Chapter 6
State Estimation

Part 4

6.4 Satellite Navigation Systems

Outline

- 6.4 Satellite Navigation Systems
  - 6.4.1 Introduction
  - 6.4.2 Implementation
  - 6.4.3 State Measurement
  - 6.4.4 Performance
  - 6.4.5 Modes of Operation
  - Summary

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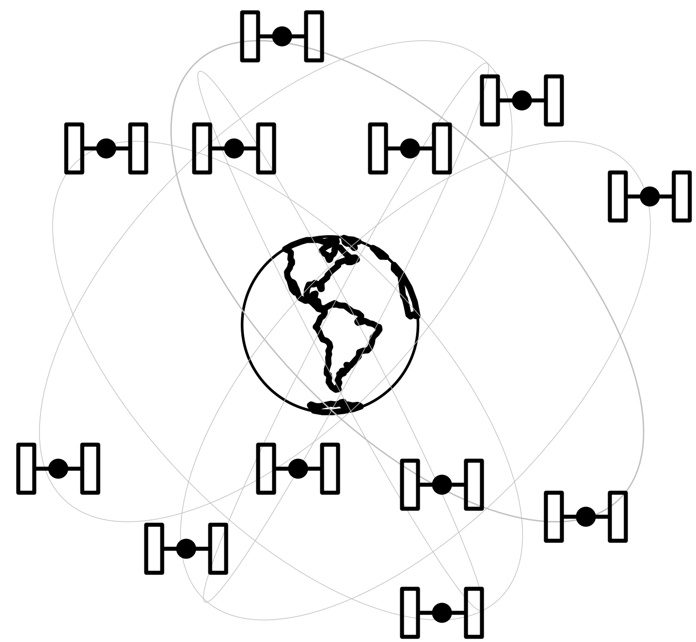
Introduction

- Satellite navigation has been called the next utility.
  - Like electric power and telephony.
  - Works continuously everywhere 24 hours a day.
- Revolutionized shipping, surveying, geophysics, all resource industries.
- Sales of GPS equipment and services are
  - expected to grow to $21.5 billion in 2008
  - up from $13 billion in 2003
- Many industries now completely dependent on it. (repercussions like power grid blackout).

History

- Marine Radio Navigation now obsolete
  - Military TRANSIT system in use since 1964
  - Civilian STARFIX system in use since 1986
- Satellite Radio Navigation
  - US Global Positioning System (developed by US DOD)
  - Soviet GLONASS (virtually identical to GPS)
  - European Galileo system launching since 2005



Availability

- Visible above mask angle of ~ 10 degrees.
- Always at least 4 visible.
- Excellent coverage of poles (but see GDOP).
- GPS requires line of sight:
  - Can't be used underground.
  - Can't be used underwater.
  - Can't be used in thick forest.
  - Can't be used around too many tall buildings.

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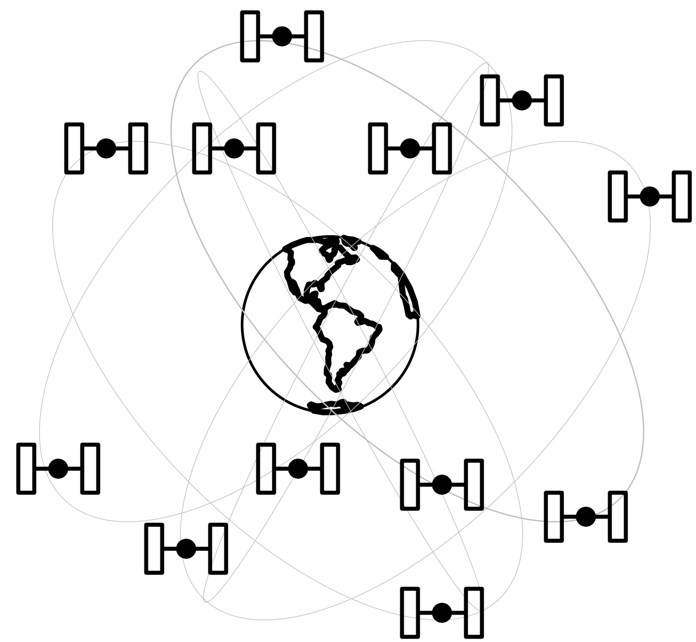
6.4.2.1 Satellites and Ground Stations

- 27 (24 plus 3 extra) earth satellites and in six orbits.
  – Actually 31 in orbit now
- Orbits inclined at 55 degrees to equatorial plane.
- Circular orbits 11,000 miles in amplitude
- Repeat exactly twice per sidereal day (12 hour orbits)



6.4.2.1 Satellites and Ground Stations

- Five ground stations spaced in longitude around the globe
- One is designated Master Control Station (MCS):
  - Tracks satellite positions very precisely.
  - Maintains overall system time standard.
- Satellites are updated on their position and clock bias for later retransmission to receivers

