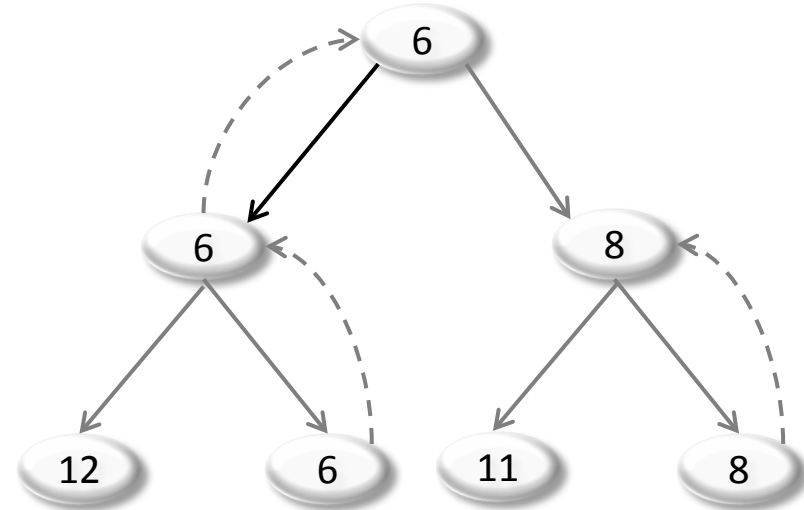
# 10.3.2.1 Purely Reactive Planning



- Search is conducted physically with the robot.
  - Bias toward goal added
- However, the right answer (above) is to move away from the goal for a while.
- Cyclic behavior is a common failure mode.

## 10.3.2.2 Depth Limited Planning

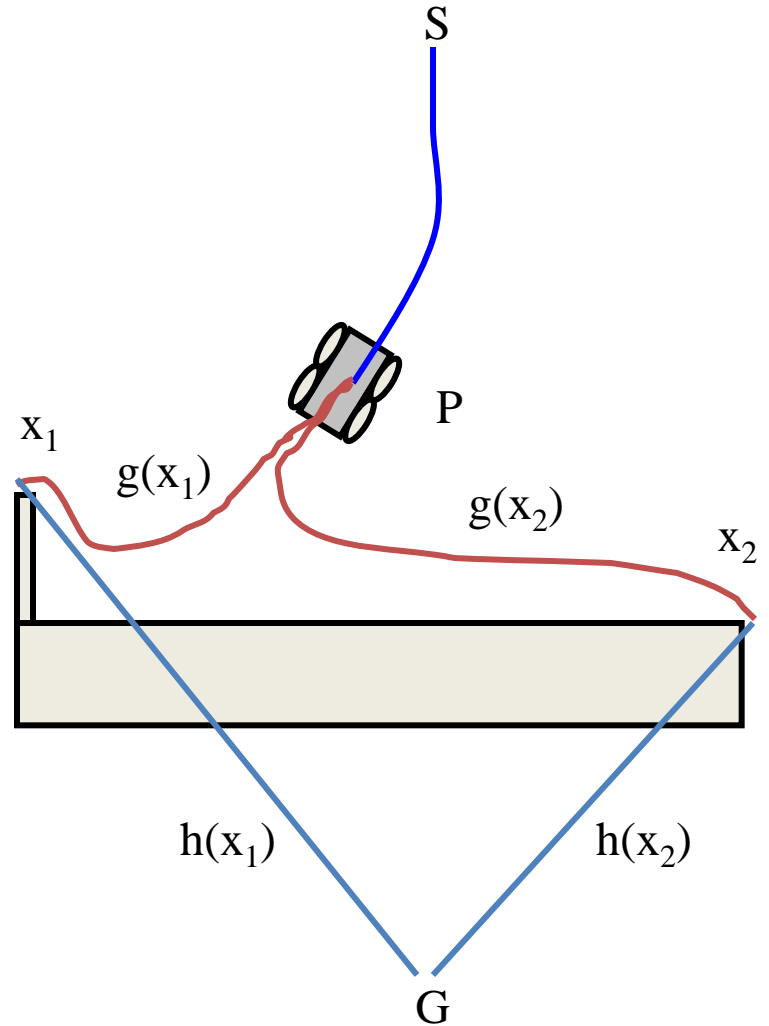
- Same as receding horizon predictive control.
- Propagate **best child** up the tree ...
- Then, takes the first step toward the best leaf.
- and repeat.





# Real Time A\* (Korf)

- In RTA\* we re-interpret  $g(X)$  to mean the cost to get **from the current state** to state  $X$  rather than the cost from the original initial state - which is irrelevant once motion takes place.
- Net effect is to **permit physical backtracking** to an earlier visited state if the benefit of doing so outweighs the cost.
- This planner and all unknown environment planners are subject to **strategy waffling** (cycles).



