# Neural Networks

## 1. Introduction

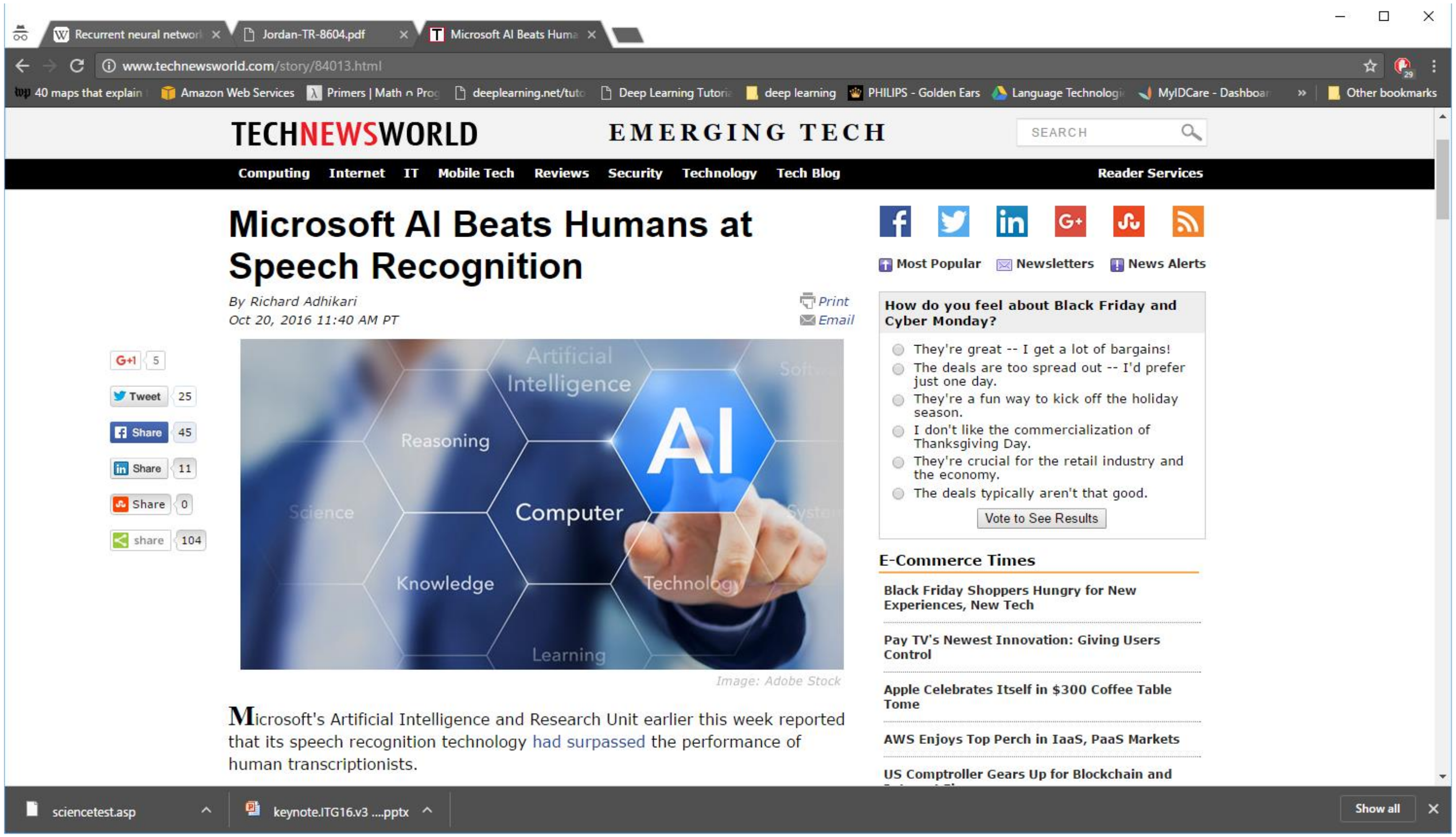
## Fall 2017

# Neural Networks are taking over!

- Neural networks have become one of the major thrust areas recently in various pattern recognition, prediction, and analysis problems

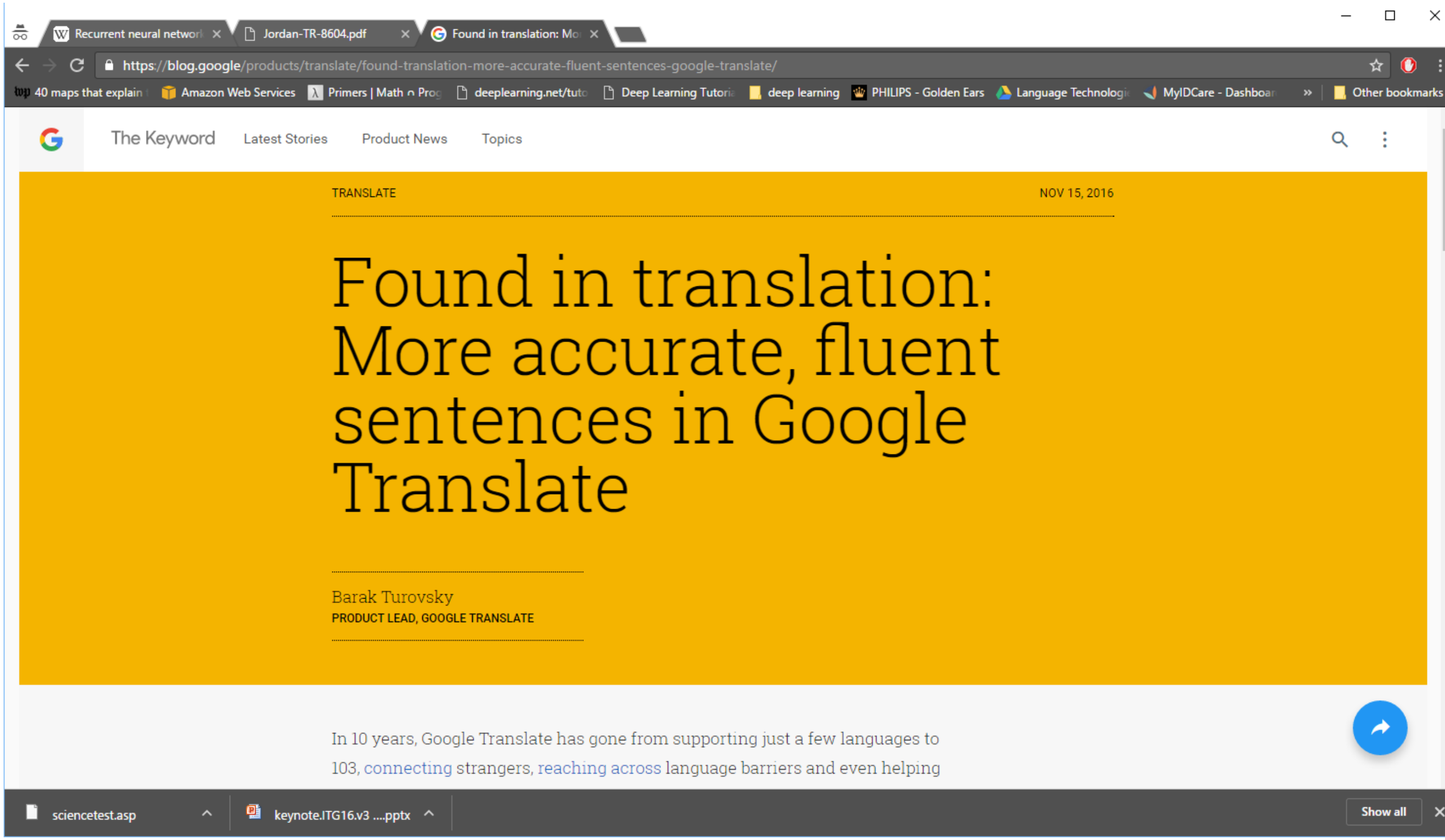
- In many problems they have established the state of the art
  - Often exceeding previous benchmarks by large margins

# Recent success with neural networks



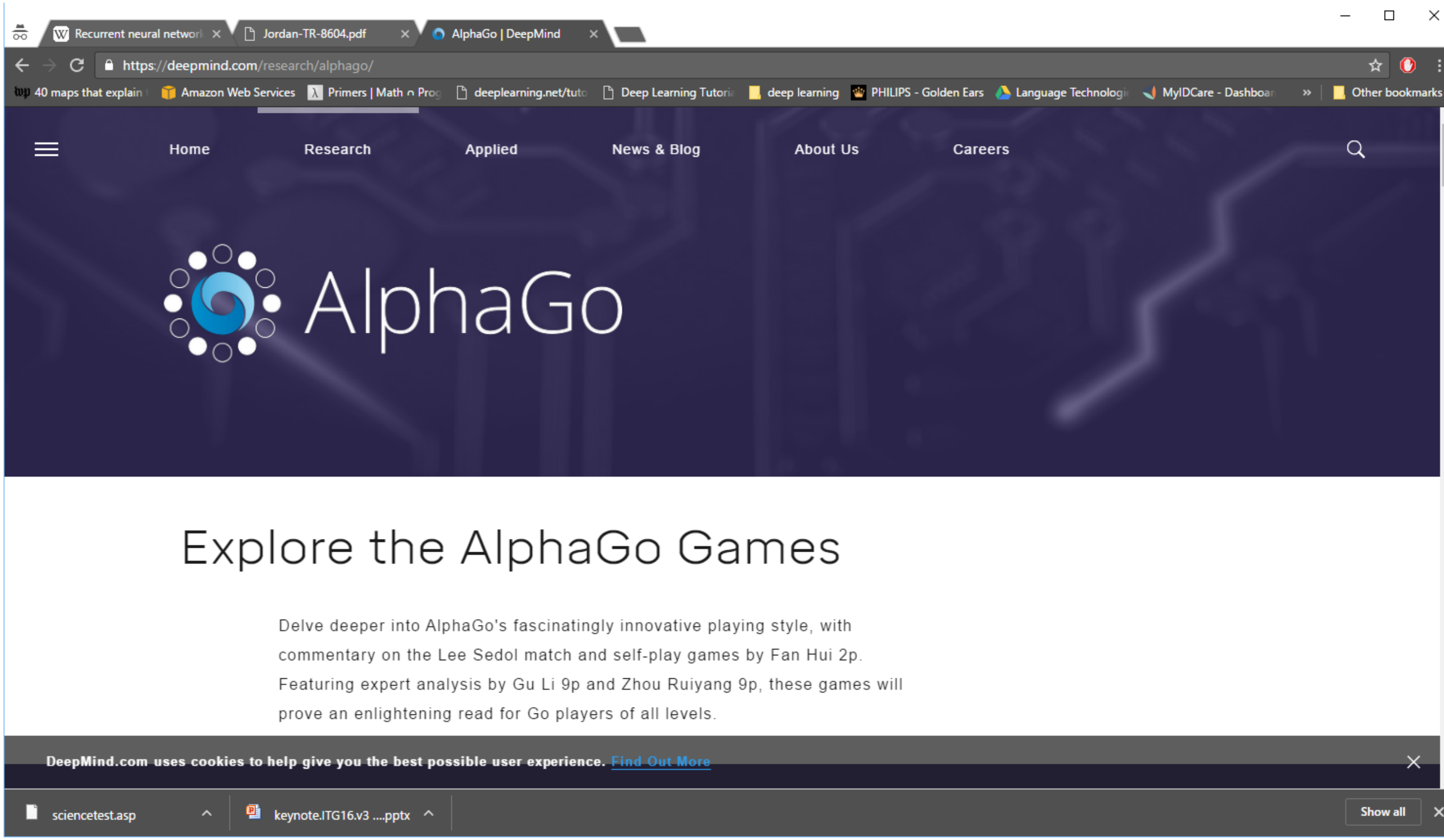

- Some recent successes with neural networks
  - A bit of hyperbole, but still..

# Recent success with neural networks



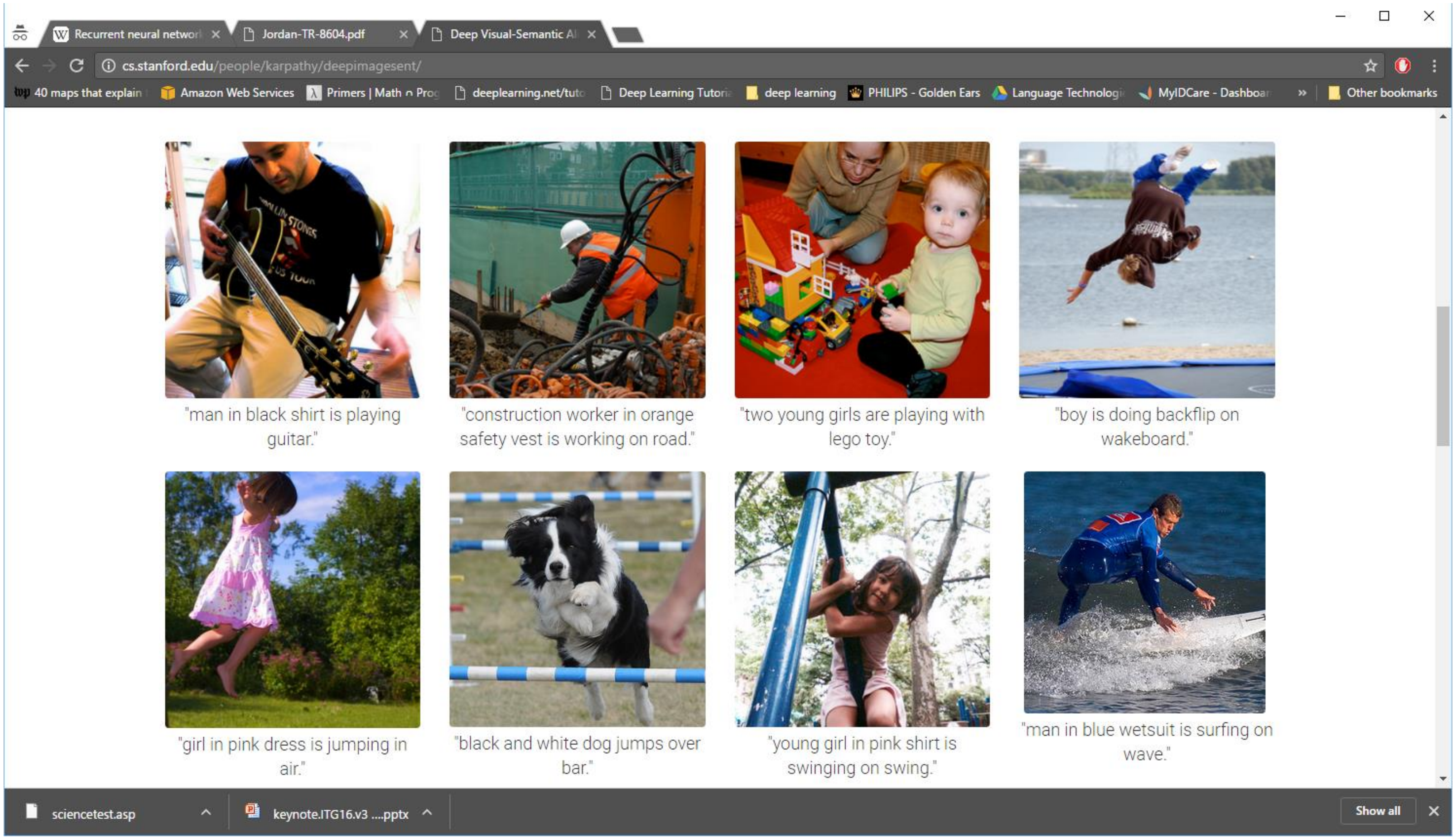

- Some recent successes with neural networks

# Recent success with neural networks




- Some recent successes with neural networks

# Recent success with neural networks




- Captions generated entirely by a neural network

# Successes with neural networks

- And a variety of other problems:
  - Image recognition
  - Signal enhancement
  - Even generating art and predicting stock markets!

