Chapter 6
State Estimation

Part 2

6.2 Sensors for State Estimation

Outline

- 6.2 Sensors for State Estimation
  - 6.2.1 Articulation Sensors
  - 6.2.2 Ambient Field Sensors
  - 6.2.3 Inertial Frames of Reference
  - 6.2.4 Inertial Sensors
  - Summary

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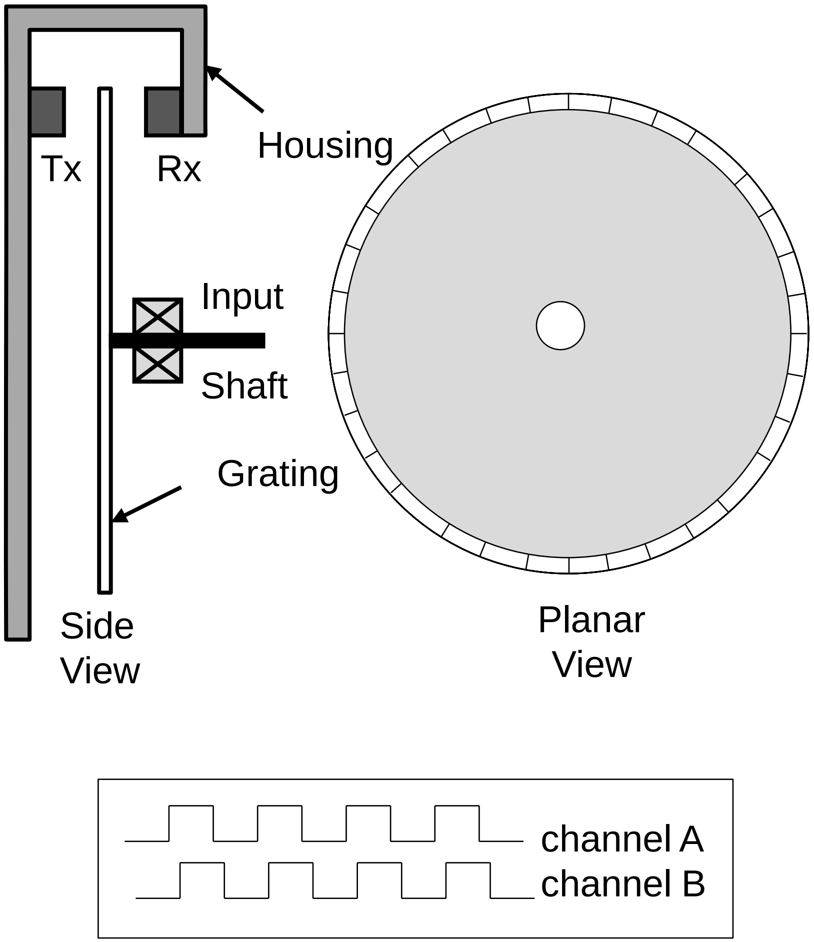
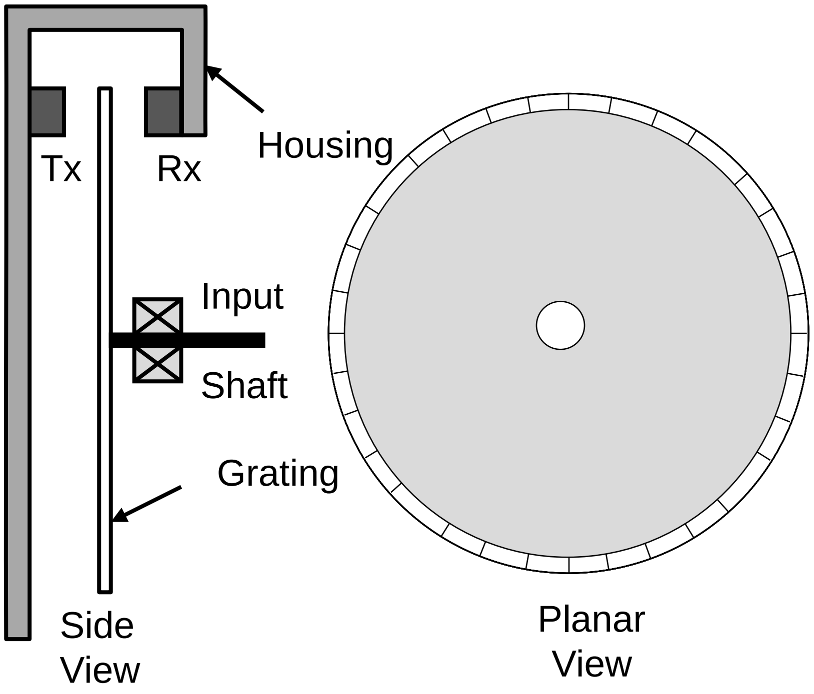
6.2.1.1 Optical Encoders

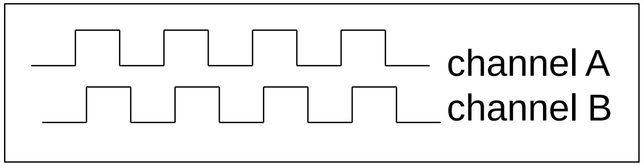
- Standard equipment on mobile robots. Used for wheel rotations, pan-tilt heads, etc.
  - Wheel slip issues mean super high accuracy encoders don’t make sense (for wheels).
- A photodetector (photodiode) and phototransmitter (laser diode) are lined up to look through the holes in a grating.



6.2.1.1 Optical Encoders

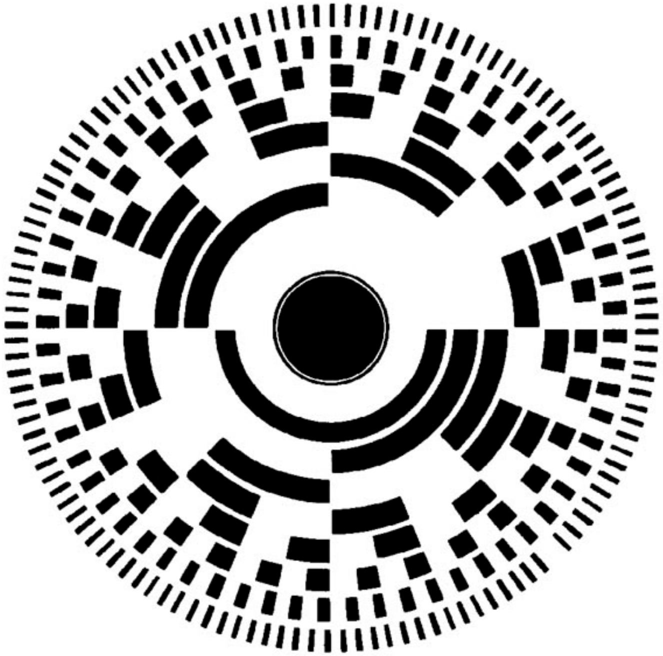
- Interrupted light source generates a series of pulses which are counted.
- A second Tx/Rx pair can generate a second set of pulses 90 degree out of phase with first.
  - Permits sensing direction of motion.
- 1000 to 10000 counts per revolution are typical.
  - Gear ratio and backlash sometimes affects output resolution.





6.2.1.2 Absolute Optical Encoders

- Relative encoders (like the above) lose count when shut off.
- Few solutions:
  - Drive physically to a home position after power is applied.
  - n concentric circles of gratings encoding binary numbers (opposite) and n recievers.
- Up to 26 bits precision is available!



