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Computational Complexity of Terrain Mapping Perception in Autonomous Mobility

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Abstract

For autonomously navigating vehicles, the automatic generation of dense geometric models of the environment is a computationally expensive process. Using first principles, it is possible to quantify the relationship between the raw throughput required of the perception system and the maximum safely achievable speed of the vehicle. We show that terrain mapping perception is of polynomial complexity in the response distance. To the degree that geometric perception consumes time, it also degrades real-time response characteristics. Given this relationship, several strategies of adaptive geometric perception arise which are practical for autonomous vehicles.

1 Introduction

The need for high throughput perception algorithms has been acknowledged for some time [1][2][3][6][8][9][10][11][12] in the field of autonomous vehicle navigation. Yet, the evidence for this need has not been based on any underlying theory. This paper is concerned with the problem of estimating the computational complexity of perception in typical outdoor mobility scenarios. It is one of three related papers [4], [5] in these proceedings of which [4] and [5] should be read first.

We will use models of vehicle response and resolution requirements to quantify the complexity of terrain mapping perception in autonomous vehicles.

2 Preliminaries

Any vehicle which attempts to navigate autonomously in the presence of unknown obstacles must exhibit performance that satisfies a basic set of requirements. At the highest level, if the system is to survive on its own, the vehicle control system must satisfy the following requirements which will be collectively referred to as the **guaranteed safety** policy:

- **guaranteed response**: It must respond fast enough to avoid an obstacle once it is perceived.
- **guaranteed throughput**: It must update its model of the environment at a rate commensurate with its speed.
- **guaranteed detection**: It must incorporate high enough resolution sensors and computations to enable it to detect the smallest event or feature that can present a hazard.

Unfortunately attempting to satisfy these requirements simultaneously can lead to a system whose speed over the ground is severely limited by the throughput of the perception system. We will call this predicament the **throughput problem** and attempt to do something about it in this paper.

2.1 Nomenclature

In this paper, the words **response** and **reaction** will be distinguished for reasons of notational convenience. Generally, response will refer to the entire autonomous system including the vehicle, and reaction will refer to the computational and control aspects of the system, only. Finally, the term **maneuver** will apply to the vehicle physical response only.

For example, if the vehicle applies the brakes, the time it took to decide to brake is the **reaction time**, the time spent stopping is the **maneuver time**, and the sum of these is the **response time**.

$$T_{response} = T_{react} + T_{maneuver}$$

2.2 Major Assumptions

We will be interested in terrain mapping perception algorithms which associate a single unique elevation with each point in a sampled representation of the groundplane. Within this context, we will find it useful to make a few assumptions.

2.2.1 Stationary Environment Assumption

The most important assumption of the analysis is the **stationary environment assumption** because this permits us to perceive a point on the groundplane only once and avoid dealing with moving obstacles. While points in the environment will certainly move relative to the vehicle, they will be assumed not be moving relative to each other.

2.2.2 Point Vehicle Assumption

The point **vehicle assumption** is the assumption that the finite extent of the vehicle can be ignored in the analysis. When adopted, it allows us to ignore the fact that the sensor is not positioned at the vehicle control point, and ignore the extension of the vehicle nose in front of the sensor. This assumption will be made for reasons of convenience and clarity only. It is not necessary for deeper reasons and does not unduly affect our results.

2.2.3 Small Incidence Angle

The **perception ratio** will be our name for the nondimensional quantity given by the ratio of the sensor height above the groundplane to the measured range. This is also the angle of incidence to the terrain, so the **small incidence angle assumption** is the assumption that this angle is shallow, and correspondingly, that the perception ratio is small.

2.3 Interactions

The satisfaction of the guaranteed safety policy requires that response, throughput, and resolution requirements all be met simultaneously. They cannot, however, be treated individually because they are all interrelated. For example:

- Throughput depends on resolution if the same area of ground is to be covered per unit time.
- Resolution depends on range through the projective geometry of the sensor.
- Sensor range depends on speed if lookahead is modulated to match or exceed response distance.