# Neural nets and the employment market


This guy didn't know about neural networks (a.k.a deep learning)

This guy learned about neural networks (a.k.a deep learning)

# Objectives of this course

- Understanding neural networks
- Comprehending the models that do the previously mentioned tasks
  - And maybe build them
- Familiarity with some of the terminology
  - What are these:
    - http://www.datasciencecentral.com/profiles/blogs/concise-visual-summary-of-deep-learning-architectures
- Fearlessly design, build and train networks for various tasks

- *You will not become an expert in one course*

# Course learning objectives: Broad level

- Concepts
  - Some historical perspective
  - Forms of neural networks and underlying ideas
  - Learning in neural networks
    - Training, concepts, practical issues
  - Architectures and applications

  - Will try to maintain balance between squiggles and concepts (concept >> squiggle)

- Practical
  - Familiarity with training
  - Implement various neural network architectures
  - Implement state-of-art solutions for some problems

- Overall: Set you up for further research/work in your research area

# Course learning objectives: Topics

- Basic network formalisms:
  - MLPs
  - Convolutional networks
  - Recurrent networks
  - Boltzmann machines

- Topics we will touch upon:
  - Computer vision: recognizing images
  - Text processing: modelling and generating language
  - Machine translation: Sequence to sequence modelling
  - Modelling distributions and generating data
  - Reinforcement learning and games
  - Speech recognition

# Reading

- List of books on course webpage
- Additional reading material also on course pages

# Instructors and TAs

- Instructor: Me
  - bhiksha@cs.cmu.edu
  - x8-9826
- TAs:
  - Daniel Schwartz
  - Alex Litzenberger
- Office hours: On webpage

- http://deeplearning.cs.cmu.edu/

# Lecture Schedule

- On website
  - The schedule for the latter half of the semester may vary a bit
    - Guest lecturer schedules are fuzzy..
- Guest lectures:
  - 25 Sep:  Mike Tarr
  - 27 Sep: Scott Fahlman
  - 30 Oct: Pulkit Agarwal
  - 8 Nov: Rich Stern
  - 13 Nov: Graham Neubig

# Grading

- Weekly multiple-choice Quizzes (25%)
- 6 homeworks (50%)
  - Basic MLPs
  - CNNs
  - RNNs
  - Sequence to sequence modelling: Speech recognition
  - Style transfer:  Generative models
  - DBM/RBM or a deep-Q network
- One project (25%)

# Weekly Quizzes

- Weekly quizzes
  - 10 multiple-choice questions
  - Relate to topics covered that week
  - Released Friday, closed Saturday night

# Additional Logistics

- Feedback group required
  - Group of 3-4 volunteers to provide feedback once a week
- Hackathons – if needed
  - Will provide Pizza
- Compute infrastructure:
  - Figuring this out
- Recitation on toolkits:
  - 1.5 hours, time TBD.  By Alex Litzenberger

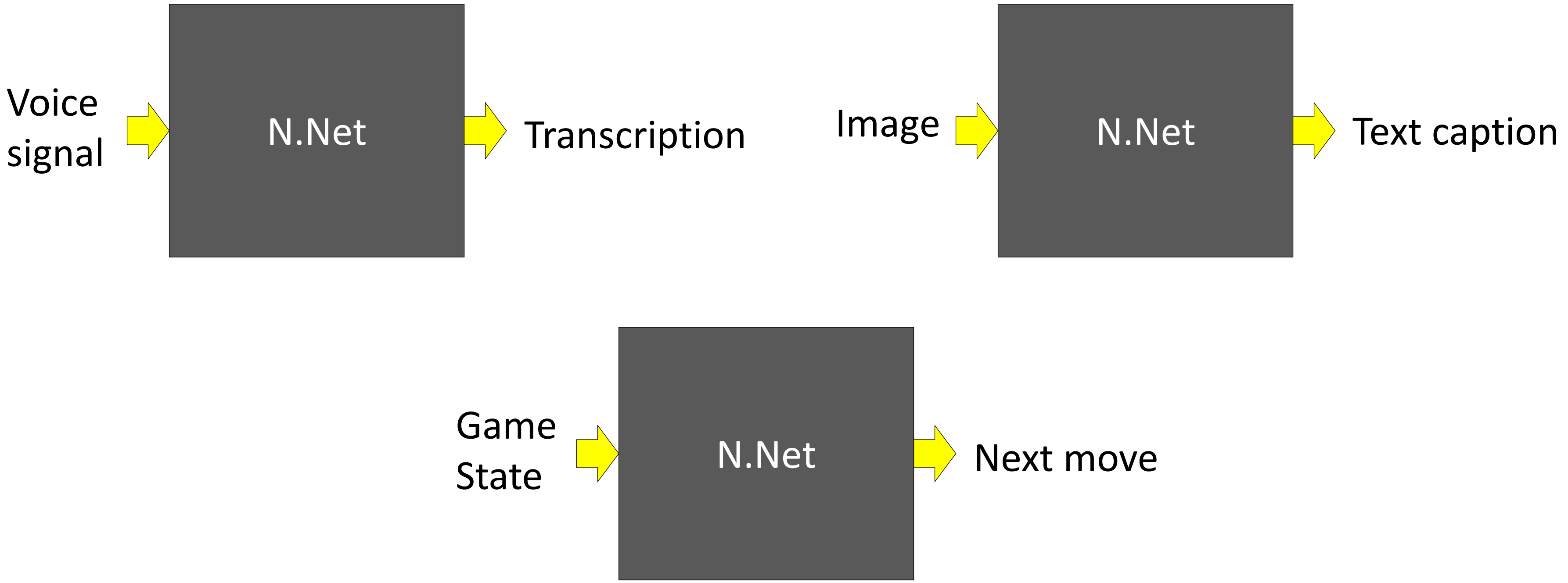
# *Perception:* From Rosenblatt, 1962..

- "Perception, then, emerges as that relatively primitive, partly autonomous, institutionalized, ratiomorphic subsystem of cognition which achieves prompt and richly detailed orientation habitually concerning the vitally relevant, mostly distal aspects of the environment on the basis of mutually vicarious, relatively restricted and stereotyped, insufficient evidence in uncertainty-geared interaction and compromise, seemingly following the highest probability for smallness of error at the expense of the highest frequency of precision. "
  - From "Perception and the Representative Design of Psychological Experiments, " by Egon Brunswik, 1956 (posthumous).

- "That's a simplification. Perception is standing on the sidewalk, watching all the girls go by."
  - From "The New Yorker", December 19, 1959

# Onward..

# So what are neural networks??

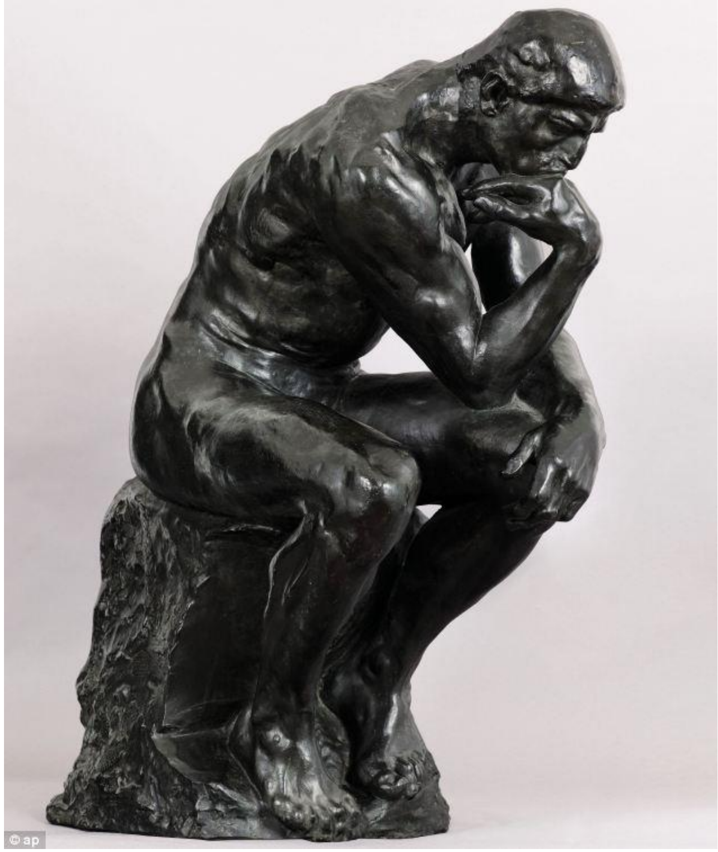



- What are these boxes?

# So what are neural networks??

- It begins with this..

# So what are neural networks??



"The Thinker!"
by Augustin Rodin

- Or even earlier.. with this..

# The magical capacity of humans

- Humans can
  - Learn
  - Solve problems
  - Recognize patterns
  - Create
  - Cogitate
  - …

Dante!

- Worthy of emulation
- But how do humans “work“?

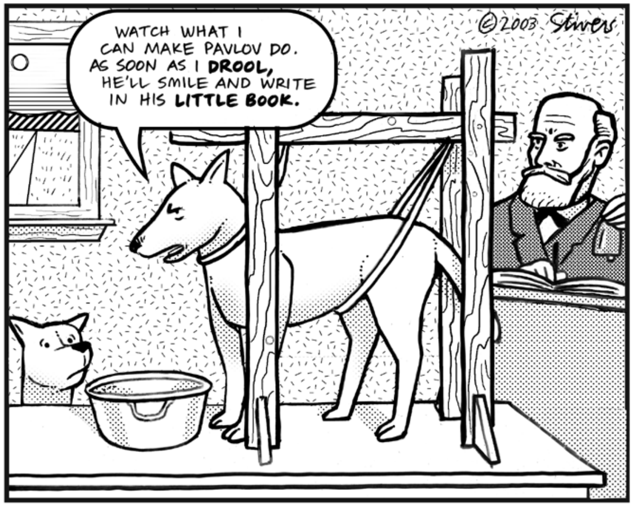
# Cognition and the brain..

- “If the brain was simple enough to be understood - we would be too simple to understand it!”
  – Marvin Minsky

# Early Models of Human Cognition

- Associationism
  - Humans learn through association
- 400BC-1900AD: Plato, David Hume, Ivan Pavlov..

# What are "Associations"

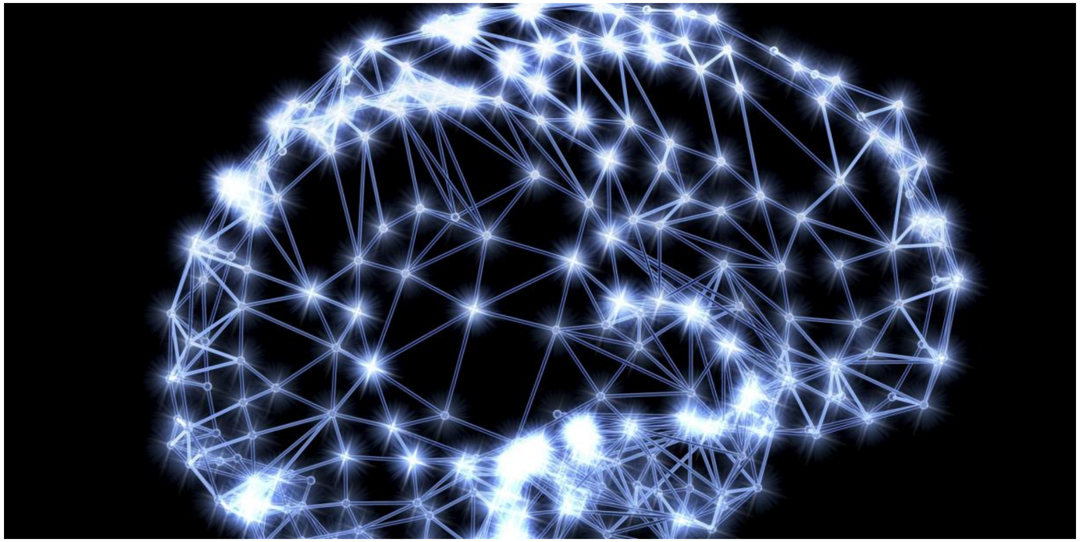



- Lightning is generally followed by thunder
  - Ergo – "hey here's a bolt of lightning,  we're going to hear thunder"
  - Ergo – "We just heard thunder; did someone get hit by lightning"?
- Association!

- **But where are the associations stored??**

- **And how?**

# Observation: ***The Brain***

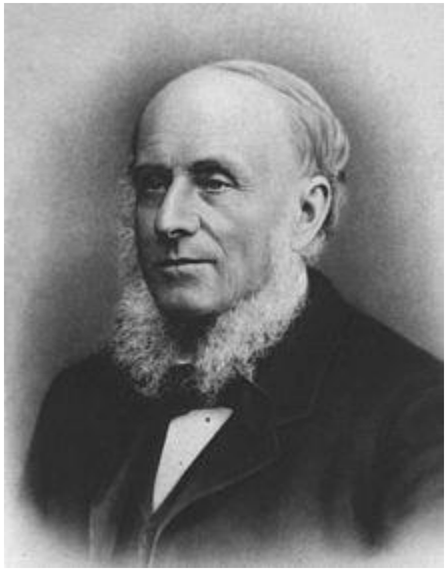


- Mid 1800s:  The brain is a mass of interconnected neurons

# Brain: Interconnected Neurons

- Many neurons connect *in* to each neuron
- Each neuron connects *out* to many neurons

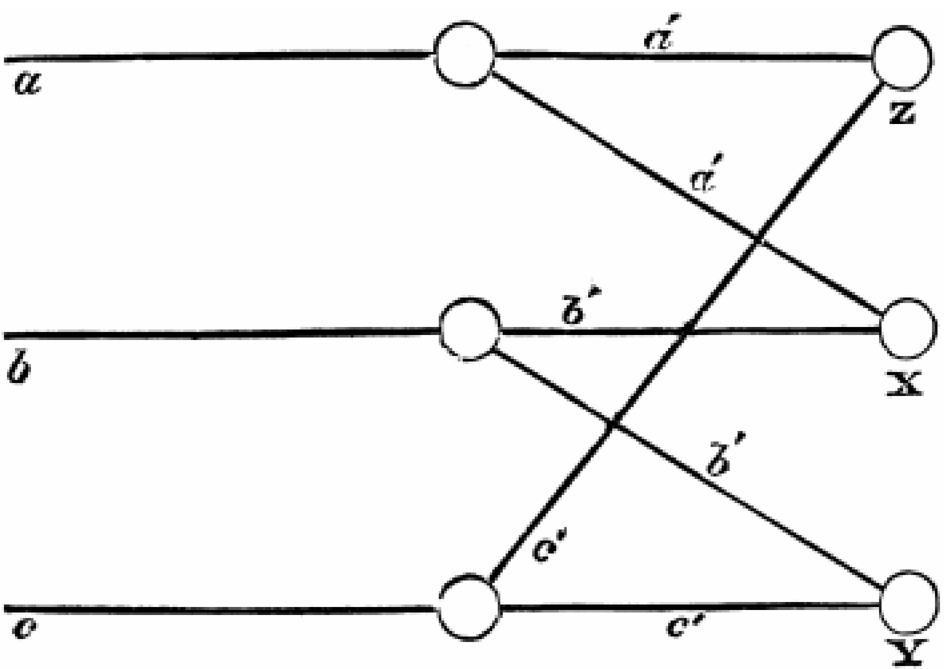
# Enter ***Connectionism***



- Alexander Bain, philosopher, mathematician, logician, linguist, professor
- 1873: The information is in the ***connections***
  - *The mind and body* (1873)

# Bain's Idea 1: **Neural Groupings**

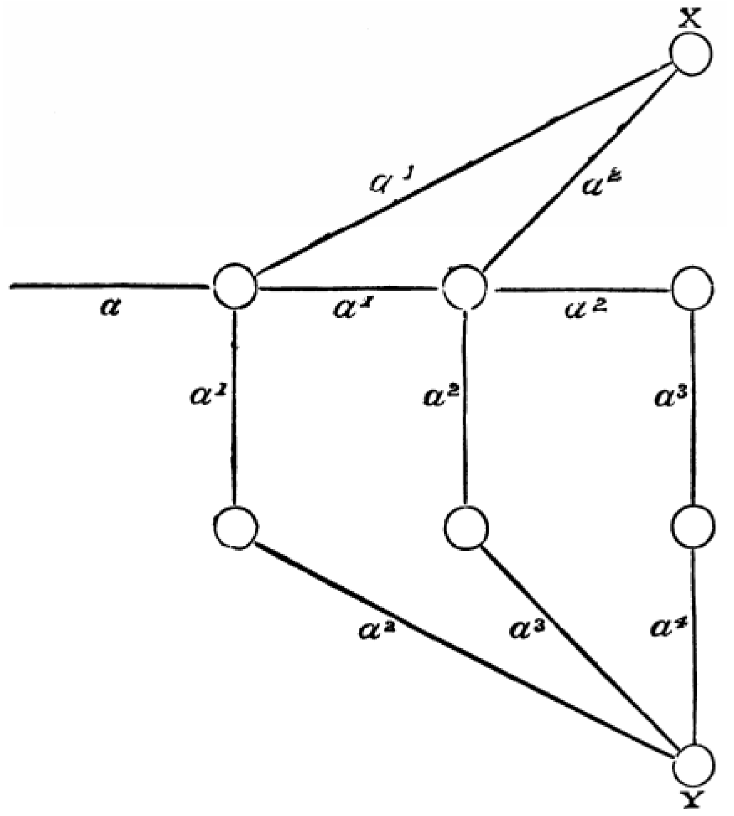
- Neurons excite and stimulate each other
- Different combinations of inputs can result in different outputs



# Bain's Idea 1: **Neural Groupings**

- Different intensities of activation of A lead to the differences in when X and Y are activated

- Even proposed a learning mechanism..



# Bain’s Doubts

- “***The fundamental cause of the trouble is that in the modern world the stupid are cocksure while the intelligent are full of doubt.***”
  - Bertrand Russell

- In 1873, Bain postulated that there must be one million neurons and 5 billion connections relating to 200,000 “acquisitions”

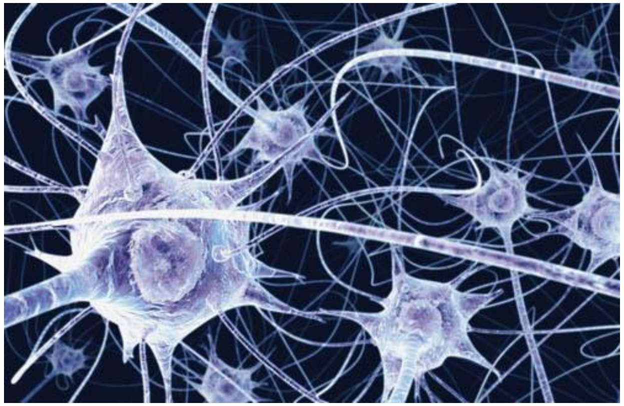
- In 1883, Bain was concerned that he hadn’t taken into account the number of “partially formed associations” and the number of neurons responsible for recall/learning

- By the end of his life (1903), recanted all his ideas!
  - Too complex; the brain would need too many neurons and connections

# Connectionism lives on..

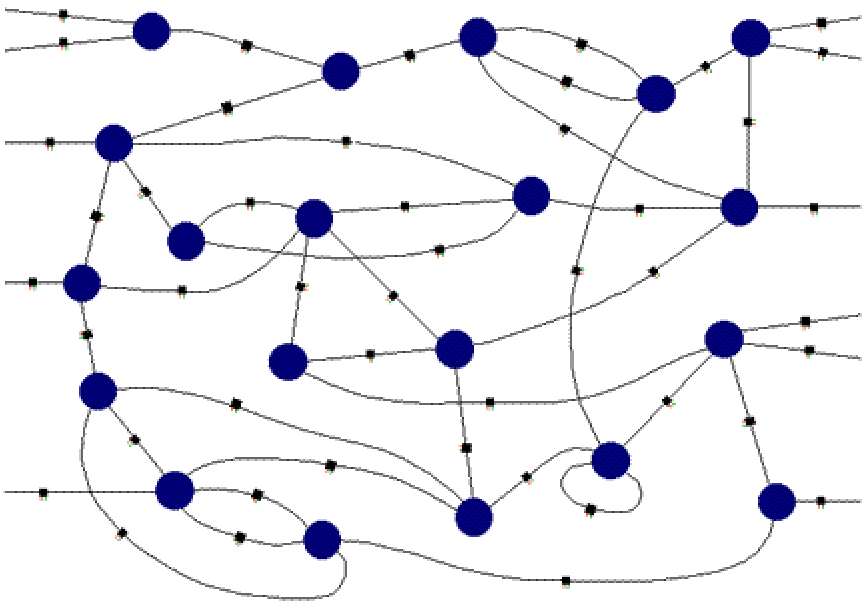
- The human brain is a connectionist machine
  - Bain, A. (1873). *Mind and body. The theories of their relation.* London: Henry King.
  - Ferrier, D. (1876). *The Functions of the Brain.* London: Smith, Elder and Co