Global Positioning System (GPS) Master Control and Monitor Station Network

6.4.2.2 Signals

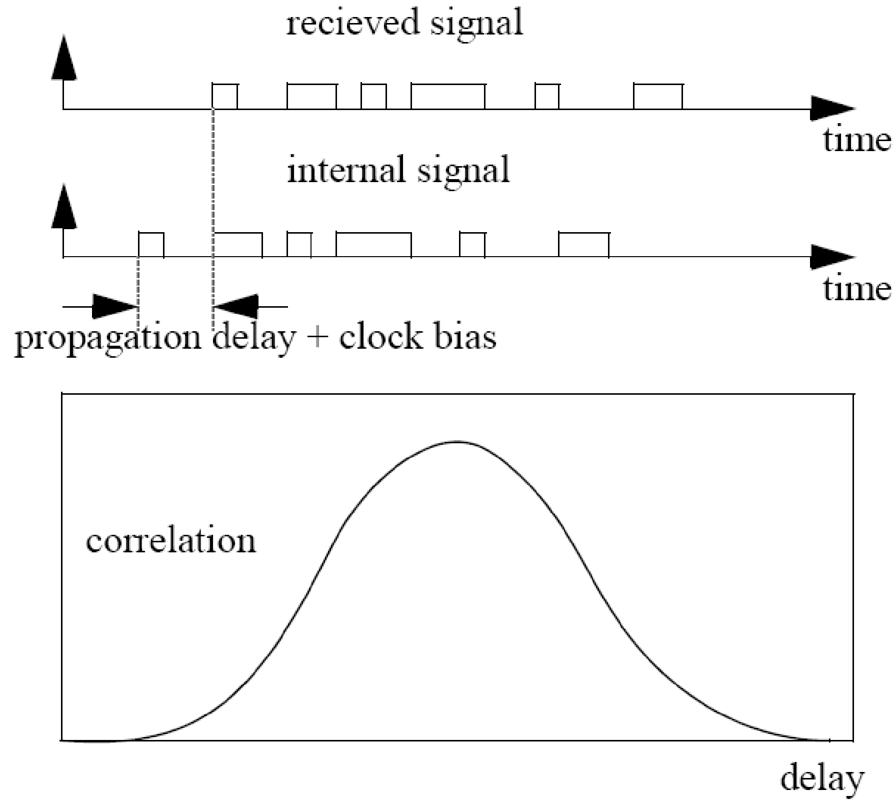
- Like most radio, the signals are modulated carrier signals.
- Two carriers are used, designated L1 (1575.42 MHz), and L2 (1227.60 MHz).
  - This allows measurement of atmospheric delay
- Modulators include many things:
  - C/A (coarse acquisition) PRN (pseudorandom noise) codes
  - P (precise) PRN code
  - secret Y code
  - Navigation message
  - Each satellite has its own distinct codes.
- Nav Message includes:
  - Handover word (system time of week) to aid in P code acquisition.
  - Accurate ephemeris for the satellite.
  - Less accurate ephemeris for all other satellites.

	C/A	P	NAV
L1	x	x	x
L2		x *	x

* P is encripted on L2
And known as P(Y)

6.4.2.3 Receiver Operation

- Receiver duplicates PRN codes internally to match with received signals.
- Correlation peaks at correct delay.
- PRN codes allow use of small antennae and handheld receivers are available.
- Receiver is a matched filter.
- Some receivers function solely as precision time or frequency standards.



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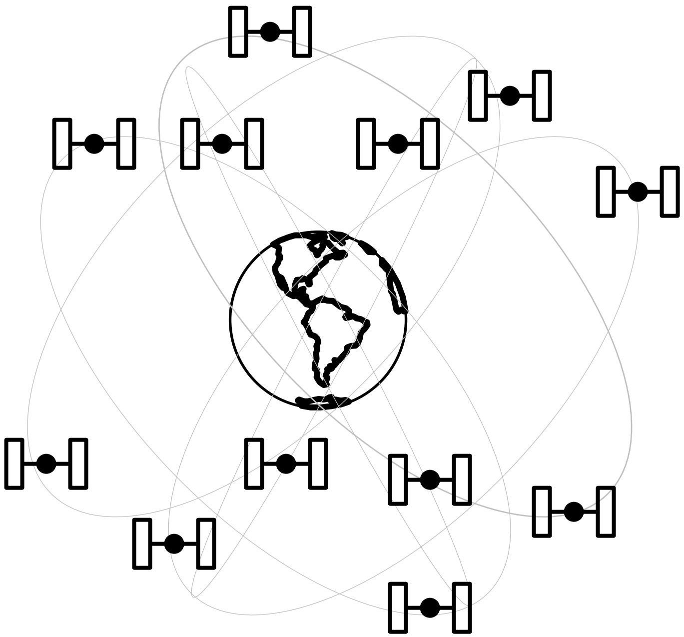
6.4.3.1 Position Measurement

(Principle of Operation)

- Basic idea is range triangulation.
- Constellation of satellites in earth orbit.
- Receivers pick up satellite radio transmissions on the ground.
- Satellites broadcast signals. This has military advantages:
  - Receivers do not answer - stealthy.
  - No limit on number of users.
- This also means anyone can use it.
  - Civilian use is part of the federal plan.
  - Usage is free once you buy the equipment.



