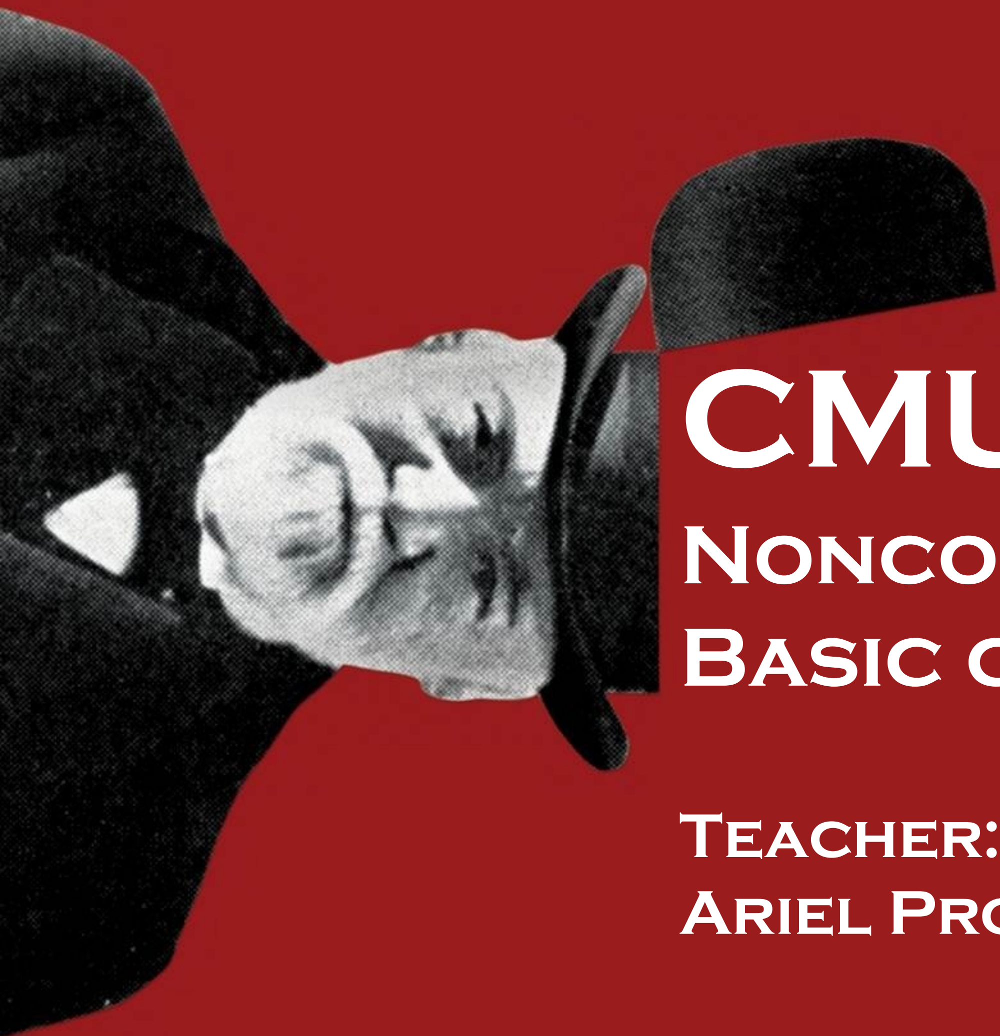


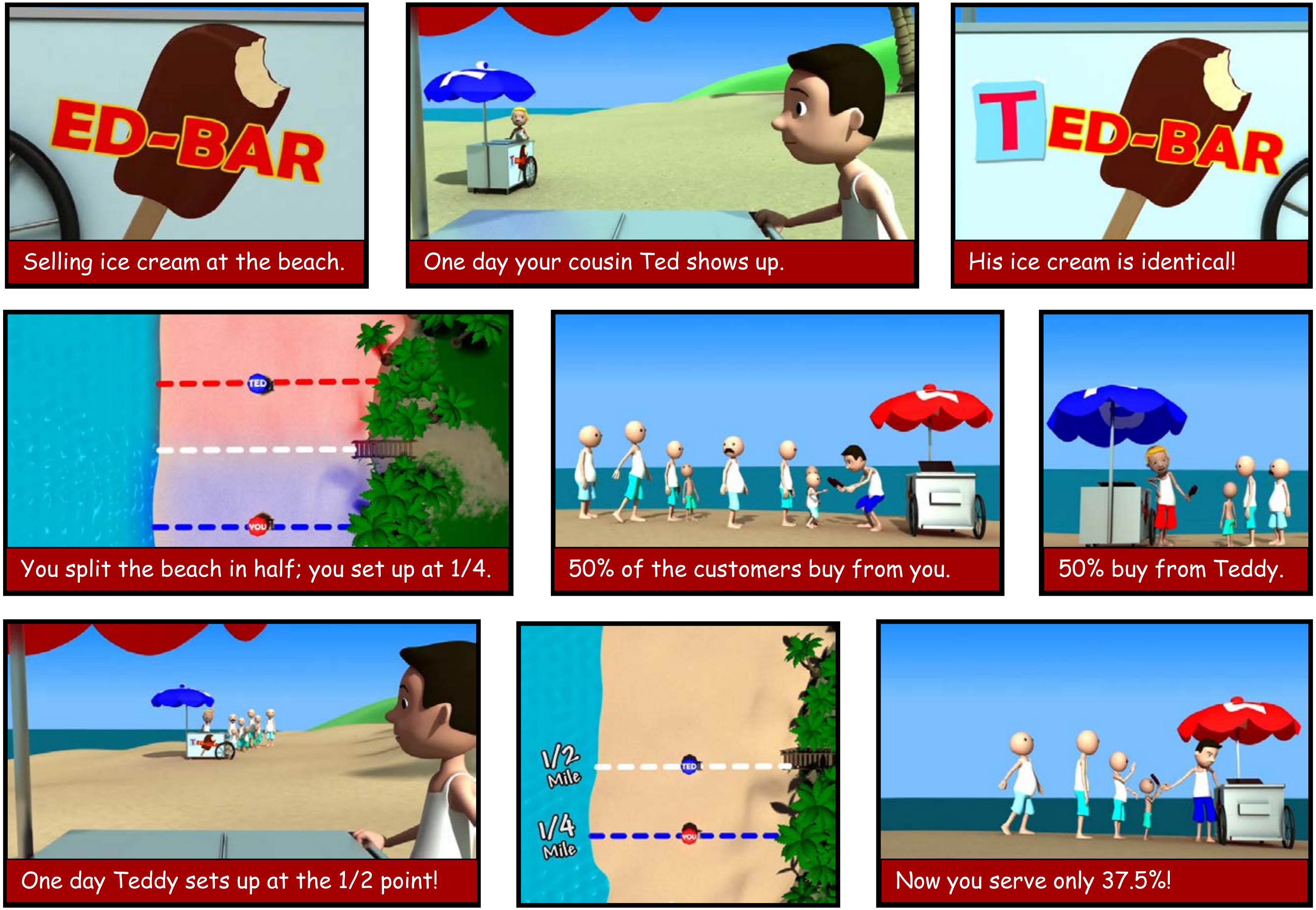
CMU 15-896

Noncooperative games 1: Basic concepts

Teacher:
Ariel Procaccia

Normal-Form Game

- A game in normal form consists of:
  - Set of players $N = \{1, \dots, n\}$
  - Strategy set S
  - For each $i \in N$, utility function $u_i : S^n \to \mathbb{R}$: if each j $\in N$ plays the strategy $s_j \in S$, the utility of player i is $u_i(s_1, \dots, s_n)$
- Next example created by taking screenshots of http://youtu.be/jILgxeNBK_8


ED-BAR
Selling ice cream at the beach.
One day your cousin Ted shows up.
TED-BAR
His ice cream is identical!
TED
YOU
You split the beach in half; you set up at 1/4.
50% of the customers buy from you.
50% buy from Teddy.
One day Teddy sets up at the 1/2 point!
1/2 Mile
TED
1/4 Mile
YOU
Now you serve only 37.5%!

The Ice Cream Wars

- $N = \{1,2\}$
- $S = [0,1]$
- $u_i(s_i, s_j) = \begin{cases} \frac{s_i + s_j}{2}, & s_i < s_j \\ 1 - \frac{s_i + s_j}{2}, & s_i > s_j \\ \frac{1}{2}, & s_i = s_j \end{cases}$
- To be continued...

The prisoner's dilemma

- Two men are charged with a crime
- They are told that:
  - If one rats out and the other does not, the rat will be freed, other jailed for nine years
  - If both rat out, both will be jailed for six years
- They also know that if neither rats out, both will be jailed for one year

The prisoner's dilemma

	Cooperate	Defect
Cooperate	-1,-1	-9,0
Defect	0,-9	-6,-6

What would you do?