Resolving the above relations leads to the conclusion that throughput depends at least on speed. Naive analysis will suggest that the problem of high speed navigation is difficult because the necessary throughput approaches impractical levels. Guaranteed safety implies that throughput is proportional to a high power of velocity because:

- Maximum range increases quadratically with speed because response distance does.
- Pixel size decreases quadratically with maximum range for constant groundplane resolution.
- Throughput increases quadratically with pixel size assuming a fixed sensor field of view.

On the other hand, if one computes the rate at which a vehicle covers ground area as it moves, any reasonable spatial resolution for sampling this area leads to a perceptual throughput that is trivial to meet. This difference has many sources which will be the core issue in our analysis.

2.4 Coordinate Conventions

The angular coordinates of a pixel will be expressed in terms of horizontal angle or **azimuth** ψ, and vertical angle or **elevation** θ. Three orthogonal axes are considered to be oriented along the vehicle body axes of symmetry. Generally, we will arbitrarily choose z up, y forward, and x to the right:

- x - **crossrange**, in the groundplane, normal to the direction of travel.
- y - **downrange**, in the groundplane, along the direction of travel.
- z - **vertical**, normal to the groundplane.

Certain vehicle dimensions that will be generally important in the analysis are summarized in the following figure. One distinguished point on the vehicle body will be designated the vehicle control point. The position of this point and the orientation of the associated coordinate system is used to designate the pose of the vehicle.

The wheelbase is L, and the wheel radius is r. The height of the sensor above the groundplane is designated h and its offset rear of the vehicle nose is p. The height of the undercarriage above the groundplane is c. Range measured from the sensor is designated R.

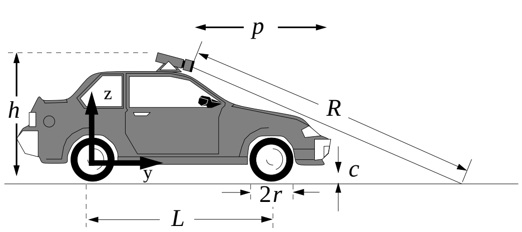

Figure 1: Important Dimensions

We will sometimes distinguish range, R measured in 3D from a range sensor, and the projection of range Y onto the groundplane. Generally, both will be measured forward from the sensor unless otherwise noted.

The velocity of the vehicle will be denoted V. The coefficient of friction between the wheels and the terrain will be μ, and the acceleration due to gravity will be g.

2.5 Acronyms

The following acronyms will be employed:

- **VFOV** - vertical field of view
- **HFOV** - horizontal field of view
- **IFOV** - instantaneous field of view
- **IFOV**$_H$ - horizontal instantaneous field of view
- **IFOV**$_V$ - vertical instantaneous field of view.

The **instantaneous field of view** will be defined as the angular width of a pixel.

3 Analysis of Guaranteed Safety

A brief analysis of the requirements arising from the policy of guaranteeing vehicle safety is presented in the accompanying papers [4][5]. These results have been presented in order to support our analysis here.

4 Assumptions of the Analysis

The following subsections will analyze the throughput problem in terms of the design of a vehicle which is optimized for some maximum speed. The pixel size is permitted to change with speed, so the following graphs represent the variation of system designs versus speed - not the throughput requirement for a single design as it drives faster.

Based on analysis of field of view that is not included here, the HFOV will be fixed because it must be chosen based on the worst case speed which is often below the maximum vehicle speed. We will employ the following assumptions:

- Horizontal field of view is fixed at 80°, 120°, 170°, and 215° for each increasing reaction time respectively based on analysis presented in [4].
- Sensor frame rate is set to 2 Hz because this is a typical value for a laser rangefinder with a wide VFOV.
- Minimum acuity will be used because this is actually the most stringent requirement beyond some range.

The estimates that are produced are underestimates for many reasons, including the following additional assumptions, all of which lower the required throughput:

- The processor load is assumed to be 50 flops per pixel when experience suggests that many times this is required in a practical system.
- Braking is chosen as the obstacle avoidance maneuver. This is viable for a system which stops when a hazard is detected. However, when a vehicle turns to avoid obstacles, sensor lookahead must exceed the stopping distance by a large factor - being based on a turning maneuver.
- The maximum range that is chosen is based on the response distance. Actually it is the minimum range which should be set to the response distance.
- The point vehicle assumption is used to avoid dealing with the offset of the vehicle nose from the sensor.
- We use an obstacle sampling factor of (2 pixels are assumed sufficient to resolve an obstacle reliably). A practical factor is perhaps between 3 and 10. This implies that the results must be multiplied by the square of a practical sampling factor.

