CMU 15-896

Social choice 1: The basics

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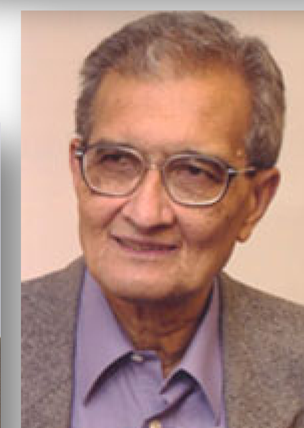
Social choice theory

- A mathematical theory that deals with aggregation of individual preferences
- Origins in ancient Greece
- Formal foundations: 18^{th} Century (Condorcet and Borda)
- 19^{th} Century: Charles Dodgson
- 20^{th} Century: Nobel prizes to Arrow and Sen



The voting model

- Set of voters $N = \{1, \ldots, n\}$
- Set of alternatives $A, |A| = m$
- Each voter has a ranking over the alternatives
- Preference profile = collection of all voters' rankings

1	2	3
a	c	b
b	a	c
c	b	a

Voting rules

- Voting rule = function from preference profiles to alternatives that specifies the winner of the election
- Plurality
  - Each voter awards one point to top alternative
  - Alternative with most points wins
  - Used in almost all political elections

More voting rules

- Borda count
  - Each voter awards $m-k$ points to alternative ranked k'th
  - Alternative with most points wins
  - Proposed in the 18th Century by the chevalier de Borda
  - Used for elections to the national assembly of Slovenia
  - Similar to rule used in the Eurovision song contest

Lordi, Eurovision 2006 winners

More voting rules

- Positional scoring rules
  - Defined by vector $(s_1, \dots, s_m)$
  - Plurality $= (1,0,\dots,0)$, Borda $= (m-1, m-2, \dots, 0)$
- x beats y in a pairwise election if the majority of voters prefer x to y
- Plurality with runoff
  - First round: two alternatives with highest plurality scores survive
  - Second round: pairwise election between these two alternatives

More voting rules

- Single Transferable vote (STV)
  - $m-1$ rounds
  - In each round, alternative with least plurality votes is eliminated
  - Alternative left standing is the winner
  - Used in Ireland, Malta, Australia, and New Zealand (and Cambridge, MA)

STV: EXAMPLE

2 voters	2 voters	1 voter
a	b	c
b	a	d
c	d	b
d	c	a

2 voters	2 voters	1 voter
a	b	c
b	a	b
c	c	a

2 voters	2 voters	1 voter
a	b	b
b	a	a

2 voters	2 voters	1 voter
b	b	b

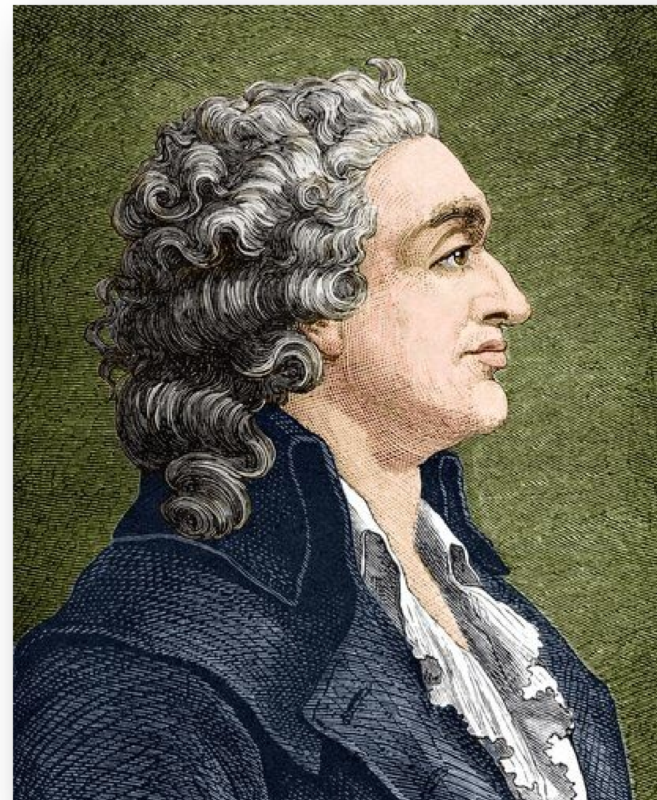
Social choice axioms

- How do we choose among the different voting rules? Via desirable properties!
- Majority consistency = if a majority of voters rank alternative x first, then x should be the winner

Which of the rules we talked about is not majority consistent?