Prisoner's dilemma on TV

http://youtu.be/S0qjK3TWZE8

The professor's dilemma

Professor \ Class	Listen	Sleep
Make effort	10^6,10^6	-10,0
Slack off	0,-10	0,0

Dominant strategies?

Nash equilibrium

- Each player's strategy is a best response to strategies of others
- Formally, a Nash equilibrium is a vector of strategies $s = (s_1 \dots, s_n) \in S^n$ such that $\forall i \in N, \forall s_i' \in S, u_i(s) \geq u_i(s_i', s_{-i})$

Nash equilibrium

http://youtu.be/CemLiSI5ox8

Russel Crowe was wrong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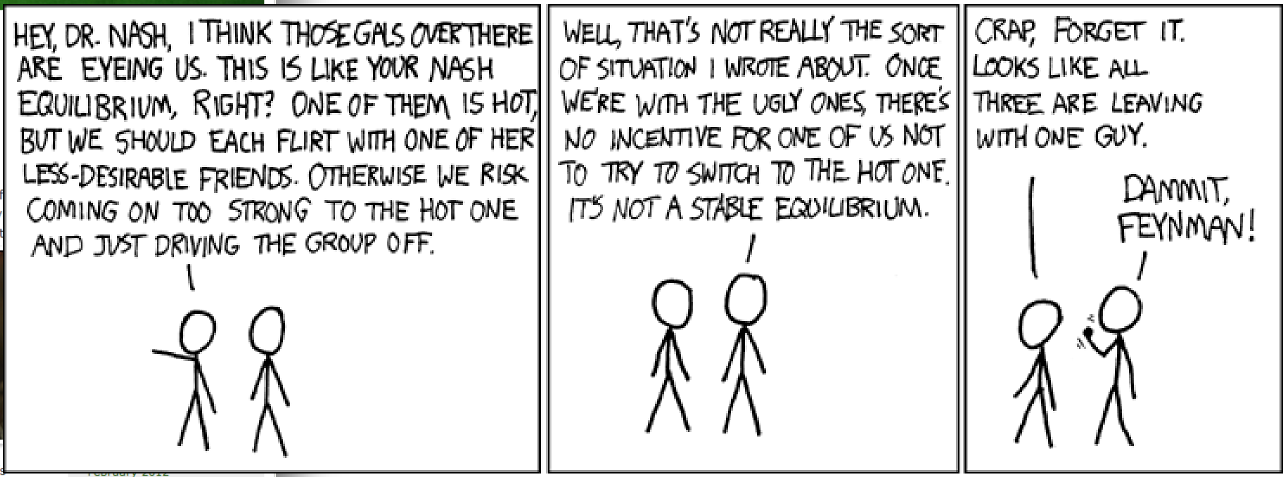
Russell Crowe was wrong
October 30, 2012 by Ariel Procaccia | Edit

Yesterday I taught the first of five algorithmic economics lectures in my undergraduate AI course. This lecture just introduced the basic concepts of game theory, focusing on Nash equilibria. I was contemplating various way making the lecture more lively, and it occurred to me that I could stand on t shoulders of giants. Indeed, didn't Russell Crowe already explain Nash's ideas in A Beautiful Mind, complete with a 1940's-style male chauvinistic example?