5 Common Throughput Expression

It will be necessary to quantify the number of operations performed per unit time in terms of the number of sensor pixels processed times the number of operations used per pixel. This section develops a basic expression which is then modified based on further assumptions.

5.1 Definitions

The following terms are defined:

5.1.1 Sweep Rate

The product of the vertical field of view and the frame rate is measures the effective angular velocity of the sensor in the vertical direction, and is known as the **sweep rate**:

$$\dot{\theta} = VFOV \times f_{images}$$

where $VFOV$ is the vertical field of view and f_{images} is the frame rate.

5.1.2 Sensor Flux

The **sensor flux** $\dot{\Psi}$ represents the solid angle subtended by the field of view and generated per unit time[1]. It can be written as:

$$\dot{\Psi} = HFOV \times VFOV \times f_{images}$$

where $HFOV$ is the horizontal field of view. Note that the sensor flux is the two dimensional analog of the sweep rate. Not surprisingly, the two are related by:

$$\dot{\Psi} = HFOV \times \dot{\theta}$$

and the sensor flux has units of angular flux - solid angle per unit time.

5.1.3 Sensor Throughput

The number of range pixels generated per unit time by a sensor will be called the **sensor throughput** f_{pixels}. If the field of view is fixed and pixels are square, the sensor throughput is given by:

$$f_{pixels} = \frac{\dot{\Psi}}{(IFOV)^2}$$

The IFOV is the angular resolution of the sensor. A sensor for which Ψ is constant is called **constant flux**, and one for which the IFOV is constant is called **constant scan**.

5.1.4 Processor Load

It is useful to define the **processor load** σ_P as the number of flops necessary to process a single range pixel.

$$\sigma_P = \frac{flops}{pixel}$$

Thus, the relationship between processing load and sensor throughput is:

$$f_{cpu} = f_{pixels} \times \sigma_P$$

5.1.5 Computational Bandwidth

The **computational bandwidth** is the number of flops required of a processor per unit time. If the geometric transforms of perception are the only aspect of the system considered, this quantity is related to the sensor bandwidth by the processor load:

$$f_{cpu} = \sigma_P f_{pixels} = \sigma_P \frac{\dot{\Psi}}{(IFOV)^2}$$

$$f_{cpu} = \sigma_P \frac{HFOV \times IFOV \times f_{images}}{(IFOV)^2}$$

When it is necessary to employ a nonsquare pixel size, the horizontal and vertical pixel dimensions can be differentiated as follows:

$$f_{cpu} = \sigma_P \frac{HFOV \times VFOV \times f_{images}}{IFOV_H IFOV_V}$$

1. In physics, flux is a measure of flow past a surface - expressed in units of area per unit time. Hence the name here.

5.2 Basic Mechanism

The basic mechanism for generating a complexity estimate is as follows:

- Choose an angular resolution that is consistent with the need to resolve obstacles at the maximum range (guaranteed detection).
- Choose a maximum range consistent with the need to stop if necessary (guaranteed response).
- Choose a fixed field of view and frame rate (because sensors are designed that way).
- Throughput is then the number of pixels generated per unit time times the cost of processing one pixel.

Guaranteed detection is enforced by substituting for the IFOV from the minimum acuity rule developed earlier:

$$IFOV = \frac{1}{2}\frac{Lh}{R^2}$$

Guaranteed response is enforced by substituting for the maximum range based on the expression derived in an earlier section for the stopping distance in terms of the braking coefficient:

$$Y_{min} = p + T_{react}V[1 + \bar{b}]$$

We will invoke the point vehicle assumption and eliminate p and the small incidence angle assumption to equate Y to R:

$$R = T_{react}V[1 + \bar{b}]$$

Complexity is estimated by noting that the braking coefficient does not approach 1 for the speed regimes of current research, so it can be neglected. Under this assumption, the stopping distance is the product of speed and reaction time - the reaction distance.

The resulting complexity estimate represents the minimum computational throughput necessary in order to meet guaranteed response, throughput, and detection simultaneously. Any system which cannot supply this throughput must either:

- reduce resolution and violate guaranteed detection.
- reduce field of view and violate guaranteed throughput.
- reduce lookahead and violate guaranteed response.