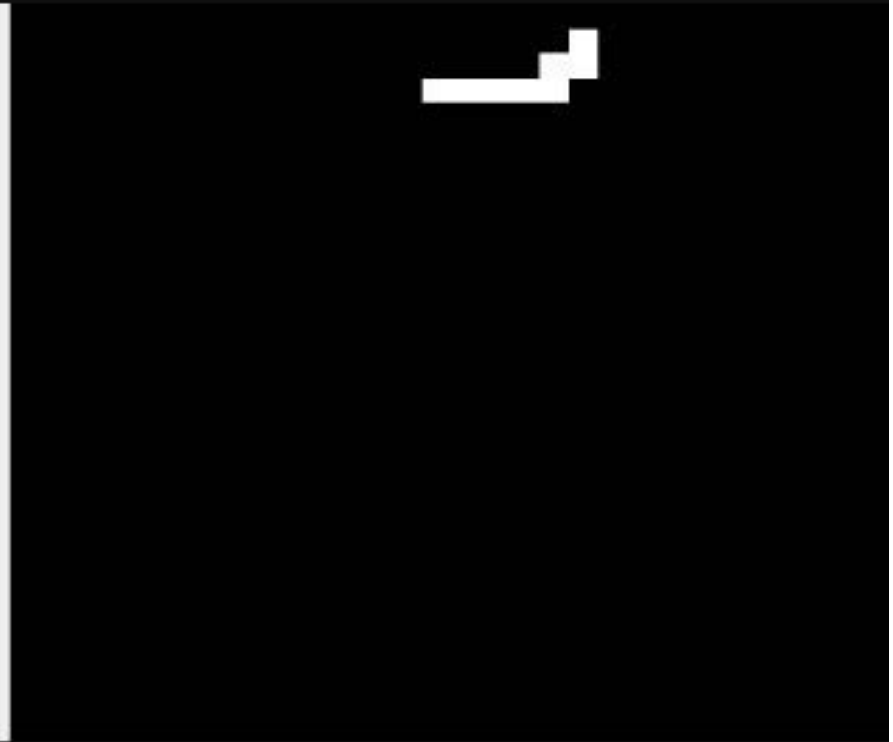
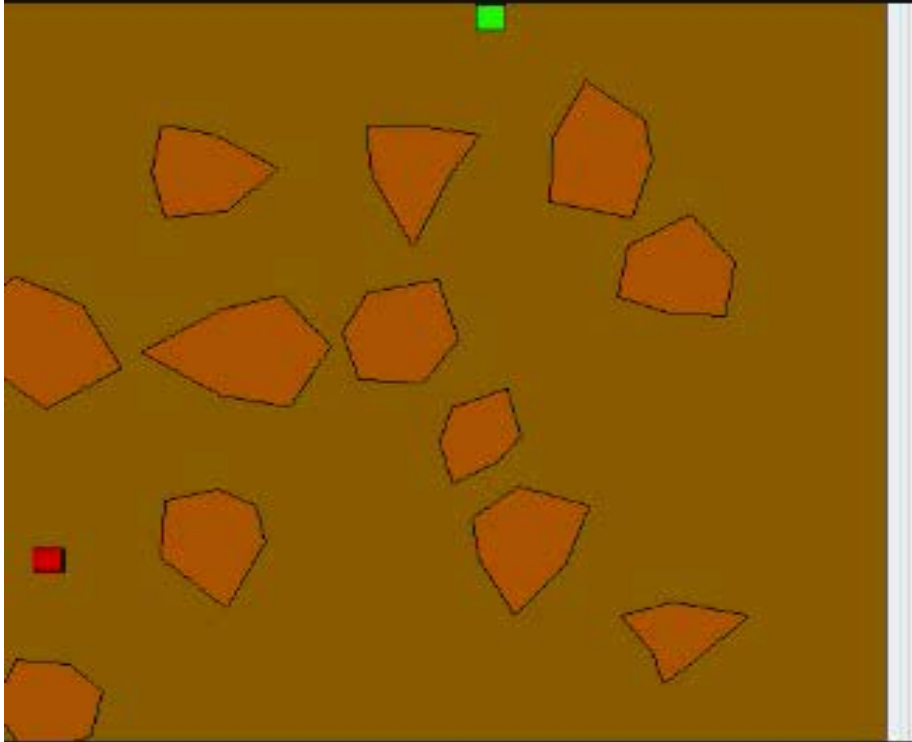
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  - 10.3.3 Anytime Approaches - Skip
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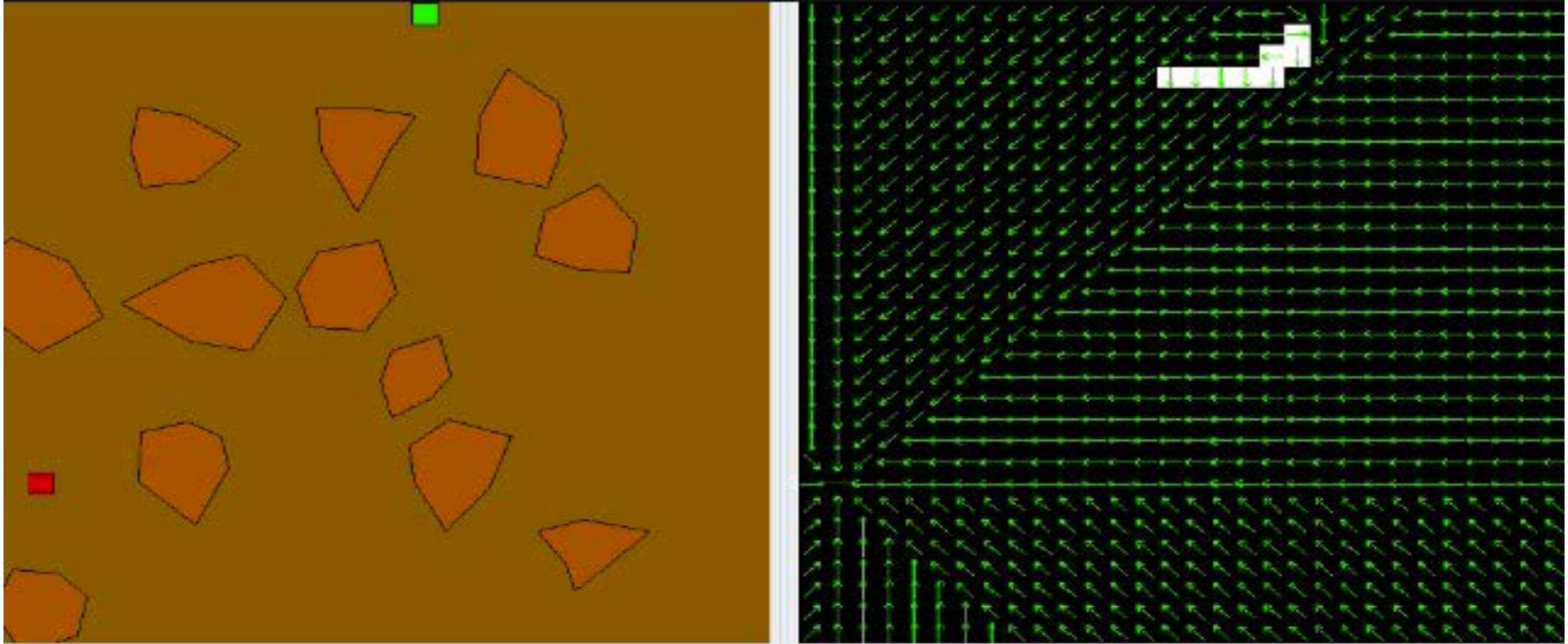
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# A\* Replanning is Still Too Slow



