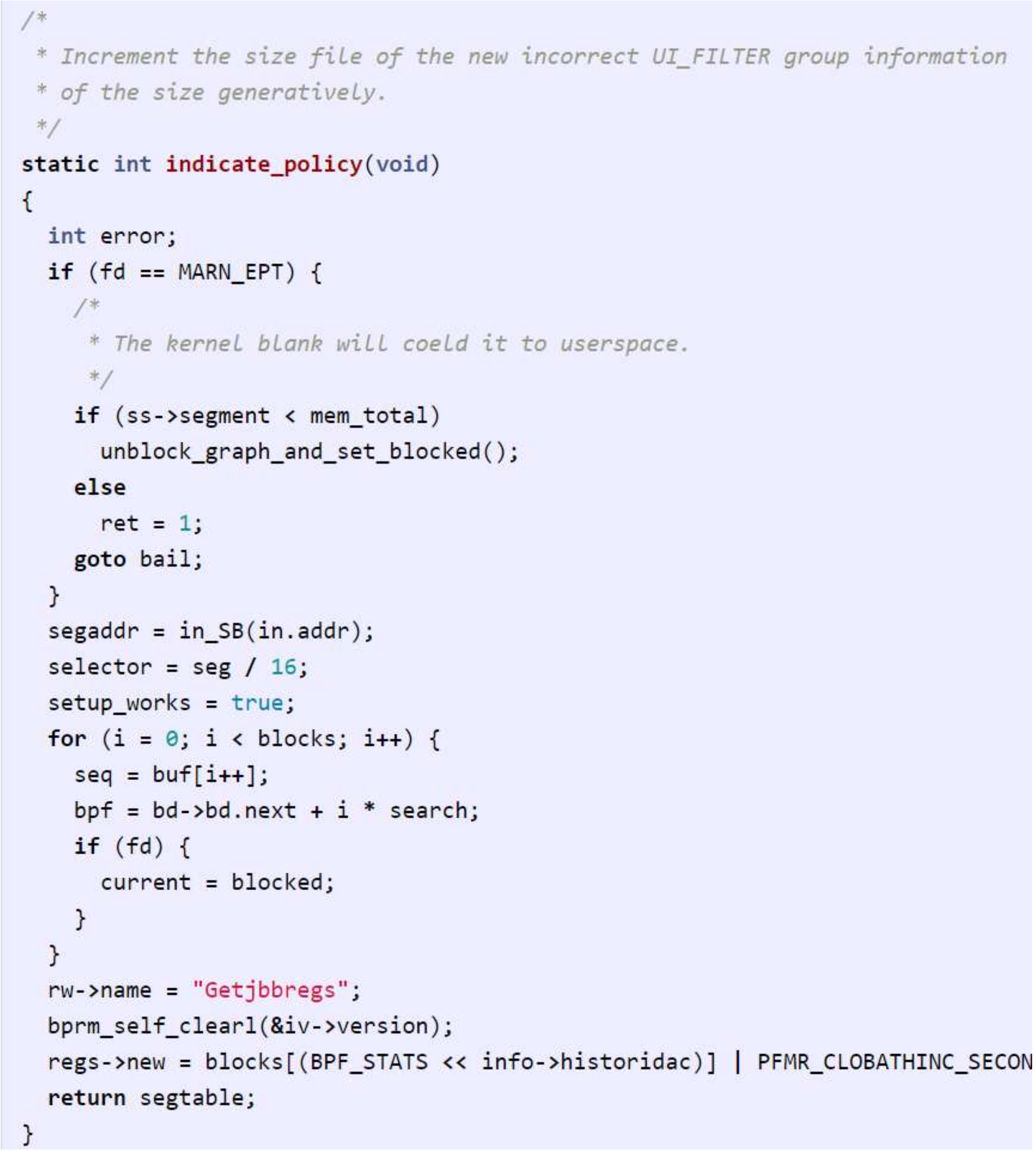
Deep Learning
Recurrent Networks : 1
Fall 2019