6 Constant Flux

A real sensor usually has a fixed field of view and fixed frame rate, so the sensor flux Ψ is constant. It is straightforward to compute the throughput required to keep up with the sensor. Throughput under guaranteed detection is obtained by substituting the acuity expression into the basic throughput expression:

$$f_{cpu} = \sigma_P f_{pixels} = \sigma_P \left(\frac{4R^4}{(Lh)^2}\right)\dot{\Psi}$$

Substituting the stopping distance for range gives:

$$f_{cpu} = \sigma_P f_{pixels} = \sigma_P \left[\frac{4(T_{react}V[1 + \bar{b}])^4}{(Lh)^2}\right]\dot{\Psi}$$

In the kinematic braking regime, the following result for the computational complexity is obtained:

$$f_{cpu} \sim \sigma_P O([T_{react}V]^4)$$

The following graph indicates the variation of throughput with speed when square pixel size is chosen to satisfy the minimum acuity resolution requirement at the maximum range. The processing rates required are substantial - even under the liberal assumptions mentioned earlier.

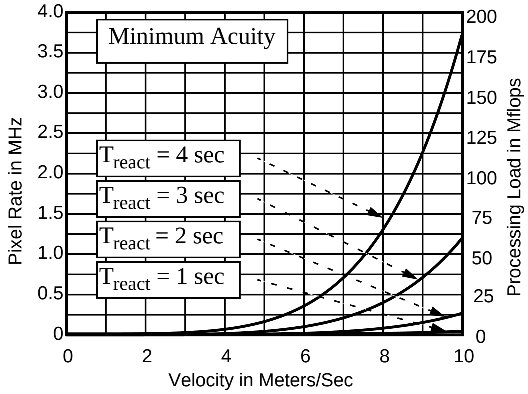

Figure 2: Throughput for Constant Flux

7 Adaptive Sweep

Adaptive sweep will be defined as the process of modulating the sweep rate of the sensor to generate an imaging density of unity and thereby barely satisfy guaranteed throughput. This does not compromise guaranteed response and it leads to significant reduction in throughput.

The basic throughput expression under guaranteed detection is:

$$f_{cpu} = \sigma_P f_{pixels} = \sigma_P\left(\frac{4R^4}{(Lh)^2}\right)\dot{\Psi}$$

The sensor flux is, again, the solid angle measured per unit time. Thus:

$$\dot{\Psi} = VFOV \times HFOV \times f_{images}$$

The complexity expression is now:

$$f_{cpu} = \sigma_P\left(\frac{4R^4}{(Lh)^2}\right)VFOV \times HFOV \times f_{images}$$

An earlier expression which relates the vertical field of view to its projection on the groundplane is:

$$VFOV = h\frac{VT_{cyc}}{\rho_{throughput}Y^2}$$

Guaranteed throughput is implemented by setting the throughput ratio to unity. Also, by definition:

$$T_{cyc} = 1/f_{images}$$

Which gives the sweep rate as:

$$VFOV \times f_{images} = h\frac{V}{Y^2}$$

This gives:

$$f_{cpu} = \sigma_P\left(\frac{4R^4}{(Lh)^2}\right)h\frac{V}{Y^2}HFOV$$

Cancelling an R^2 and a Y^2 and substituting the stopping distance for R gives:

$$f_{cpu} = \sigma_P\left[\frac{4(T_{react}V[1+\bar{b}])^2}{L^2h}\right](V)(HFOV)$$

In the kinematic braking regime, the following result for the computational complexity is obtained:

$$f_{cpu} \sim \sigma_P O([T_{react}V]^2[V])$$

which is less than the previous result by the factor $T^2_{react}V$.

This result leads to the conclusion that adjusting the vertical field of view based on vehicle speed can significantly reduce computational throughput requirements.

The following graph indicates the variation of throughput with speed when the vertical field of view is computed from the above expressions and square pixel size is chosen to satisfy the resolution requirement at the maximum range.