Absolute Encoder
(No need to count pulses now)

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Field (Radiation) Sensors

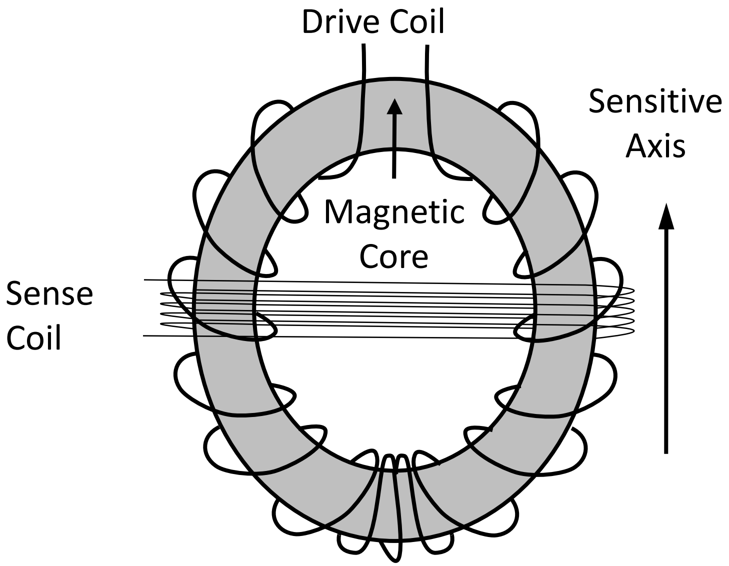
- Moving in a field of known characteristics is equivalent to having a map.
- Cheap, low tech sensors for measuring attitude on the earth such as compasses and inclinometers rely on the earth’s fields (magnetic and gravitational).
- This same principle is used in radio navigation but the field is generated artificially.
- All vision-based navigation uses the same principle.
- Attitude can be computed from the relative orientations of the vehicle body and two known non-collinear vectors.
  - Why Two?
  - Two are required because one does not constrain rotation about its direction.

6.2.1.1 Compasses and Magnetometers

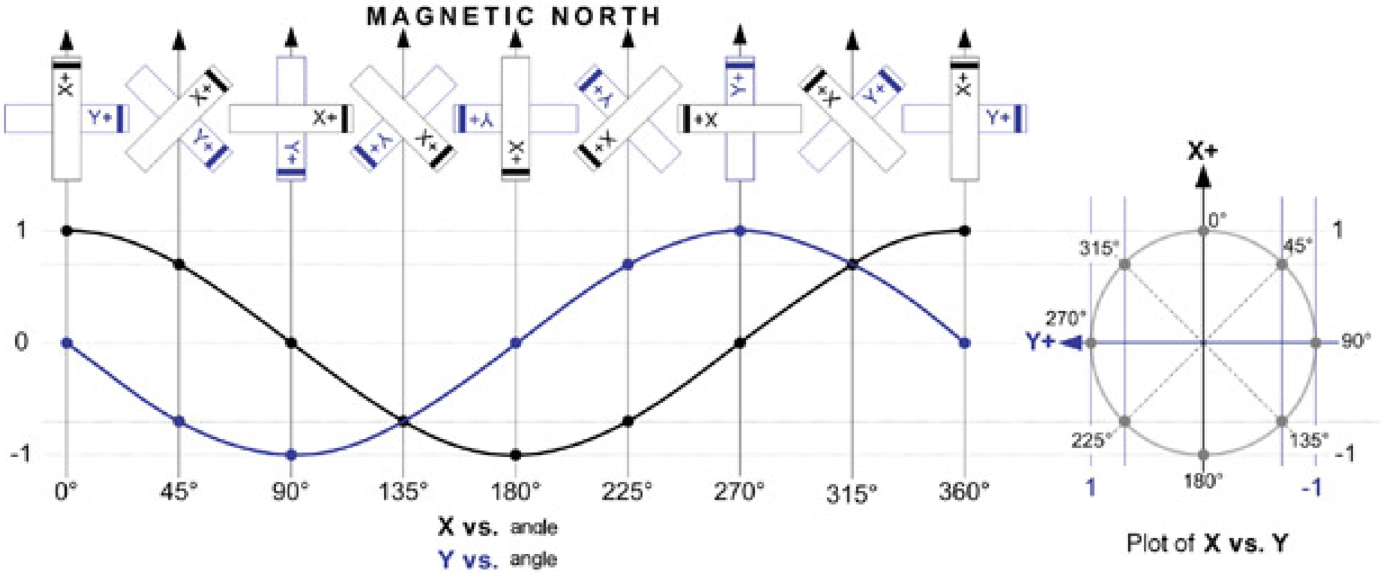
- Compass = a device that measures orientation with respect to the Earth's poles.
- Gyrocompasses have been used on ships
  - sense the true direction of the earth's spin axis.
- Only the magnetic compass is used on mobile robots.
  - sense earth's magnetic field as an orientation reference
- Often occur on mobile robots in the form of a (fluxgate) magnetometer.
  - sense the <u>magnitude</u> of the ambient magnetic field in one or more directions.

Fluxgate Magnetometer

- Compare the drive-coil current needed to saturate the core in one direction as opposed to the opposite direction.
- Output the difference
  - Which is equal to external field magnitude.
- Saturation detected as nonlinearity.



(2 axis) Magnetometer Response



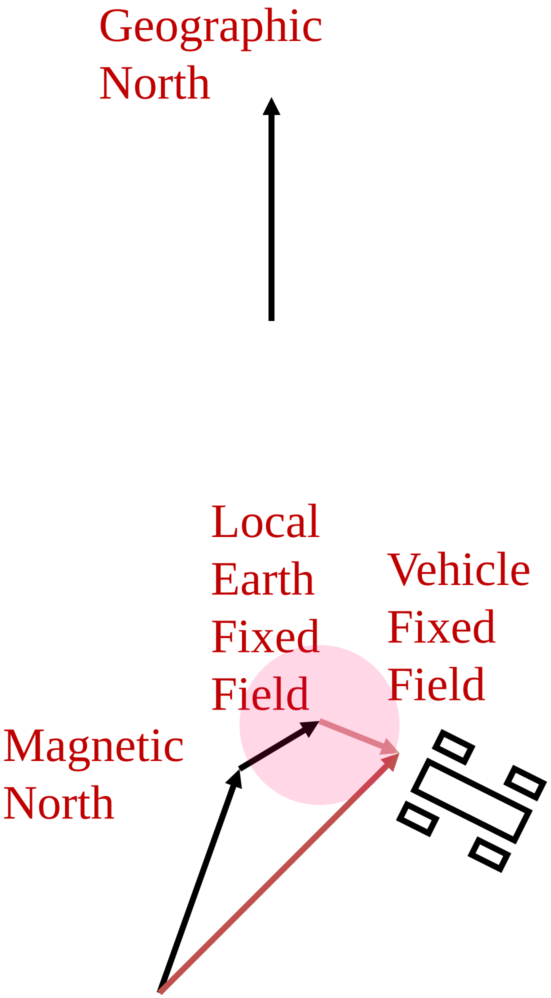
- Theta = atan2(y,x)
- Real devices often have slow response.
- Their alignment on the vehicle also needs to be calibrated.

Earth’s magnetic field: Variation

- Flow of liquid ferromagnetic material in Earth’s core causes a very very weak field.
- Mariners used to rely on it and have charted all its many quirks.
  - Not aligned with geographic north (direction to earth’s spin axis). Known as variation.
  - Magnetic north pole is in Canada, magnetic south pole is off cost of Australia.
  - Entire east coast of US is 15 degrees east variation.

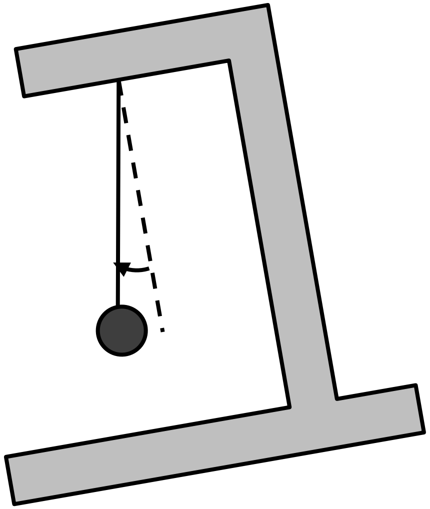
Local Field: Deviation

- Some metals have magnetic permeability which redirects the ambient field.
- Permanent and electromagnets create fields which redirect the ambient field.
- Some of these things are fixed to the robot and some are fixed to the ground.
  - Some may be fixed to other robots.
- Can calibrate the vehicle field out by spinning in a circle and measuring difference from ground truth rotation.