6.4.3.1 Position Measurement

(Position Measurement)

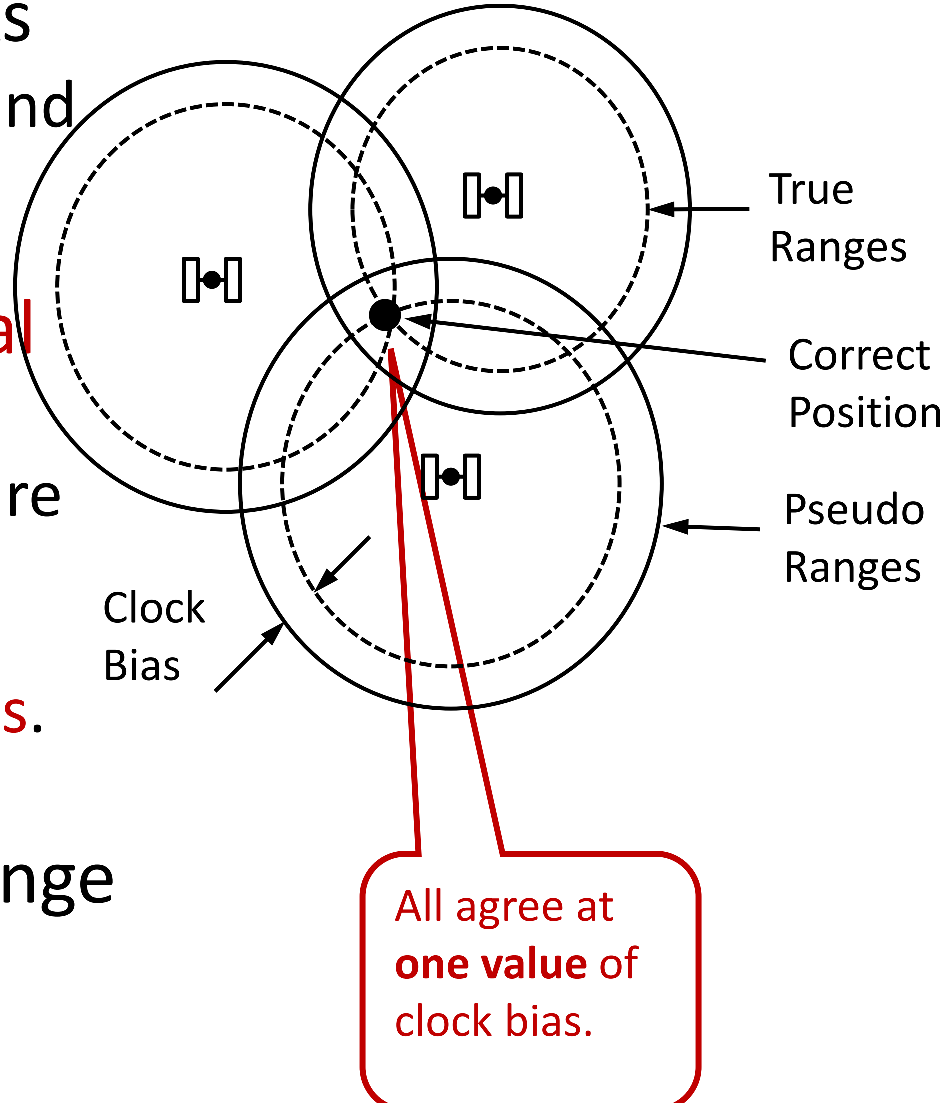
- Receivers measure range to satellites.
- Satellites transmit their positions, called "ephemeris data".

$$r1 = \sqrt{(x - x1)^2 + (y - y1)^2}$$
$$r2 = \sqrt{(x - x2)^2 + (y - y2)^2}$$

- 3D case is based on intersections of spheres.
- Ambiguity is resolved in several ways.
- Range is measured as time of transit of radio signal times wave speed.
- Sensitivity is about 1 meter in 50 million!

6.4.3.2 Time Measurement

- Satellites use atomic clocks
  - synchronized with the ground stations to the nearest nanosecond or so.
- Receivers use cheap crystal oscillator clocks.
  - Hence the receiver clocks are always off.
  - Use one extra satellite to measure this user clock bias.
- Clock bias causes range measurement errors so range measurements are called pseudoranges



4D Situation

- Since (GPS) satellites are synchronized, clock errors cause identical equivalent range errors for all satellites.

$$r1 = \sqrt{(x - x1)^2 + (y - y1)^2 + (z - z1)^2} + c\Delta t$$

$$r2 = \sqrt{(x - x2)^2 + (y - y2)^2 + (z - z2)^2} + c\Delta t$$

$$r3 = \sqrt{(x - x3)^2 + (y - y3)^2 + (z - z3)^2} + c\Delta t$$

$$r4 = \sqrt{(x - x4)^2 + (y - y4)^2 + (z - z4)^2} + c\Delta t$$

Equation A

GPS 4D Triangulation Equations

- Solve these for x, y, z, Δt ! No big deal.