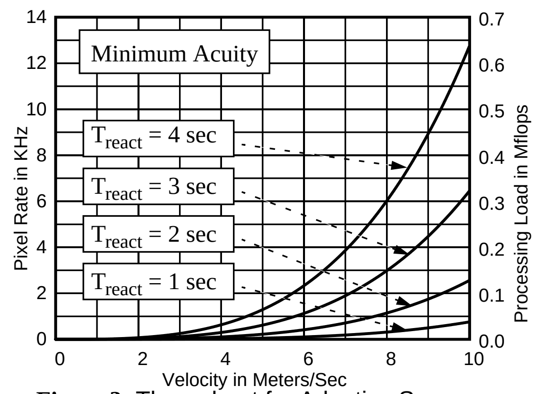

Figure 3: Throughput for Adaptive Sweep

8 Adaptive Sweep/Scan

Adaptive scan will be defined as the process of modulating the aspect ratio of image pixels in order to achieve roughly the same crossrange and downrange resolution on the groundplane. This process does not compromise guaranteed detection and it leads to further reduction in throughput.

Recall that the cpu load required to process all sensory data is given by:

$$f_{cpu} = \sigma_P f_{pixels} = \sigma_P\frac{\dot{\Psi}}{(IFOV)^2}$$

In practical adaptive scan, the pixel aspect ratio is adjusted to be a constant over the field of view and equal to the **perception ratio**. The vertical and horizontal instantaneous field of view then have different expressions at minimum acuity:

$$IFOV_V = \frac{1}{2}\frac{Lh}{R^2} \qquad IFOV_H = \frac{1}{2}\frac{Lh}{R^2}\left(\frac{R}{h}\right) = \frac{1}{2}\frac{L}{R}$$

where the horizontal image resolution was multiplied by the factor R/h in order to implement adaptive scan[1].

The throughput required to process all sensory data is then given by:

1. Note that it may be appropriate to determine crossrange resolution from considerations of minimum obstacle width rather than vehicle wheelbase. In our analysis, we are attempting to show that complexity is high - even in the best case of minimum acuity. The terrain smoothness assumption is a huge assumption that we employ here for illustrative purposes only. The real situation is far worse than our calculations bear out.

$$f_{cpu} = \sigma_P\left(\frac{4R^3}{L^2h}\right)VFOV \times HFOV \times f_{images}$$

As before, the sweep rate under guaranteed throughput is:

$$VFOV \times f_{images} = h\frac{V}{Y^2}$$

This gives:

$$f_{cpu} = \sigma_P\left(\frac{4R^3}{L^2h}\right)h\frac{V}{Y^2}HFOV$$

Cancelling an R^2 and a Y^2 and substituting the stopping distance for R gives:

$$f_{cpu} = \sigma_P\left[\frac{4(T_{react}V[1+\bar{b}])}{L^2}\right](V)(HFOV)$$

In the kinematic braking regime, the following result for the computational complexity is obtained:

$$f_{cpu} \sim \sigma_P O(T_{react}V^2)$$

which is less than the previous result by the factor $T_{react}V$. The following graph indicates the variation of throughput with speed when vertical field of view is computed from the above expressions, nonsquare pixel size is chosen to satisfy the resolution requirement at the maximum range.

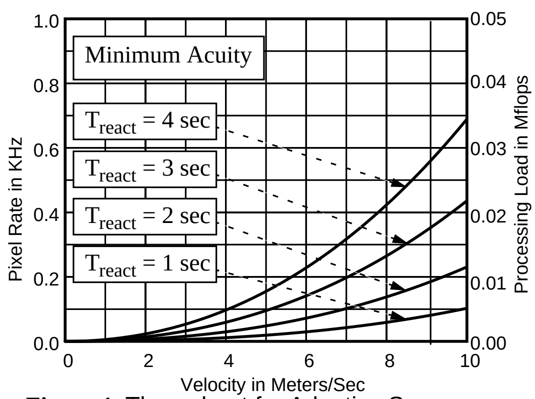

Figure 4: Throughput for Adaptive Scan

These two simple adaptive techniques have reduced the throughput requirements by 4 orders of magnitude over constant flux at speeds of 20 mph and reaction times of 4 seconds.

9 Adaptive Sweep, Uniform Scan

Uniform scan will be defined as the process of modulating pixel size and field of view in order to achieve perfect homogeneous and isotropic distribution of resolution on the groundplane. No existing sensor can provide this capability, but it is a useful theoretical approximation.