# Replanning Done Right D\* (Lite)

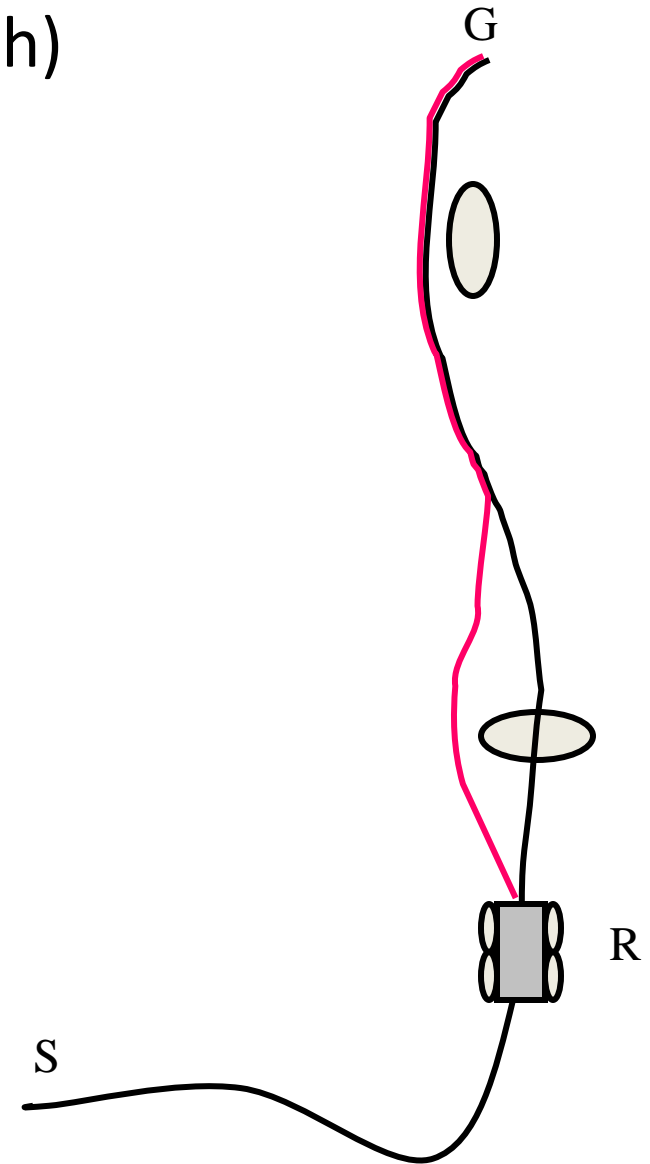




# 10.3.4 Plan Repair Approach: D\* Algorithm

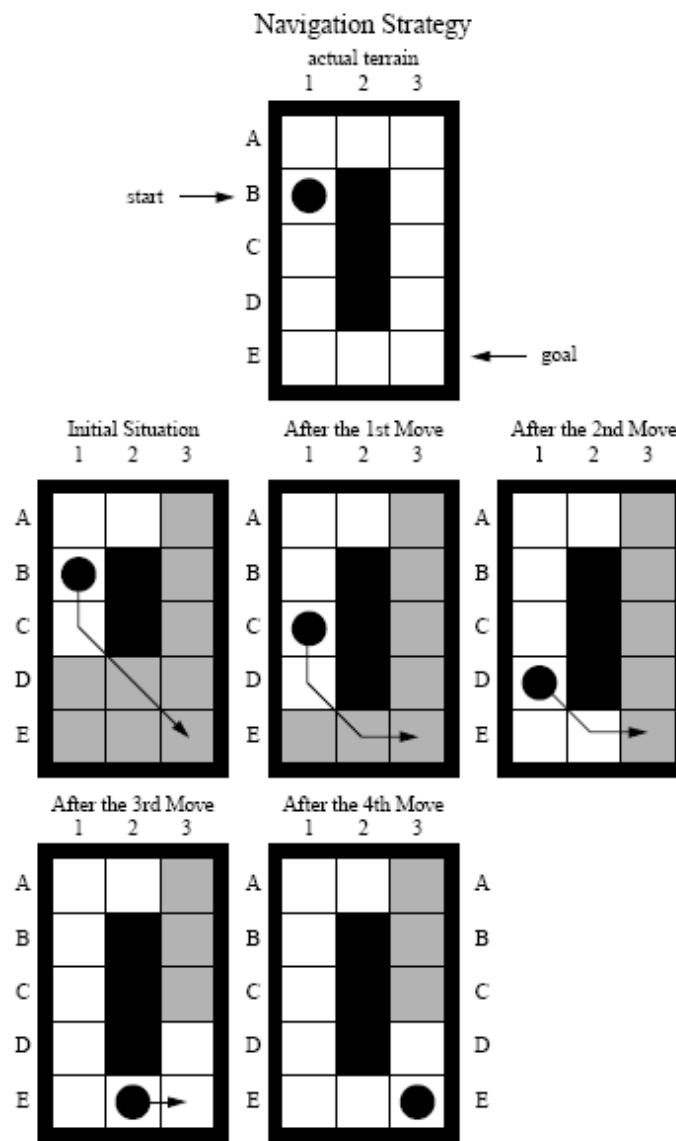
(Basic Approach)

- Construct an initial solution using A\* (or whatever).
- Continuously maintain this solution as....
  - 1: New information arrives
  - 2: The robot moves.



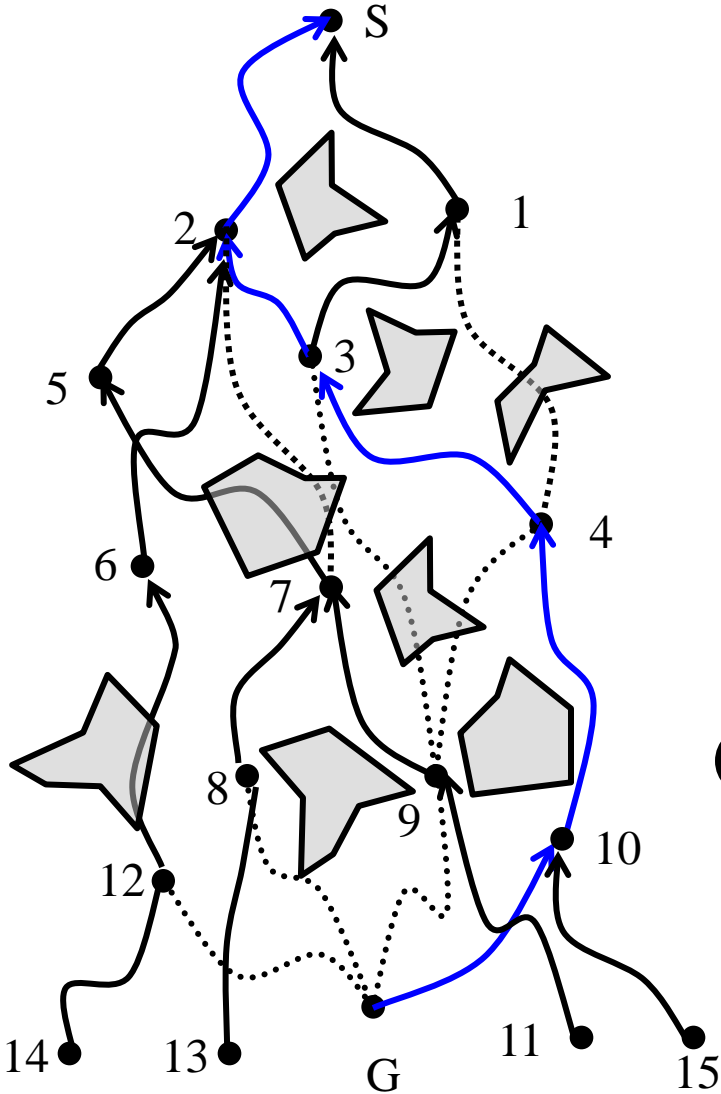
# 10.3.4 Plan Repair Approach: D\* Algorithm (Basic Approach)

- 1: Compute initial path up front.
- 2: Follow path until something new is learned.
- 3: Propagate the changes through search tree.
- 4: Compute new path
- 5: Goto 2:

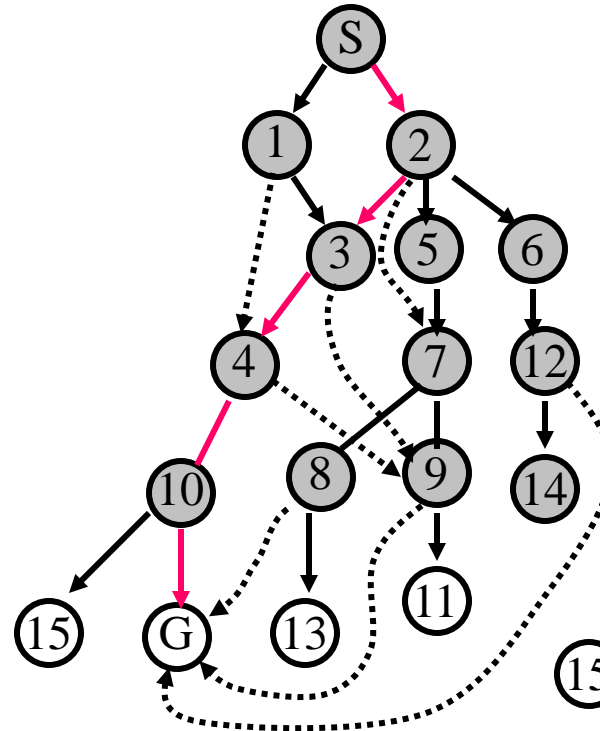


# 10.3.4 Plan Repair Approach: D\* Algorithm

## (Search Graph Vs Tree)

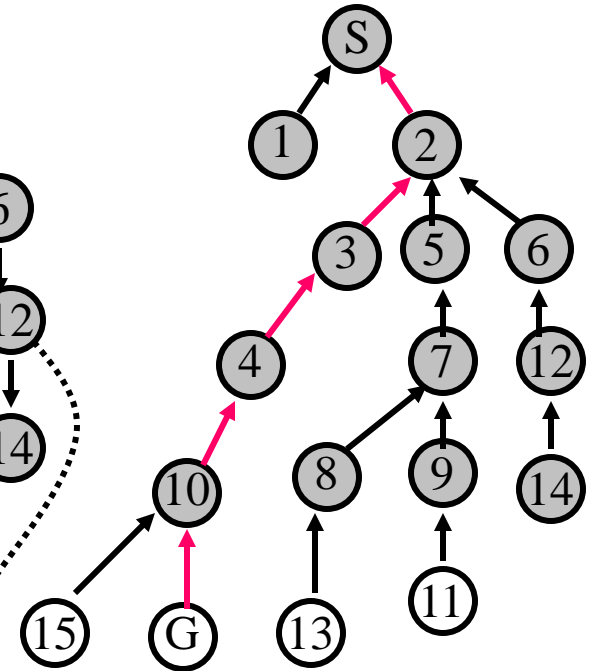


Search **Graph**



**Downward arrows are graph elaboration**  
point parent → child

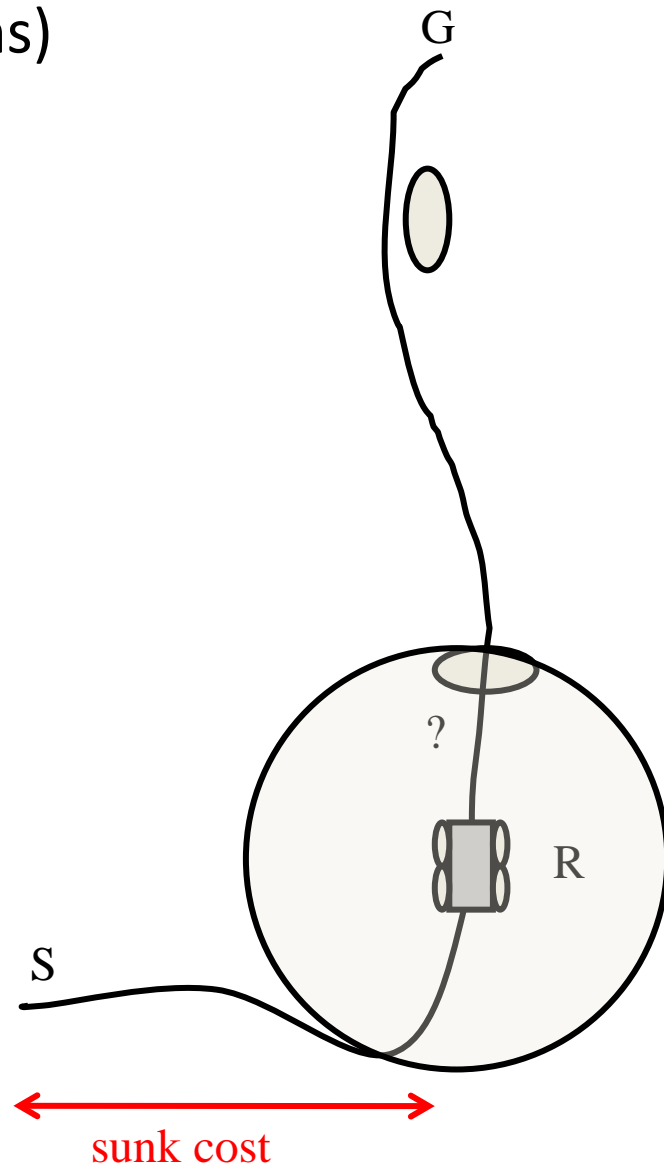
Search **Tree**



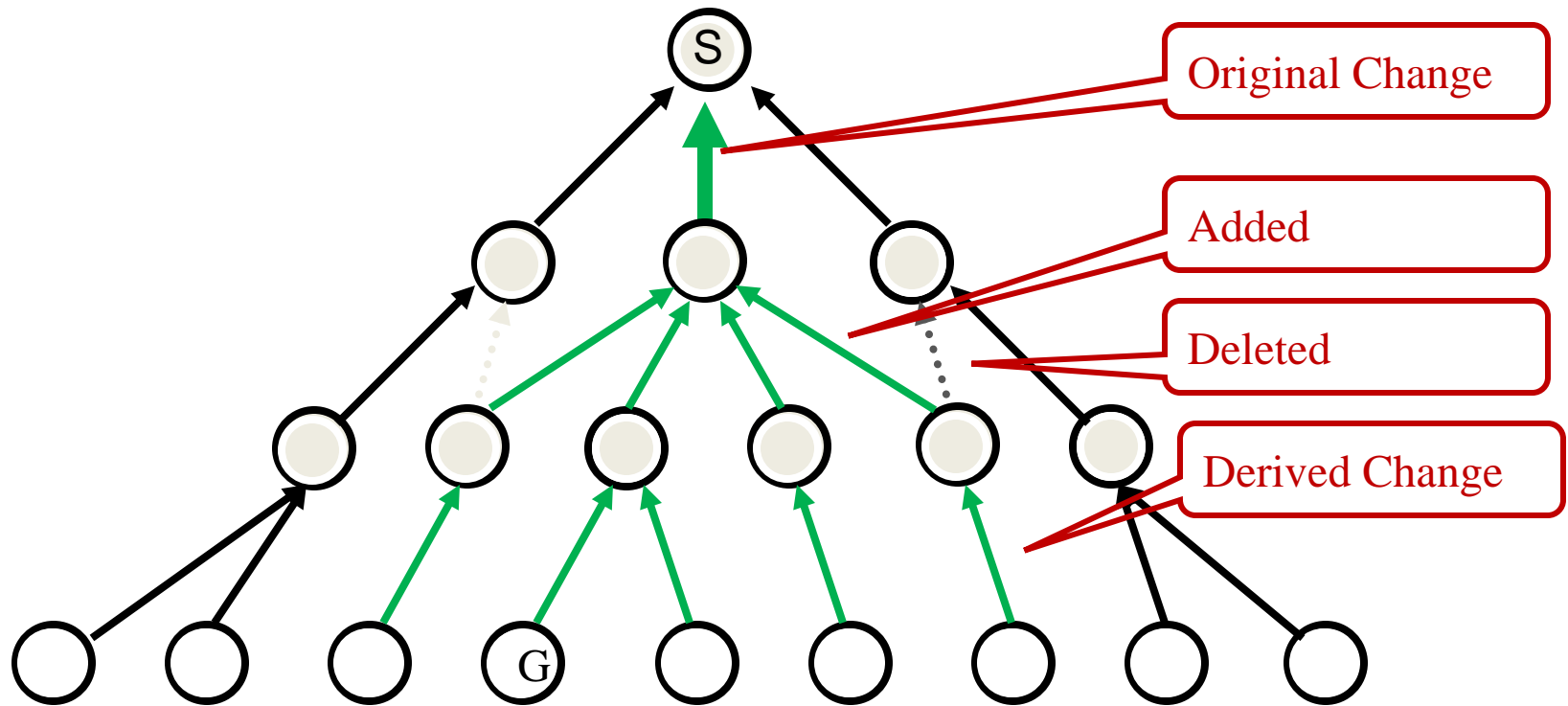
**Upward arrows are backpointers**  
point child → parent

## 10.3.4 Plan Repair Approach: D\* Algorithm (Some Observations)

1. Only the path from **here** (not from start) **to the goal** is needed.
2. Discoveries are generally made close to the robot.



## 10.3.4 Plan Repair Approach: D\* Algorithm (Propagating Cost Changes)

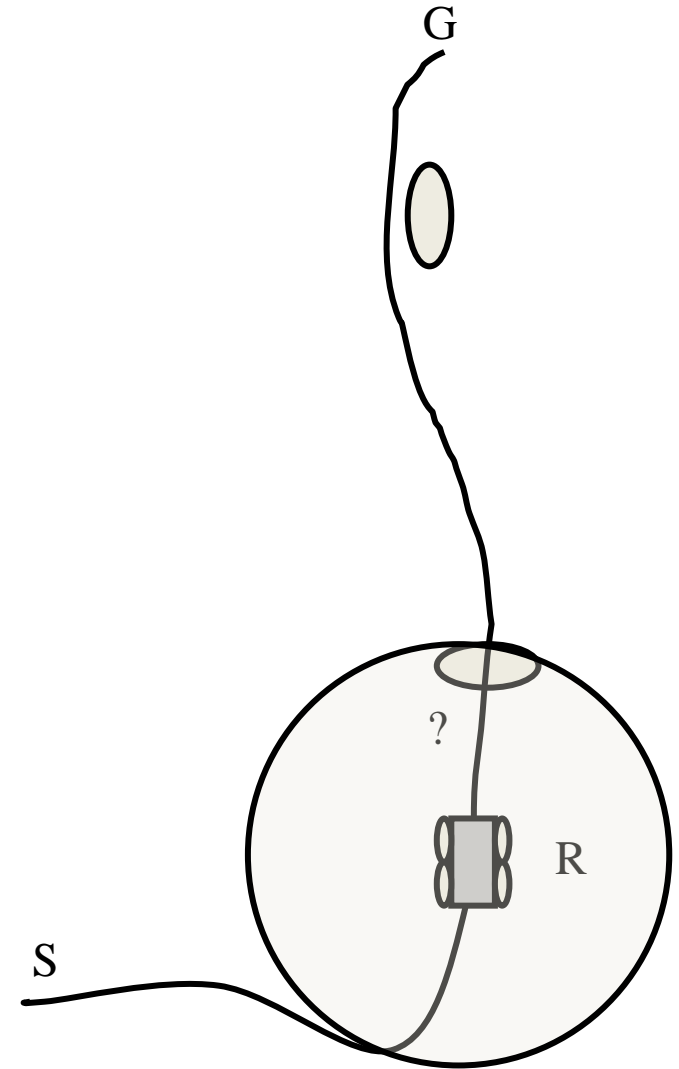


- Changes must propagate all the way to all pertinent affected leaves of the search tree