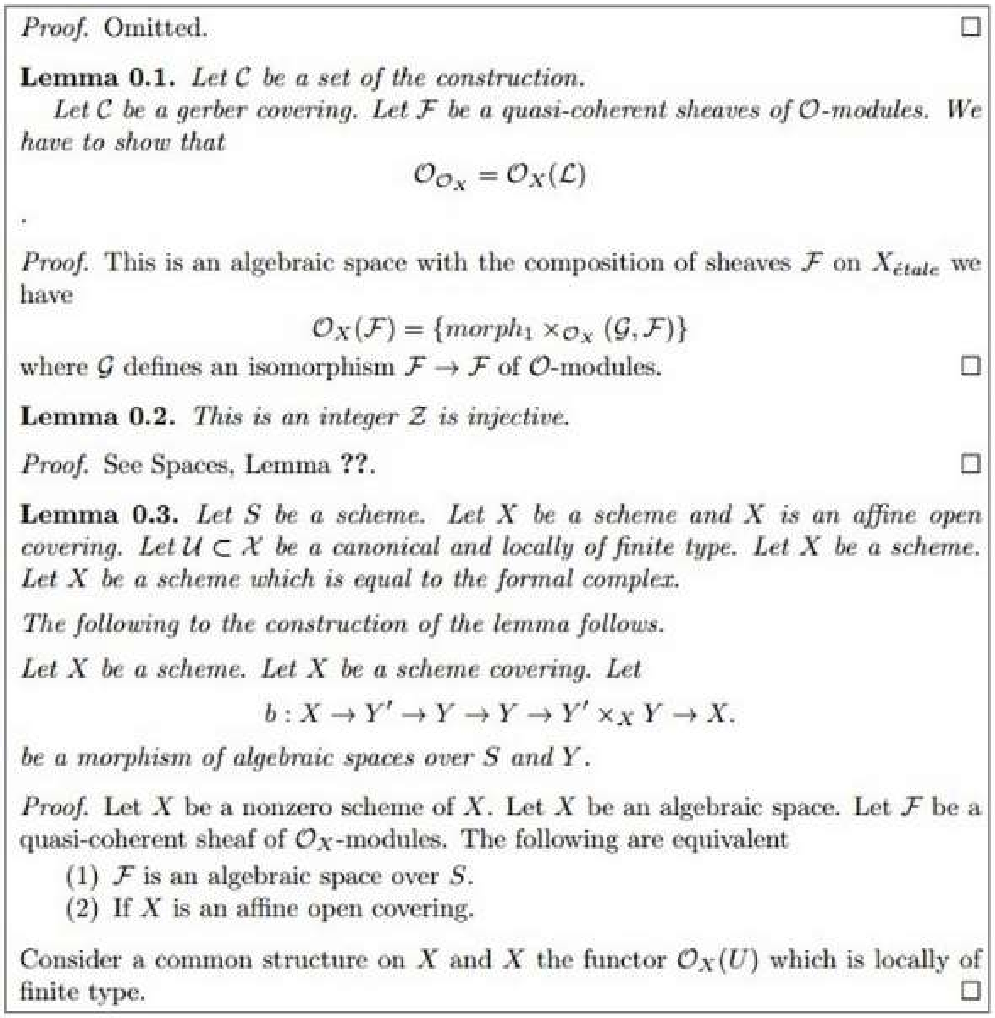
Instructor: Bhiksha Raj

Which open source project?



```
/*
 * Increment the size file of the new incorrect UI_FILTER group information
 * of the size generatively.
 */
static int indicate_policy(void)
{
  int error;
  if (fd == MARN_EPT) {
    /*
     * The kernel blank will coeld it to userspace.
     */
    if (ss->segment < mem_total)
      unblock_graph_and_set_blocked();
    else
      ret = 1;
    goto bail;
  }
  segaddr = in_SB(in.addr);
  selector = seg / 16;
  setup_works = true;
  for (i = 0; i < blocks; i++) {
    seq = buf[i++];
    bpf = bd->bd.next + i * search;
    if (fd) {
      current = blocked;
    }
  }
  rw->name = "Getjbbregs";
  bprm_self_clearl(&iv->version);
  regs->new = blocks[(BPF_STATS << info->historidac)] | PFMR_CLOBATHINC_SECON
  return segtable;
}
```

Related math. What is it talking about?



Proof. Omitted. □

Lemma 0.1. *Let $\mathcal{C}$ be a set of the construction.*

Let $\mathcal{C}$ be a gerber covering. Let $\mathcal{F}$ be a quasi-coherent sheaves of $\mathcal{O}$-modules. We have to show that

$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

.

Proof. This is an algebraic space with the composition of sheaves $\mathcal{F}$ on $X_{étale}$ we have

$$\mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$$

where $\mathcal{G}$ defines an isomorphism $\mathcal{F} \to \mathcal{F}$ of $\mathcal{O}$-modules. □

Lemma 0.2. *This is an integer $\mathcal{Z}$ is injective.*

Proof. See Spaces, Lemma ??. □

Lemma 0.3. *Let S be a scheme. Let X be a scheme and X is an affine open covering. Let $\mathcal{U} \subset \mathcal{X}$ be a canonical and locally of finite type. Let X be a scheme. Let X be a scheme which is equal to the formal complex.*

The following to the construction of the lemma follows.