6.2.2.2 Inclinometers

- Measure direction of specific force.
- Often work on identical principles as pendulous accelerometers.
  - Measure angle of proof mass deflection.
  - Cheap devices have bandwidths on the order or 2 Hz.
  - Rebalance devices are faster.
- Ideal solution for mobile robots that don’t move ;)
- Why?



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6.2.3 Inertial Frames

- Simply, a frame where Newton's laws hold.
- Don't those laws hold everywhere in the universe?
  - Nope.
  - F only = ma in an inertial frame of reference.
- Remember the moving observer stuff? There were two accelerations measured. Which is right?
  - Both are.
  - But the a times the m only equals the F if the observer is in an inertial frame.
  - Otherwise the motion seems inconsistent with the F due to apparent other forces.

6.2.3 Inertial Frames

- A frame which is:
  - Far removed from any massive object (no gravitation), or...
  - Under no net gravitational force, or...
  - Free to move under the influence of gravitation (freefall)
- Third case subsumes the other two.
- Examples include:
  - Intergalactic space (far removed)
  - Center of the earth (no net g’s)
  - Spacecraft in orbit (why are you “weightless” if you are being held in orbit by the earth’s gravity?)
  - Aircraft in freefall (NASA trains astronauts in these)
- Anything that rotates wrt the stars is not inertial.

6.2.3.1 Apparent Forces

- Reverse our classical acceleration transform…

$$\vec{a}_o^{\,m} = \vec{a}_o^{\,f} \underset{\textcolor{red}{\uparrow}}{-} (\vec{a}_m^{\,f} + 2\vec{\omega}_m^{\,f} \times \vec{v}_o^{\,m} + \vec{\alpha}_m^{\,f} \times \vec{r}_o^{\,m} + \vec{\omega}_m^{\,f} \times [\vec{\omega}_m^{\,f} \times \vec{r}_o^{\,m}])$$

- If the f frame is inertial, only the $\vec{a}_0^{\,f}$ term can be explained by applied forces.
- Yet, the moving observer sees motion that indicates other forces are being applied.
  - What “force” pushes you against the wall of a car in a turn?
  - The other terms are said to be caused by apparent forces.
  - 4 terms inside the brackets above are, in order, Einstein, Coriolis, Euler, Centrifugal

6.2.3.2 Noninertial Earth Fixed Frame of Reference

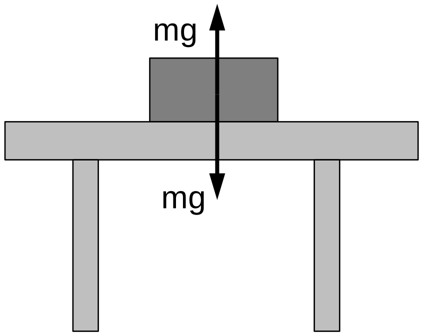
- Earth-fixed observer says table reaction equals mg.

$$\sum \vec{F} = \vec{R} + m\vec{g} = 0$$

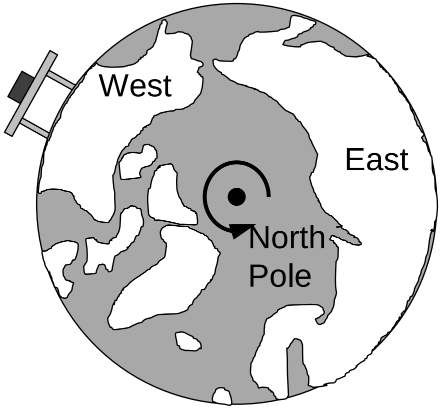
- Inertial observer says table reaction is slightly less – explains circular motion.

$$\sum \vec{F} = \vec{R} + \vec{G} = m\vec{a}$$

- Therefore ! $m\vec{g} = \vec{G} - m\vec{a}$
- Gravity (g) is more than gravitation (G).
  - Contains apparent forces !
- BTW: g depends on latitude



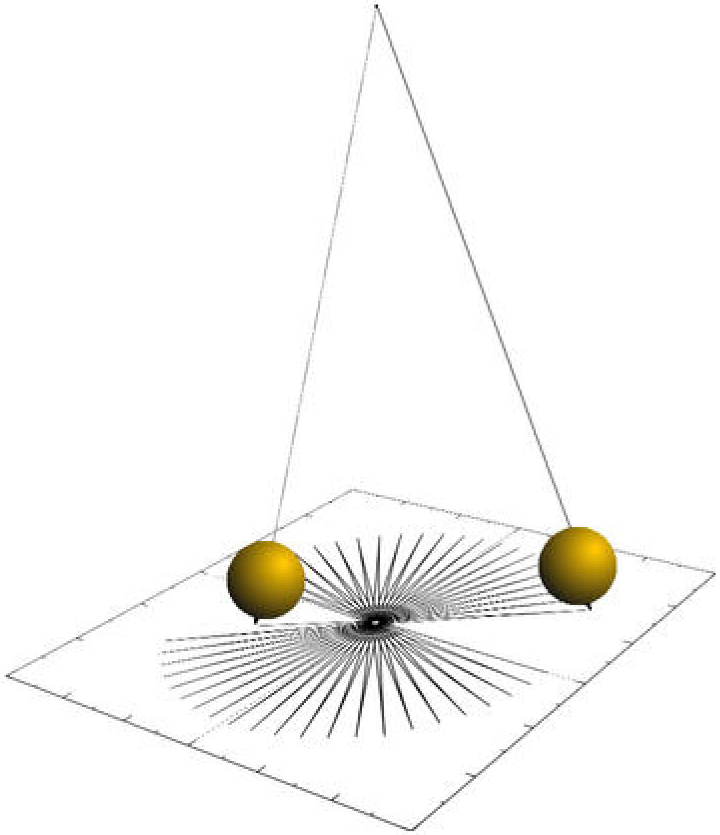
Earth-Fixed Observer



Inertial Observer

6.2.3.2.1 Effect of Earth's Rotation

- The surface of the earth is not an inertial frame.
- Almost every f=ma you wrote in high school was wrong.
- Instruments can detect this. Some examples:
  - Gyroscope at equator pointing "up" rotates once per day.
  - Faucault pendulum in UN building in New York.
  - Objects don't fall in straight lines.
  - Gyrocompass oscillates due to earth's rotation.



Faucault Pendulum

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    - <u>6.2.4.1 Inertial Sensor Performance</u>
    - 6.2.4.2 Accelerometers
    - 6.2.4.3 Gyroscopes
    - 6.2.4.1 Inertial Measurrment Units
  - Summary

6.2.4 Inertial Sensors

- Higher tech sensors such as accelerometers and gyroscopes are based on inertial principles.
- Accelerometers and inclinometers can each do the other’s job:
  - accelerometers can provide attitude (w.r.t gravity) when acceleration is known.
  - inclinometers can provide acceleration when attitude is known.

6.2.4.1 Inertial Sensor Performance

- Devices are sensitive to temperature, shock, and vibration.
- A simple model of performance is:

$$f_{meas} = \underset{\substack{\nearrow \\ \text{Scale Factor}}}{k} f_{true} + \underset{\substack{\nearrow \\ \text{Bias}}}{b} + \underset{\substack{\nearrow \\ \text{Noise}}}{w}_{noise}$$

- Because autocalibration is the rule, it's the stability of bias and scale factors that matters most.