# 10.3.4 Plan Repair Approach: D\* Algorithm

(Conclusions)

- Since changes to a search tree **must propagate all the way to the leaves to fully understand their implications** .....
- Search from the goal **BACKWARD** to the robot.
  - Root = Goal
  - Leaf = Robot

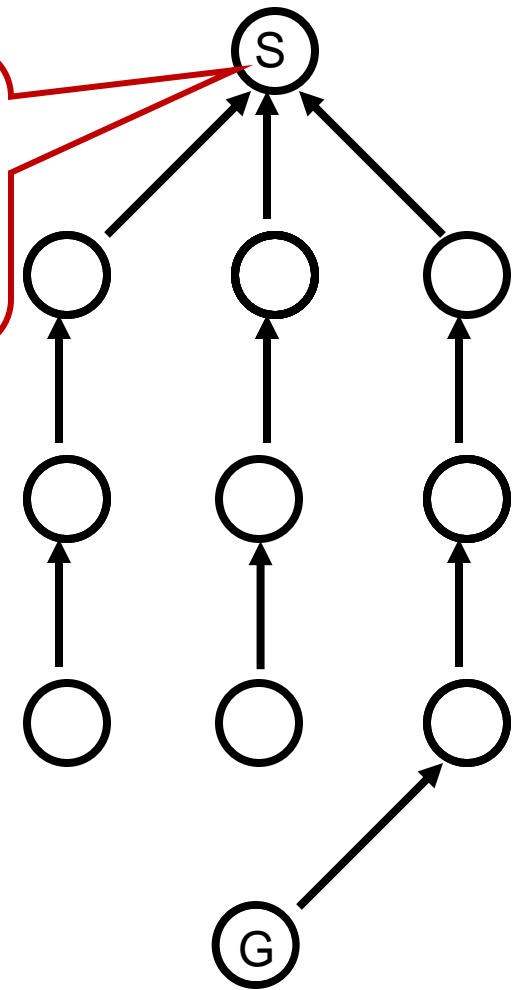


# 10.3.4 Plan Repair Approach: D\* Algorithm

(Compute initial Path)

- Use A\* from G (goal) to R (robot). Save  $f()$  values of every node opened.
- Dstar: Work in terms of  $h()$  and  $k()$  where:
  - $h()$  is same as  $f()$  in A\*
  - $k()$  is the minimum value  $h()$  has ever had since it was placed on OPEN.
- DstarLite: Work in terms of  $g()$  and  $rhs()$  where:
  - $g$  is same as before
  - $rhs$  is **best possible  $g$  RIGHT NOW** based on all possible neighbors

Now the GOAL is the root of the search tree but I call it S for cleaner code.

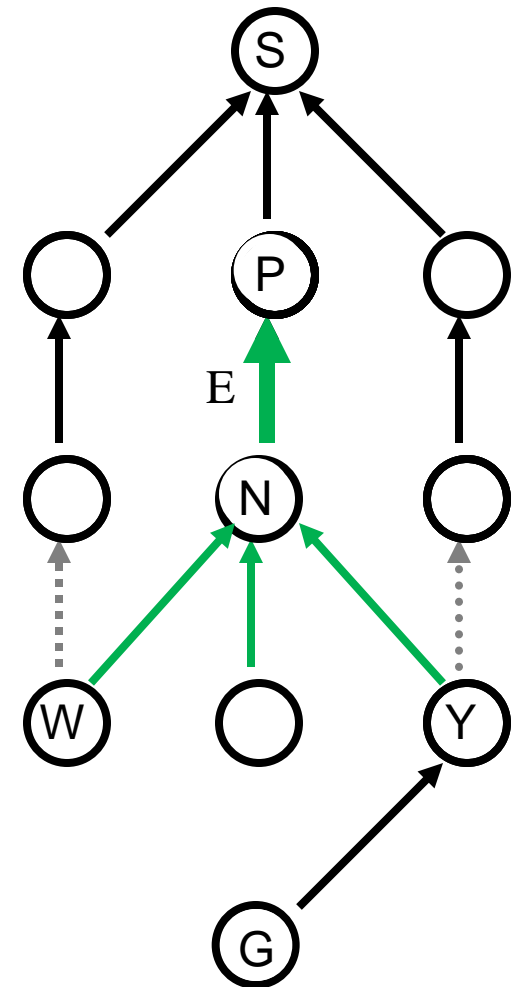


This is the search tree → spanning tree encoded in backpointers.

# 10.3.4.1.2 Implications of Edge Changes

(Lowered Cost)

- Suppose an edge E **gets cheaper**....
- Nodes W and Y may want to **abandon their parents** in favor of N.



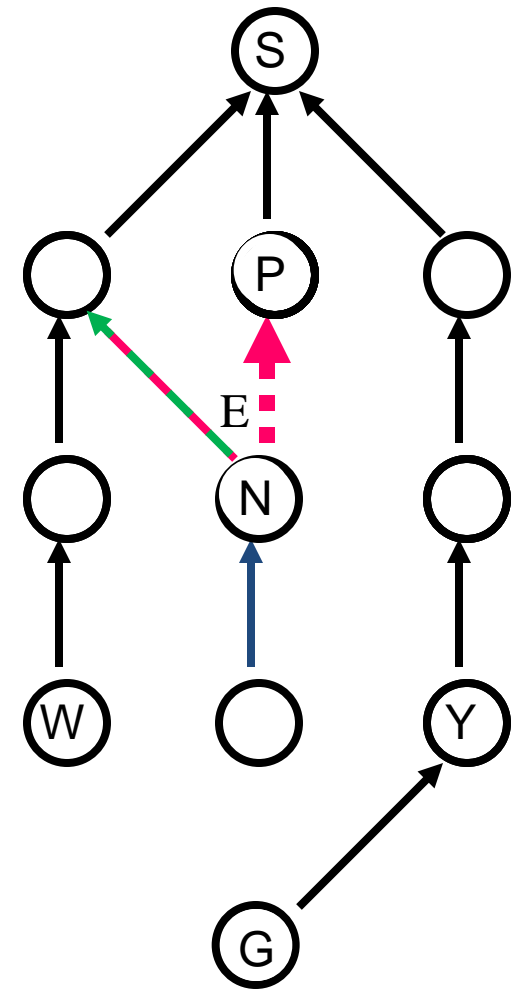
This is the search tree → spanning tree encoded in backpointers.



# 10.3.4.1.2 Implications of Edge Changes

(Raised Cost)

- Suppose an edge **gets costlier**....
- Node N may **want a different parent**.