6.4.3.3 Wave Speed Prediction and Measurement

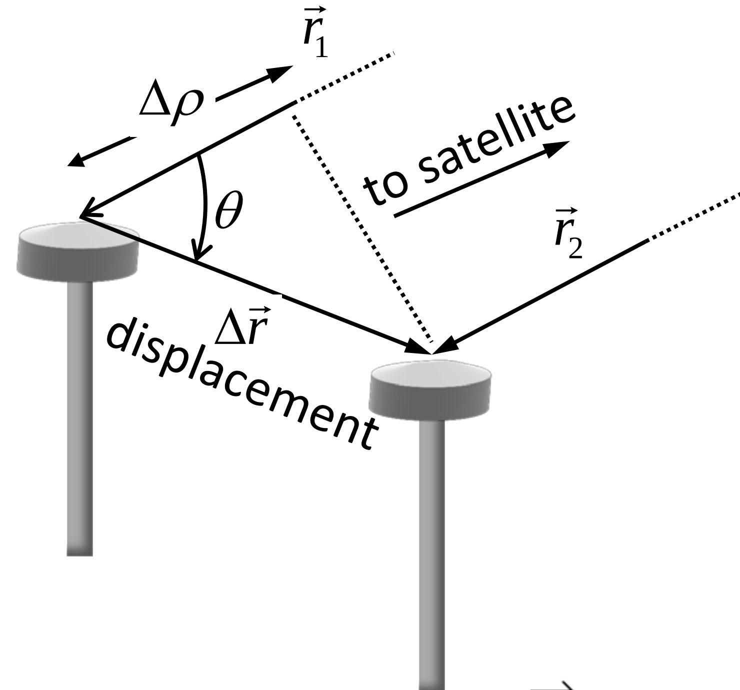
- Deviation from the nominal ‘c’ is significant enough to matter greatly.
- Two methods are used:
  - Prediction: Mathematical models of atmospheric delay (satellites broadcast model coefficients).
  - Measurement: Differential delay measurement for two carrier frequencies (delay is frequency dependent).

6.4.3.4 Velocity Measurement

- Principle is the Doppler frequency shift.
- Frequency shift is proportional to "range rate" caused by:
  - Velocity of satellite on orbit.
  - Velocity of earth's surface caused by earth's spin.
  - Velocity of vehicle on the earth's surface.
- Differentiate equation A to see how to convert four range rate observations into geocentric Cartesian velocity.

6.4.3.5 Orientation Measurement

- Principle is measurement of differential positions of several antennae fixed to a rigid vehicle.
- Measure differential carrier phase - “codeless” operation
- Each of three satellites provides a projection of the baseline vector (between the two antenna) onto each satellite beam axis.
- Solve for Δr vector in world frame.
- Need second baseline to determine rotation around this Δr.



$$\Delta\phi_1 = \Delta r \cos\theta_1 = (\overrightarrow{\Delta r} \cdot \hat{r}_1)/|\hat{r}_1|$$

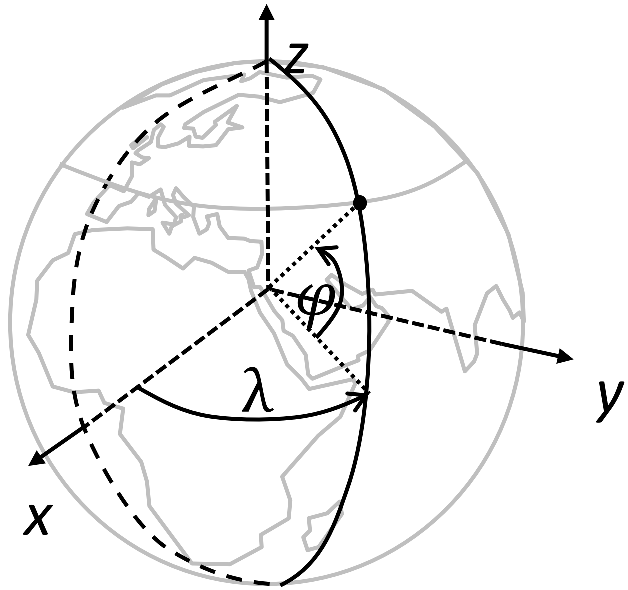
$$\Delta\phi_2 = \Delta r \cos\theta_2 = (\overrightarrow{\Delta r} \cdot \hat{r}_2)/|\hat{r}_2|$$

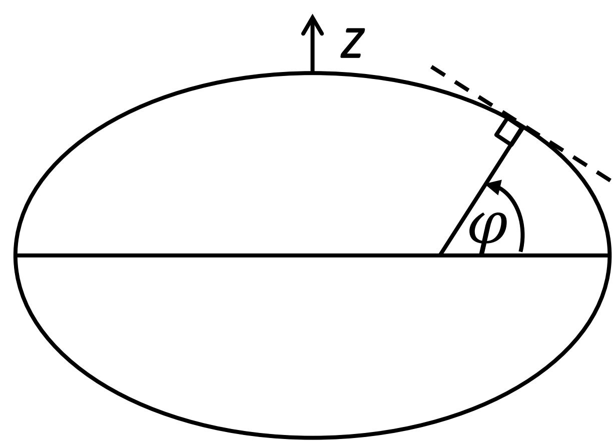
$$\Delta\phi_3 = \Delta r \cos\theta_3 = (\overrightarrow{\Delta r} \cdot \hat{r}_3)/|\hat{r}_3|$$

$$\begin{bmatrix} \hat{r}_{1x} & \hat{r}_{1y} & \hat{r}_{1z} \\ \hat{r}_{2x} & \hat{r}_{2y} & \hat{r}_{2z} \\ \hat{r}_{3x} & \hat{r}_{3y} & \hat{r}_{3z} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} \Delta\phi_1 \\ \Delta\phi_2 \\ \Delta\phi_3 \end{bmatrix}$$

6.4.3.6 Geodetic Coordinate Systems

- GPS uses the WGS-84 Earth-Centered Earth-Fixed (ECEF) System.
- Centered at Earth's mass center.
  - Easy to express satellite orbits.
- Earth polar radius is 21 Km less than equatorial.
  - Latitude is defined as shown opposite.