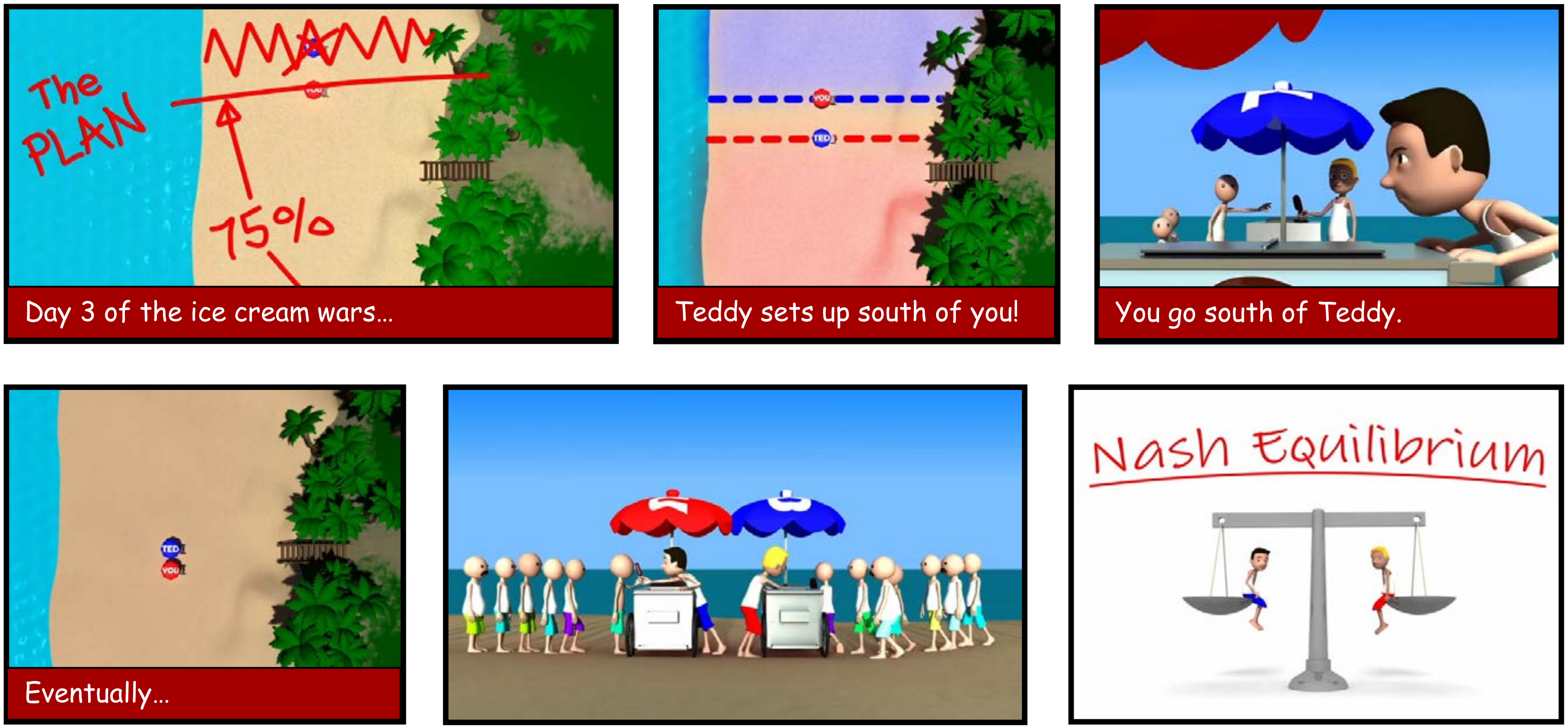
The first and last time I watched the movie was when it was released in 2001. Back then I was an undergrad freshman, working for 20+ hours a week on the programming exercises of Hebrew U's Intro to CS course, which was taught by some guy called Noam Nisan. I didn't know anything about game theory, and Crowe's explanation made a lot of sense at the time.

I easily found the relevant scene on youtube. In the scene, Nash's friends are trying to figure out how to seduce a beautiful blonde and her less beautiful friends. Then Nash/Crowe has an epiphany. The hubbub of the seedy Princeton bar is drowned by inspirational music, as Nash announces:

January 2012
December 2011
November 2011
October 2011
September 2011
August 2011
July 2011



End of the Ice Cream Wars



Rock-paper-scissors

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Nash equilibrium?

Mixed strategies

- A mixed strategy is a probability distribution over (pure) strategies
- The mixed strategy of player $i \in N$ is x_i, where

$$x_i(s_i) = \Pr[i \text{ plays } s_i]$$

- The utility of player $i \in N$ is

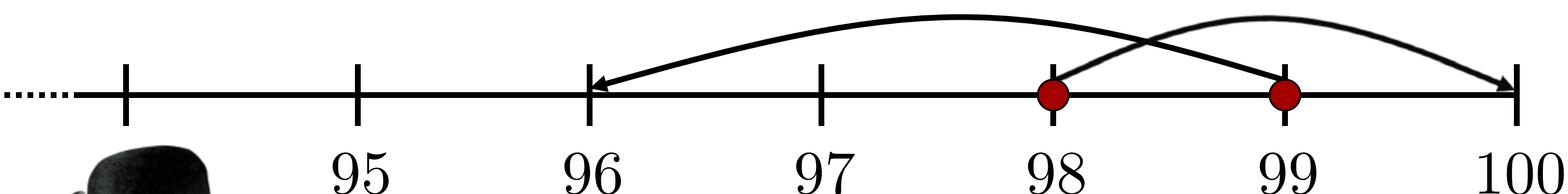
$$u_i(x_1, \ldots, x_n) = \sum_{(s_1,\ldots,s_n) \in S^n} u_i(s_1, \ldots, s_n) \cdot \prod_{j=1}^{n} x_j(s_j)$$

Nash's Theorem

- Theorem [Nash, 1950]: if everything is finite then there exists at least one (possibly mixed) Nash equilibrium
- We'll talk about computation some other time

Does NE make sense?