Marquis de Condorcet

- 18th Century French Mathematician, philosopher, political scientist
- One of the leaders of the French revolution
- After the revolution became a fugitive
- His cover was blown and he died mysteriously in prison



Condorcet winner

- Recall: x beats y in a pairwise election if a majority of voters rank x above y
- Condorcet winner beats every other alternative in pairwise election
- Condorcet paradox = cycle in majority preferences

1	2	3
a	c	b
b	a	c
c	b	a

Condorcet consistency

- Condorcet consistency = select a Condorcet winner if one exists

Which of the rules we talked about is Condorcet consistent?

Condorcet consistency

Poll: What is the relation between majority consistency and Condorcet consistency?

1. Majority cons. $\Rightarrow$ Condorcet cons.
2. Condorcet cons. $\Rightarrow$ Majority cons.
3. Equivalent
4. Incomparable

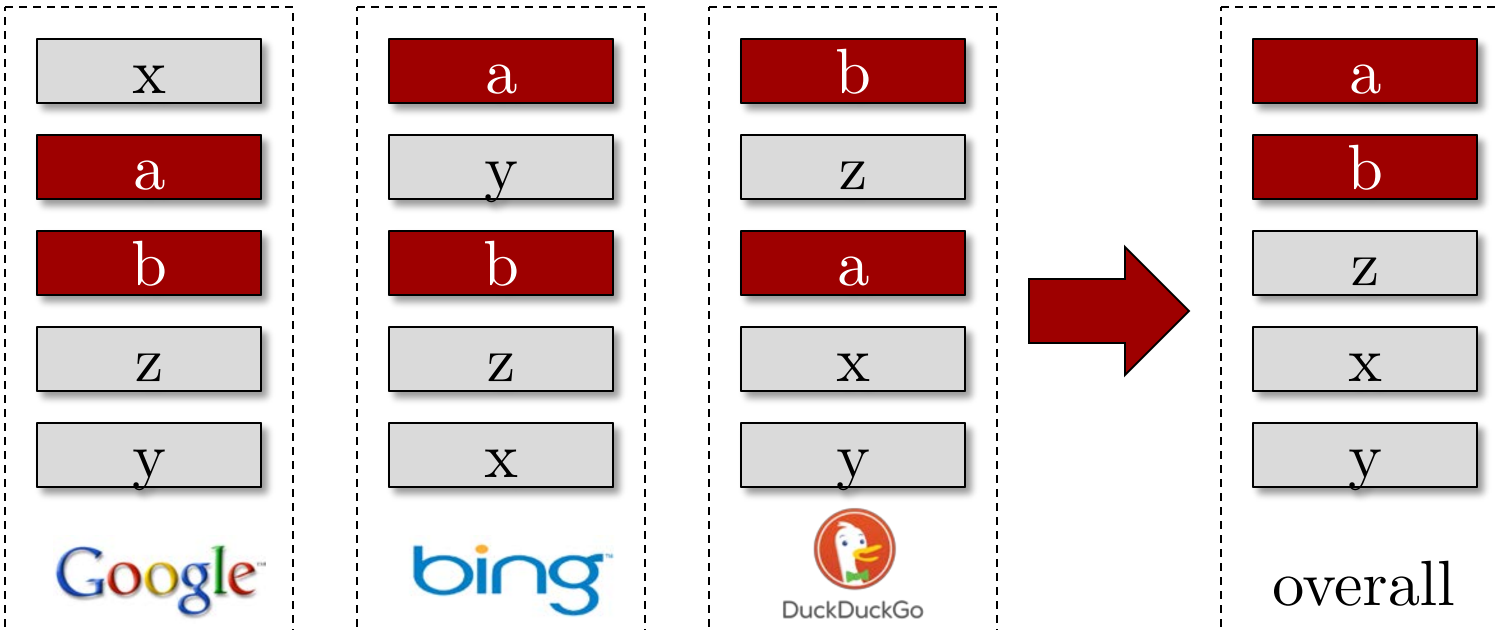
More voting rules

- Copeland
  - Alternative's score is #alternatives it beats in pairwise elections
  - Why does Copeland satisfy the Condorcet criterion?
- Maximin
  - Score of x is $\min_y |\{i \in N: x \succ_i y\}|$
  - Why does Maximin satisfy the Condorcet criterion?