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6.4.4.1 Sources of Error

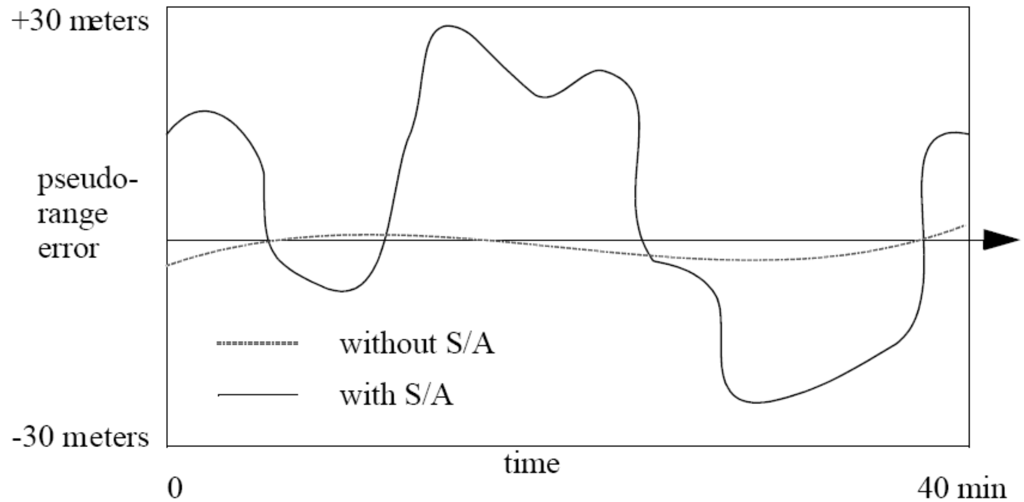
- Pseudorange error sources add in rms sense.
- Then, multiply result by GDOP.

Table 6.3: Pseudorange Error Sources (1 sigma)

Error Source	Nominal Value
Atmospheric Delays	4 meters
Satellite Clock & Ephemeris	3 meters
Multipath	1 meters
Receiver electronics and vehicle dynamics	1 meters
Total	5.2 meters (nominal)

Selective Availability

- Turned off in 2000.
- SPS error was reduced from 100m to 20 m.



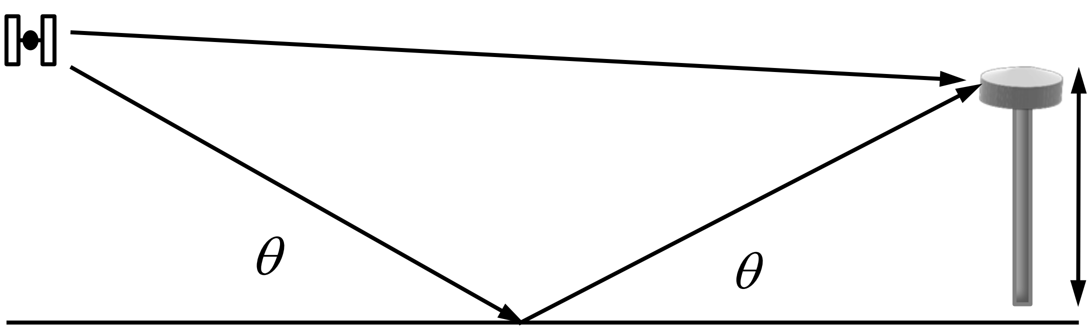
- Deliberate accuracy degradation by the DOD based on two techniques:
  - Adding “noise” to the ephemeris data.
  - Dithering the satellite clock.
- S/A is very long in period so it cannot be filtered in real time. It is robust to most ideas you may have for eliminating it - except differential techniques.
- Pseudorange error for stationary receiver might look as shown above.

6.4.4.1.1 Atmospheric ("Group") Delay

- Worth up to 30 meters range error if you do nothing.
  - Varies with time and position.
  - Different for each satellite at any time.
- Two kinds, ionospheric & tropospheric.
- Ionospheric
  - caused by charged particles causing diffraction (bending).
  - Varies by factor of 5 from day to night.
  - Varies by factor of 3 due to elevation angle.
  - Affected by solar magnetic activity.
  - Greatest at poles and equator.
  - Cannot be modeled adequately.
- Tropospheric
  - caused by water content changing index of refraction
  - 2.3 meters at zenith.
  - 25 meters at horizon.
  - Can be easily modeled.

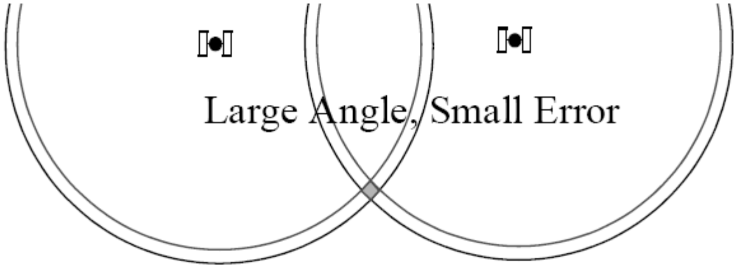
6.4.4.1.2 Multipath

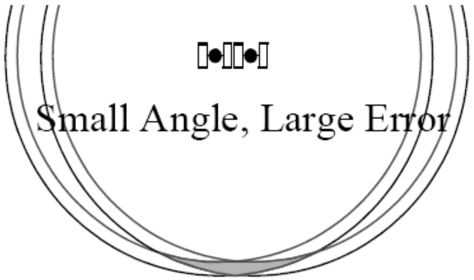
- Additional signal arrives through reflected non-line-of-sight path which is then out of phase with the direct signal.
- Result is destructive interference
- Substantial above water (water is an L band radio mirror).
- Usually worth less than 1 meter range error.
- Pronounced when receiver is close to surface.
- Moral is to mount antennae as high as possible.