Measuring Noise

- Can be characterized in terms of the spectral amplitude:
  - For gyros get "density" in deg/sec/Hz1/2 or (deg/sec)2/Hz
- Angle Random Walk (ARW): Describes the error magnitude expected from integrating a noisy rate measurement.
  - After removing effects of bias and scale.
  - Slope of the integrated velocity noise process curve in deg/root(hr) (stdv) or deg^2/hr (variance).
- Velocity Random Walk: Integral of an assumed acceleration white noise term.

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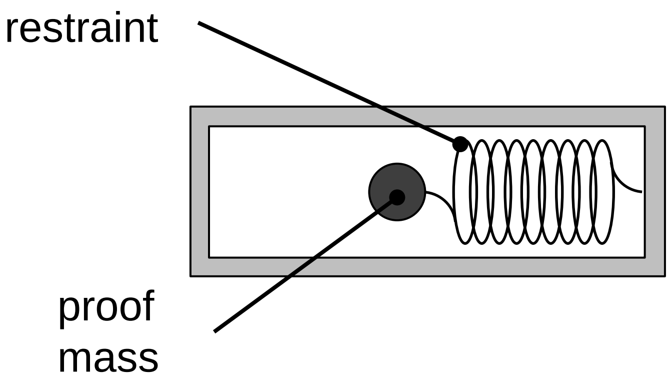
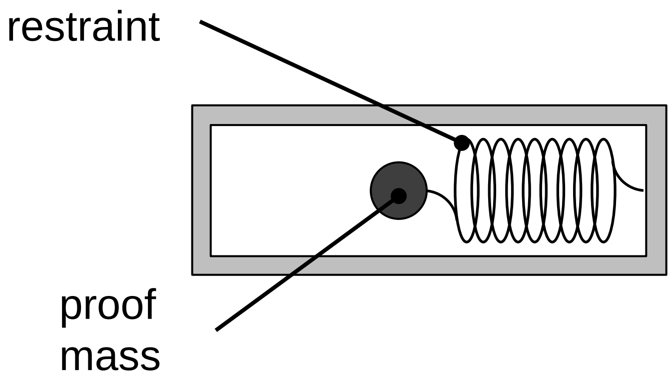
6.2.4.2 Accelerometers



- Accelerometers are typically divided into two classes, depending upon their intended use.
- Guidance accelerometers are those intended for use in measuring the steady state accelerations of rigid bodies. One might use a guidance accelerometer for measuring the acceleration of an automobile.
- Vibratory or seismic accelerometers are those intended to measure sinusoidal accelerations. They are used to measure vibrations in applications as varied as structural testing, and earthquake and tsunami detection.

6.2.4.2.1 Operating Principle



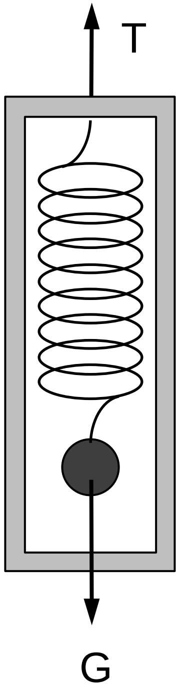
Deformable objects will deform **under acceleration or when transmitting forces**

- All accelerometers operate on the same principle, that of measuring the relative displacement of a small mass, called a proof or seismic mass, constrained within an accelerating case.
  - An accelerometer is therefore a physical manifestation of Einstein's famous box from general relativity.
- The restraint
  - may be elastic, viscous, or electromagnetic.
  - is a transducer that returns a signal proportional to the displacement of the proof mass.
- Typically, the mass is only allowed a single degree of freedom which may be either linear or rotary.

6.2.4.2.2 Specific Force

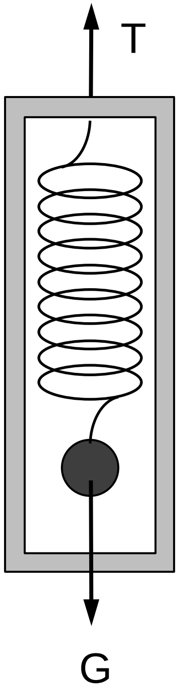
- Accelerometers measure specific force, not acceleration and there is nothing we can do about it.
- Case 1: When the case is accelerated, the spring will deflect - allowing us to measure the acceleration.
- Case 2: When the case is placed upright on a table, it will measure the influence of gravitation because the spring will deflect with the weight of the mass.

"Specific" means per unit mass.



6.2.4.2.2 Specific Force

(Subtleties)



- The deflection of the elastic restraint is directly influenced by the tension in the spring - not by the mass's acceleration.
- From Newton's Second Law; the inertial acceleration of the mass (a^i) is proportional to the total force applied to the mass - in this case, the gravitational force (Fg) and the elastic restraining force (T).

$$\vec{T} + \vec{G} = m\vec{a}^i$$

Acceleration proportional to net force, not T.

- Specific force:

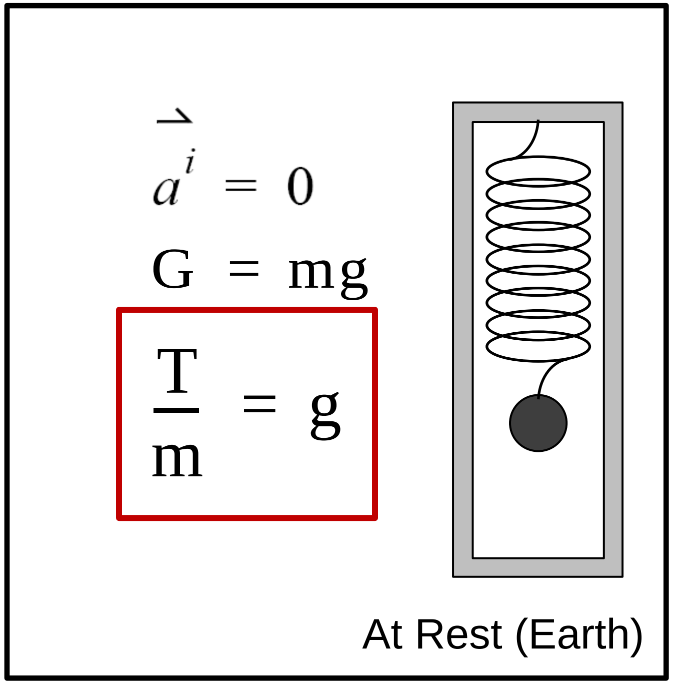
$$\frac{\vec{T}}{m} = \vec{a}^i - \frac{\vec{G}}{m}$$

Spring deflection is proportional NOT to a, but to a-g.

6.2.4.2.2 Specific Force

(Equivalence of Gravitation and Acceleration)

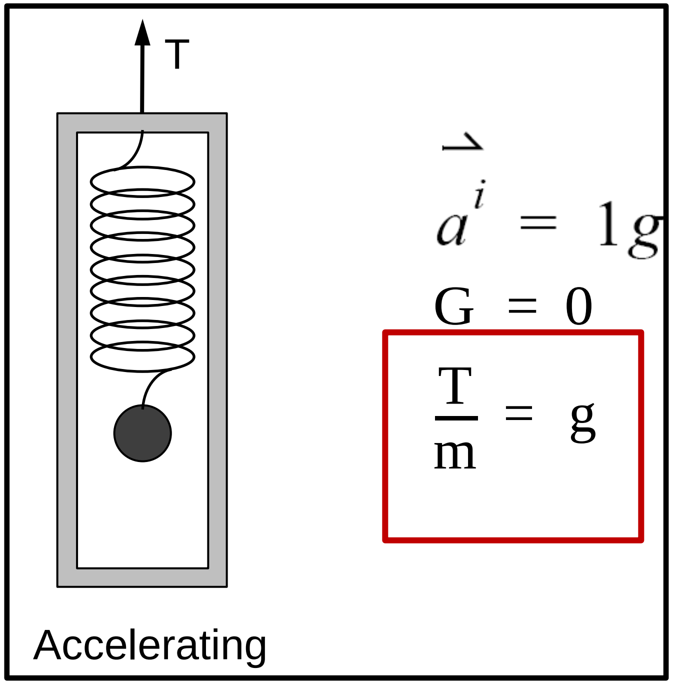
- An accelerometer cannot tell the difference between these two effects.



$$\vec{a}^{\,i} = 0$$

$$G = mg$$

$$\boxed{\frac{T}{m} = g}$$



$$\vec{a}^{\,i} = 1g$$

$$G = 0$$

$$\boxed{\frac{T}{m} = g}$$

6.2.4.2.2 Specific Force

(Accelerometers as Dynamic Systems)

- All accelerometers have some stiffness and some damping.
  - Modulating enables tuning the response.
  - Guidance accelerometers are low pass.
  - Vibratory accelerometers are bandpass.
- Devices with rebalance loops actively oppose motion and report the effort (current) required to do so.
  - These generally have much faster response times.