Let X be a scheme. Let X be a scheme covering. Let

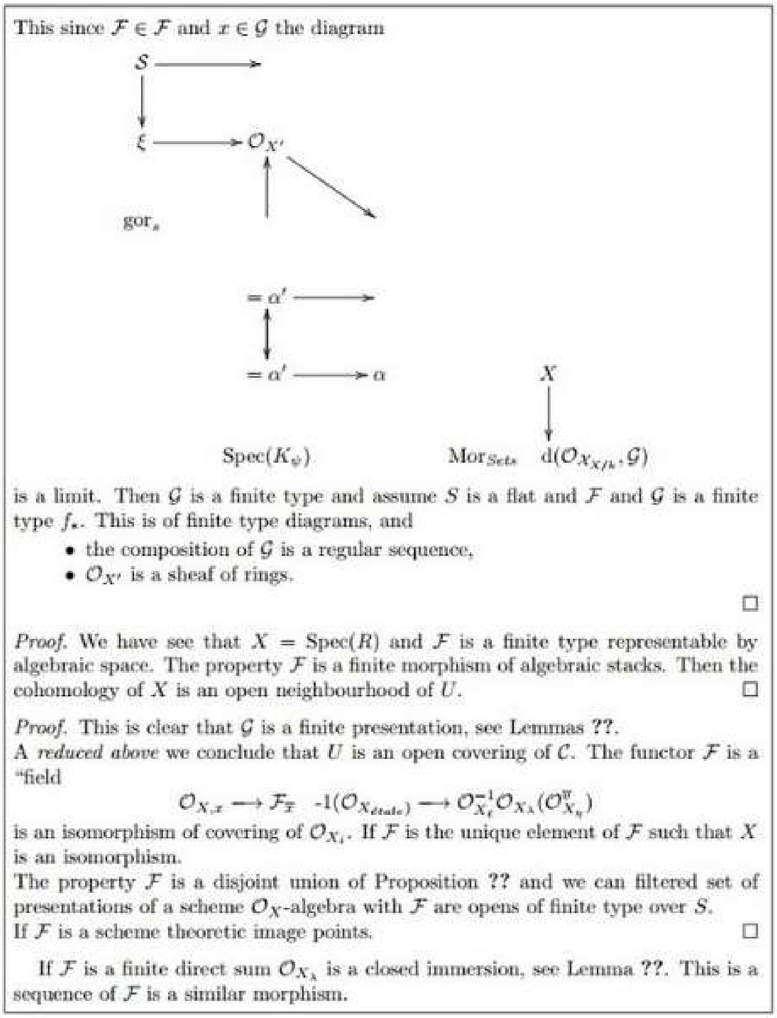
$$b : X \to Y' \to Y \to Y \to Y' \times_X Y \to X.$$

be a morphism of algebraic spaces over S and Y.

Proof. Let X be a nonzero scheme of X. Let X be an algebraic space. Let $\mathcal{F}$ be a quasi-coherent sheaf of $\mathcal{O}_X$-modules. The following are equivalent

(1) $\mathcal{F}$ is an algebraic space over S.
(2) If X is an affine open covering.

Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of finite type. □



This since $\mathcal{F} \in \mathcal{F}$ and $x \in \mathcal{G}$ the diagram

$$\begin{tikzcd}
\mathcal{S} \arrow[r] \arrow[d] & {} & & \\
\xi \arrow[r] & \mathcal{O}_{X'} \arrow[dr] & & \\
\mathrm{gor}_s & {} \arrow[u] & {} & \\
& =\alpha' \arrow[r] \arrow[d, leftrightarrow] & {} & \\
& =\alpha' \arrow[r] & \alpha & X \arrow[d] \\
& \mathrm{Spec}(K_\psi) & \mathrm{Mor}_{Sets} & \mathrm{d}(\mathcal{O}_{X_{X/k}}, \mathcal{G})
\end{tikzcd}$$

is a limit. Then $\mathcal{G}$ is a finite type and assume S is a flat and $\mathcal{F}$ and $\mathcal{G}$ is a finite type f_*. This is of finite type diagrams, and

- the composition of $\mathcal{G}$ is a regular sequence,
- $\mathcal{O}_{X'}$ is a sheaf of rings.

□

Proof. We have see that $X = \mathrm{Spec}(R)$ and $\mathcal{F}$ is a finite type representable by algebraic space. The property $\mathcal{F}$ is a finite morphism of algebraic stacks. Then the cohomology of X is an open neighbourhood of U. □