$$\Delta L = c\Delta t \approx 2h\sin\theta$$

6.4.4.1.3 Geometric Dilution of Precision





- Usually from 4 to 6
- As high as 20.
- Very poor at poles (all satellites at horizon)
- In the context of GPS, five terms are defined:
  - TDOP - time dilution of precision (range equivalent)
  - PDOP - position dilution of precision (3D)
  - HDOP - horizontal dilution of precision
  - VDOP - vertical dilution of precision
  - GDOP - geometric dilution of precision
- Related by:

$$GDOP = \sqrt{(PDOP)^2 + (TDOP)^2}$$

$$PDOP = \sqrt{(HDOP)^2 + (VDOP)^2}$$

6.4.4.2 Measures of "Accuracy"

- Circular Error Probable (CEP) (related to Spherical Error Probable, Probable Error) is NOT the most probable error.
  - 50 % of measurements have errors above, 50 % below.
- 2Drms is twice the standard deviation. 95% of measurements should be less than 2drms.

$$2 \times \text{drms} \approx 2.5 \times \text{CEP}$$

- Vendors quote anywhere from 1 mm to 1/10 kilometer accuracy. Need to look deeper to understand why.

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6.4.5 Modes of Operation

- Tradeoff exists between accuracy, frequency of operation, and excursion over which it applies.
- All modes of operation trade one of these against the others.
  – Coded modes – use the PRN codes
  – Codeless modes – use the carrier directly

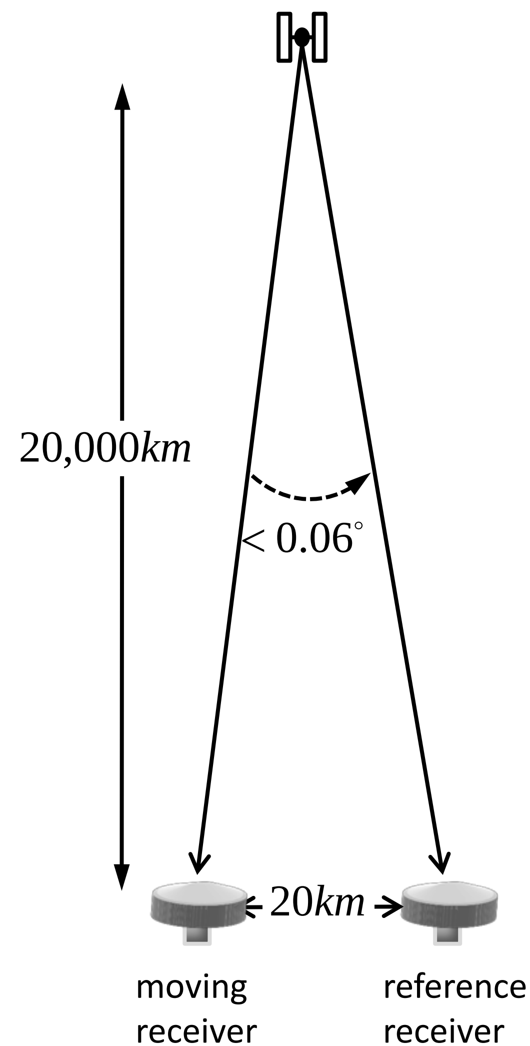
6.4.5.1 Coded and Codeless Modes (Coded Modes)

- Coded Modes
  - Highest update rate is a few Hz.
  - Relies on the PRN codes.
  - Accuracy depends on which code you use:
  - C/A code is 300 meter pulse wavelength
  - P code is 30 meter pulse wavelength
- Absolute
  - Adopt geocentric coordinate system.
  - Accuracies about 10 meters (called Standard Positioning Service or SPS).
- Relative
  - This is the error in the difference of two measurements taken by the same receiver separated in space and time.
  - Can only remove position and time independent bias.
  - Relative accuracy degrades to SPS in only a few minutes.
- Repetitive
  - This is the difference in two measurements taken at the same place at different times.
  - Can only remove time independent bias.