- Neurons connect to other neurons. The processing/capacity of the brain is a function of these connections



- Connectionist machines emulate this structure

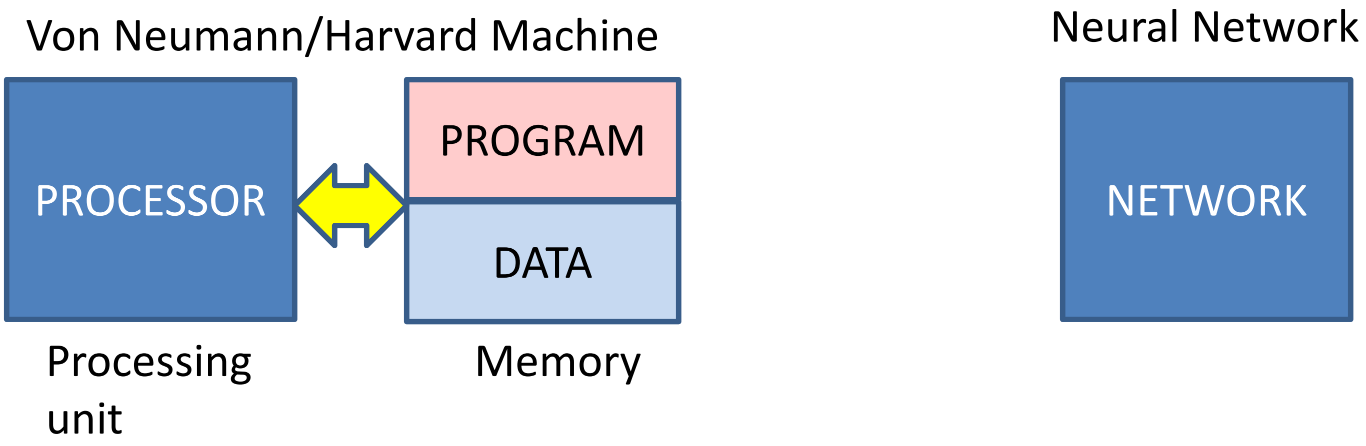
# Connectionist Machines



- Network of processing elements
- **All world knowledge is stored in the *connections* between the elements**

# Connectionist Machines

- Neural networks are *connectionist* machines
  - As opposed to Von Neumann Machines

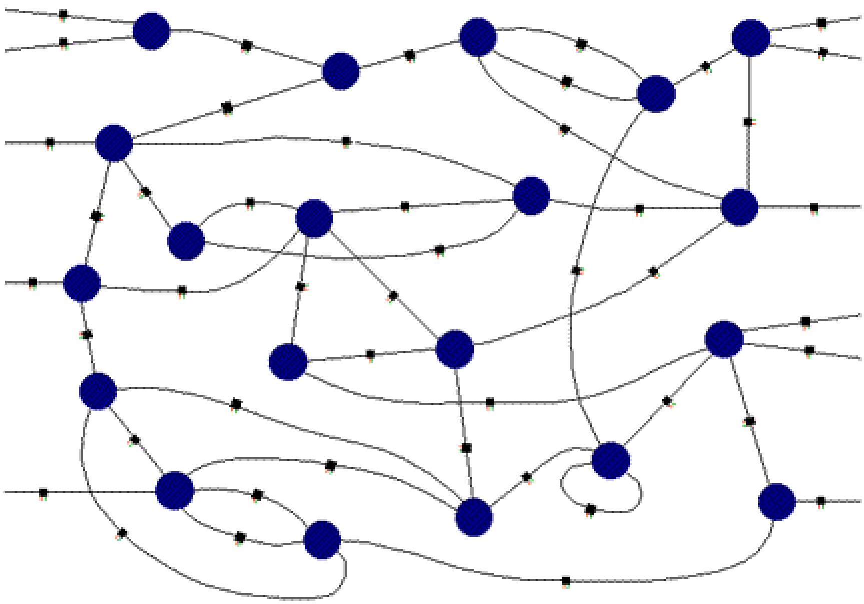



- The machine has many non-linear processing units
  - The program is the connections between these units
    - Connections may also define memory

# Recap

- Neural network based AI has taken over most AI tasks
- Neural networks originally began as computational models of the brain
  - Or more generally, models of cognition
- The earliest model of cognition was *associationism*
- The more recent model of the brain is *connectionist*
  - Neurons connect to neurons
  - The workings of the brain are encoded in these connections
- Current neural network models are *connectionist machines*

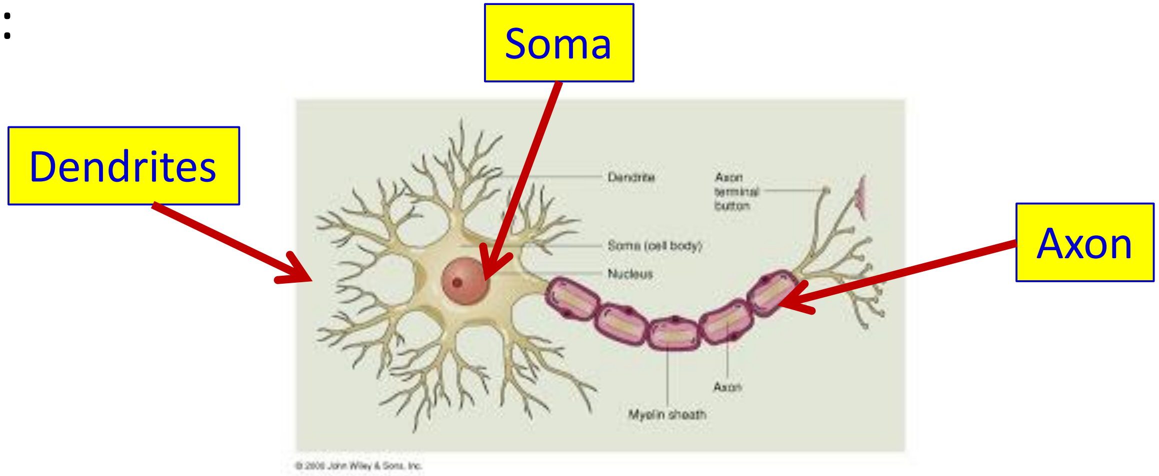
# Connectionist Machines



- Network of processing elements
  - All world knowledge is stored in the *connections* between the elements
- *But what are the individual elements?*

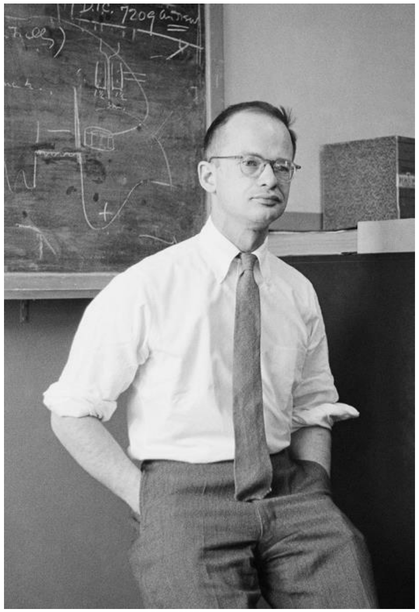
# Modelling the brain

- What are the units?
- A neuron:



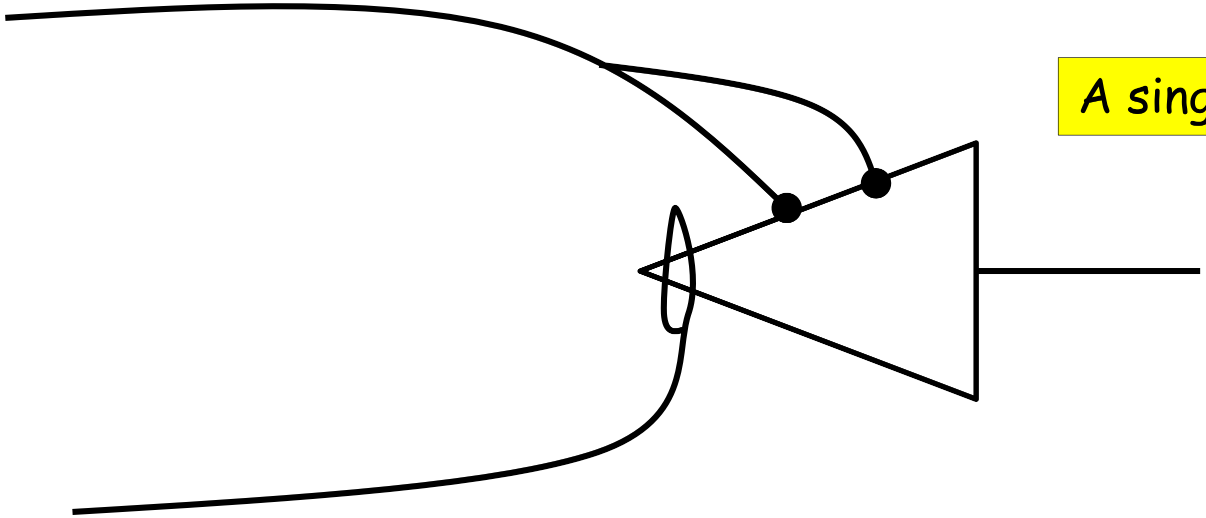

- Signals come in through the dendrites into the Soma
- A signal goes out via the axon to other neurons
  - Only one axon per neuron
- Factoid that may only interest me: Adult neurons do not undergo cell division

# McCullough and Pitts



- The Doctor and the Hobo..
  - Warren McCulloch:  Neurophysician
  - Walter Pitts: Homeless wannabe logician who arrived at his door

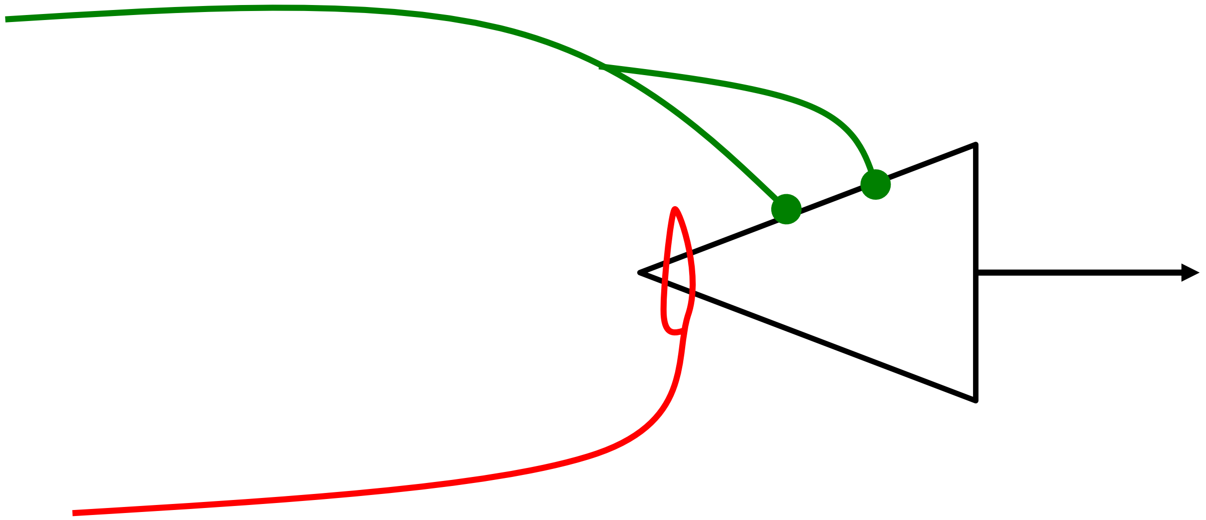
# The McCulloch and Pitts model



A single neuron

- A mathematical model of a neuron
  - McCulloch, W.S. & Pitts, W.H. (1943). A Logical Calculus of the Ideas Immanent in Nervous Activity, Bulletin of Mathematical Biophysics, 5:115-137, 1943
    - Pitts was only 20 years old at this time

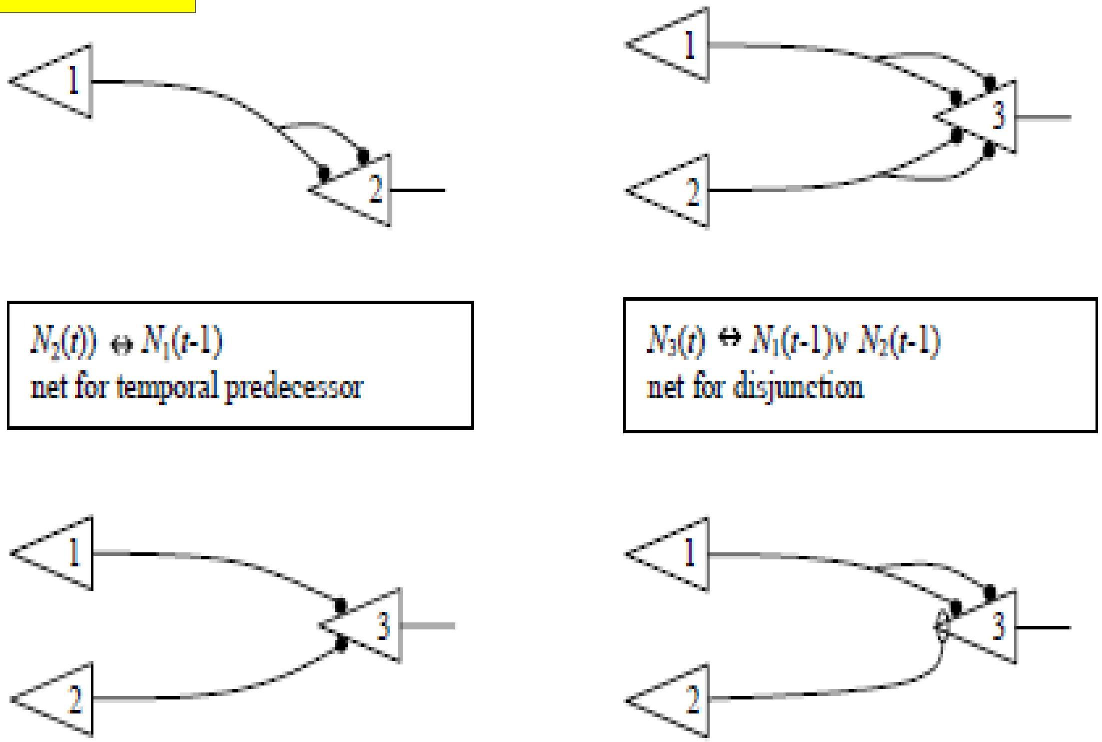
# Synaptic Model



- ***Excitatory synapse:*** Transmits weighted input to the neuron
- ***Inhibitory synapse:*** Any signal from an inhibitory synapse forces output to zero
  - The activity of any inhibitory synapse absolutely prevents excitation of the neuron at that time.
    - Regardless of other inputs

Simple "networks" of neurons can perform Boolean operations

# Boolean Gates



$N_2(t)) \leftrightarrow N_1(t\text{-}1)$
net for temporal predecessor

$N_3(t) \leftrightarrow N_1(t\text{-}1) \vee N_2(t\text{-}1)$
net for disjunction

$N_3(t) \leftrightarrow N_1(t\text{-}1) \& N_2(t\text{-}1)$
net for conjunction

$N_3(t) \leftrightarrow N_1(t\text{-}1) \& {\sim} N_2(t\text{-}1)$
net for conjunction and negation

*Figure 1.* Diagrams of McCulloch and Pitts nets. In order to send an output pulse, each neuron must receive two excitory inputs and no inhibitory inputs. Lines ending in a dot represent excitatory connections; lines ending in a hoop represent inhibitory connections.

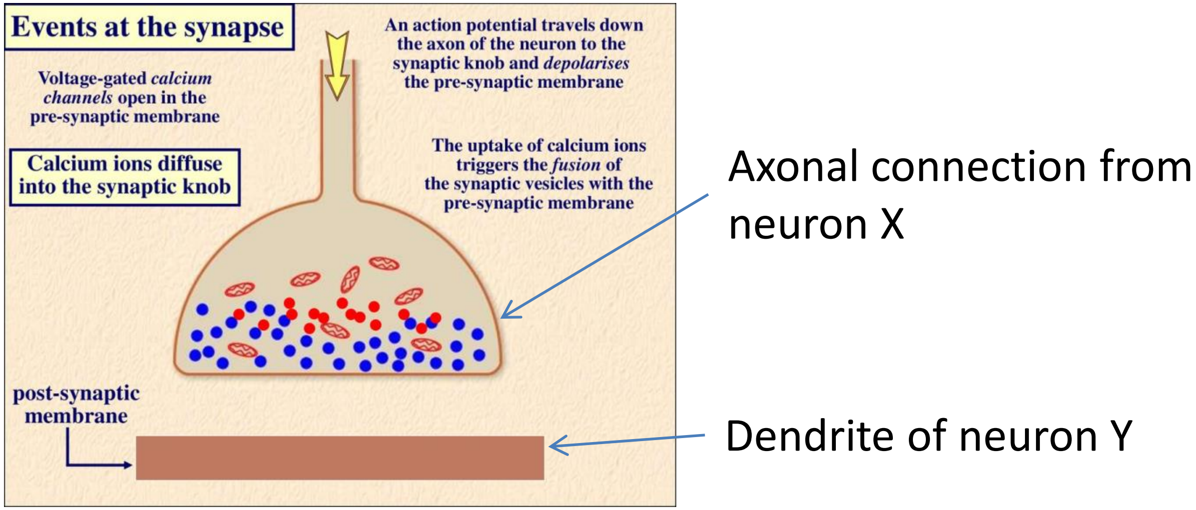
# Criticisms

- They claimed that their nets
  - should be able to compute a small class of functions
  - also if tape is provided their nets can compute a richer class of functions.
    - additionally they will be equivalent to Turing machines
    - Dubious claim that they're Turing complete
  - They didn't prove any results themselves
- Didn't provide a learning mechanism..