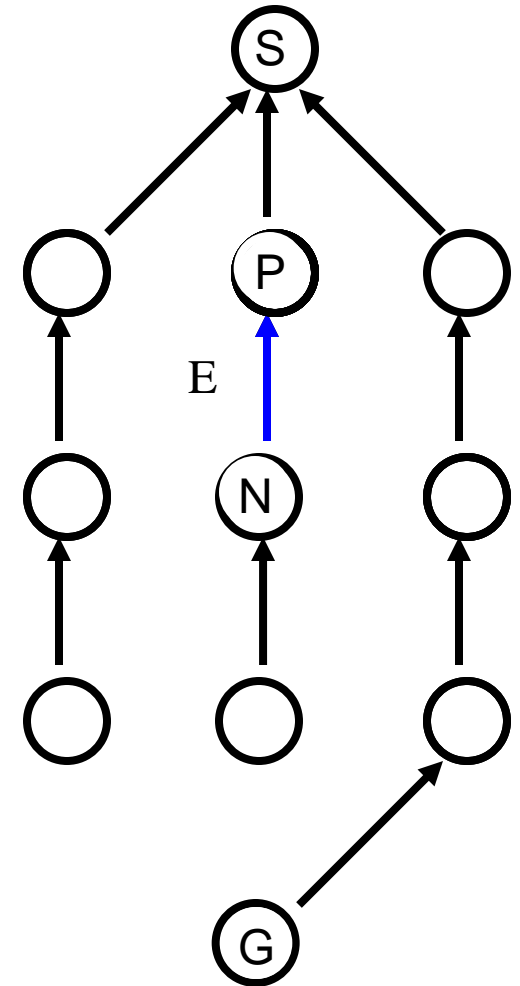


This is the search tree → spanning tree encoded in backpointers.

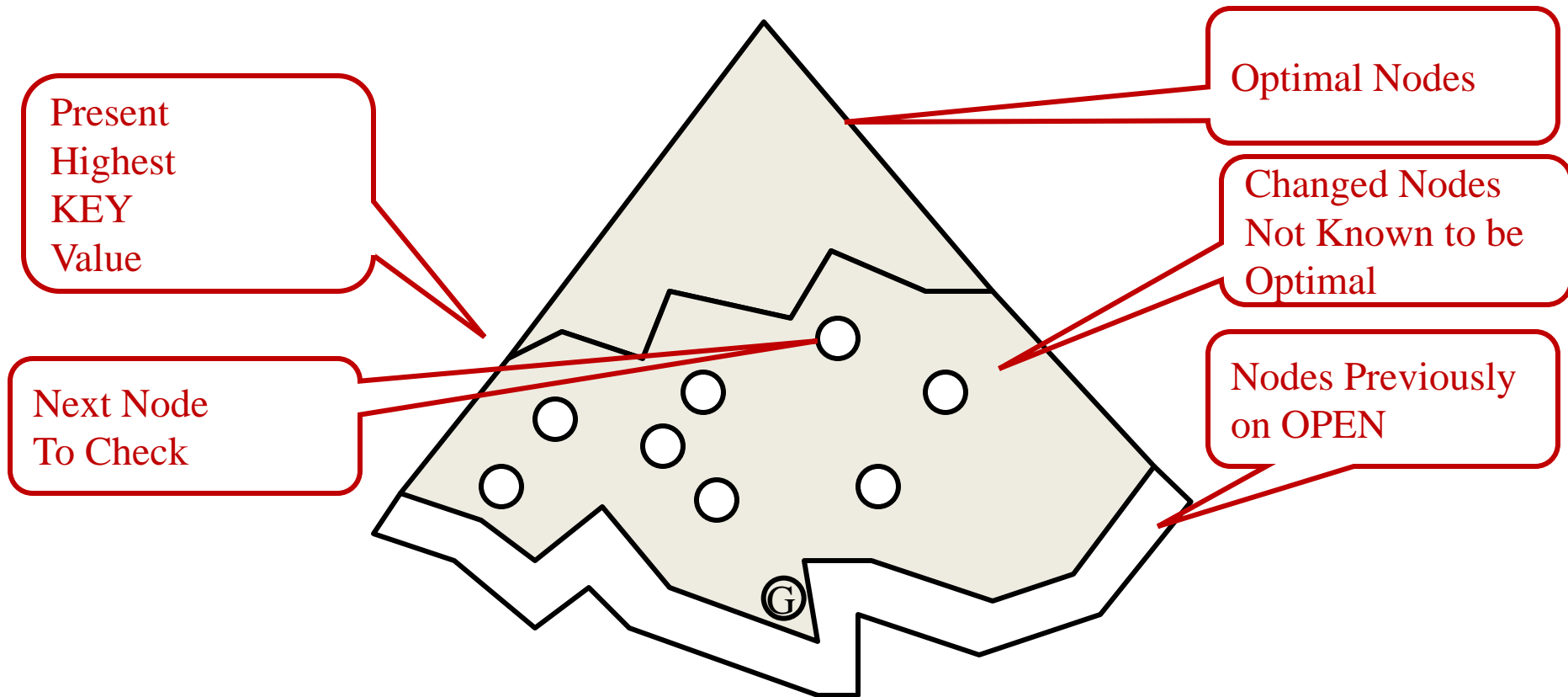
# 10.3.4.1.2 Implications of Edge Changes

(Efficient Propagation)

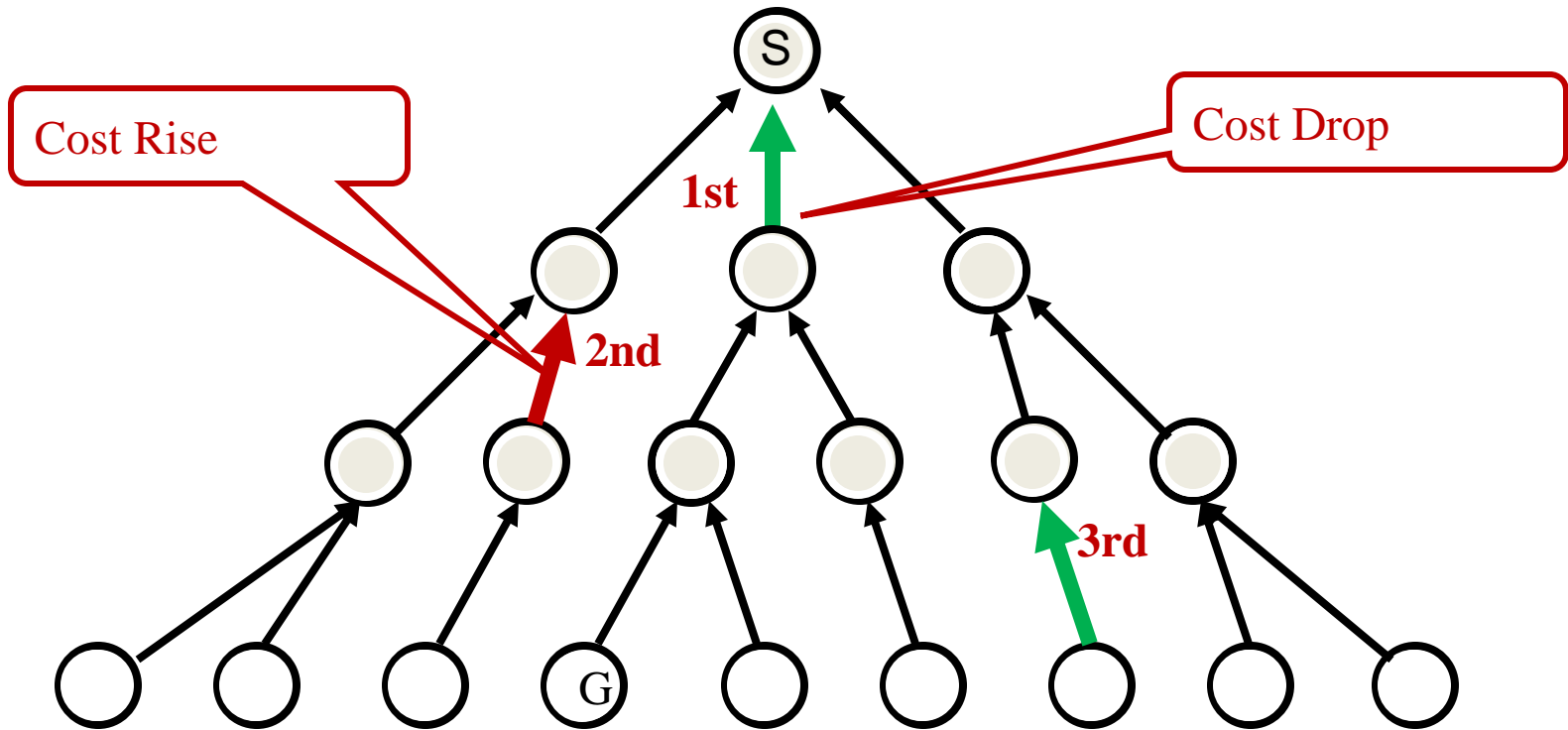
- (Almost) Brute force approach:
  - Go back in time....
  - Remove all nodes from OPEN or CLOSED for which  $f(\text{Node}) > f(P)$ .
  - Mark remaining leaves as open
  - Rerun Astar.
- Efficient?
  - Touches every node between P and G in the solution tree.
  - Many end up unchanged from last time.
- Not efficient.
- BUT: Placing affected nodes on OPEN is a good idea.
  - See next figure to visualize.