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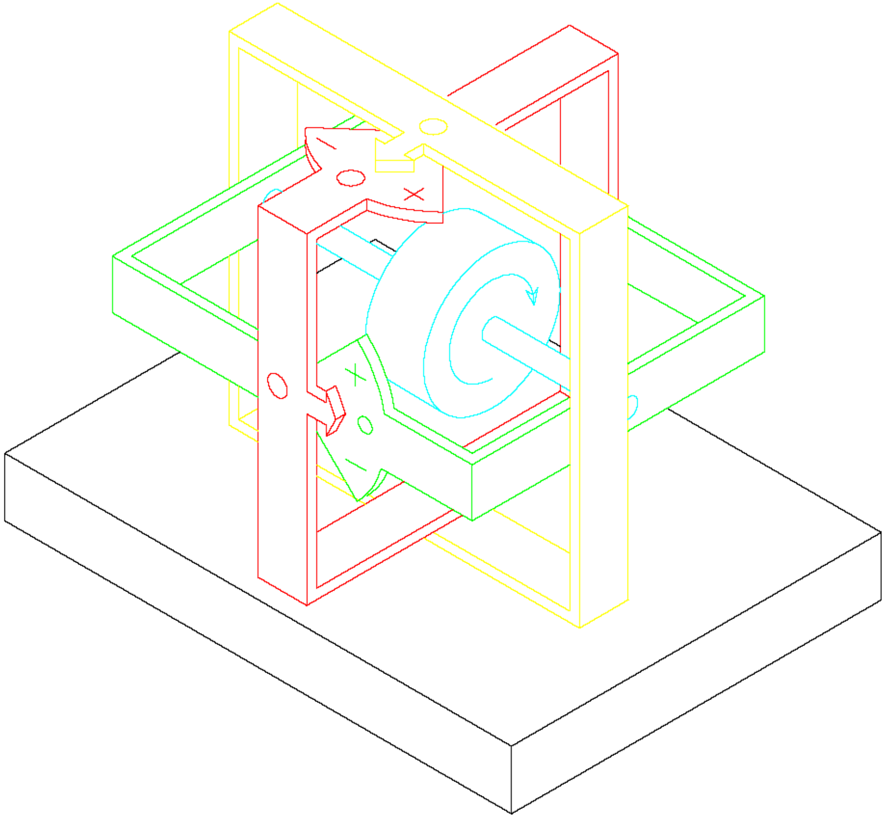
    - <u>6.2.4.3 Gyroscopes</u>
    - 6.2.4.1 Inertial Measurement Units
  - Summary

6.2.4.3 Gyroscopes

- Word coined by Foucault in 1852.
- A spinning rotor
  - Rotor => has an axis of symmetry
  - Spinning => you know
- Two properties of interest
  - Rigidity: Angular momentum means hard to rotate
    - Not useful
  - Precession: Angular momentum means attempts to rotate one way cause rotations another way.



6.2.4.3.1 Mechanical Gyroscope

(Precession)

- Consider Euler's equation:

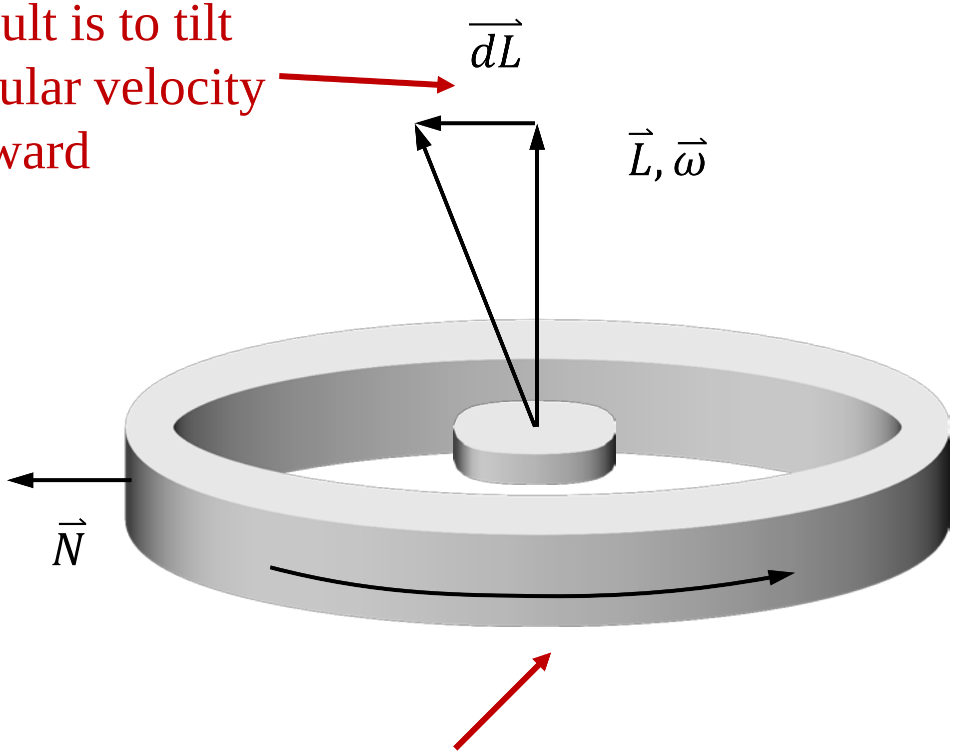
$$\vec{N} = \frac{d\vec{L}^i}{dt} = \frac{d(I^i\vec{\omega})}{dt}$$

- Inertia is diagonal so.

$$d\vec{\omega} \parallel \vec{N}$$

- Hence ω (not the disk) must rotate in the direction of N !

$$d\vec{L} = d(I_i\vec{\omega})$$



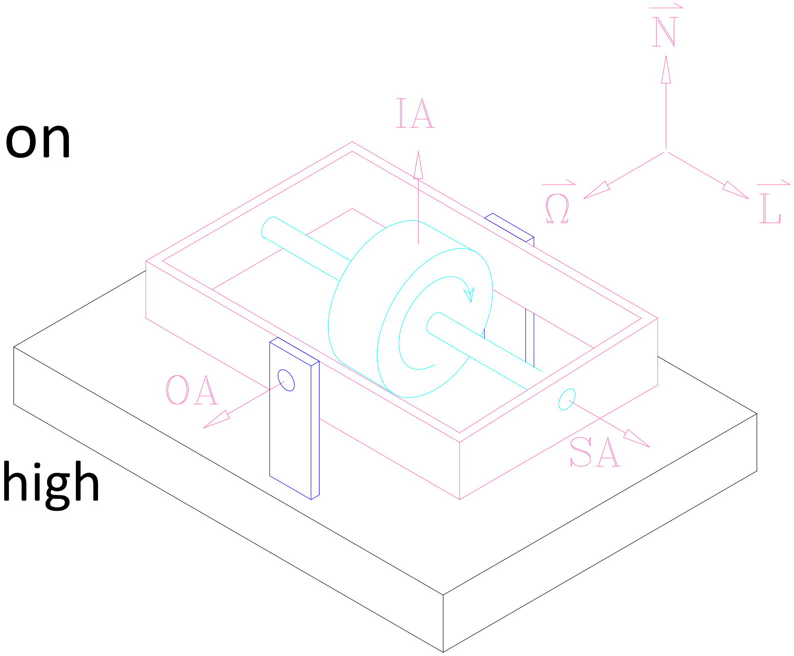
Result is to tilt angular velocity forward