# Donald Hebb

- "Organization of behavior", 1949
- A learning mechanism:
  - "When an axon of cell *A* is near enough to excite a cell *B* and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that *A*'s efficiency, as one of the cells firing *B*, is increased."
    - As *A* repeatedly excites *B*, its *ability* to excite *B* improves
  - *Neurons that fire together wire together*

# Hebbian Learning




- If neuron $x_i$ repeatedly triggers neuron $y$, the synaptic knob connecting $x_i$ to $y$ gets larger
- In a mathematical model:

$$w_i = w_i + \eta x_i y$$

  - Weight of $i^{\text{th}}$ neuron's input to output neuron $y$
- This simple formula is actually the basis of many learning algorithms in ML

# Hebbian Learning

- **Fundamentally unstable**
  - Stronger connections will enforce themselves
  - No notion of "competition"
  - No *reduction* in weights
  - Learning is unbounded

- Number of later modifications, allowing for weight normalization, forgetting etc.
  - E.g. Generalized Hebbian learning, aka Sanger's rule

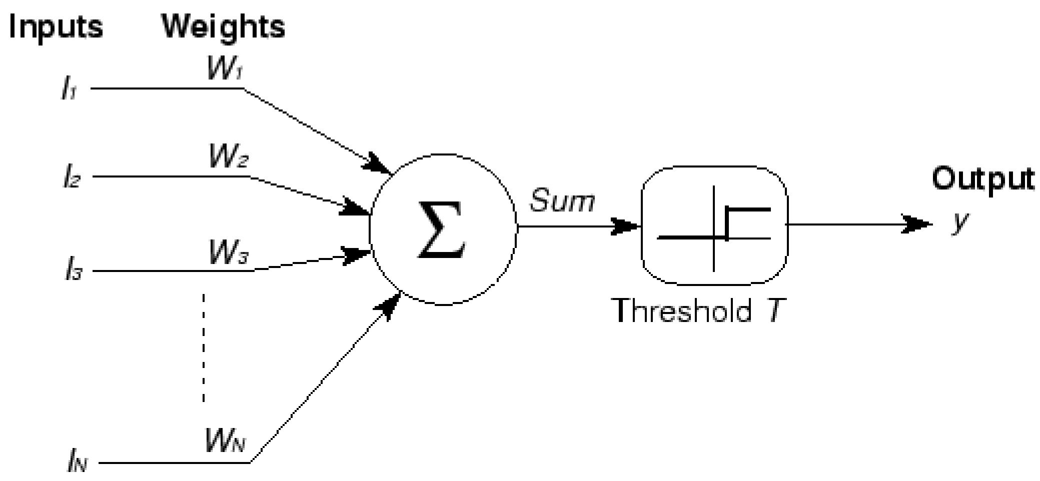
$$w_{ij} = w_{ij} + \eta y_j \left( x_i - \sum_{k=1}^{j} w_{ik} y_k \right)$$

  - The contribution of an input is incrementally *distributed* over multiple outputs..

# A better model


- Frank Rosenblatt
  - Psychologist, Logician
  - Inventor of the solution to everything, aka the Perceptron (1958)

# Simplified mathematical model

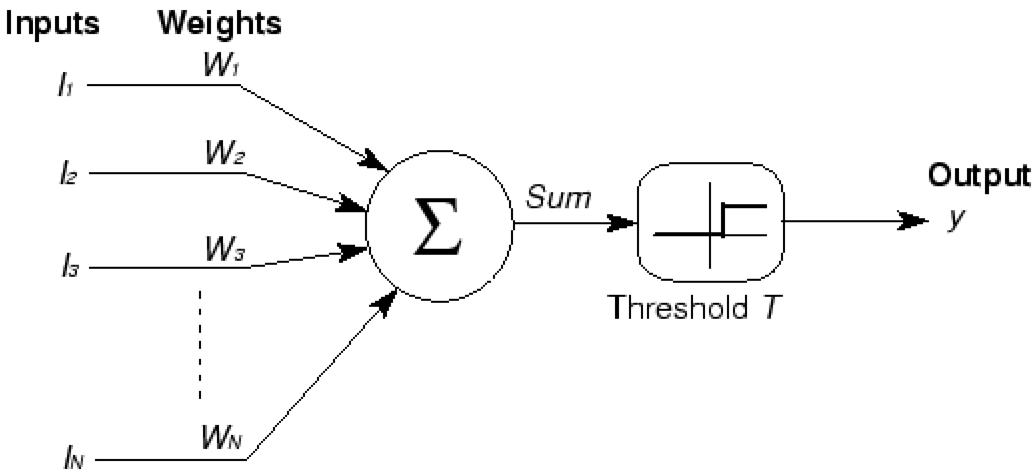



- Number of inputs combine linearly
  - Threshold logic: Fire if combined input exceeds threshold

$$Y = \begin{cases} 1 \;\; if \; \sum_i w_i x_i - T > 0 \\ 0 \qquad\qquad\qquad else \end{cases}$$

# His “Simple” Perceptron

- Originally assumed could represent *any* Boolean circuit and perform any logic
  - *“the embryo of an electronic computer that [the Navy] expects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence,”* New York Times (8 July) 1958
  - *“Frankenstein Monster Designed by Navy That Thinks,”* Tulsa, Oklahoma Times 1958



# Also provided a learning algorithm

$$\mathbf{w} = \mathbf{w} + \eta(d(\mathbf{x}) - y(\mathbf{x}))\mathbf{x}$$

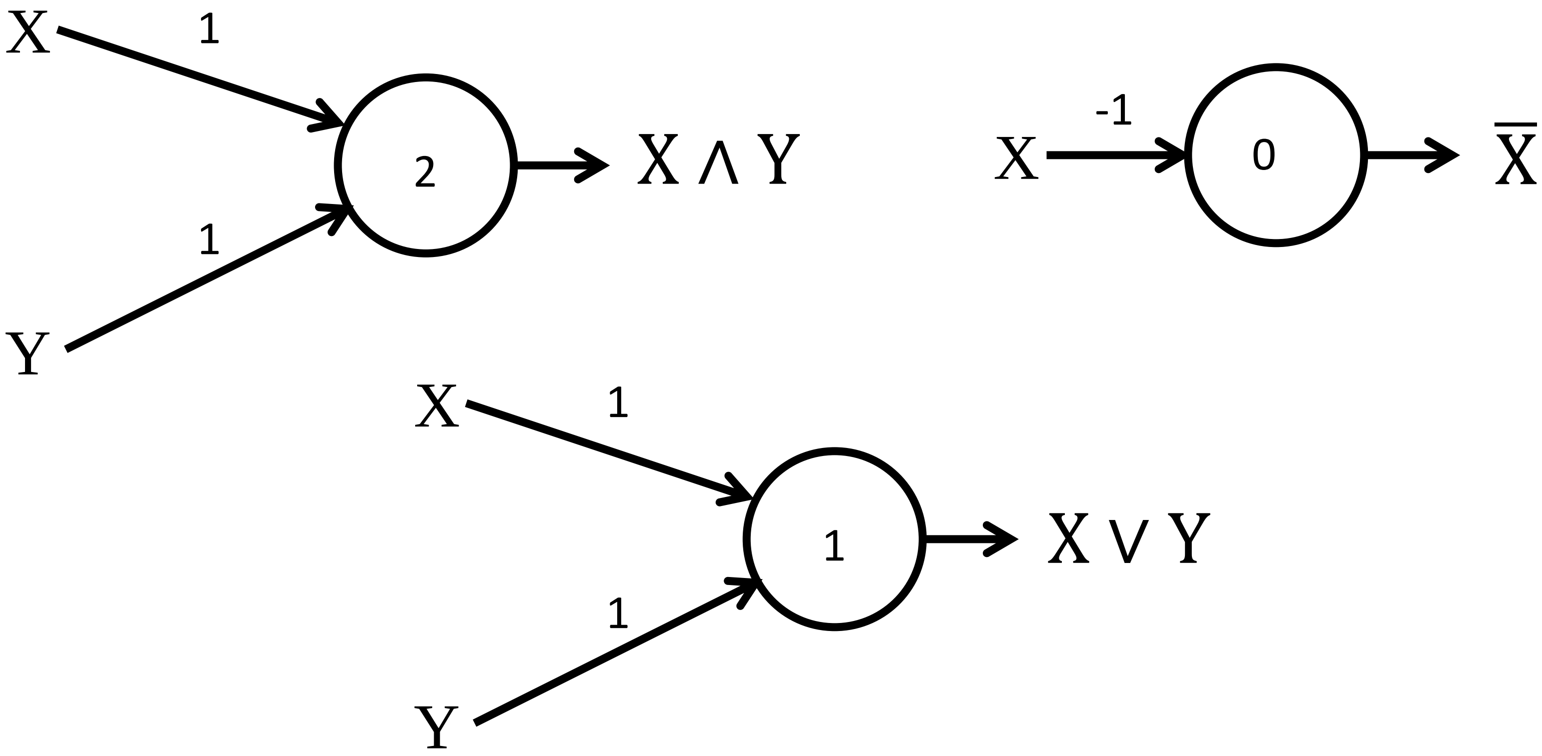
Sequential Learning:

$d(x)$ is the desired output in response to input $x$

$y(x)$ is the actual output in response to $x$

- Boolean tasks
- Update the weights whenever the perceptron output is wrong
- Proved convergence

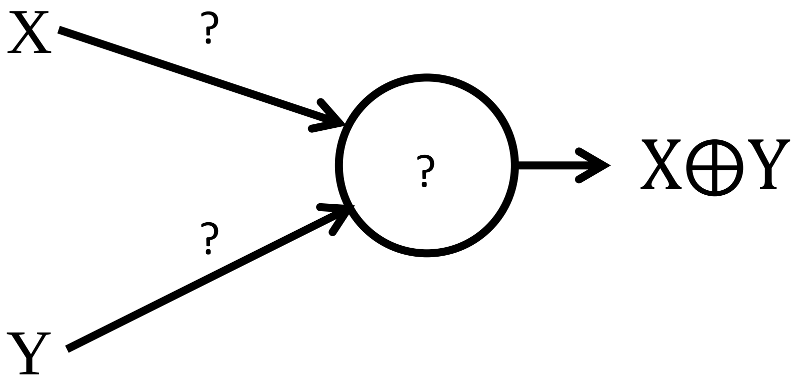
# Perceptron




- Easily shown to mimic any Boolean gate
- But…

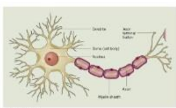
# Perceptron

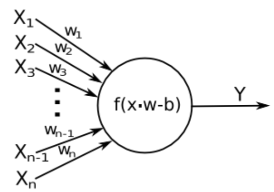
No solution for XOR!
Not universal!

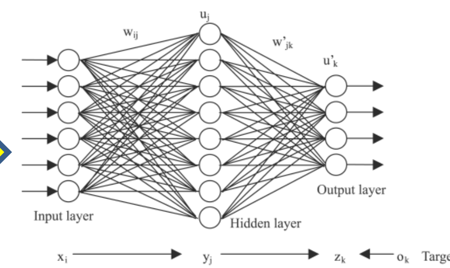



- Minsky and Papert, 1968

# A single neuron is not enough

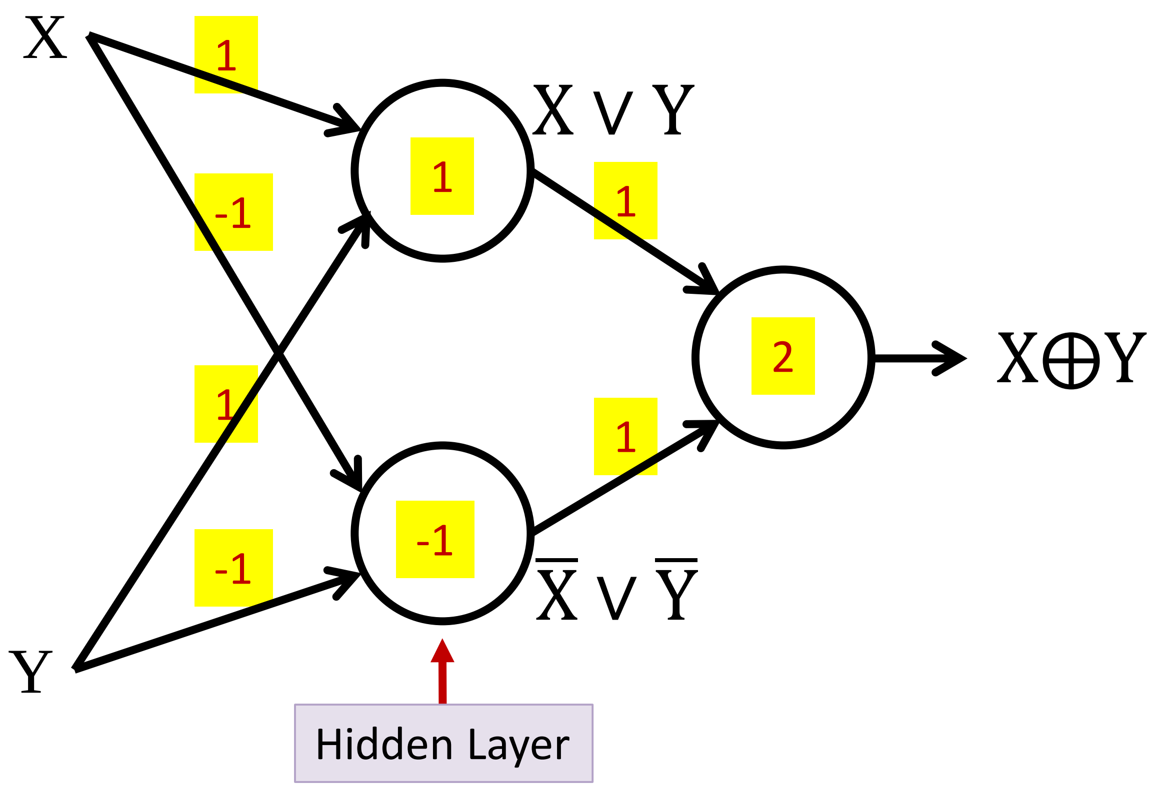







- Individual elements are weak computational elements
  - Marvin Minsky and Seymour Papert, 1969, *Perceptrons: An Introduction to Computational Geometry*
- *Networked* elements are required

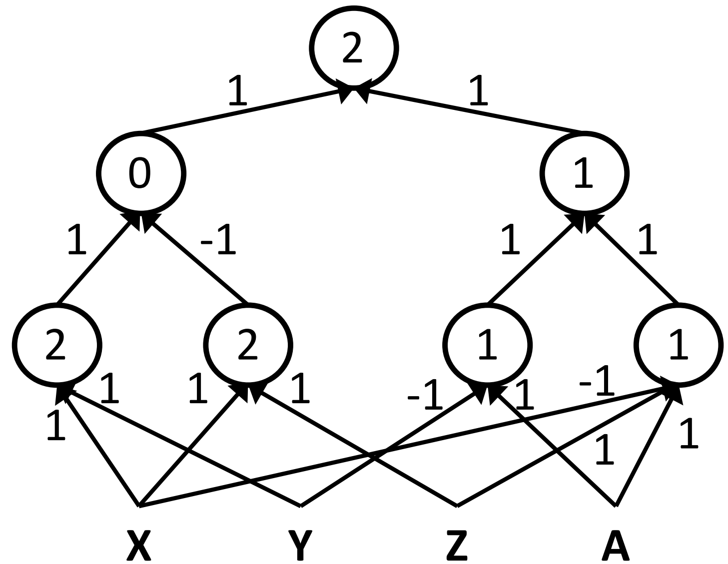
# Multi-layer Perceptron!




- XOR
  - The first layer is a "hidden" layer
  - Also originally suggested by Minsky and Paper 1968

# A more generic model

$$((A \& \bar{X} \& Z) | (A \& \bar{Y})) \& ((X \& Y) | \overline{(X \& Z)})$$

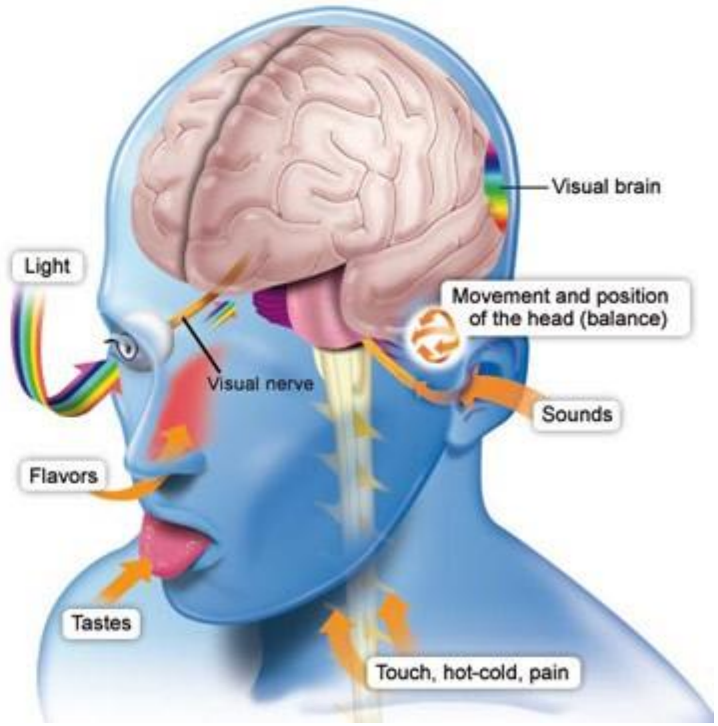



- A "multi-layer" perceptron
- Can compose arbitrarily complicated Boolean functions!
  - In cognitive terms: Can compute arbitrary Boolean functions over sensory input
  - More on this in the next class