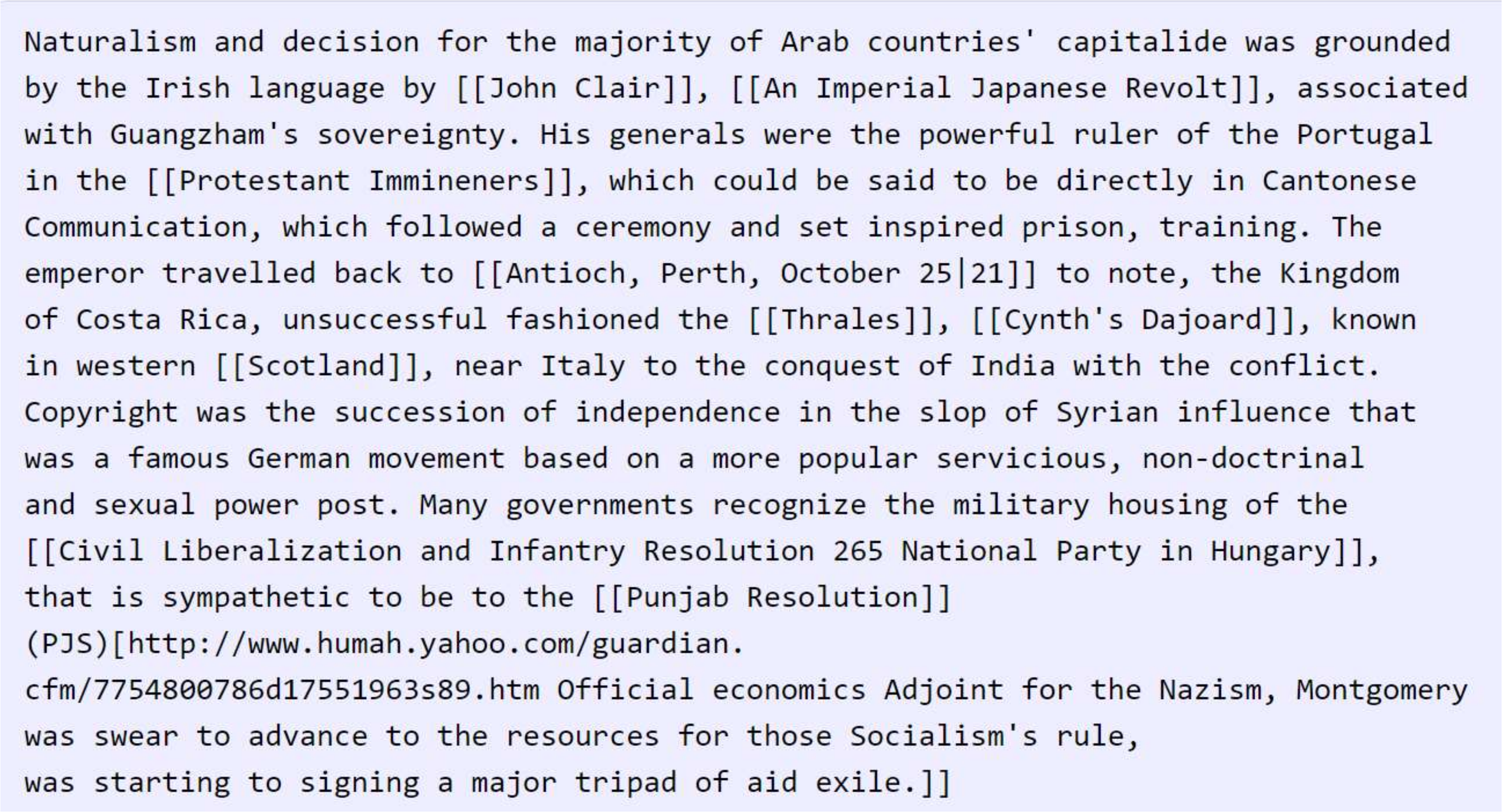
Proof. This is clear that $\mathcal{G}$ is a finite presentation, see Lemmas ??.
A *reduced above* we conclude that U is an open covering of $\mathcal{C}$. The functor $\mathcal{F}$ is a "field

$$\mathcal{O}_{X,x} \longrightarrow \mathcal{F}_{\overline{x}} \quad \text{-1}(\mathcal{O}_{X_{étale}}) \longrightarrow \mathcal{O}_{X_\ell}^{-1}\mathcal{O}_{X_\lambda}(\mathcal{O}_{X_\eta}^{\overline{v}})$$

is an isomorphism of covering of $\mathcal{O}_{X_i}$. If $\mathcal{F}$ is the unique element of $\mathcal{F}$ such that X is an isomorphism.
The property $\mathcal{F}$ is a disjoint union of Proposition ?? and we can filtered set of presentations of a scheme $\mathcal{O}_X$-algebra with $\mathcal{F}$ are opens of finite type over S.
If $\mathcal{F}$ is a scheme theoretic image points. □

If $\mathcal{F}$ is a finite direct sum $\mathcal{O}_{X_\lambda}$ is a closed immersion, see Lemma ??. This is a sequence of $\mathcal{F}$ is a similar morphism.