Application: web search

- Generalized Condorcet: if there is a partition X, Y of A such that a majority prefers every $x \in X$ to every $y \in Y$, then X is ranked above Y
- Assumption: spam website identified by a majority of search engines
- When aggregating results from different search engines, spam websites will be ranked last [Dwork et al., WWW 2001]

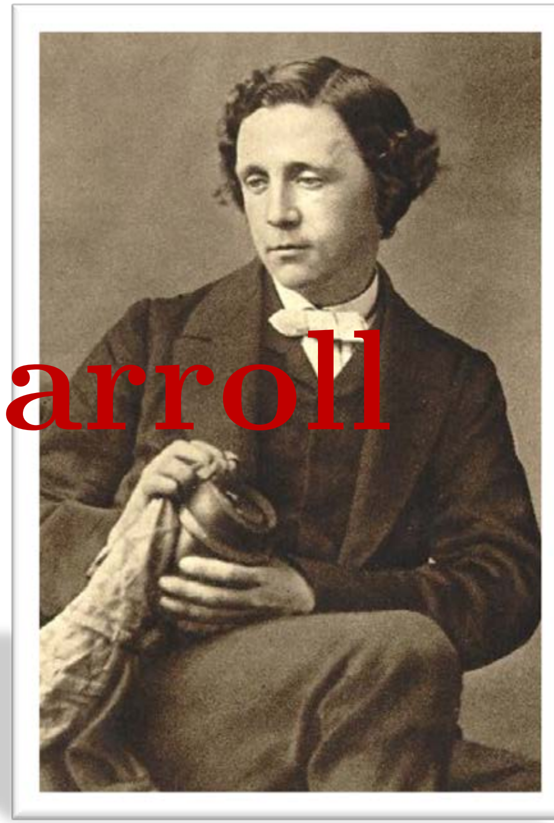
Application: Web Search



Metamorphosis

rles
idge
gson

Lewis Carroll



Dodgson's Rule

- Distance function between profiles: #swaps between adjacent candidates
- Dodgson score of x = the min distance from a profile where x is a Condorcet winner
- Dodgson's rule: select candidate that minimizes Dodgson score
- The problem of computing the Dodgson score is NP-complete!

Dodgson Unleashed

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
a	b	e	e	b
b	a	b	c	e
c	c	c	d	d
d	d	a	a	a
e	e	d	b	c

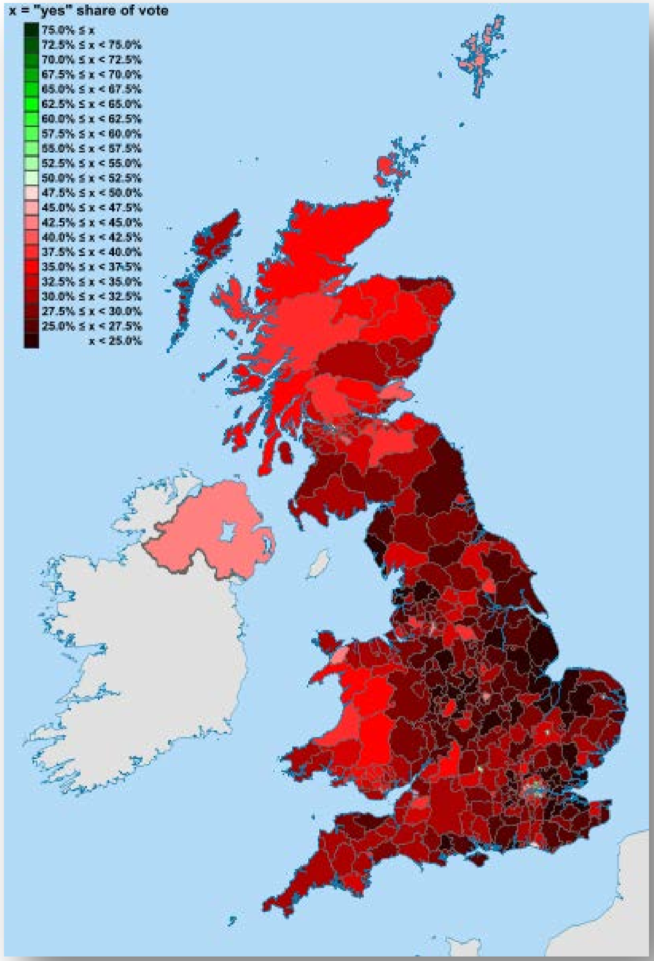
Awesome example

- Plurality: *a*
- Borda: *b*
- Condorcet winner: *c*
- STV: *d*
- Plurality with runoff: *e*

33 voters	16 voters	3 voters	8 voters	18 voters	22 voters
a	b	c	c	d	e
b	d	d	e	e	c
c	c	b	b	c	b
d	e	a	d	b	d
e	a	e	a	a	a

Is social choice practical?