Torque is trying to lift the front up

6.2.4.3.1 Mechanical Gyroscope

(Gyro Configurations)

- Some forms of drift are reduced by high ω so rpms are usually very high.
- Rebalance loops used in precision devices.
- Packaging of one or more:
  - "rate" gyros: high spring, low damping (rate in -> rate out)
  - "integrating" gyros: low spring, high damp (angle in-> angle out)
  - "directional" gyros hold their azimuth, indicate heading (not gyrocompasses)
  - "vertical" gyros indicate pitch and roll (from gimbals) by staying vertical



6.2.4.3.3 Optical Gyroscopes

- Are replacing mechanical ones in most applications.
  - Used on all new civil transport aircraft (757, 767, Airbus 310)
  - FOGs becoming common on robots.
- Based on Sagnac Effect discovered in 1913.
  - Considered obscure at the time.
- Characteristics:
  - solid state or nearly so
  - More reliable, rugged
  - Good in harsh environments
  - Linearity of parts per billion
  - Large dynamic range

6.2.4.3.3 Optical Gyroscopes

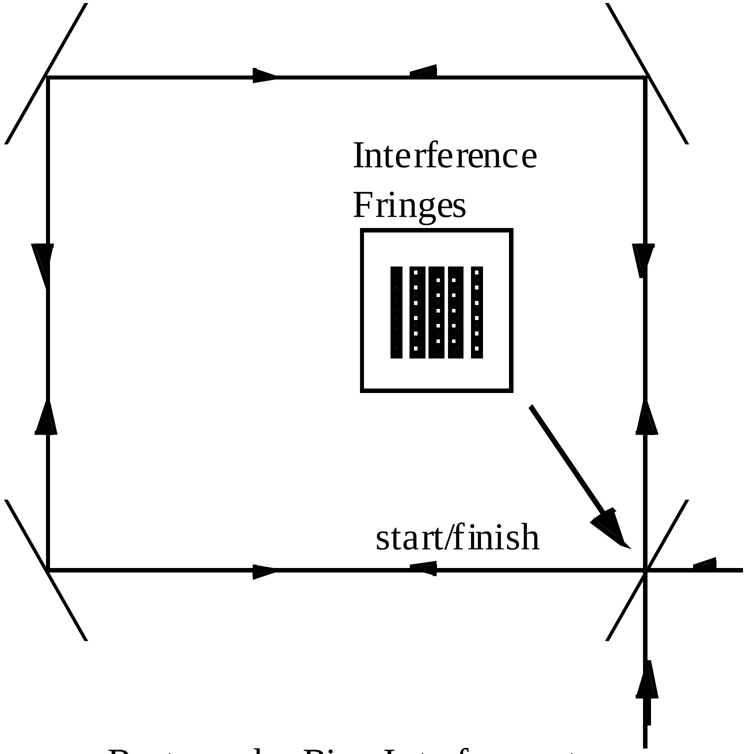
(Types)

- Ring Laser Gyros (RLGs)
  - Not solid state yet.
  - Becoming very popular
  - Will replace most inertial mechanical gyros
- Fiber Optic Gyros (FOGs)
  - Solid state
  - Newer to the market.
  - Will ultimately replace both mechanical and RLG’s

6.2.4.3.3 Optical Gyroscopes

(Sagnac Effect)

- While optical measurement of inertial translation is impossible (Michelson-Morely), optical measurement of inertial rotation is possible.
- Physical rotation of the ring interferometer gives rise to an apparent path length difference for two counter-rotating coherent beams of light.
- Effect can be understood in terms of:
  - different time of flight
  - different path length
  - different Doppler shifts



Rectangular Ring Interferometer

6.2.4.3.3 Optical Gyroscopes

(Sagnac Effect : Intuitive Derivation)

- Transit times for each beam in terms of itself:

$t = s / v$

$$t_+ = \frac{2\pi r + r\Omega t_+}{c} \qquad t_- = \frac{2\pi r - r\Omega t_-}{c}$$

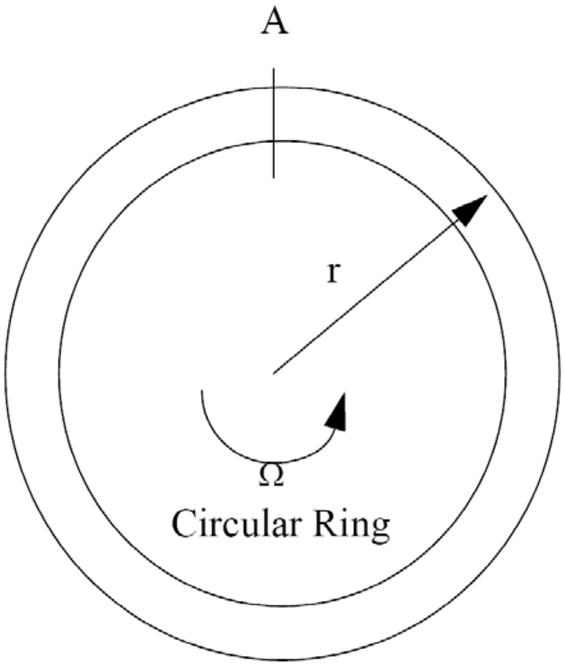
- Solve for times and take difference:

$$\Delta t = t_+ - t_- = \frac{4\pi r^2\Omega}{c^2 - r^2\Omega^2}$$

- For practical values of r: $r\Omega^2 \ll c^2$ so:

$$\boxed{\Delta t = \frac{4\pi r^2\Omega}{c^2}}$$

$c^2 = 10^{16}$!

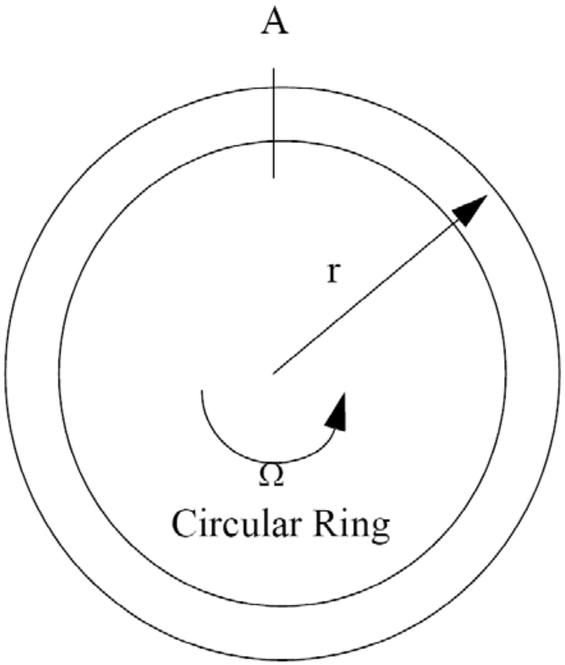


6.2.4.3.3 Optical Gyroscopes

(Sagnac Effect : Intuitive Derivation)

- So time difference is proportional to Ω.

$$\boxed{\Delta t = \frac{4\pi r^2 \Omega}{c^2}}$$



- Alternatively, the path length difference is:

$$\boxed{\Delta L = c\Delta t = \left(\frac{4\pi r^2}{c}\right)\Omega}$$

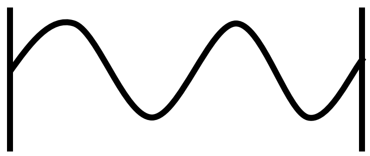
- For rotation of 1 degree/sec and r = 0.1 meter, the Sagnac effect gives 1/100 nanometer (1X10-11 m) of deviation.