And a Wikipedia page explaining it all



```
Naturalism and decision for the majority of Arab countries' capitalide was grounded
by the Irish language by [[John Clair]], [[An Imperial Japanese Revolt]], associated
with Guangzham's sovereignty. His generals were the powerful ruler of the Portugal
in the [[Protestant Immineners]], which could be said to be directly in Cantonese
Communication, which followed a ceremony and set inspired prison, training. The
emperor travelled back to [[Antioch, Perth, October 25|21]] to note, the Kingdom
of Costa Rica, unsuccessful fashioned the [[Thrales]], [[Cynth's Dajoard]], known
in western [[Scotland]], near Italy to the conquest of India with the conflict.
Copyright was the succession of independence in the slop of Syrian influence that
was a famous German movement based on a more popular servicious, non-doctrinal
and sexual power post. Many governments recognize the military housing of the
[[Civil Liberalization and Infantry Resolution 265 National Party in Hungary]],
that is sympathetic to be to the [[Punjab Resolution]]
(PJS)[http://www.humah.yahoo.com/guardian.
cfm/7754800786d17551963s89.htm Official economics Adjoint for the Nazism, Montgomery
was swear to advance to the resources for those Socialism's rule,
was starting to signing a major tripad of aid exile.]]
```

The unreasonable effectiveness of recurrent neural networks..

- All previous examples were *generated* blindly by a *recurrent* neural network..
  - With simple architectures

- http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Modern text generation is a lot more sophisticated that that

- One of the many sages of the time, the Bodhisattva Bodhisattva Sakyamuni (1575-1611) was a popular religious figure in India and around the world. This Bodhisattva Buddha was said to have passed his life peacefully and joyfully, without passion and anger. For over twenty years he lived as a lay man and dedicated himself toward the welfare, prosperity, and welfare of others. Among the many spiritual and philosophical teachings he wrote, three are most important; the first, titled the "Three Treatises of Avalokiteśvara"; the second, the teachings of the "Ten Questions;" and the third, "The Eightfold Path of Discipline.“
  - Entirely randomly generated

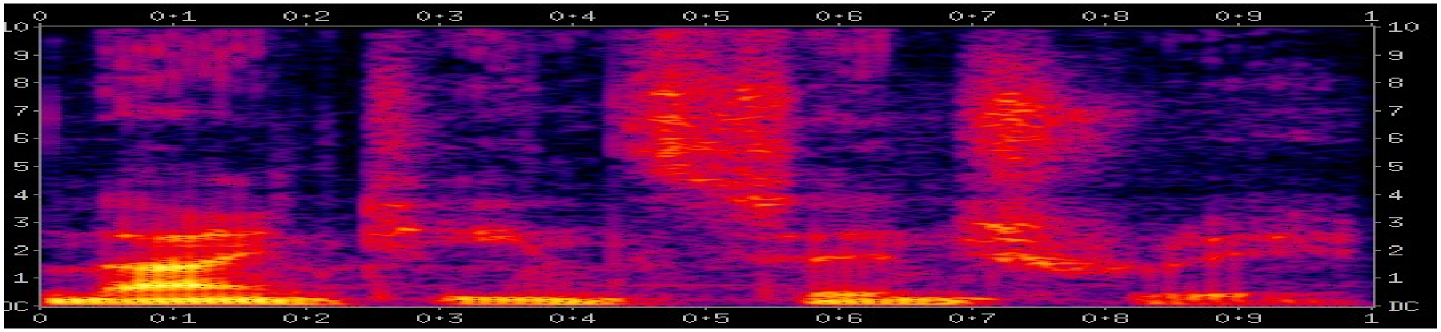
Modelling Series

- In many situations one must consider a *series* of inputs to produce an output
  - Outputs too may be a series

- Examples: ..

What did I say?

“To be” or not “to be”??



- Speech Recognition
  - Analyze a series of spectral vectors, determine what was said
- Note: Inputs are vectors. Output is a classification result

What is he talking about?

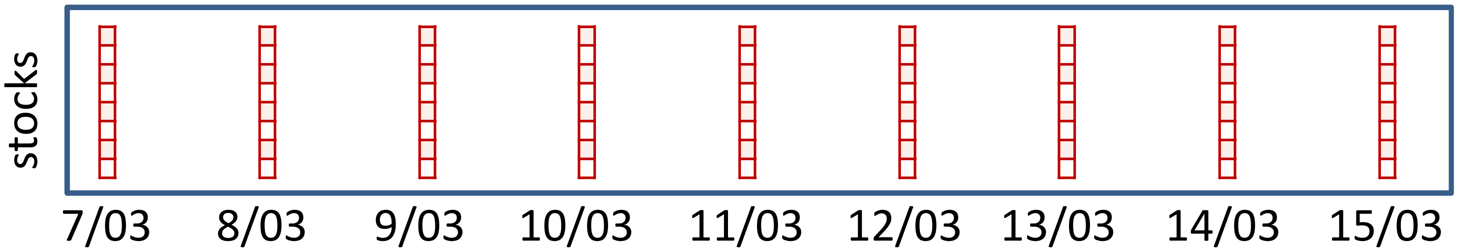
"Football" or "basketball"?