# 10.3.4.1.2 Implications of Edge Changes (D\* Processing Wavefront)



# 10.3.4.1.4 Processing Multiple Changes

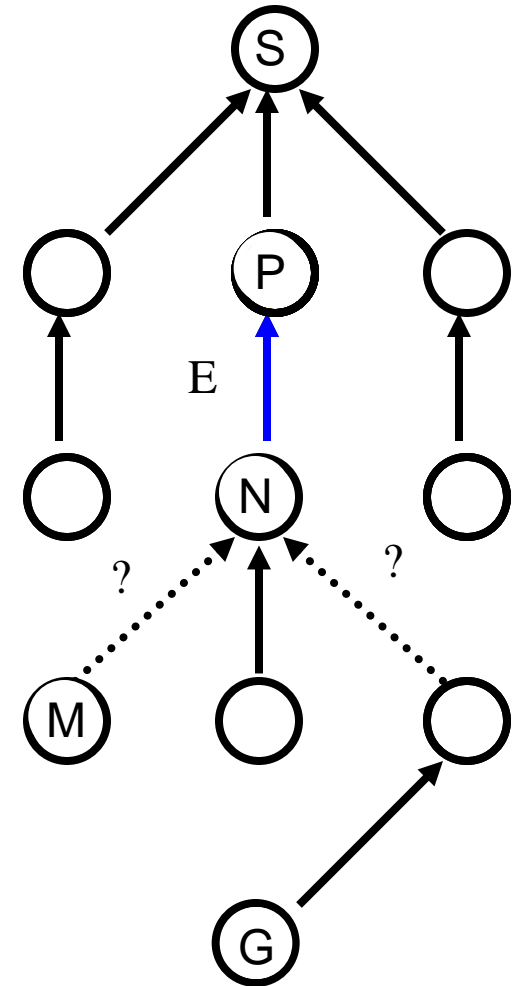


- Propagate changes **downward** in one pass **committing** as you go
  - Hence sort changed nodes (perhaps on OPEN? )
- **Lowered** states may need to move up the tree
  - Their sort key is their **new cost** (move up before the slot closes)
- **Raised** states may need to move down the tree
  - Their sort key is their **old cost** (move down before you get stuck)

# 10.3.4.1.4 Propagating Cost Changes

(Will Rerunning Astar Work?)

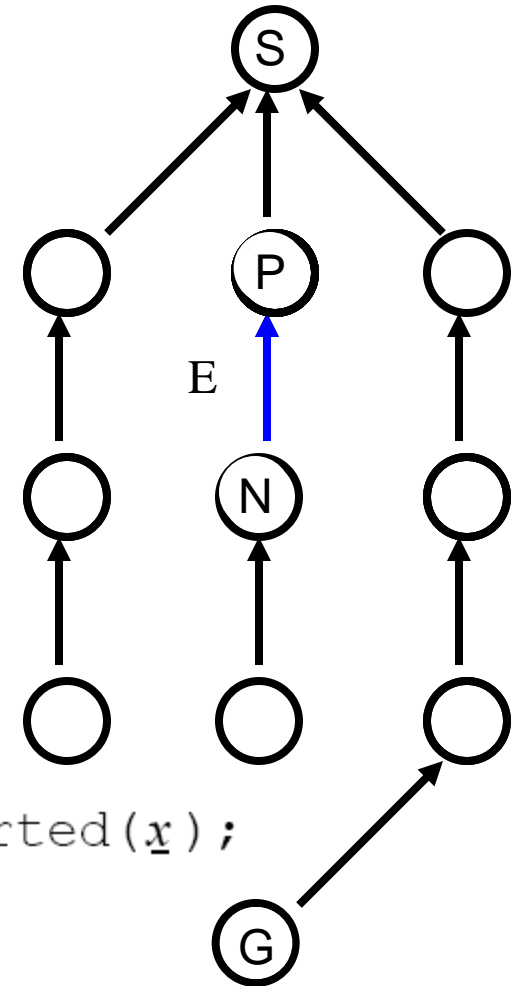
- Place N on OPEN and propagate changes downward (reopening closed nodes)
- **Does not work.**
  - In Astar, nodes on OPEN compete to be the parents of neighboring nodes.
  - The resulting subtree **must have N as its root** (N is like start).
  - So, every changed node will have a path that goes through N.
  - **No mechanism for M to route around N** if Edge E increased in cost.



# Idea: Propagate Inconsistency

- Node  $N$  is inconsistent if it does not point to its “best” parent.
- Remove this node and, if it is not optimal, reinsert in  $O$  in correct place.

```
00:  updateVertex( $\underline{x}$ ) {  
01:      if(  $\underline{x} \neq \underline{x}_{start}$  ) getRhs( $\underline{x}$ );  
02:      if(  $\underline{x} \in O$  )  $O.remove(\underline{x})$ ;  
03:      if(  $g(\underline{x}) \neq rhs(\underline{x})$  )  $O.insertSorted(\underline{x})$ ;  
04:  }
```



# Dstar Lite Goodies

- “Right Hand Side”

$$rhs(s) = \begin{cases} 0 & \text{if } s = s_{start} \\ \min_{s' \in Pred(s)} (g(s') + c(s', s)) & \text{otherwise.} \end{cases}$$

Detects  
Inconsistent  
Nodes

- It’s the **cost a node would** have if one level of lookahead was resolved.

- “Key” (f value)

$$[\min(g(s), rhs(s)) + h(s, s_{goal}); \min(g(s), rhs(s))]$$

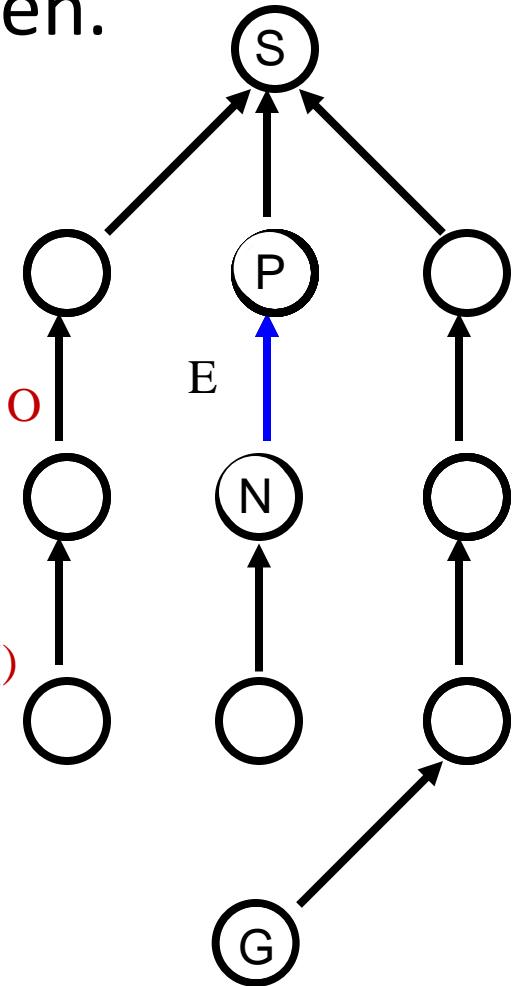
Sorts  
Properly  
For One Pass  
Resolution

- Cost a node will have as soon as its neighbors are told they need to change.
  - **Break ties** with second key.