- Two players, strategies are $\{2, \dots, 100\}$
- If both choose the same number, that is what they get
- If one chooses s, the other t, and $s < t$, the former player gets $s + 2$, and the latter gets $s - 2$
- Poll 1: what would you choose?



Correlated equilibrium

- Let $N = \{1,2\}$ for simplicity
- A mediator chooses a pair of strategies (s_1, s_2) according to a distribution p over S^2
- Reveals s_1 to player 1 and s_2 to player 2
- When player 1 gets $s_1 \in S$, he knows that the distribution over strategies of 2 is

$$\Pr[s_2|s_1] = \frac{\Pr[s_1 \wedge s_2]}{\Pr[s_1]} = \frac{p(s_1, s_2\ )}{\sum_{s_2' \in S} p(s_1, s_2')}$$

Correlated equilibrium

- Player 1 is best responding if for all $s_1' \in S$

$$\sum_{s_2 \in S} \Pr[s_2|s_1]\, u_1(s_1, s_2) \geq \sum_{s_2 \in S} \Pr[s_2|s_1]\, u_1(s_1', s_2)$$

- Equivalently,

$$\sum_{s_2 \in S} p(s_1, s_2) u_1(s_1, s_2) \geq \sum_{s_2 \in S} p(s_1, s_2) u_1(s_1', s_2)$$

- p is a correlated equilibrium (CE) if both players are best responding

Game of chicken

http://youtu.be/u7hZ9jKrwvo

Game of chicken

- Social welfare is the sum of utilities
- Pure NE: (C,D) and (D,C), social welfare = 5
- Mixed NE: both (1/2,1/2), social welfare = 4
- Optimal social welfare = 6

	Dare	Chicken
Dare	0,0	4,1
Chicken	1,4	3,3

Game of chicken

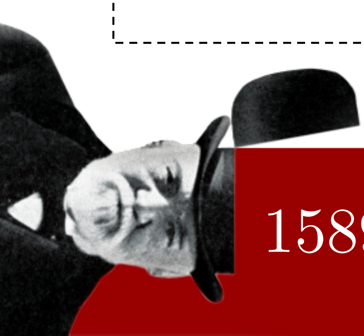
- Correlated equilibrium:
  - (D,D): 0
  - (D,C): $\frac{1}{3}$
  - (C,D): $\frac{1}{3}$
  - (C,C): $\frac{1}{3}$
- Social welfare of CE $= \frac{16}{3}$

	Dare	Chicken
Dare	0,0	4,1
Chicken	1,4	3,3

Implementation of CE

- Instead of a mediator, use a hat!
- Balls in hat are labeled with "chicken" or "dare", each blindfolded player takes a ball

Which balls implement the distribution of the previous slide?



CE vs. NE

- Poll 2: What is the relation between CE and NE?
  1. CE $\Rightarrow$ NE
  2. NE $\Rightarrow$ CE
  3. NE $\Leftrightarrow$ CE
  4. NE $\|$ CE

CE As LP

- Can compute CE via linear programming in polynomial time!

find $\forall s_1, s_2 \in S, p(s_1, s_2)$

s.t. $\forall s_1, s_1', s_2 \in S, \sum_{s_2 \in A} p(s_1, s_2) u_1(s_1, s_2) \geq \sum_{s_2 \in A} p(s_1, s_2) u_1(s_1', s_2)$

$\forall s_1, s_2, s_2' \in S, \sum_{s_1 \in A} p(s_1, s_2) u_2(s_1, s_2) \geq \sum_{s_1 \in A} p(s_1, s_2) u_2(s_1, s_2')$

$$\sum_{s_1, s_2 \in S} p(s_1, s_2) = 1$$

$\forall s_1, s_2 \in S, p(s_1, s_2) \in [0,1]$