The Steelers, meanwhile, continue to struggle to make stops on defense. They've allowed, on average, 30 points a game, and have shown no signs of improving anytime soon.

- Text analysis
  - E.g. analyze document, identify topic
    - Input series of words, output classification output
  - E.g. read English, output French
    - Input series of words, output series of words

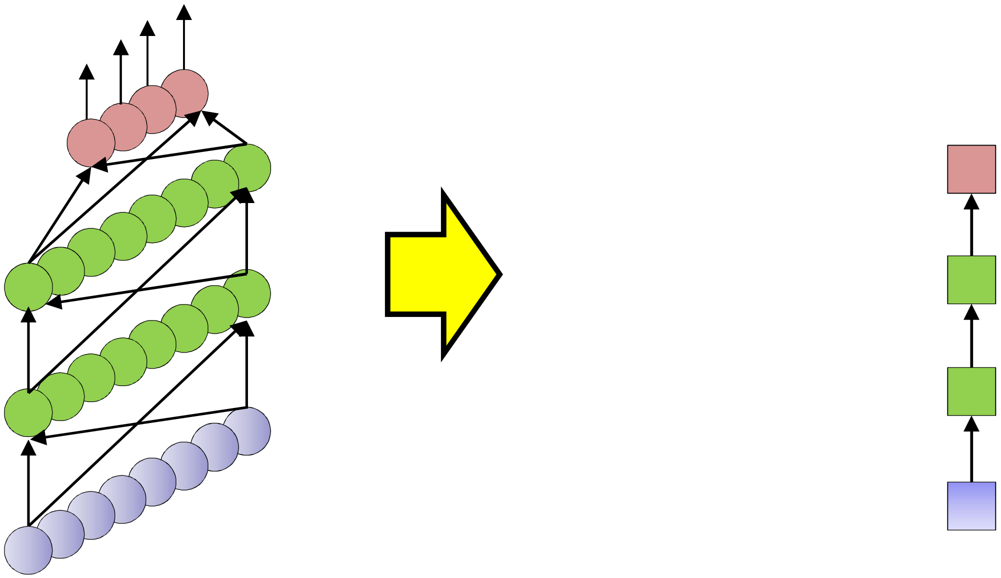
Should I invest..

To invest or not to invest?



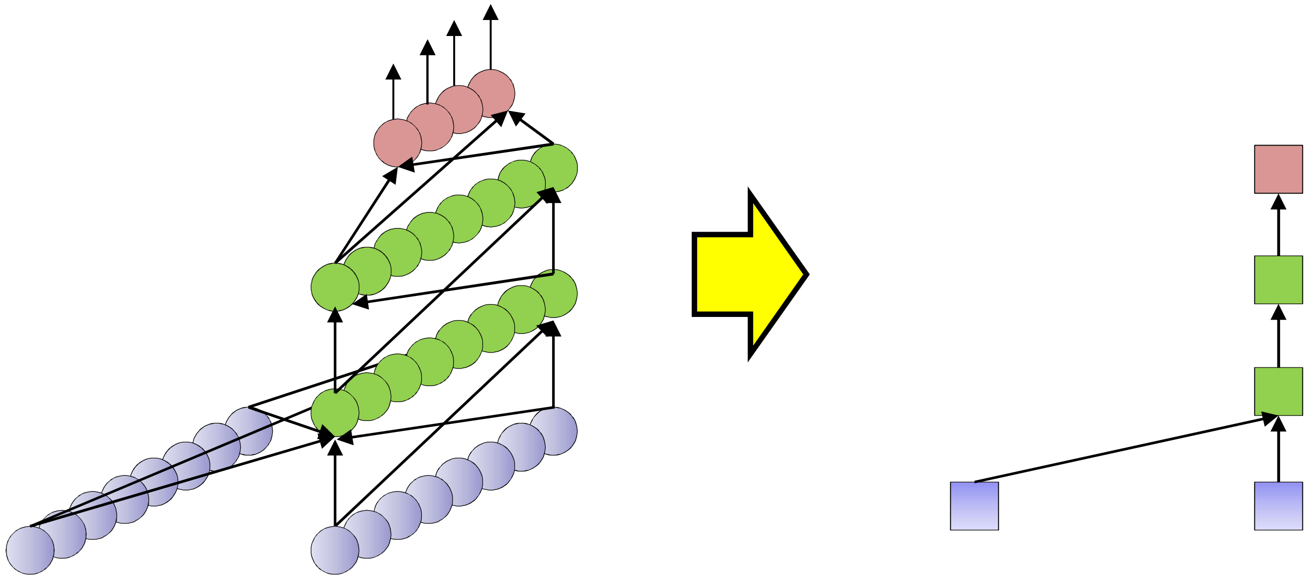
- Stock market
  - Must consider the series of stock values in the past several days to decide if it is wise to invest today
    - Ideally consider *all* of history
- Note: Inputs are vectors. Output may be scalar or vector
  - Should I invest, vs. should I invest in X