# Story so far

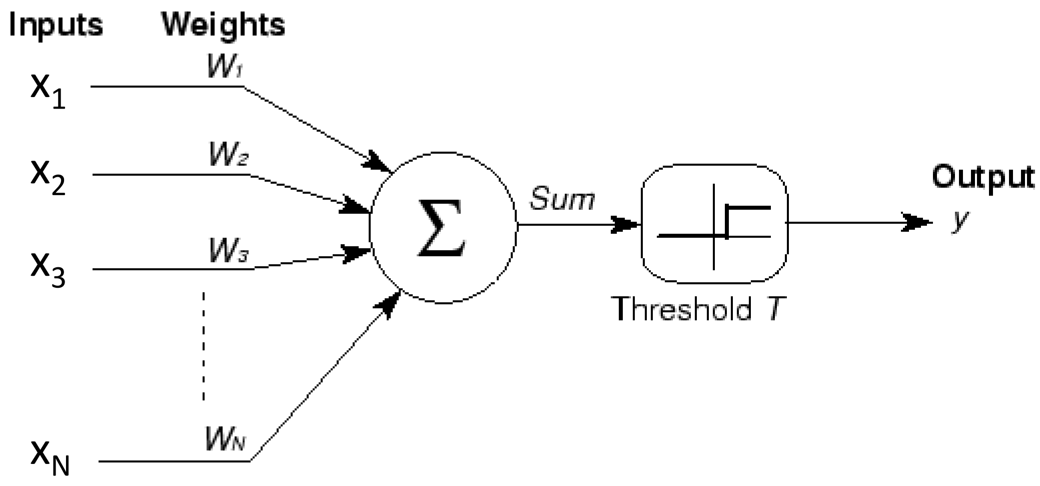
- Neural networks began as computational models of the brain
- Neural network models are *connectionist machines*
  - The comprise networks of neural units
- McCullough and Pitt model: Neurons as Boolean threshold units
  - Models the brain as performing propositional logic
  - But no learning rule
- Hebb's learning rule: Neurons that fire together wire together
  - Unstable
- Rosenblatt's perceptron : A variant of the McCulloch and Pitt neuron with a provably convergent learning rule
  - But individual perceptrons are limited in their capacity (Minsky and Papert)
- Multi-layer perceptrons can model arbitrarily complex Boolean functions

# But our brain is not Boolean



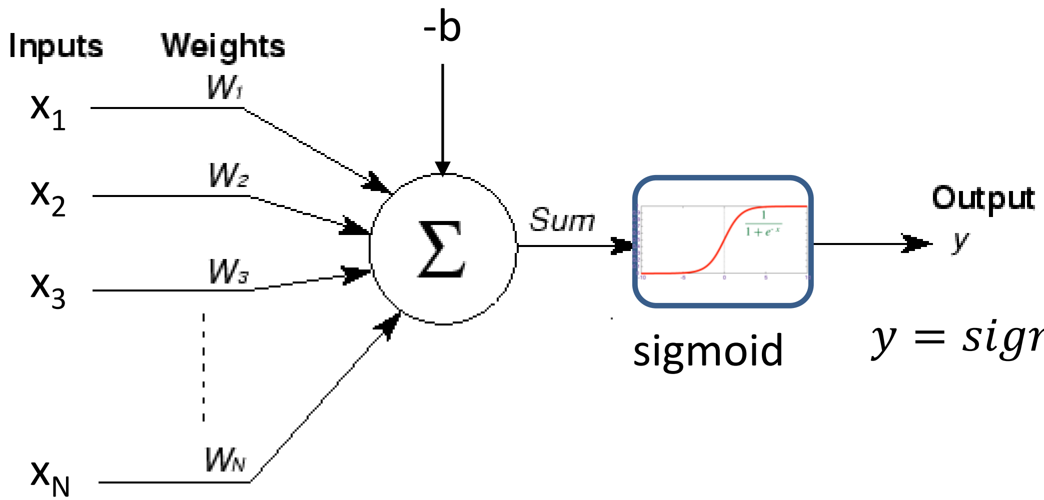

- We have real inputs
- We make non-Boolean inferences/predictions

# The perceptron with *real* inputs




- $x_1 \ldots x_N$ are real valued
- $W_1 \ldots W_N$ are real valued
- Unit "fires" if weighted input exceeds a threshold

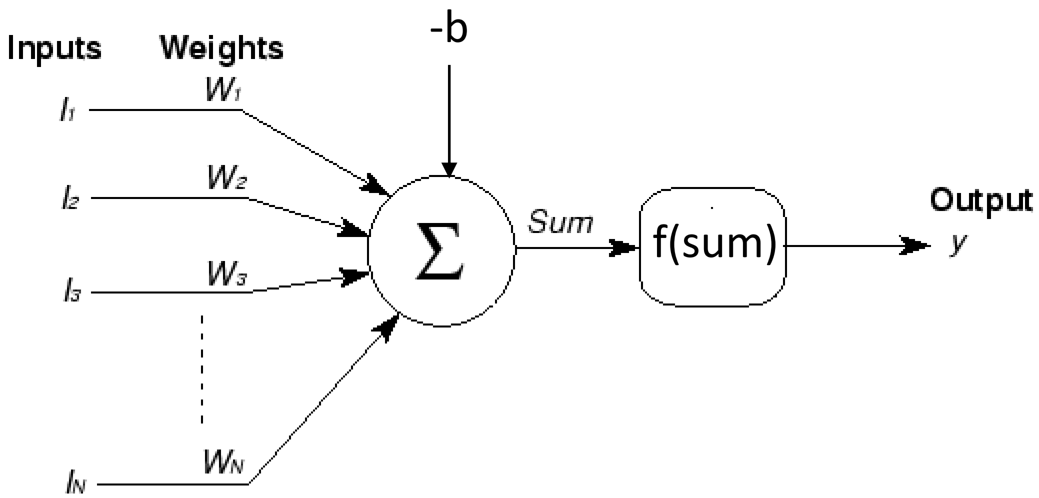
# The perceptron with *real* inputs and a real *output*




$$y = sigmoid(\sum_i w_i x_i - b)$$

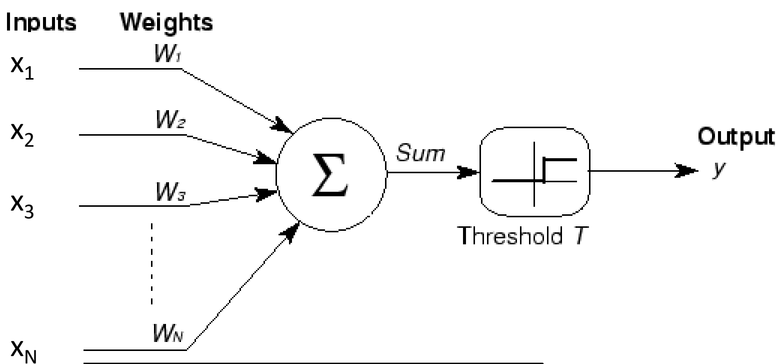
- $x_1 \ldots x_\mathrm{N}$ are real valued
- $W_1 \ldots W_\mathrm{N}$ are real valued
- The output $y$ can also be real valued
  - Sometimes viewed as the “probability” of firing

# The “real” valued perceptron

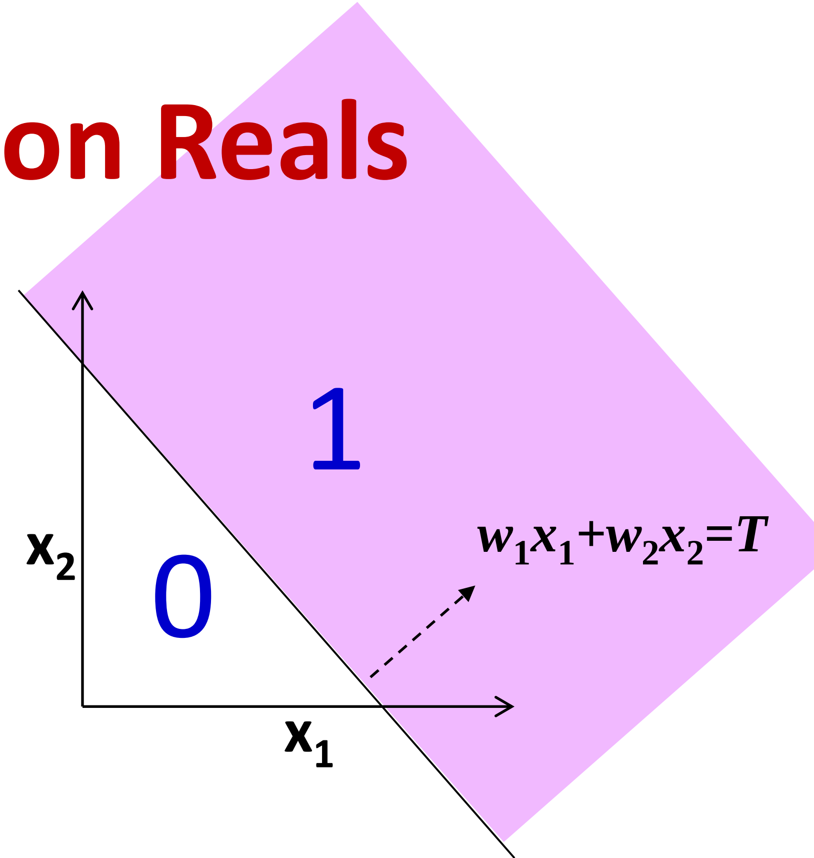


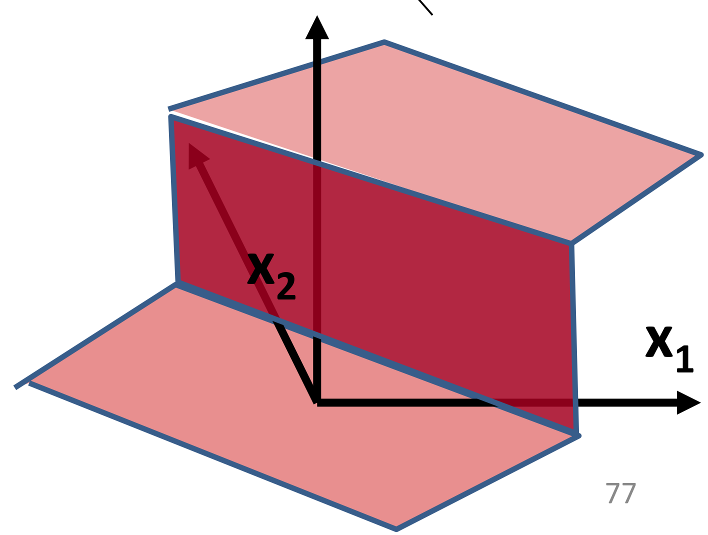

- Any real-valued “activation” function may operate on the weighted-sum input
  - We will see several later
  - Output will be real valued
- The perceptron maps real-valued inputs to real-valued outputs
- *Is useful to continue assuming Boolean outputs though, for interpretation*

# A Perceptron on Reals



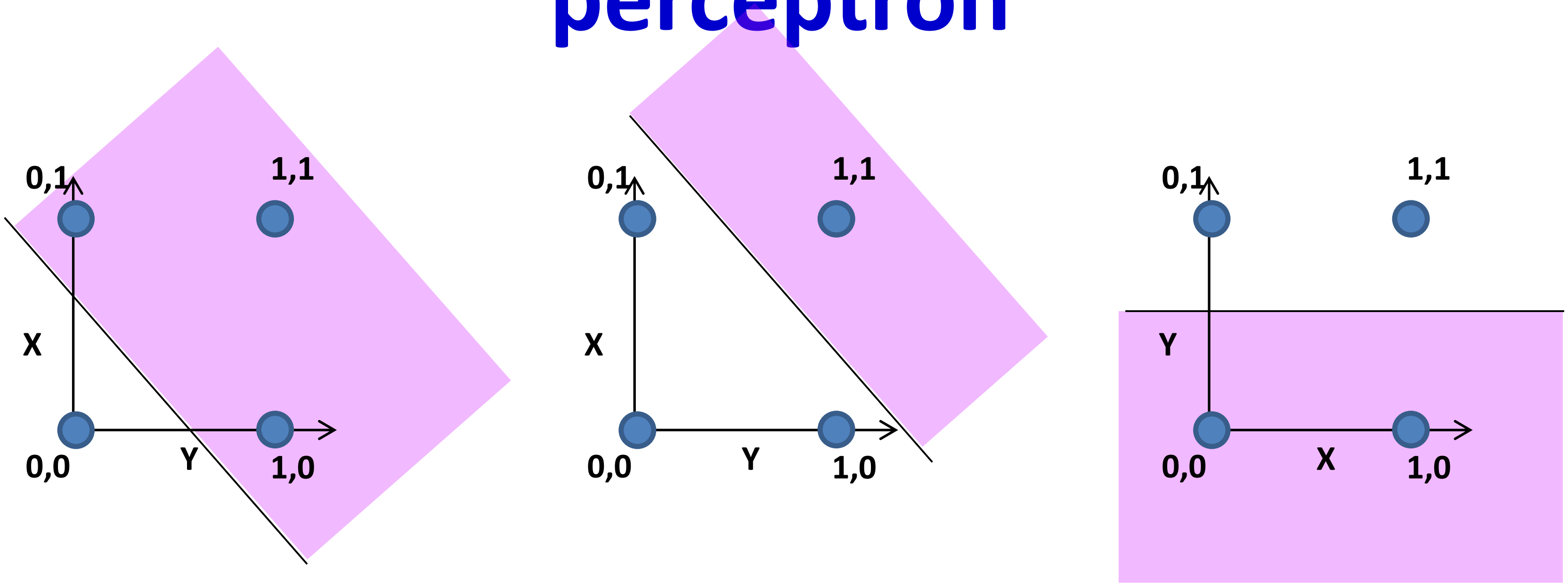

$$y = \begin{cases} 1 \; if \; \sum_i w_i x_i \geq T \\ 0 \; else \end{cases}$$



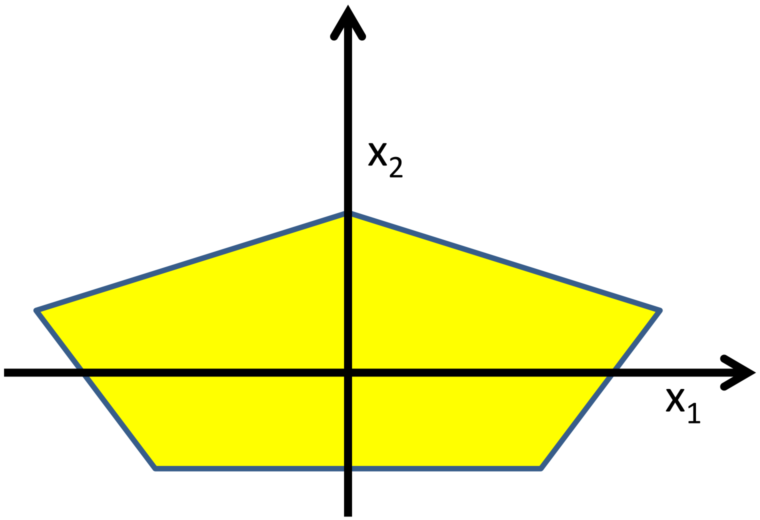

- A perceptron operates on *real*-valued vectors
  - **This is a *linear classifier***

# Boolean functions with a real perceptron




- Boolean perceptrons are also linear classifiers
  - Purple regions have output 1 in the figures
  - What are these functions
  - Why can we not compose an XOR?

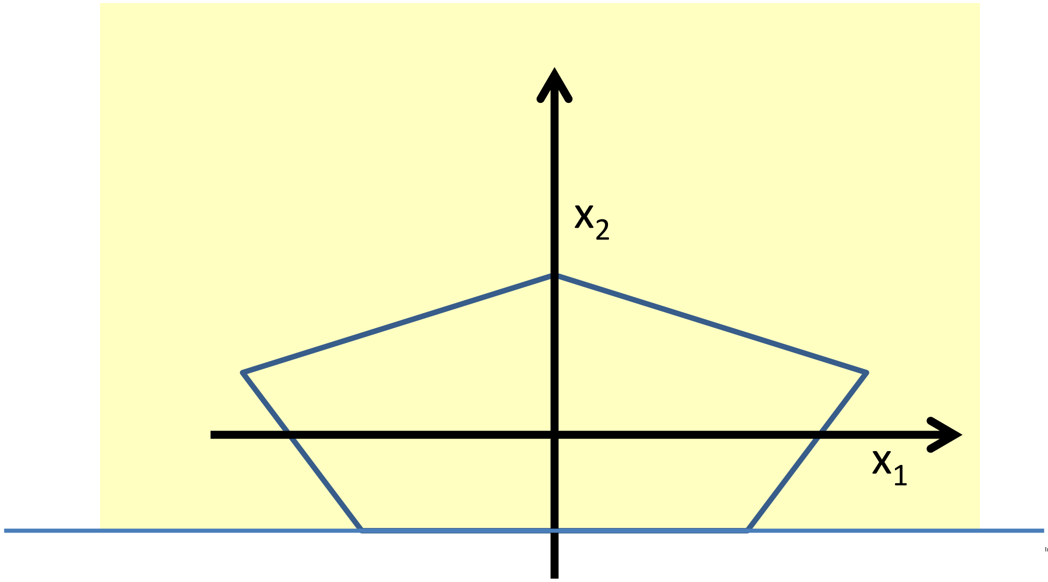
# Composing complicated "decision" boundaries

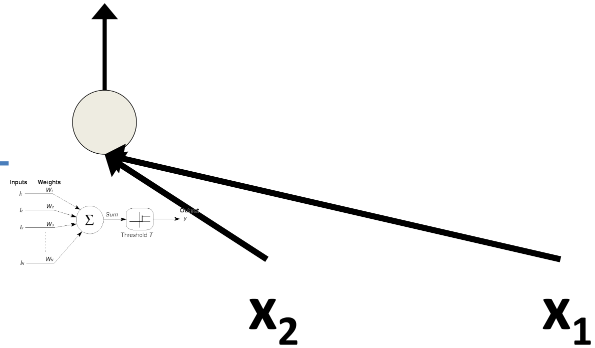



Can now be composed into "networks" to compute arbitrary classification "boundaries"

- Build a network of units with a single output that fires if the input is in the coloured area