- UK referendum: Choose between plurality and STV as a method for electing MPs
- Academics agreed STV is better...
- ... but STV seen as beneficial to the hated Nick Clegg
- Hard to change political elections!



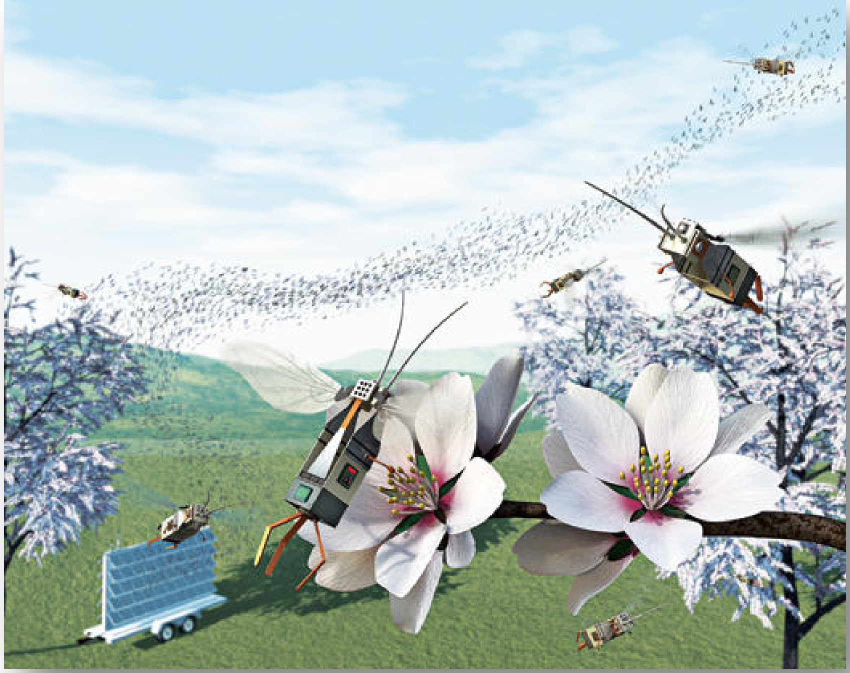
Computational social choice

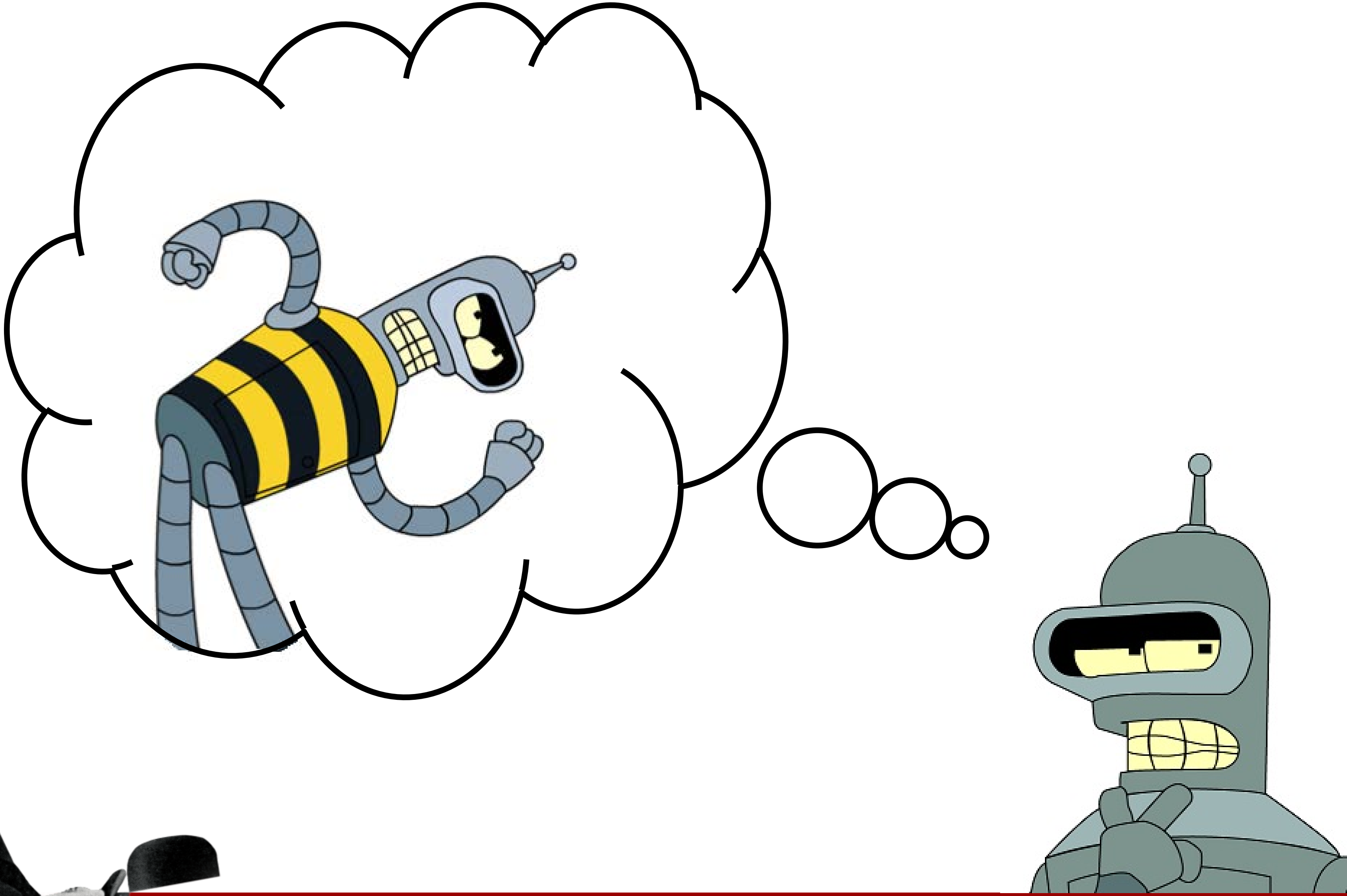
- However:
  - in human computation systems...
  - in multiagent systems...