This analysis considers the fundamental acuity and throughput requirements of perception. As a minimum requirement, any sensor must generate geometry at a rate that is consistent with the rate at which the vehicle consumes geometry through its motion. Consider that the motion of the vehicle consumes a swath of geometry directly in front of it as shown below:

In the simplest case, this consumed area to the left must be replaced by adding new information to the right. The area con-

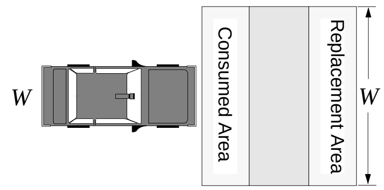

Figure 5: Area Consumption

sumed per second, expressed in appropriate units, is the required absolute minimum throughput of a perception system under guaranteed throughput.

Let the width of the swath be W, the velocity of the vehicle be V, and the required spatial resolution be δ. This minimum rate is given by:

$$f_{cells} = \frac{WV}{\delta^2}$$

In previous sections it was shown that, under guaranteed response, the maximum range can be determined from the stopping distance. Let L be the vehicle wheelbase. Setting the width of the swath to twice the maximum range gives:

$$f_{cells} = \frac{2s_{response}V}{L^2} = \frac{2T_{react}V^2[1+\bar{b}]}{L^2}$$

Putting all of these results together, gives the following expression for the processing load:

$$f_{cpu} = \sigma_P f_{cells} = \sigma_P\frac{2T_{react}V^2[1+\bar{b}]}{L^2}$$

In the kinematic braking regime, the following result for the computational complexity is obtained:

$$f_{cpu} \sim \sigma_P O(T_{react}V^2)$$

which is, in complexity terms, equal to the adaptive sweep, adaptive scan expression. There is a multiplicative constant difference of $2 \times HFOV$ between this minimum requirement and the adaptive sweep, adaptive scan expression because the whole image is processed at the same nonsquare pixel resolution in adaptive scan and the HFOV is fixed. This relationship is plotted below for minimum acuity spatial resolution of 3.3 meters versus vehicle velocity for various values of system reaction time.

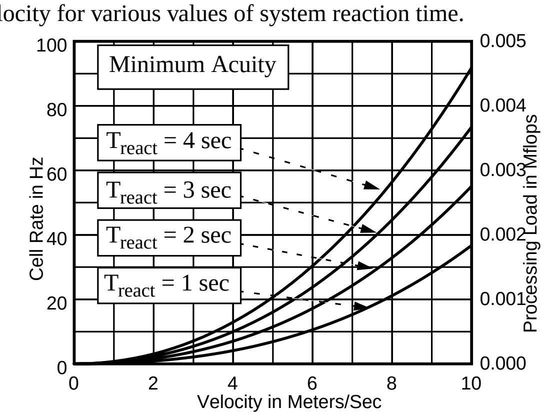

Figure 6: Throughput for Uniform Scan

10 Throughput for All Algorithms

Recall that the preceding complexity estimates are all consistently based on a kinematic braking regime assumption. The true power of velocity is actually squared as speeds increase. Identical assumptions of minimum acuity, 4 second reaction time, and 10 meter / second speed, have led to the following throughput estimates for different image processing algorithms:

Table 1: Throughput Estimates

Algorithm	Estimate	Complexity
constant flux	250 Mflops	$O(T_{react}^4 V^4)$
adaptive sweep	0.7 Mflops	$O(T_{react}^2 V^3)$
adaptive scan	0.035 Mflops	$O(T_{react} V^2)$
ideal	0.0045 Mflops	$O(T_{react} V^2)$

The actual data for all 4 second reaction time curves is plotted below on a logarithmic vertical scale.