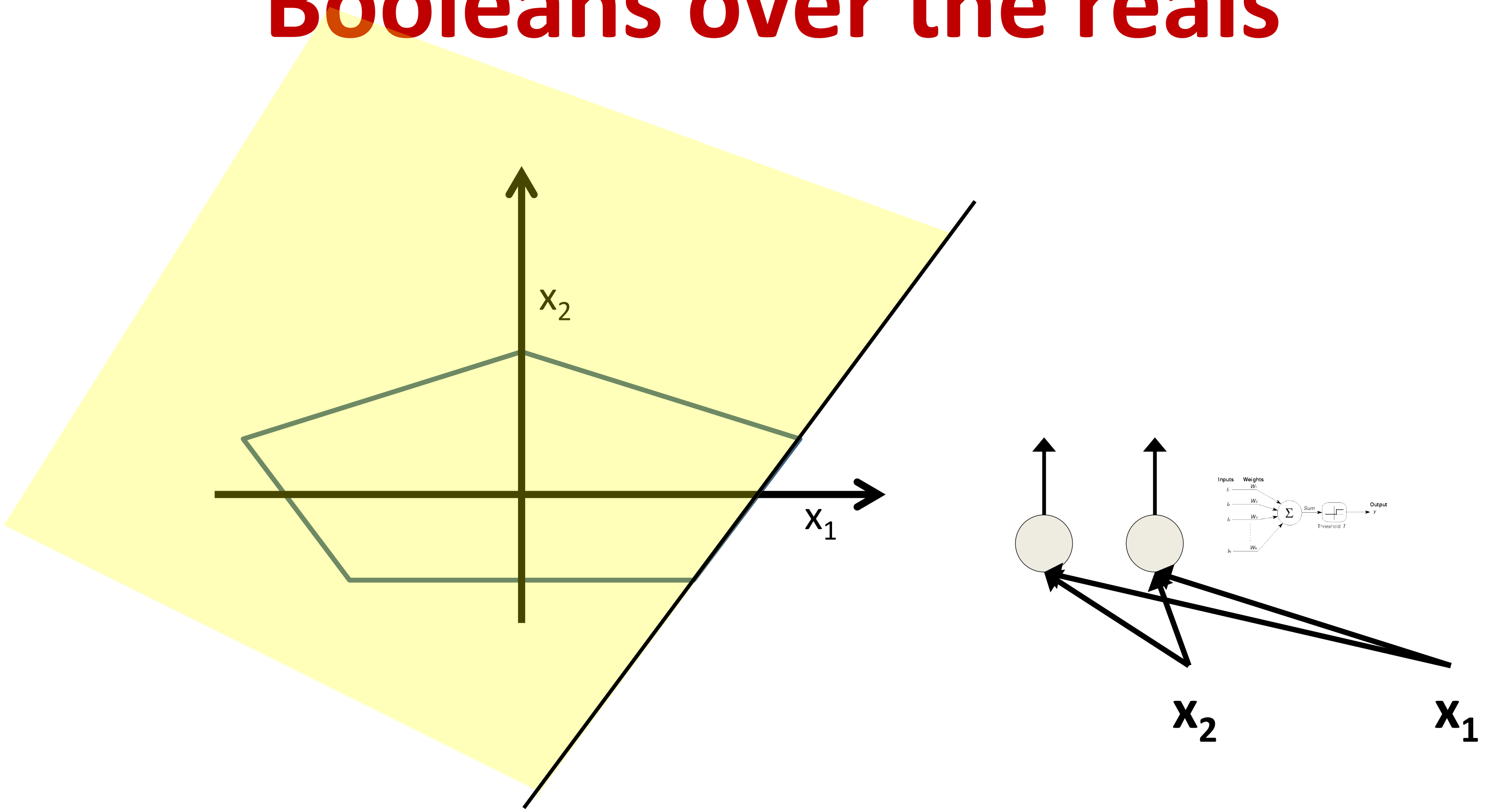
# Booleans over the reals



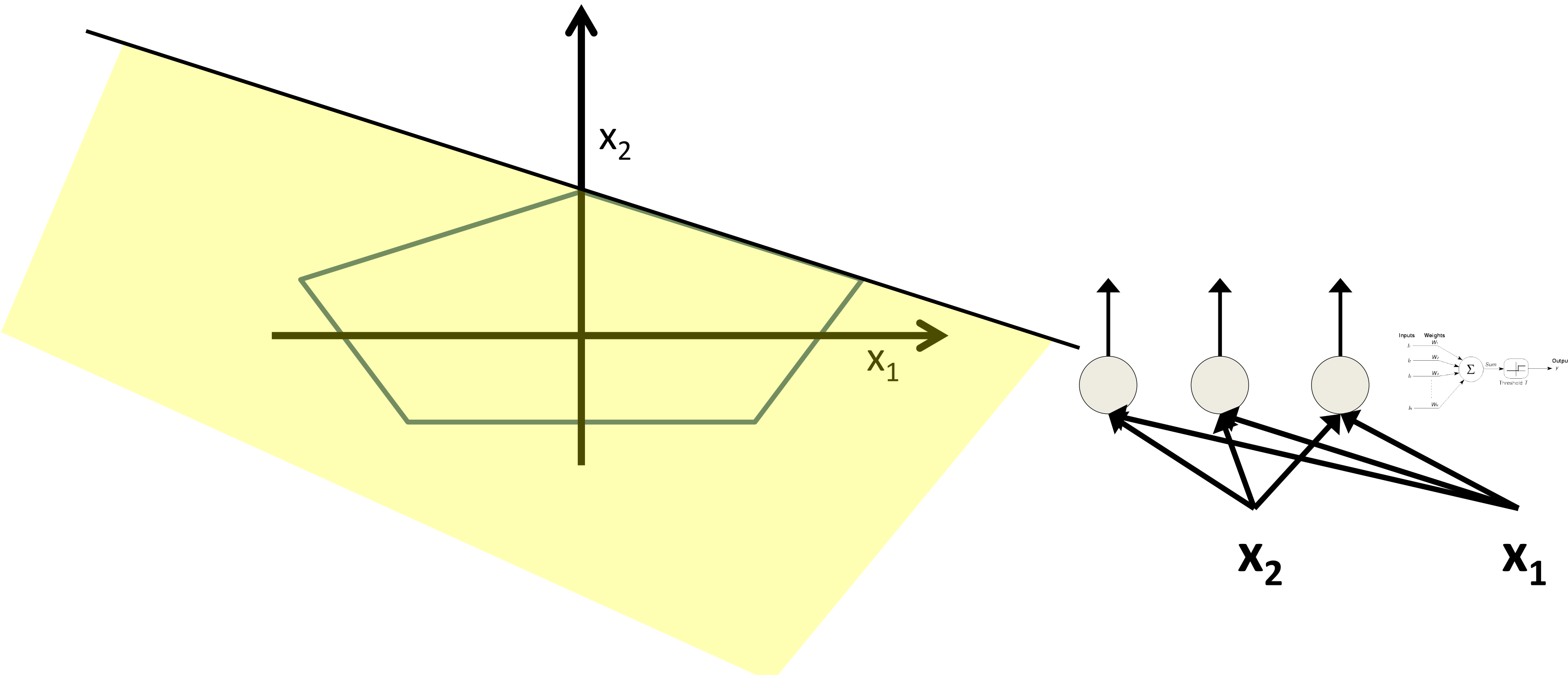

- The network must fire if the input is in the coloured area

# Booleans over the reals



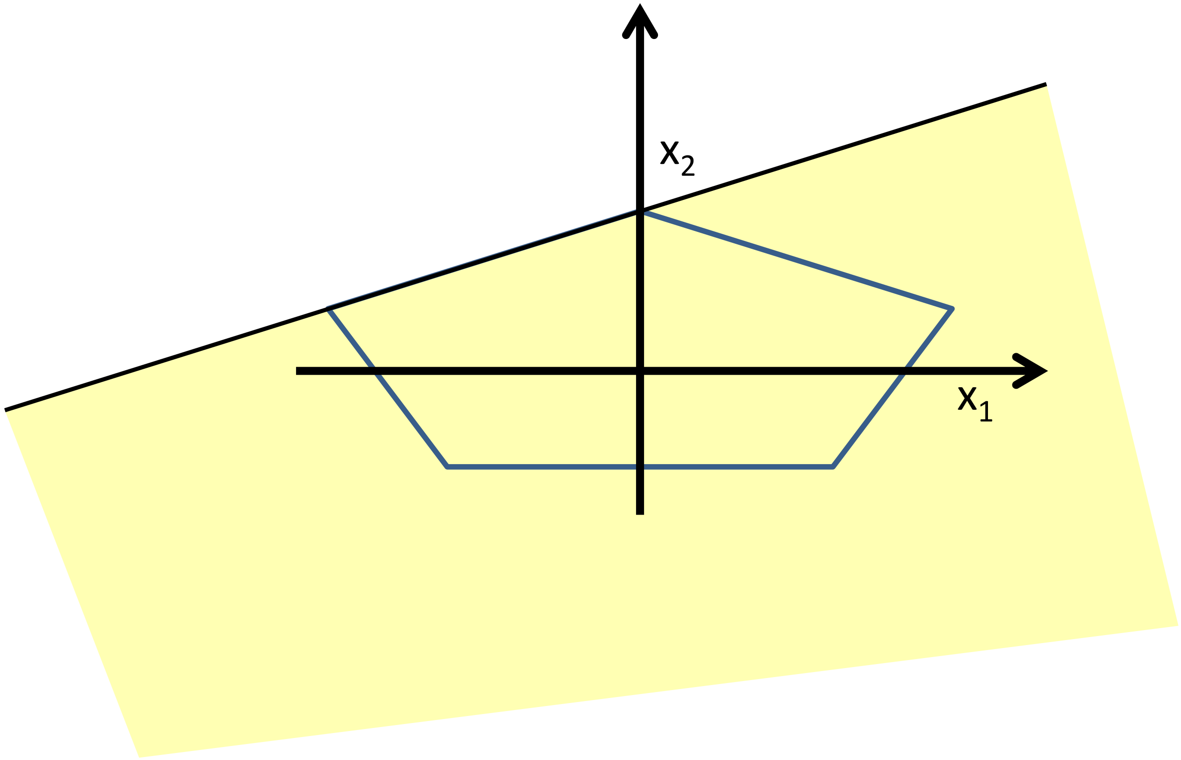

- The network must fire if the input is in the coloured area

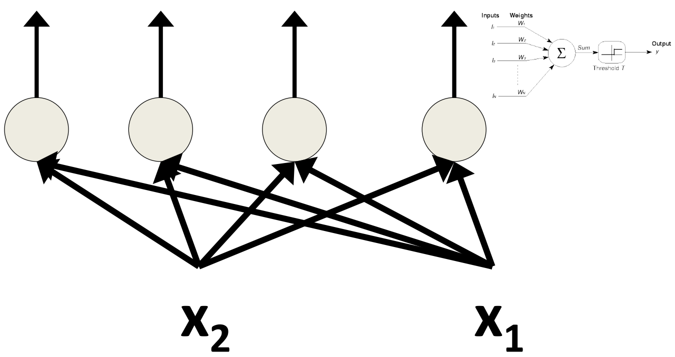
# Booleans over the reals




- The network must fire if the input is in the coloured area

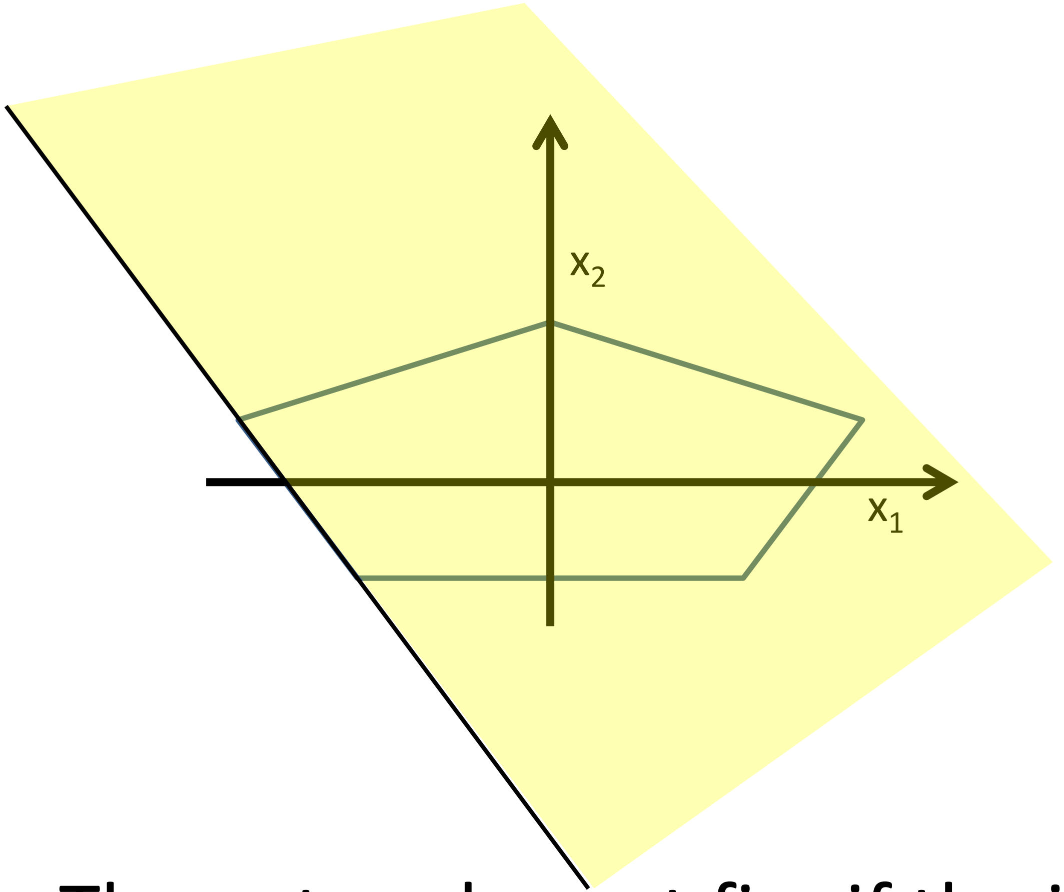
# Booleans over the reals

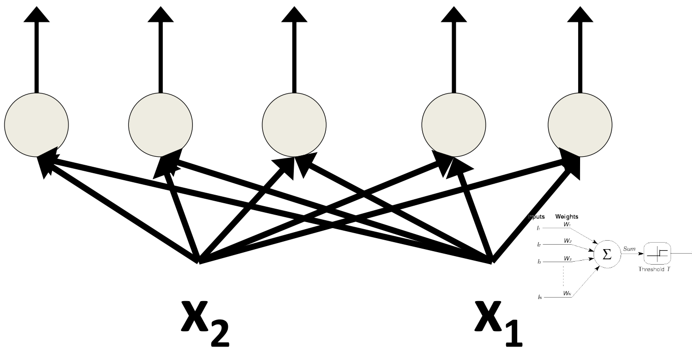



- The network must fire if the input is in the coloured area

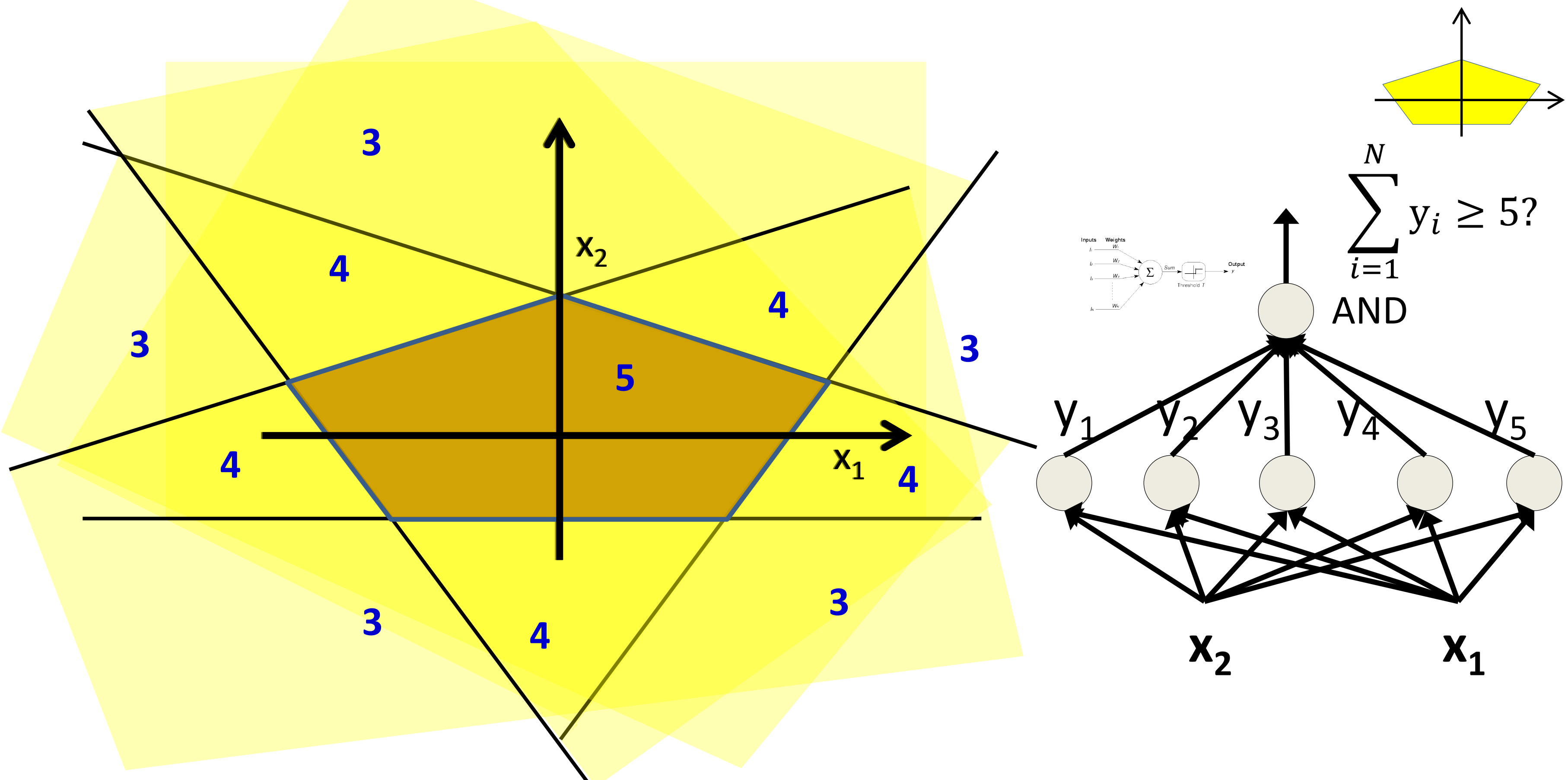
# Booleans over the reals



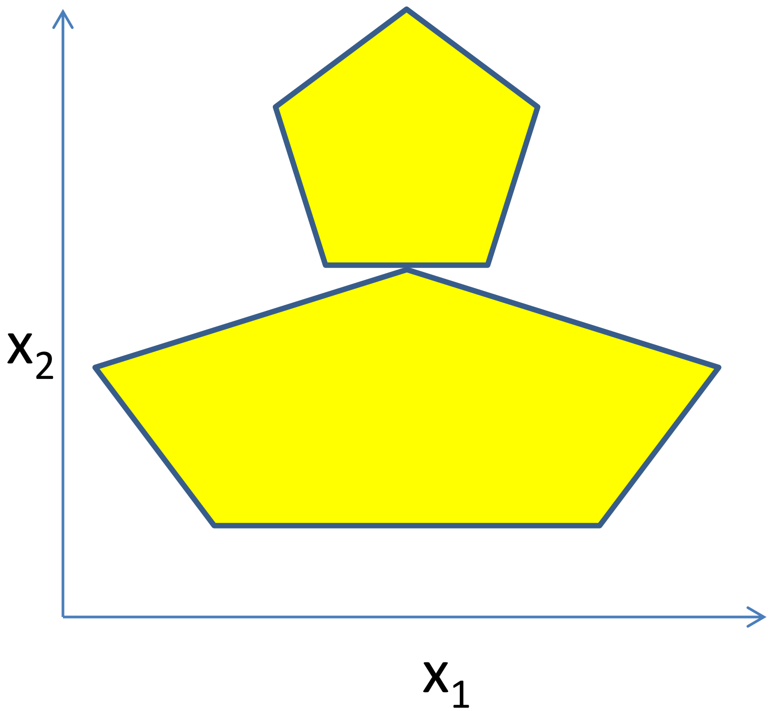

- The network must fire if the input is in the coloured area

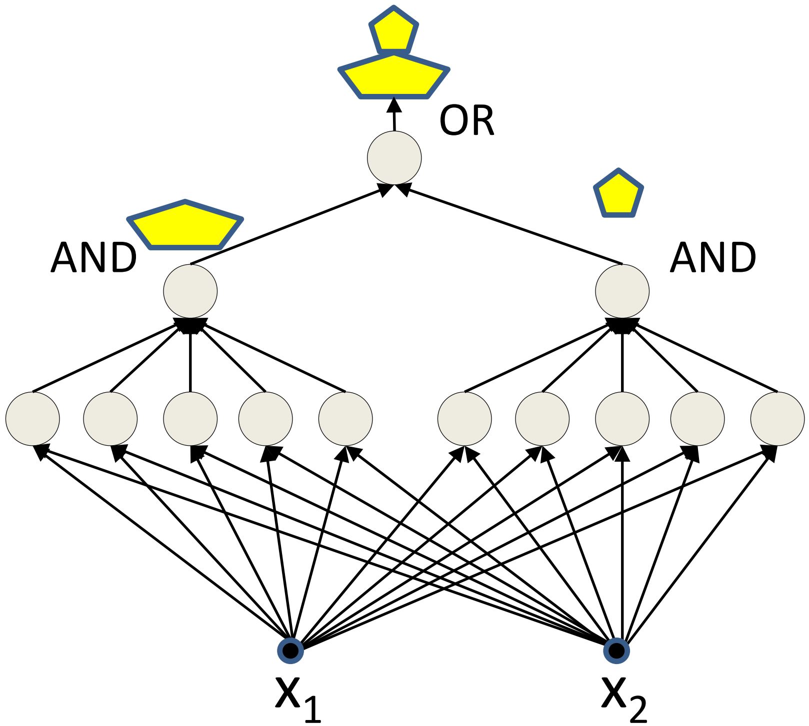
# Booleans over the reals




- The network must fire if the input is in the coloured area

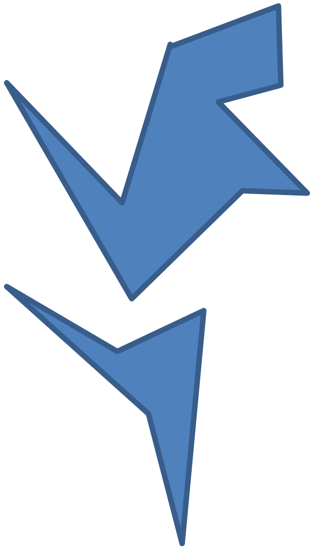
# More complex decision boundaries

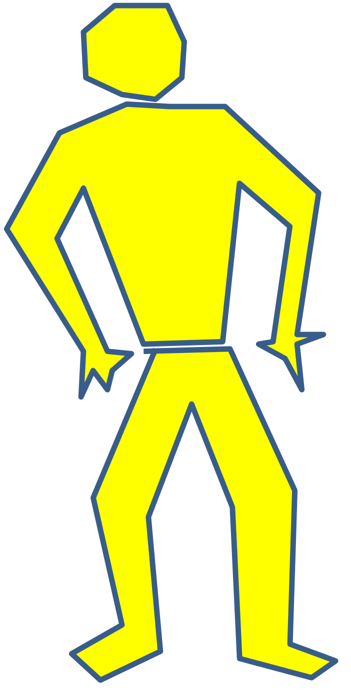


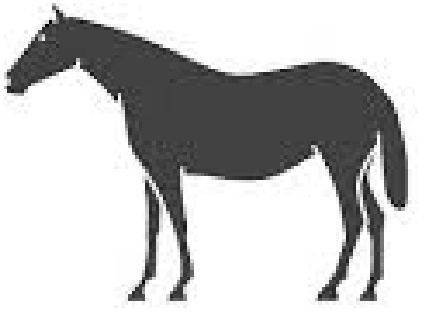




- Network to fire if the input is in the yellow area
  – "OR" two polygons
  – A third layer is required

# Complex decision boundaries



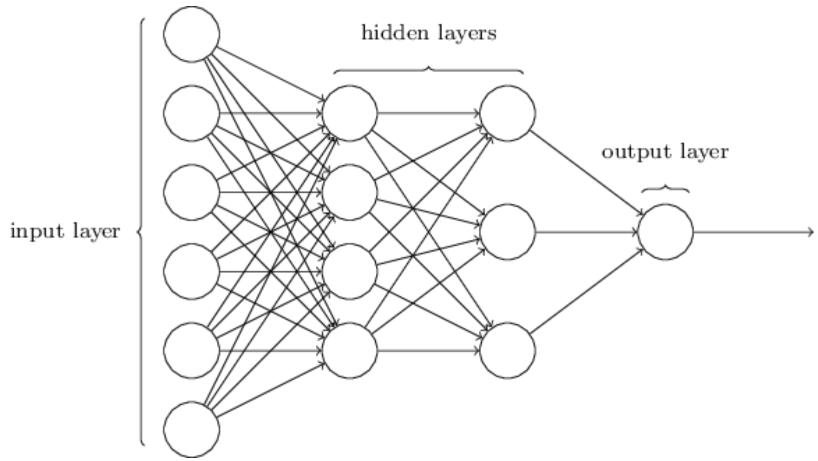


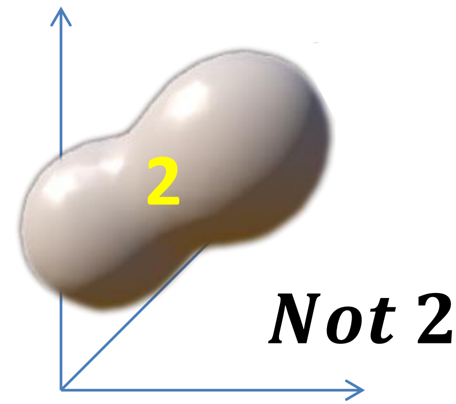


- Can compose very complex decision boundaries
  - How complex exactly? More on this in the next class

# Complex decision boundaries

784 dimensions
(MNIST)



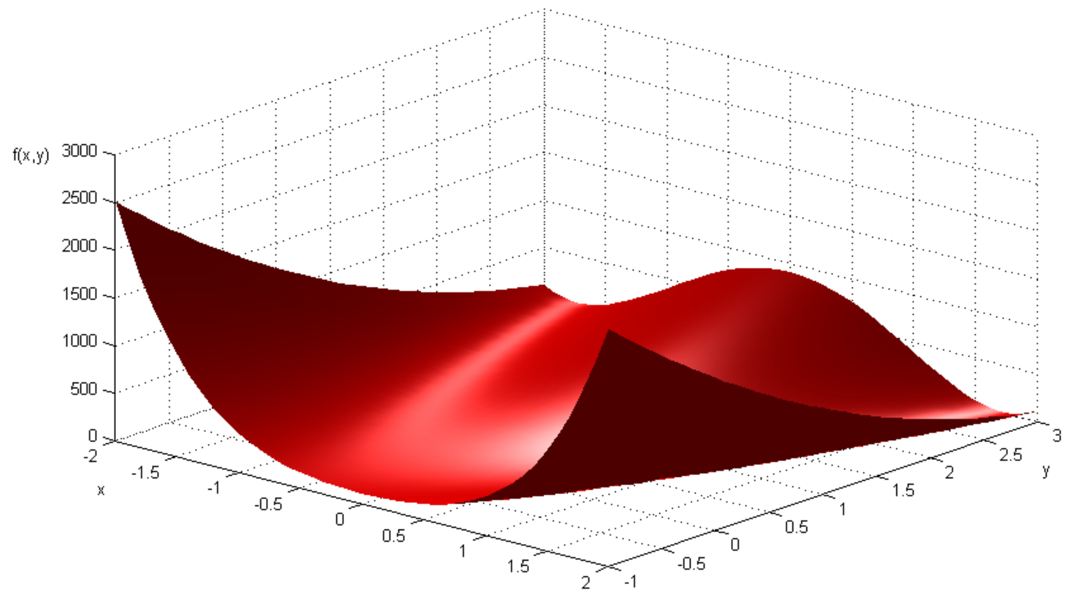

784 dimensions

- Classification problems: finding decision boundaries in high-dimensional space
  - Can be performed by an MLP
- MLPs can *classify* real-valued inputs

# Story so far

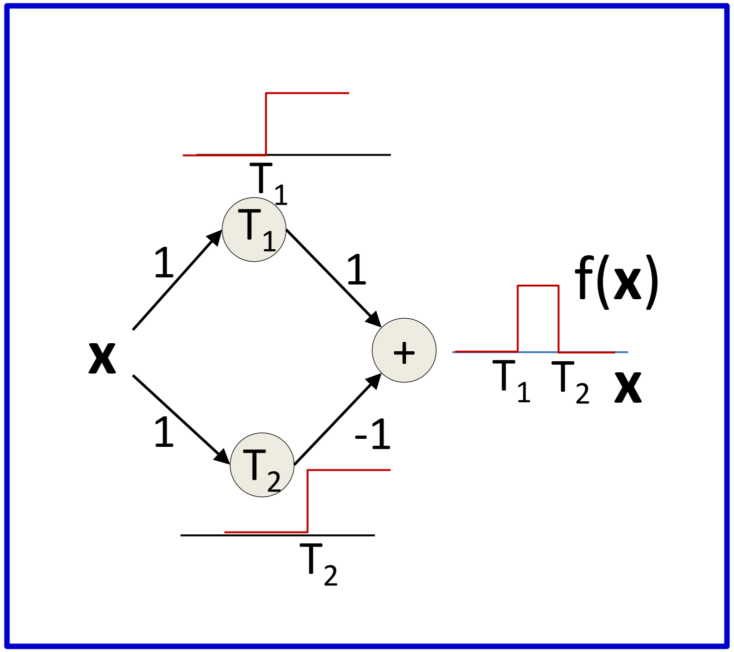
- **MLPs are connectionist computational models**
  - Individual perceptrons are computational equivalent of neurons
