CMU 15-896

Social networks 3: It's a Small world

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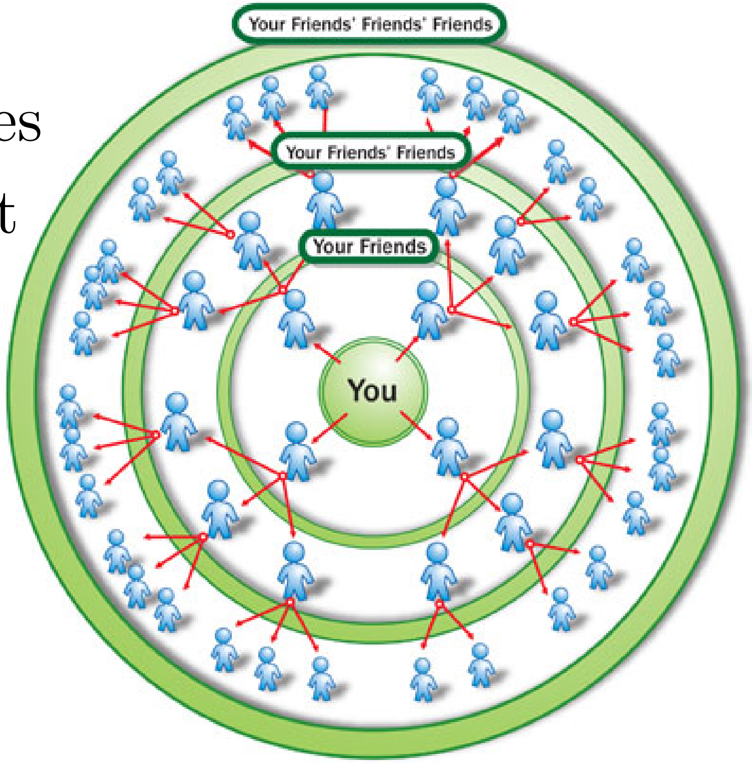
Six degrees of separation

- Stanley Milgram's famous experiments (1960's)
- Find short chains of acquaintances
- Source person in Nebraska, target in MA
- Deliver letter by passing to a friend
- Anyone receiving the letter gets same instructions
- Avg number of hops between 5 and 6

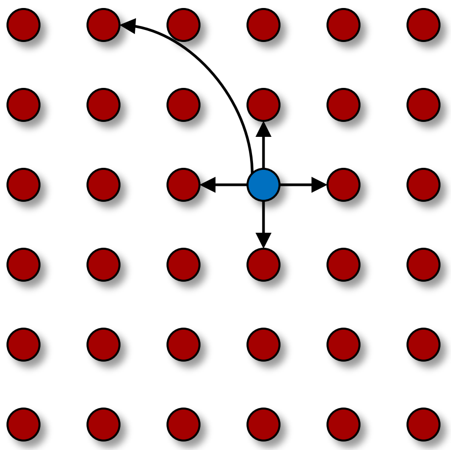


Questions

- Why should there exist short chains linking arbitrary pairs of strangers?
  - This question has been well understood for a long time
- Why should arbitrary pairs of strangers be able to find short chains that link them?
  - Short chains may exist, but no "decentralized" alg may find them
  - This is the topic of [Kleinberg, 2000]

Model

- Nodes reside on an $n \times n$ lattice
$$V = \{(i,j) | i,j \in \{1,\ldots,n\}\}$$
- Each node has a directed edge to each of its 4 neighbors
- The Manhattan distance is
$$d\big((i,j),(k,l)\big) = |i-k| + |j-l|$$
- Each node u has an additional long-distance edge v with probability $\frac{d(u,v)^{-r}}{\sum_{v'} d(u,v')^{-r}}$, where $r \geq 0$ is a parameter
- The source s and target t are chosen u.a.r.



Decentralized algs

- A local decentralized alg knows only:
  1. The underlying grid structure
  2. The lattice coordinates of t and nodes that held the message
  3. The coordinates of long-range contacts of nodes that held the message
- 3 may seem restrictive, but it just strengthens negative result, positive result doesn't use it

A negative result

- Consider the case of $r = 0$: the long-distance neighbor is chosen uniformly at random
- Fact: $\mathbb{E}[d(s,t)] = O(\log n)$
- Theorem [Kleinberg, 2000]: When $r = 0$, the expected number of hops under any local decentralized alg is $\Omega\left(n^{2/3}\right)$
- We prove the theorem on the board

A positive result

- Greedy alg: forward the message to the neighbor that minimizes distance from target
- The alg doesn't use information about long-distance neighbors of previous nodes
- Theorem [Kleinberg 2000]: When $r = 2$ the greedy alg requires $O(\log^2 n)$ hops in expectation
- We prove the theorem on the board

More generally

- $r = 2$ is the only value for which there is a local decentralized alg that requires polylogarithmic #hops
- Theorem [Kleinberg 2000]:
  1. For $0 \le r < 2$, expected #hops is $\Omega\left(n^{(2-r)/3}\right)$
  2. For $r > 2$, expected #hops is $\Omega\left(n^{(r-2)/(r-1)}\right)$