6.4.5.2 Code Phase Differential GPS

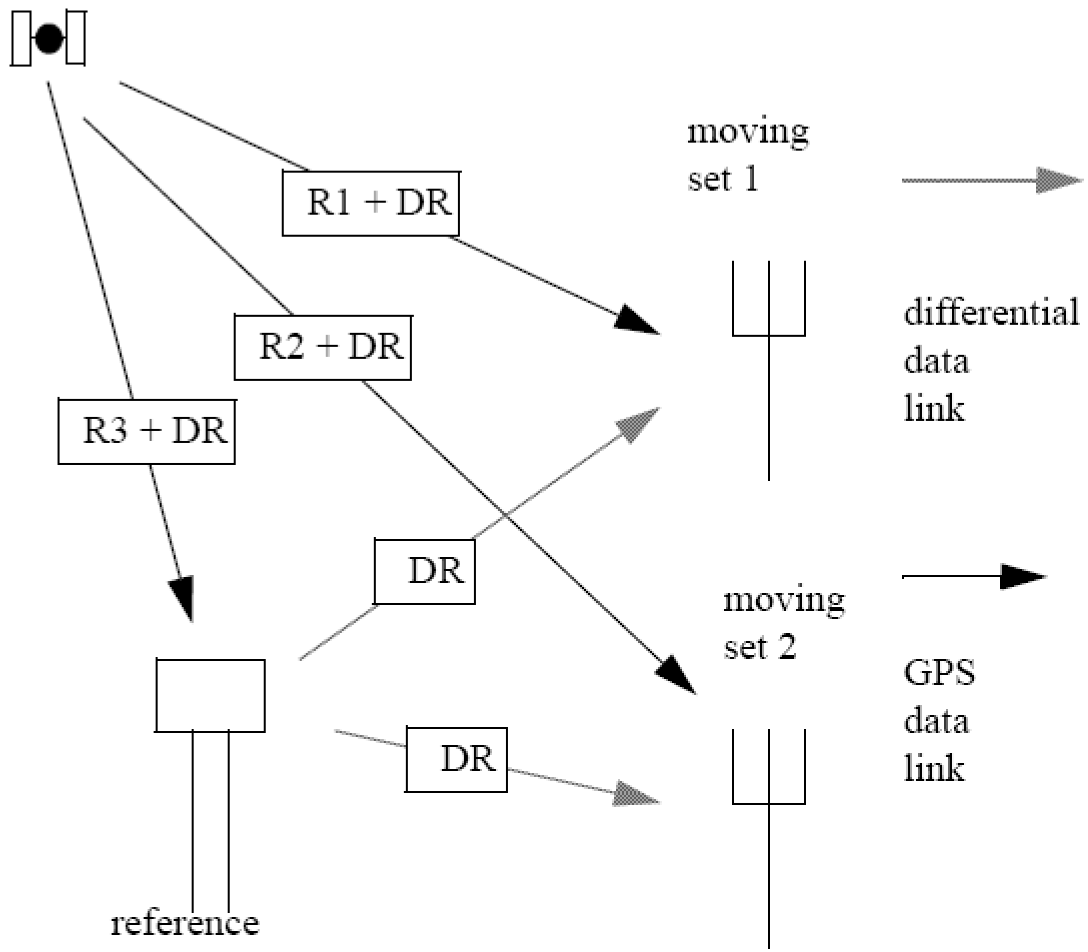
- Use two receivers, near each other, simultaneously with a communication link between them.
  – Relies on redundant communications channel.
  – Creates another opportunity for signal occlusion.
- Relies on reference "all in view" set called the base station.
- Pseudorange error is known at the reference site since it does not move (and its position is surveyed or considered the origin).
- Transmit this to the moving set.
- 3-5 meter accuracy possible up to 20 Km away.

Geometry is the Key



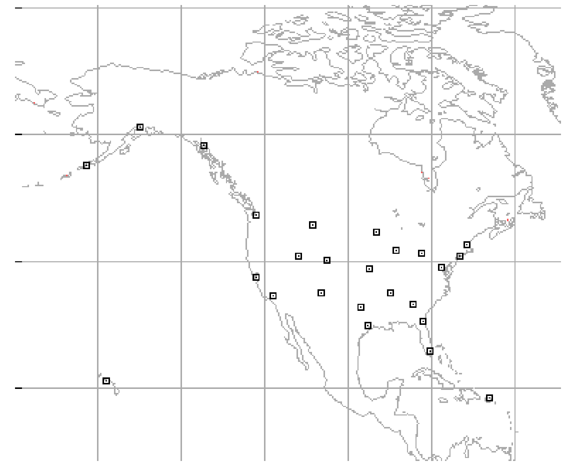
6.4.5.2 Code Phase Differential GPS

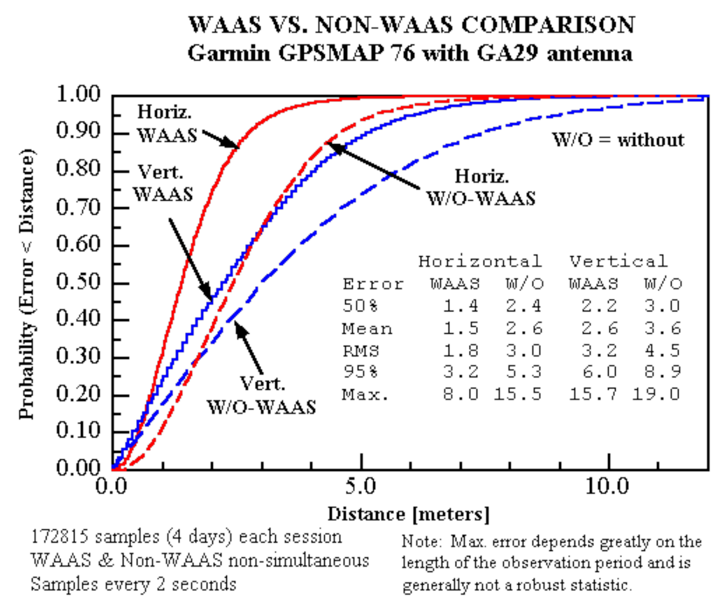
- Basic comms topology:



3.4.5.4 GPS Augmentation (WAAS)

- Wide Area Augmentation System
  - Available in Western Hemisphere.
  - Europe, India, Japan have their own
- Geostationary satellites used to boost the effective range of dedicated base stations.
- WAAS Satellites send corrections for 3 most significant error sources:
  - clock
  - ephemeris
  - atmosphere corrections





STARFIRE

NAVCOM
SF-2050G STARFIRE RECEIVER
$8550 (2009)

- Clock and ephemeris corrections distributed by satellite – like WAAS
- Global coverage based on 25 ground stations around the world.
- Expensive dual-channel receiver reads Y code on L1 and L2 frequencies
- Computes Ionosphere delay
  - Does not decrypt the military Y code
  - Just measures phase difference
- Accuracies / Repeatabilities
  - SF1 service (1 σ) → 1 m / 15 cm
  - SF2 service (1 σ) → 4.5 cm / 2.5 cm

Codeless Differential Modes

- Techniques used in surveying. Available everywhere.
- All are "differential" modes.
- Based on carrier phase measurements.
- Accuracies of few millimeters plus 2 ppm excursion possible.
- Static Surveying
  - Most reliable and most accurate method.
  - One receiver in known position.
  - Other one in unknown position.
  - Need 1 to 3 hours dwell per point.
  - Answer available only after postprocessing.
  - No lock on carrier required.
  - 5 mm accuracy achievable.

- Kinematic Surveying
  - Occupy point for fraction of a second.
  - Continuous lock required on 4 satellites.
  - This is now real time for robotics applications. 1cm accuracy typical when signal is strong.

Overall Accuracy

- A Note about Accuracy Measurements
  - Accuracy depends on confidence measure -- CEP (SEP) (50%), 2 drms (95%), 3drms (99%), etc.
  - Varies with location, even time of day (GDOP, atmospheric delays)
- Numbers below are for 2 drms, i.e. 95% confidence measure

Standard Positioning System (SPS)

- 3 meter horizontal accuracy (95% confidence)
- 5 meter vertical accuracy (95% confidence)
- 167 nanosecond time accuracy

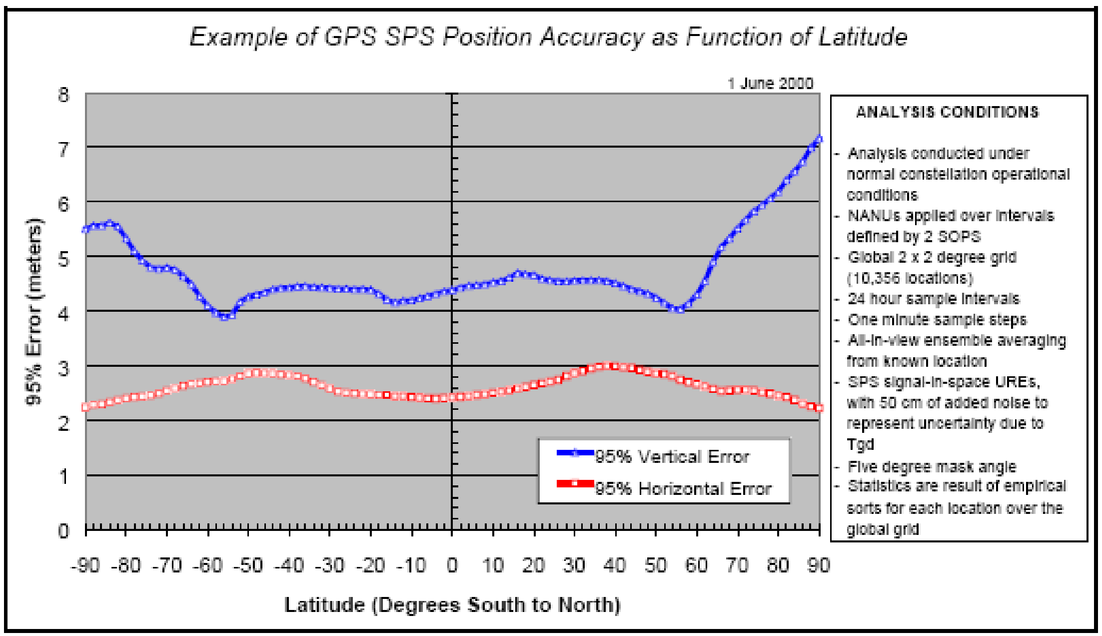


Figure A-5-1. Example of GPS SPS Position Accuracy as Function of Latitude

2drms
(2 sigma)
All-in-view
1 sec averaging
24 hour test

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Summary

- GPS provides continuous (2Hz) radio-triangulated position everywhere on the planet and in the near space region where line of sight to the sky is available.
- Receivers measure range to the satellites and time is considered a 4th unknown.
- Civilians are presently denied access to the more precise absolute navigation signals and therefore get an order of magnitude reduction of performance.
  – However, use of differential techniques makes this irrelevant in most applications.

Summary

- There are many signals available and many processing techniques which lead to a universe of performance specs with vary based on:
  - use of code correlation, code phase, or carrier phase
  - which code (C/A, P, Y), if any, is used
  - which carrier (L1, L2, Both) is used
  - availability of differential corrections
  - frequency of updates
  - extent of excursion
  - dynamics of host vehicle