  - The MLP is a layered composition of many perceptrons
- **MLPs can model Boolean functions**
  - Individual perceptrons can act as Boolean gates
  - Networks of perceptrons are Boolean functions
- **MLPs are Boolean *machines***
  - They represent Boolean functions over linear boundaries
  - They can represent arbitrary decision boundaries
  - They can be used to *classify* data

# But what about continuous valued *outputs?*



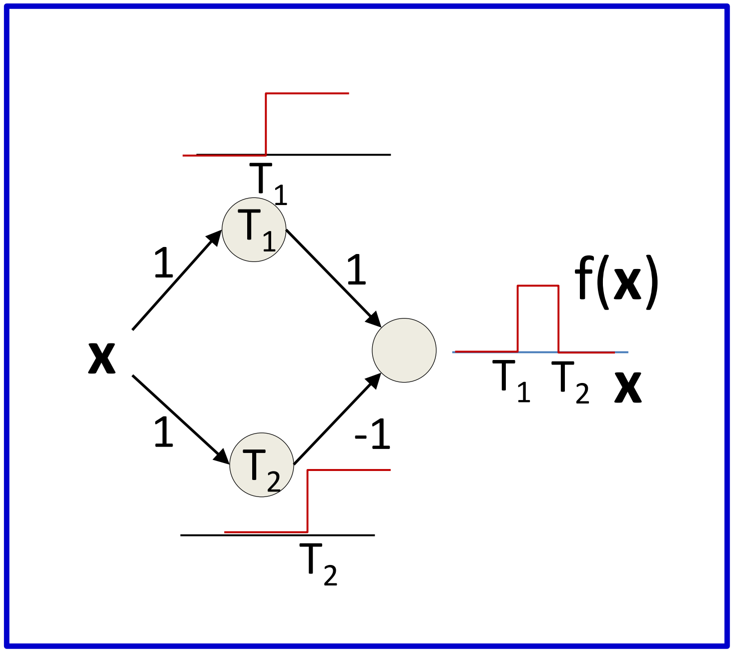

- Inputs may be real valued
- Can outputs be continuous-valued too

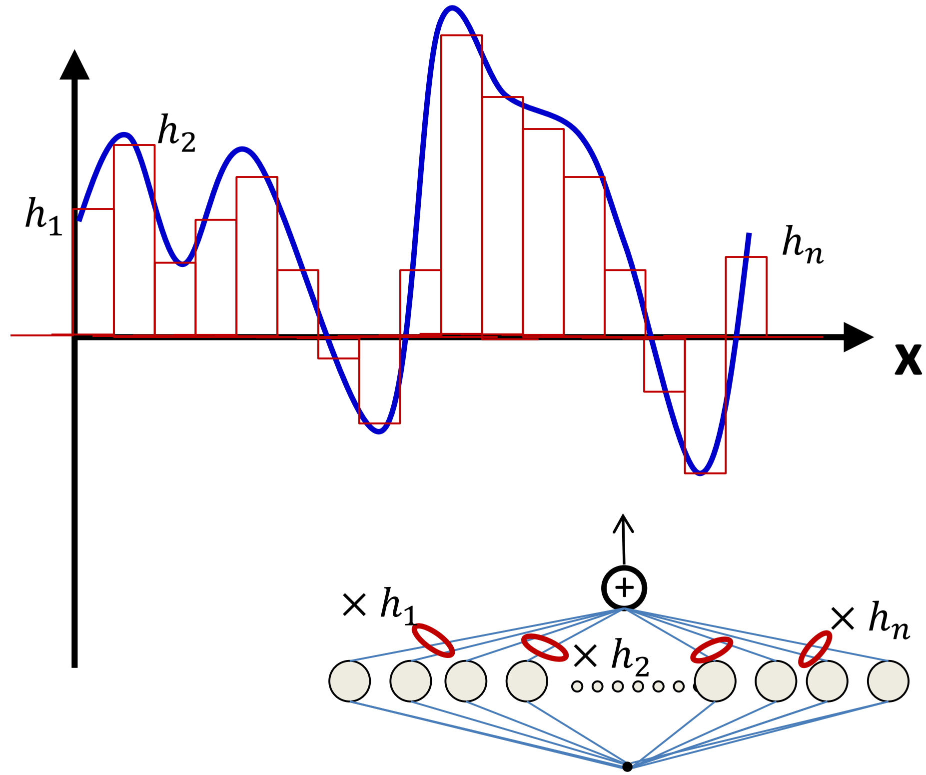
# MLP as a continuous-valued regression




- A simple 3-unit MLP with a "summing" output unit can generate a "square pulse" over an input
  - Output is 1 only if the input lies between $T_1$ and $T_2$
  - $T_1$ and $T_2$ can be arbitrarily specified

# MLP as a continuous-valued regression




- A simple 3-unit MLP can generate a "square pulse" over an input
- **An MLP with many units can model an arbitrary function over an input**
  - To arbitrary precision
    - Simply make the individual pulses narrower
- This generalizes to functions of any number of inputs (next class)

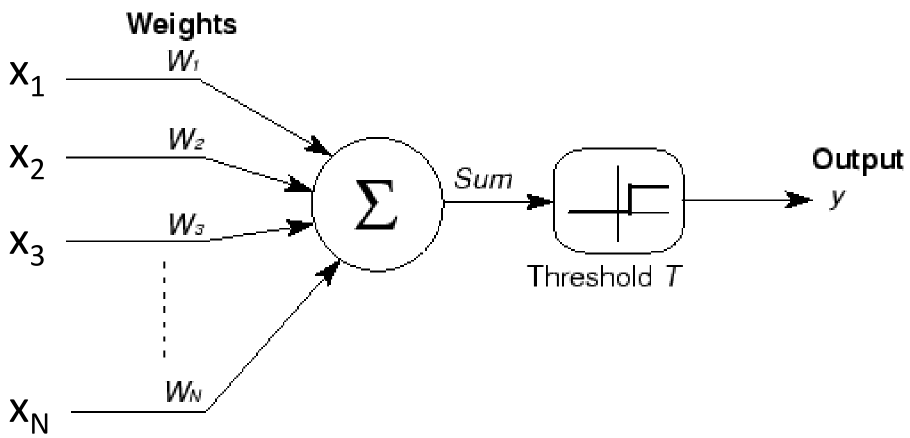
# Story so far

- **Multi-layer perceptrons are connectionist computational models**
- **MLPs are *classification engines***
  - They can identify classes in the data
  - Individual perceptrons are feature detectors
  - The network will fire if the combination of the detected basic features matches an "acceptable" pattern for a desired class of signal
- **MLP can also model continuous valued functions**

# So what does the perceptron really model?

- Is there a “semantic” interpretation?
  – Cognitive version: Is there an interpretation beyond the simple characterization as Boolean functions over sensory inputs?