Representational shortcut



- Input at each time is a *vector*
- Each layer has many neurons
  - Output layer too may have many neurons
- But will represent everything by simple boxes
  - Each box actually represents an entire *layer with many units*

Representational shortcut



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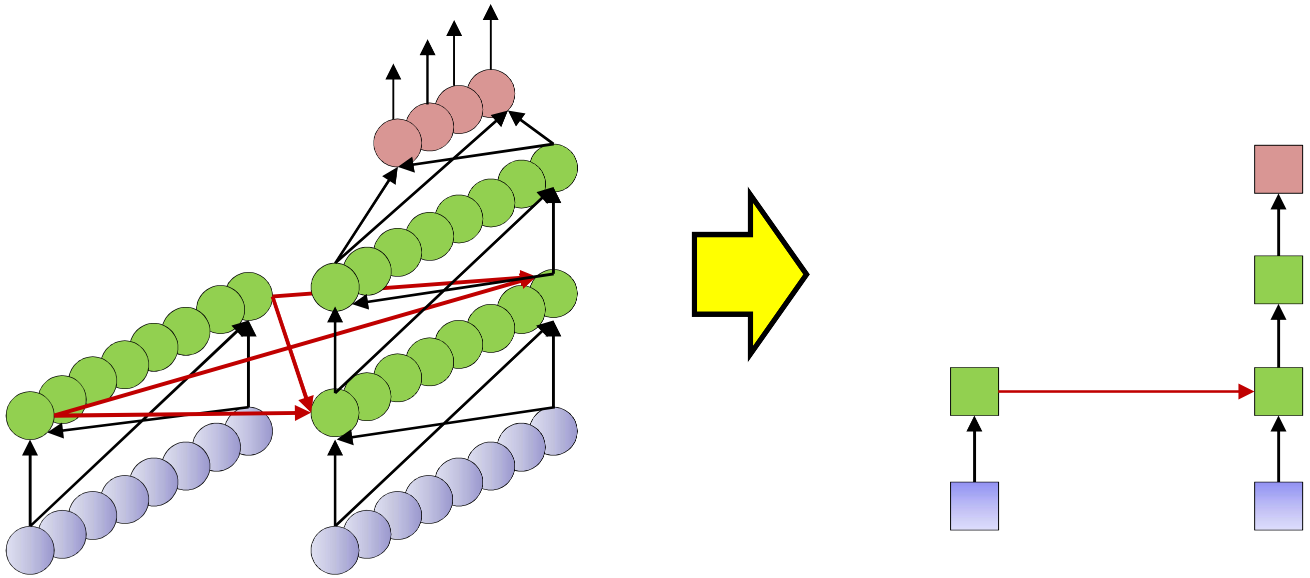