# Idea: Propagate Inconsistency

- Check all neighbors of nodes on open.

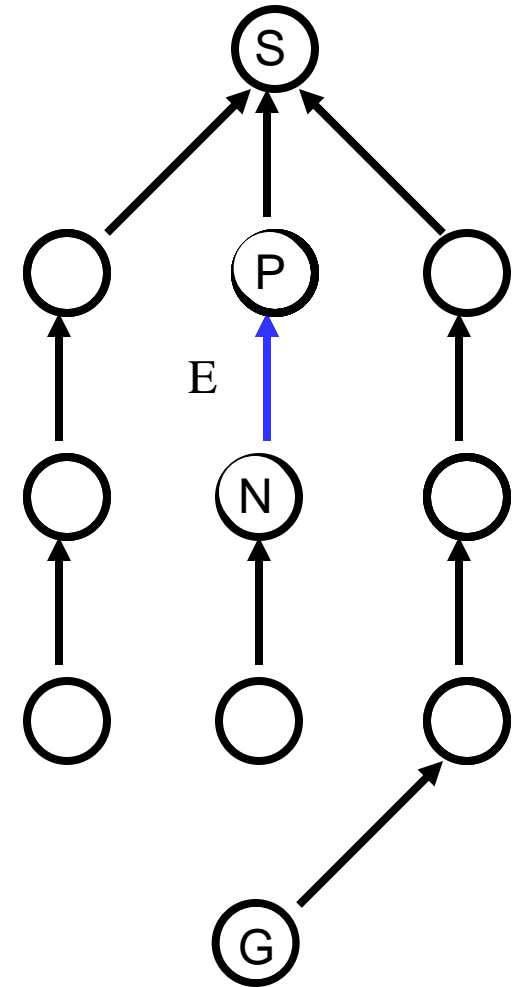
```
00: computeShortestPath( $\underline{x}$ ) {
01:   while ( $f[O.peek()] < f(\underline{x}_{goal}) \parallel g(\underline{x}_{goal}) \neq rhs(\underline{x}_{goal})$ )
02:      $\underline{x}_{next} = O.pop()$ ; Remove from O
03:     if ( $g(\underline{x}_{next}) > rhs(\underline{x}_{next})$ ) { g is too high
04:        $g(\underline{x}_{next}) = rhs(\underline{x}_{next})$ ; Correct it, don't put it back on O
05:       for (each  $\underline{x}_{neigh} \in pred(\underline{x}_{next})$ ) Check all neighbors
06:         updateVertex( $\underline{x}_{neigh}$ );
06:     } else { g is OK or too low
07:        $g(\underline{x}_{next}) = \infty$  Force it on O with key based on rhs()
08:       for (each  $\underline{x}_{neigh} \in pred(\underline{x}_{next}) \cup \{\underline{x}_{next}\}$ )
09:         updateVertex( $\underline{x}_{neigh}$ ); Check all neighbors
10:     }
11: }
```





# Termination

- Terminate when:
  - lowest  $f()$  on OPEN  $>$   $f(\text{robot})$
  - Robot node is then optimal.
- Often need to compensate for roundoff:
  - lowest  $f()$  on OPEN  $>$   $f(\text{robot}) + e$



# Entire Algorithm

```
procedure CalcKey(s)
{01'} return [min(g(s), rhs(s)) + h(sstart, s); min(g(s), rhs(s))];
```

For Sorting OPEN

```
procedure Initialize()
{02'} U = ∅;
{03'} for all s ∈ S rhs(s) = g(s) = ∞;
{04'} rhs(sgoal) = 0;
{05'} U.Insert(sgoal, CalcKey(sgoal));
```

Initialize

```
procedure UpdateVertex(u)
{06'} if (u ≠ sgoal) rhs(u) = mins' ∈ Succ(u)(c(u, s') + g(s'));
{07'} if (u ∈ U) U.Remove(u);
{08'} if (g(u) ≠ rhs(u)) U.Insert(u, CalcKey(u));
```

Perception Info

```
procedure ComputeShortestPath()
{09'} while (U.TopKey() < CalcKey(sstart) OR rhs(sstart) ≠ g(sstart))
{10'}   u = U.Pop();
{11'}   if (g(u) > rhs(u))
{12'}     g(u) = rhs(u);
{13'}     for all s ∈ Pred(u) UpdateVertex(s);
{14'}   else
{15'}     g(u) = ∞;
{16'}     for all s ∈ Pred(u) ∪ {u} UpdateVertex(s);
```

Plan Paths

```
procedure Main()
{17'} Initialize();
{18'} ComputeShortestPath();
{19'} while (sstart ≠ sgoal)
{20'}   /* if (g(sstart) = ∞) then there is no known path */
{21'}   sstart = arg mins' ∈ Succ(sstart)(c(sstart, s') + g(s'));
{22'}   Move to sstart;
{23'}   Scan graph for changed edge costs;
{24'}   if any edge costs changed
{25'}     for all directed edges (u, v) with changed edge costs
{26'}       Update the edge cost c(u, v);
{27'}       UpdateVertex(u);
{28'}     for all s ∈ U
{29'}       U.Update(s, CalcKey(s));
{30'}   ComputeShortestPath();
```

Initial A\* - Like call

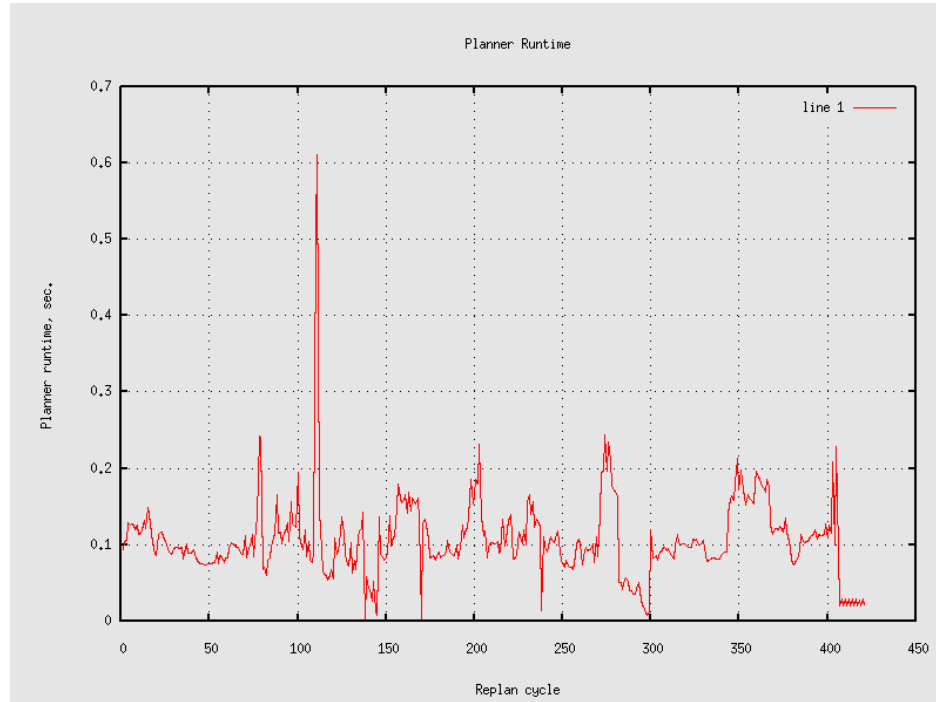
Move, Percieve, Replan

# Beware

- Code switches GOAL and START for Dstar only.
- That means **they are switched relative to the DstarLite paper.**

# Random Observations

- Runtime is not constant



- The alternative is worse.

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# Summary

- The real motion planning problem is that of planning in dynamic and uncertain environments.
  - Maps are never completely accurate.
- Computational techniques are mature in the abstract case of grids.
- The real motion planning problems therefore become:
  - Understanding mobility
  - Adequate perception