# Lets look at the weights

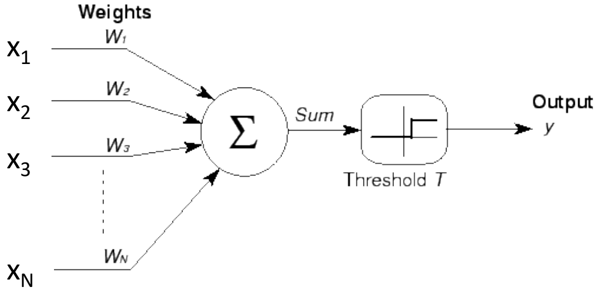



$$y = \begin{cases} 1 \; if \; \sum_i w_i x_i \geq T \\ 0 \; else \end{cases}$$

$$y = \begin{cases} 1 \; if \; \mathbf{x}^T \mathbf{w} \geq T \\ 0 \; else \end{cases}$$

- What do the *weights* tell us?
  - The neuron fires if the inner product between the weights and the inputs exceeds a threshold

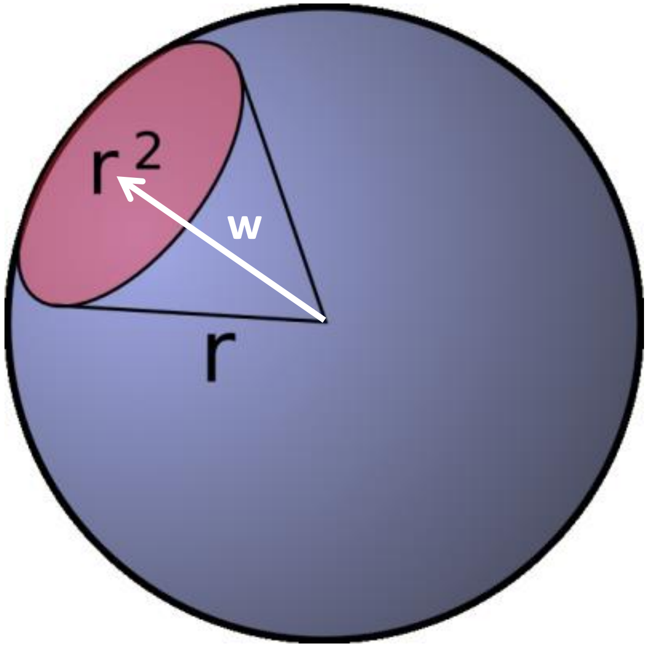
# The weight as a "template"




$$X^T W > T$$

$$\cos\theta > \frac{T}{|X|}$$

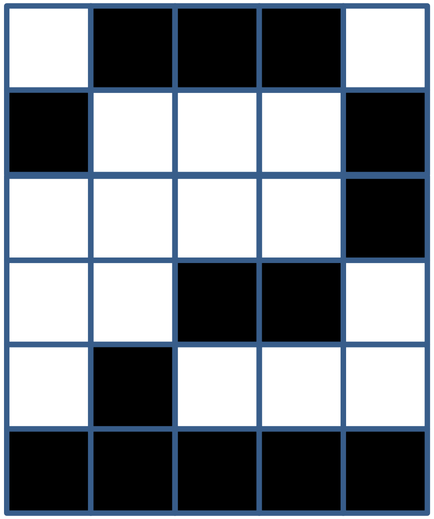
$$\theta < \cos^{-1}\left(\frac{T}{|X|}\right)$$



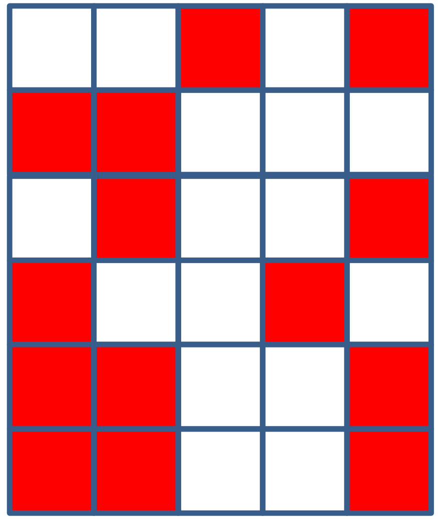

- The perceptron fires if the input is within a specified angle of the weight
- Neuron fires if the input vector is close enough to the weight vector.
  - If the input pattern matches the weight pattern closely enough

# The weight as a template

W

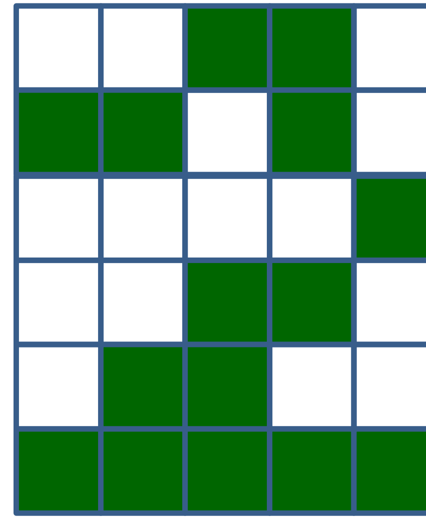


X



Correlation = 0.57

X

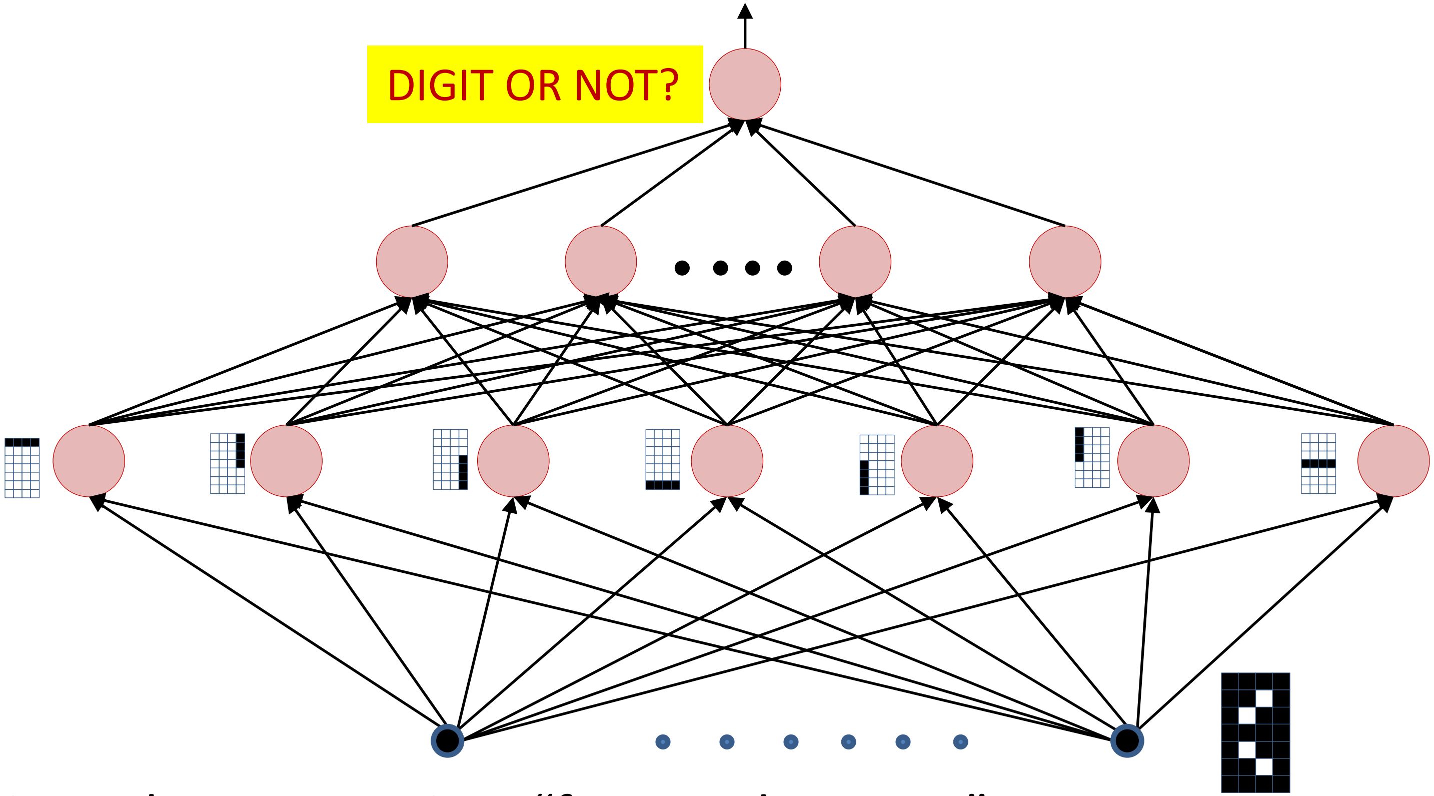


Correlation = 0.82

$$y = \begin{cases} 1\ if\ \sum_i w_i x_i \geq T \\ 0\ else \end{cases}$$

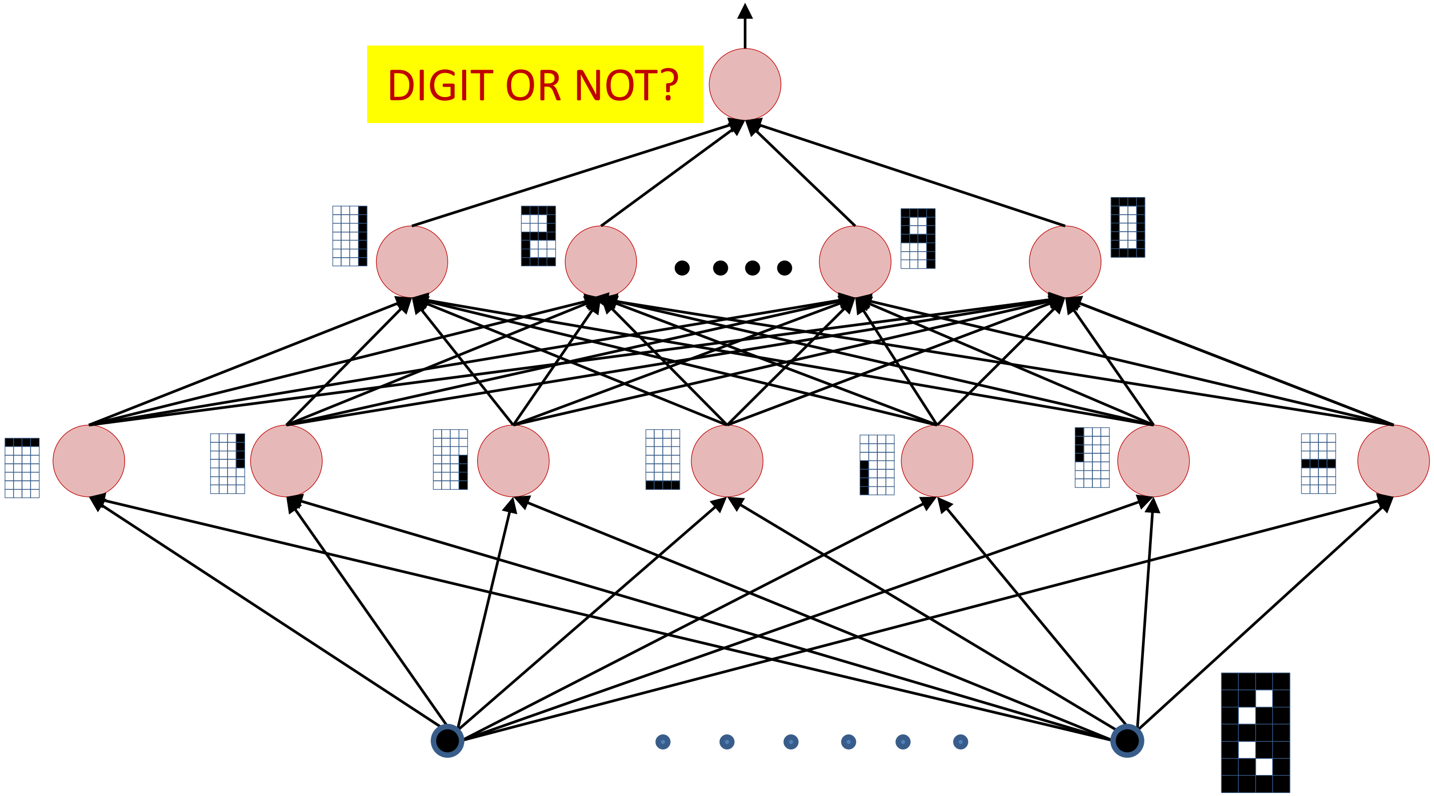
- If the *correlation* between the weight pattern and the inputs exceeds a threshold, fire
- The perceptron is a *correlation filter!*

# The MLP as a Boolean function over feature detectors




- The input layer comprises "feature detectors"
  - Detect if certain patterns have occurred in the input
- The network is a Boolean function over the feature detectors
- I.e. it is important for the *first* layer to capture relevant patterns

# The MLP as a cascade of feature detectors



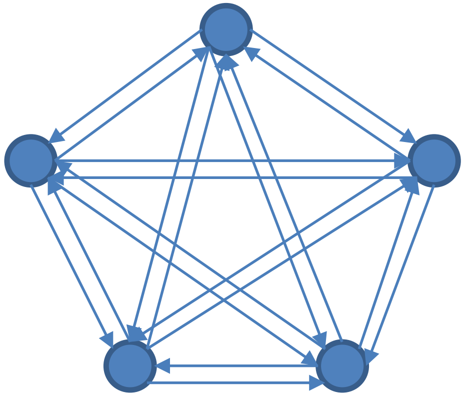

- The network is a cascade of feature detectors
  - Higher level neurons compose complex templates from features represented by lower-level neurons

# Story so far

- **Multi-layer perceptrons are connectionist computational models**
- **MLPs are Boolean *machines***
  - They can model Boolean functions
  - They can represent arbitrary decision boundaries over real inputs
- **MLPs can approximate continuous valued functions**
- **Perceptrons are *correlation filters***
  - They detect patterns in the input

# Other things MLPs can do

- Model memory
  - Loopy networks can "remember" patterns
    - Proposed by Lawrence Kubie in 1930, as a model for memory in the CNS

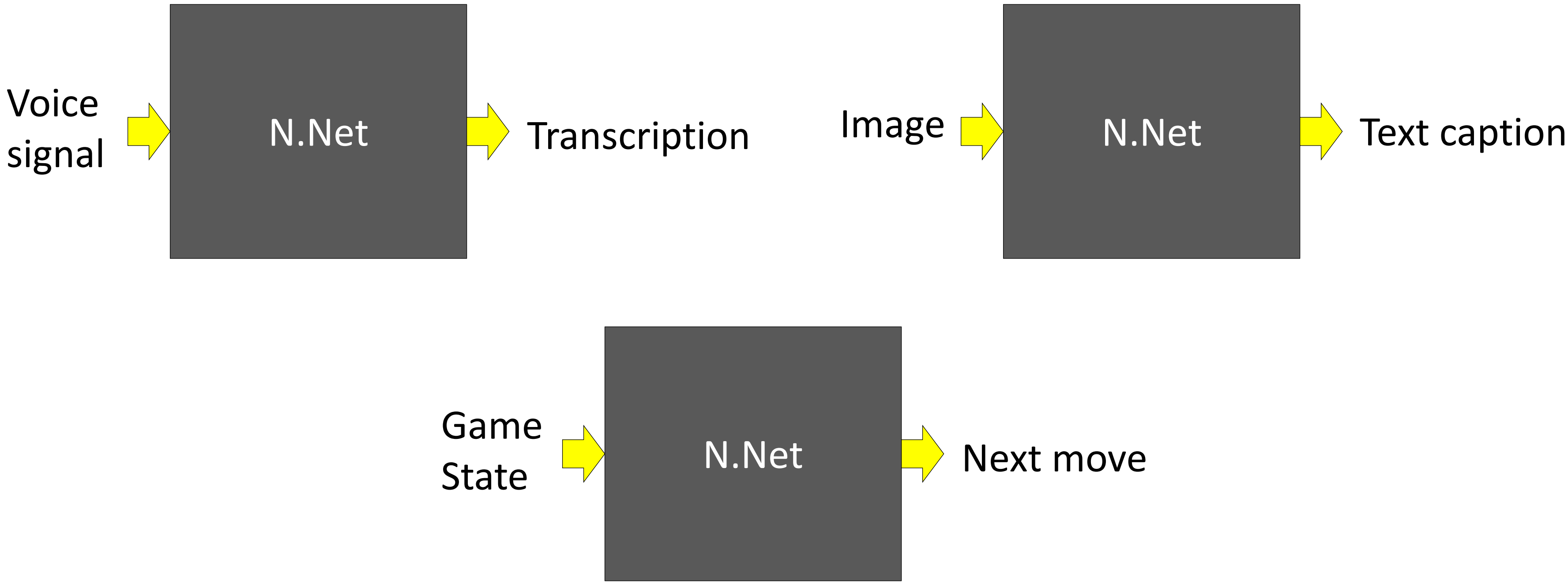


- Represent probability distributions
  - Over integer, real and complex-valued domains
  - MLPs can model both *a posteriori* and *a priori* distributions of data
    - A posteriori conditioned on other variables
- They can rub their stomachs and pat their heads at the same time..

# NNets in AI

- The network is a function
  - Given an input, it computes the function layer wise to predict an output

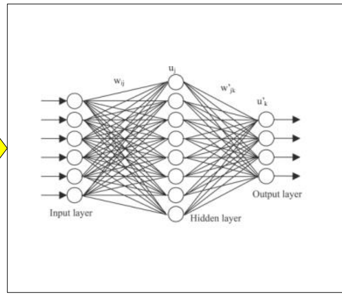
# These tasks are *functions*




- Each of these boxes is actually a function
  - E.g f: Image → Caption

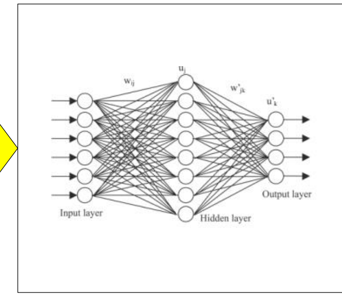
# These tasks are *functions*

Voice signal



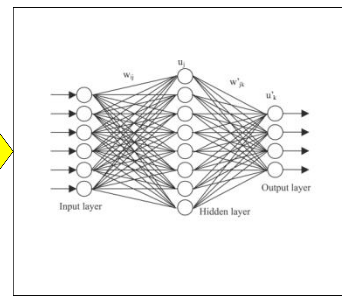

Transcription

Image




Text caption

Game State




Next move

- Each box is actually a function
  - E.g  f: Image → Caption
  - It can be approximated by a neural network

# Story so far

- **Multi-layer perceptrons are connectionist computational models**
- **MLPs are *classification engines***
- **MLP can also model continuous valued functions**
- **Interesting AI tasks are functions that can be modelled by the network**

# Next Up

- More on neural networks as universal approximators
  - And the issue of depth in networks