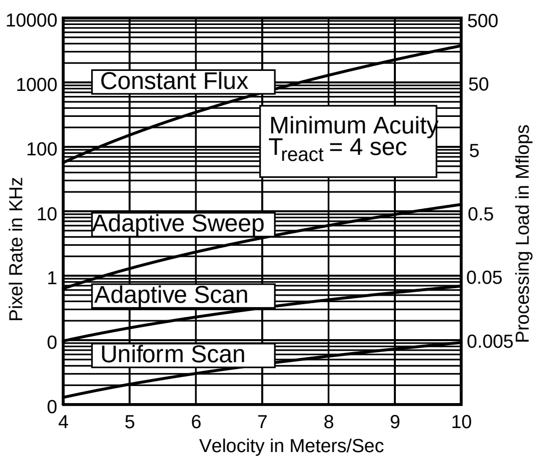

Figure 7: Throughput for All Algorithms

Notice that the complexity in all of the above cases contains a constant times a power of the product $T_{react}V$. That is:

$$f_{cpu} \sim \sigma_P O([T_{react}V]^N [V]^M)$$

There are a few ways to read this result. If throughput is fixed, then speed is inversely proportional to reaction time. If speed is fixed, throughput required grows with the nth power of reaction time. If reaction time is fixed, throughput grows with the (n+m)th power of speed. In general, the complexity of terrain mapping perception is polynomial in the vehicle reaction distance.

11 Acknowledgments

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12 Conclusions

As speeds increase, the redundant measurement of the same geometry that happens when images overlap on the groundplane becomes more of an efficiency concern because the imaging density increases without bound. We have proposed a technique called adaptive sweep which deliberately modulates the vertical field of view and shown two orders of magnitude reduction in required perceptual throughput.

Another technique which improves computational requirements is the modulation of the range pixel aspect ratio in order to precisely meet the groundplane resolution requirement. We have shown an order of magnitude reduction in required perceptual throughput under certain operating conditions.

In general, the complexity of terrain mapping perception is polynomial in the vehicle reaction distance. This complexity result quantifies the perceptual throughput problem of autonomous mobility and identifies the **fundamental trade-off** associated with the use of finite computing resources. This trade-off is one of resolution for speed, or equivalently, reliability for speed. Finite computing resources establish a limit on vehicle performance which can be expressed as either high speed and low reliability or vice versa.

13 References

[1] M. Daily et al., “Autonomous Cross Country Navigation with the ALV”, In Proc. of the 1988 IEEE International Conference on Robotics and Automation, Philadelphia, Pa, April 1988; pp. 718-726.

[2] H. P. Moravec, “Obstacle Avoidance and Navigation in the Real World by a Seeing Robot Rover”, Ph. D. Thesis, Stanford University, 1980.

[3] A. J. Kelly, “An Intelligent Predictive Control Approach to the High-Speed, Cross Country Autonomous Navigation Problem”, Ph. D. thesis, Robotics Institute, Carnegie Mellon University, June 1995.

[4] A. Kelly, and A. Stentz, “Analysis of Requirements of Rough Terrain Autonomous Mobility: Part I: Throughput and Response”, IEEE International Conference on Robotics and Automation, Albuquerque, New Mexico, April 1997.

[5] A. Kelly, and A. Stentz, “Analysis of Requirements of Rough Terrain Autonomous Mobility: Part II: Resolution and Accuracy”, IEEE International Conference on Robotics and Automation, Albuquerque, New Mexico, April 1997.

[6] L. Matthies, A. Kelly, T. Litwin, “Obstacle Detection for Unmanned Ground Vehicles: A Progress Report.”, to appear in IEEE International Symposium on Intelligent Vehicles.

[7] L. Matthies, “Stereo Vision for Planetary Rovers”, International Journal of Computer Vision, 8:1, 71-91, 1992.

[8] R. T. Dunlay and D. G. Morgenthaler, “Obstacle Detection and Avoidance from Range Data”, In Proc. SPIE Mobile Robots Conference, Cambridge, Mass., 1986

[9] E.D. Dickmanns. “Dynamic computer vision for mobile robot control”, Proceedings of the 19th International Symposium and Exposition on Robots, pp. 314-27;

[10] K. E. Olin and David Tseng. “Autonomous Cross Country Navigation”, IEEE Expert, August 1991.

[11] M. Hebert, T. Kanade, and I. Kweon. “3-D Vision Techniques for Autonomous Vehicles”, Technical Report CMU-RI-TR-88-12, The Robotics Institute, Carnegie Mellon University, 1988

[12] In So Kweon, “Modelling Rugged Terrain by Mobile Robots with Multiple Sensors”, CMU PhD Thesis, 1990