6.2.4.3.3 Optical Gyroscopes (Sagnac Effect : Amplification)

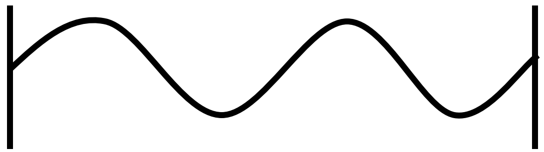
- Two modern technologies amplify the Sagnac effect:
- Development #1: lasers
  - RLG converts Sagnac path length difference into a beat frequency.
- Development #2: fiber optics
  - FOG increases Sagnac path length by using several miles of fibre optic cable coiled into small package.

6.2.4.3.3 Optical Gyroscopes (Ring Laser Gyros)

- Place a lasing medium in the path of the light beams.
  - It's a laser and a gyro in one package.
- Cavity resonance condition determines the laser wavelength:
  - End up with two different frequency lasers when rotating.



Short cavity, short waves

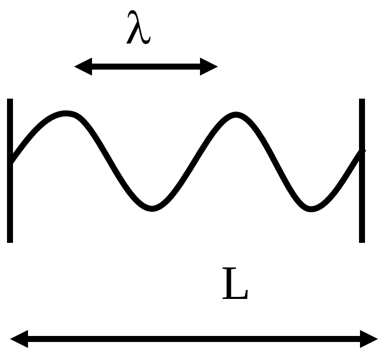


Long cavity, long waves

6.2.4.3.3 Optical Gyroscopes (Ring Laser Gyros)

- Resonance condition:

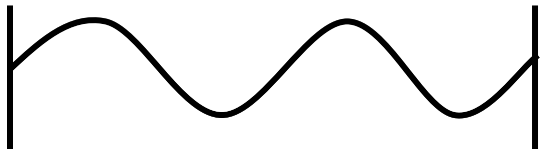
$$m\lambda = L$$



- Resonant frequency:

$$\omega = \frac{mc}{L}$$

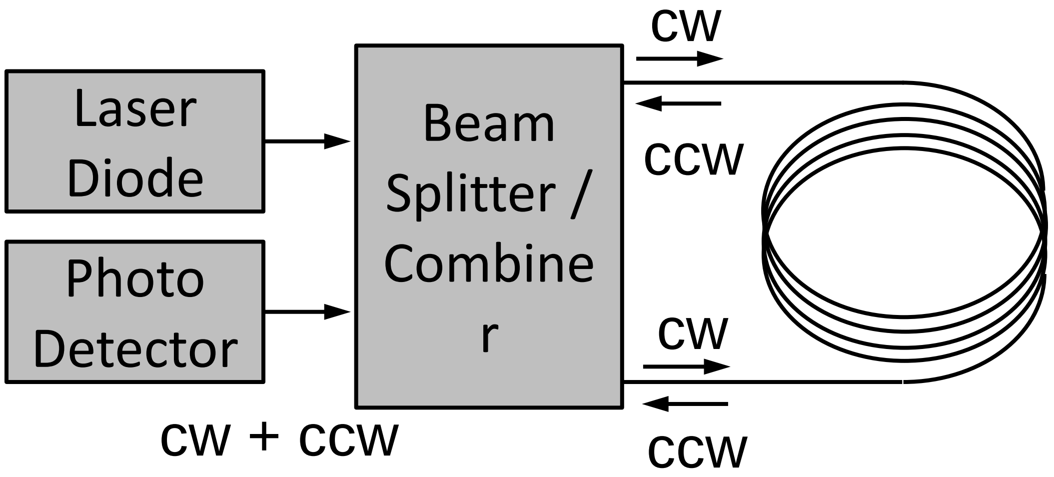
- Frequency difference is now:



$$\Delta\omega = \omega_{+} - \omega_{-} = \frac{mc}{L_{+}} - \frac{mc}{L_{-}} \approx \frac{mc\Delta L}{L^{2}} = \frac{\omega\Delta L}{L}$$

- Light frequencies ω are in the $5\text{x}10^{14}$ range.
  - Gives $\Delta\omega$ of 50 KHz!
  - Easy to measure beat frequency.

6.2.4.3.3 Optical Gyroscopes (Fiber Optic Gyros)



Laser Diode

Photo Detector

Beam Splitter / Combiner

cw

ccw

cw

ccw

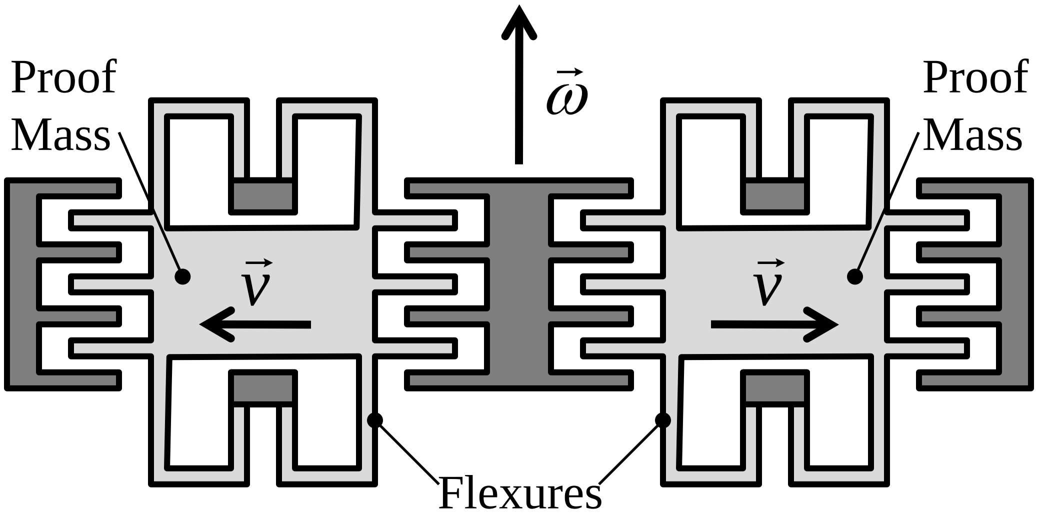
cw + ccw

- Writing the earlier path length difference as a phase difference:

$$\Delta\phi = \frac{2\pi\Delta L}{\lambda} = \frac{8\pi^2 r^2 \Omega}{c\lambda}$$

- This is multiplied by N if we coil the fiber optic cable N times.
  - Simpler and more robust technology than RLGs but not yet equal in performance.

6.2.4.3.4 MEMS Gyros



- Cheap, and becoming useful.
- Tuning fork concept is common:
  - Light grey areas are driven to oscillate left and right.
  - They vibrate out of plane when device rotates around sensitive axis due to Coriolis force.