  the designer is free to employ any voting rule!
- Computational social choice focuses on positive results through computational thinking

Example: Robobees

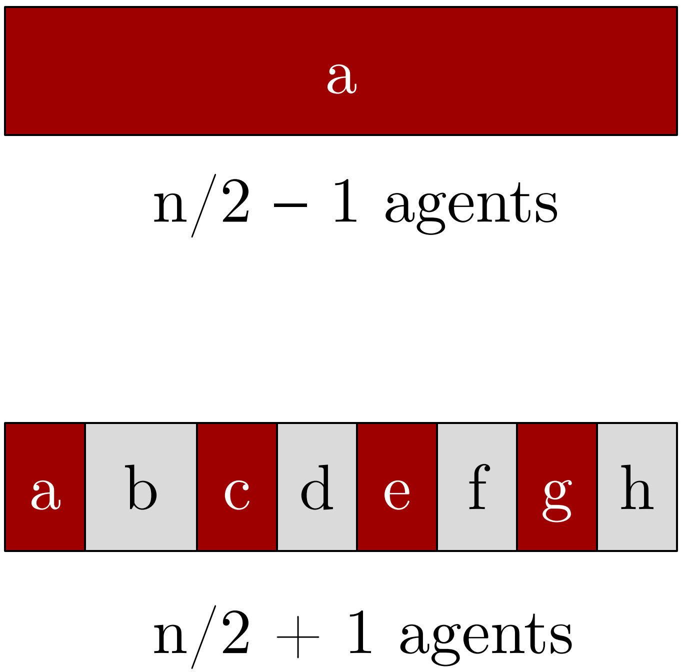
- Robobees need to decide on a joint plan (alternative)
- Many possible plans
- Each robobee (agent) has a numerical evluation (utility) for each alternative
- Want to maximize sum of utilities = social welfare
- Communication is restricted





Example: Robobees

- Approach 1: communicate utilities
  - May be infeasible
- Approach 2: each agent votes for favorite alternative (plurality)
  - $\log m$ bits per agent
  - May select a bad alternative



Example: Robobees

- Approach 3: each agent votes for an alternative with probability proportional to its utility
- Theorem [Caragiannis & P 2011]:
  if $n = \omega(m \log m)$ then this approach gives almost optimal social welfare in expectation

Example: Pnyx

	Most preferred alternative	Approved alternatives	Linear rankins	Rankings with ties	Pairwise comparisons
Unique winner	Plurality rule	Approval voting	Borda's rule	Bucket Borda's rule	Young's generalization of Borda's rule
Lottery	Random dictatorship	Nash's rule	Maximal lotteries	Maximal lotteries	Maximal lotteries
Ranking without ties	Plurality scores	Approval voting scores	Kemeny's rule	Kemeny's rule	Kemeny's rule