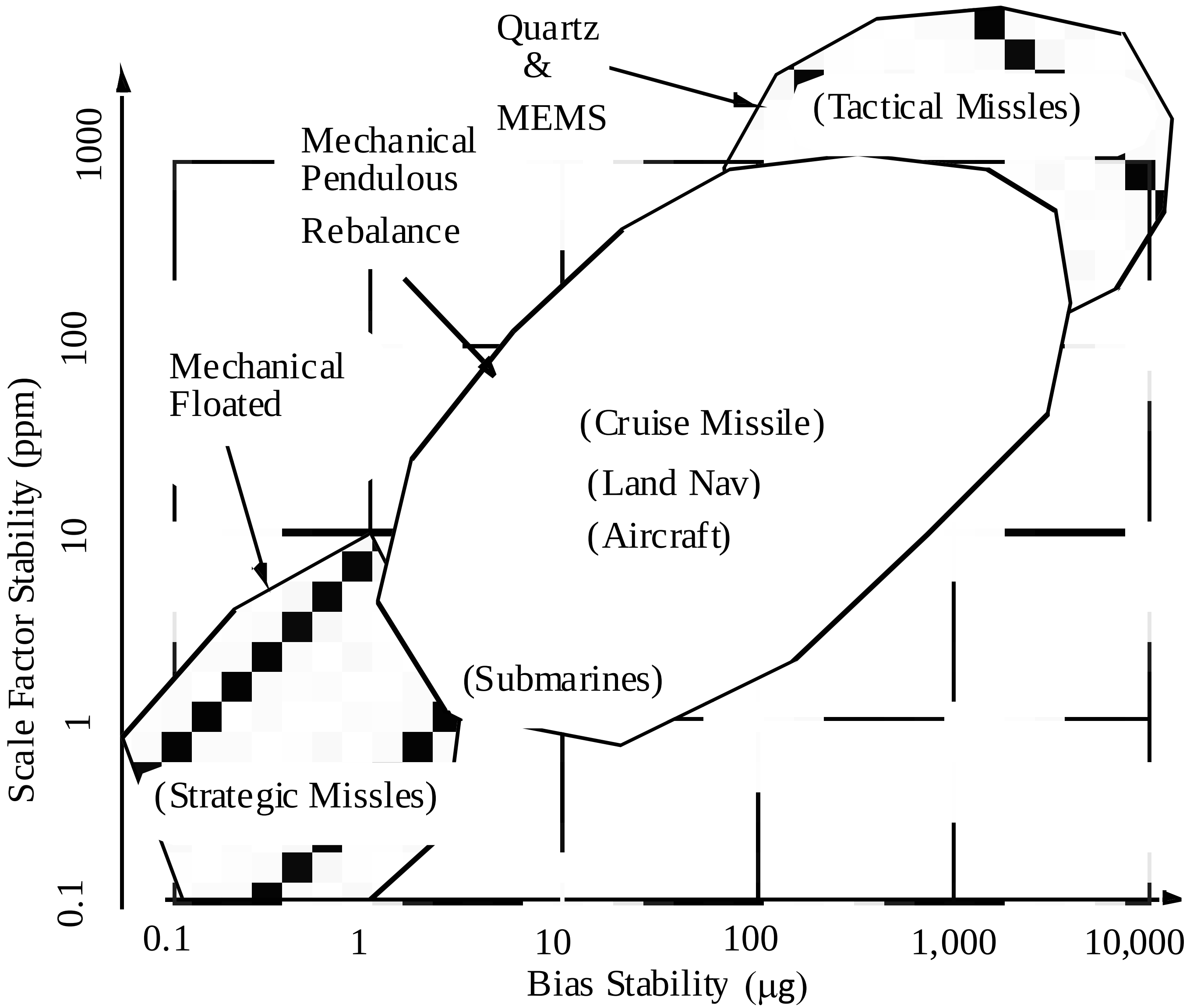
6.2.4.4. Inertial Measurement Units

- Typically 3 accels and 3 gyros (2 magnetometers) in one package.
- Price / Performance
  - 100 deg / hr = $1K
  - 1 deg / hr = $10K
  - 0.01 deg / hr = $100K
- Single chip IMUs have just happened or are about to happen.

HG 1930
Tactical Grade IMU

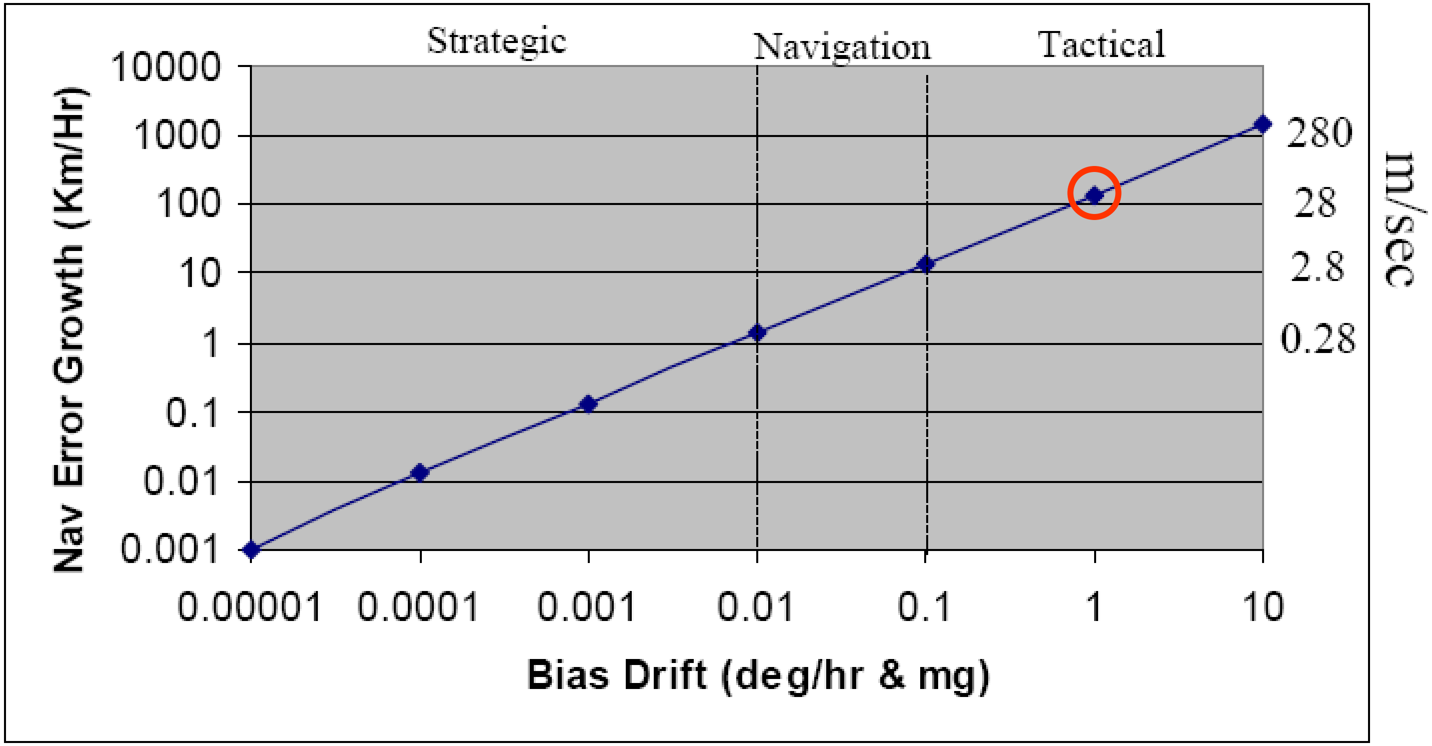
HG 9900
Navigation Grade IMU

Accelerometer Grades



Impact on Nav Error

- 1 deg/hr attitude error rate is 60 mph (100 Km/hr) position error rate.



1 deg/hr = 60 nm / hr = 100 km/hr

½ $a*t^2$ = 64 km after 1 hr

a*t = 35 m/sec after 1 hour

Outline

- 6.2 Sensors for State Estimation
  - 6.2.1 Articulation Sensors
  - 6.2.2 Ambient Field Sensors
  - 6.2.3 Inertial Frames of Reference
  - 6.2.4 Inertial Sensors
  - Summary

Summary

- Inertial frames are tricky to define.
- Accelerometers and inclinometers operate on similar principles
  - neither can distinguish gravitation from acceleration.
- Compasses indicate the local magnetic field.
  - not geographic north
  - Some error sources can be calibrated out.
- Gyros measure inertial rotation.
  - optical forms based on the Sagnac effect will be the ones used in the future. lasers and fiber optics made em practical.
  - MEMS devices are now very practical.