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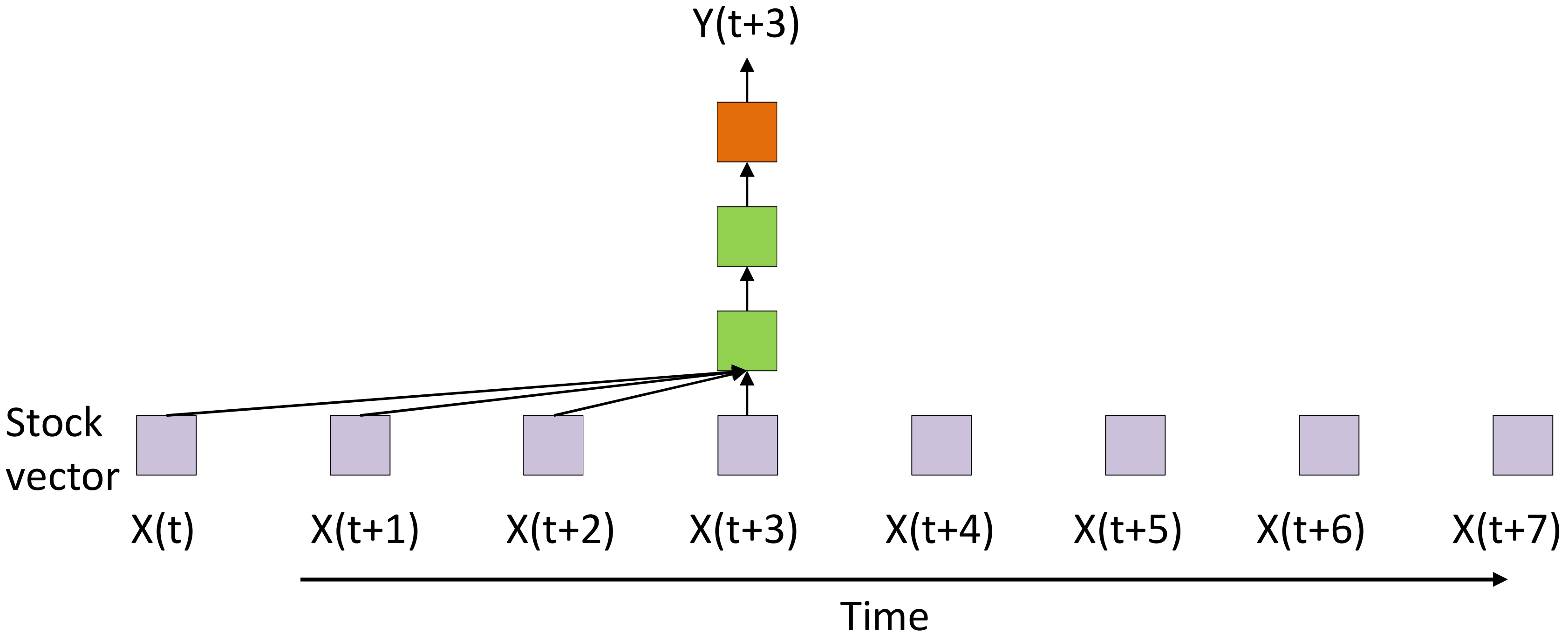
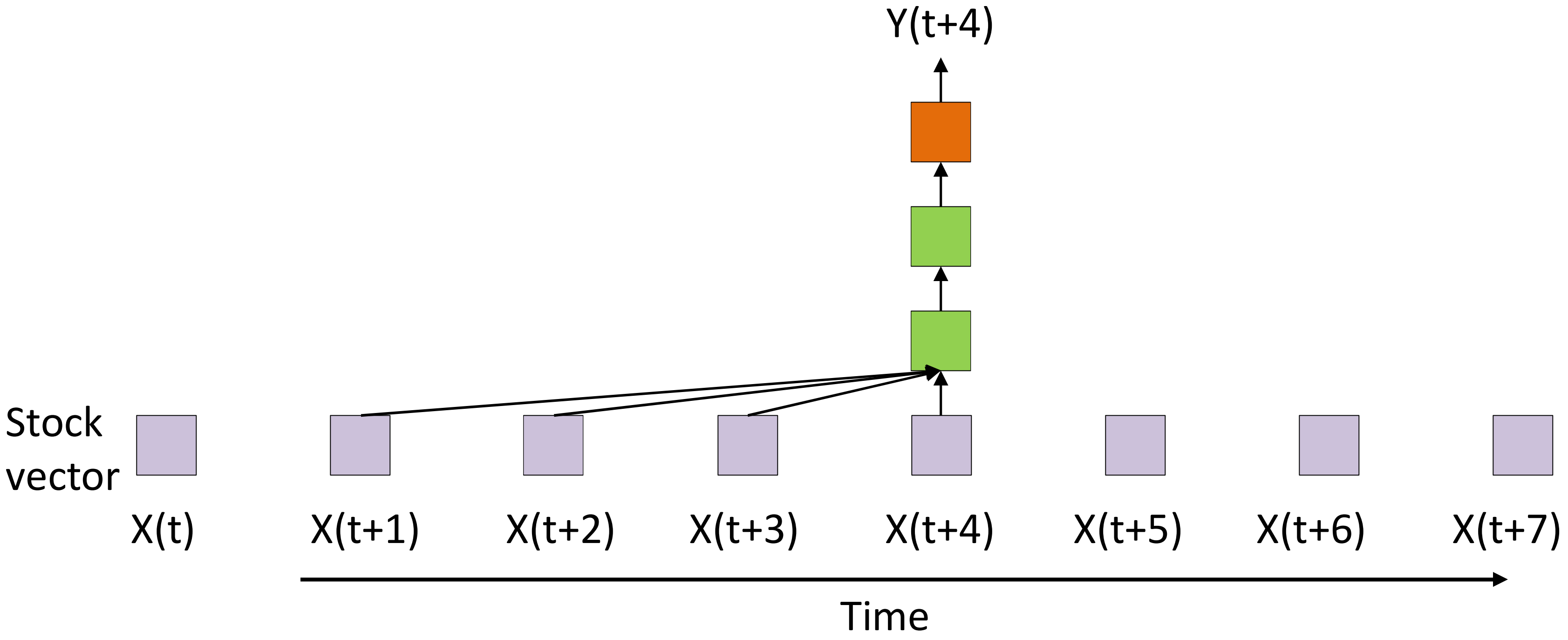
The stock predictor



- The sliding predictor
  - Look at the last few days
  - This is just a convolutional neural net applied to series data
    - Also called a *Time-Delay neural network*

The stock predictor



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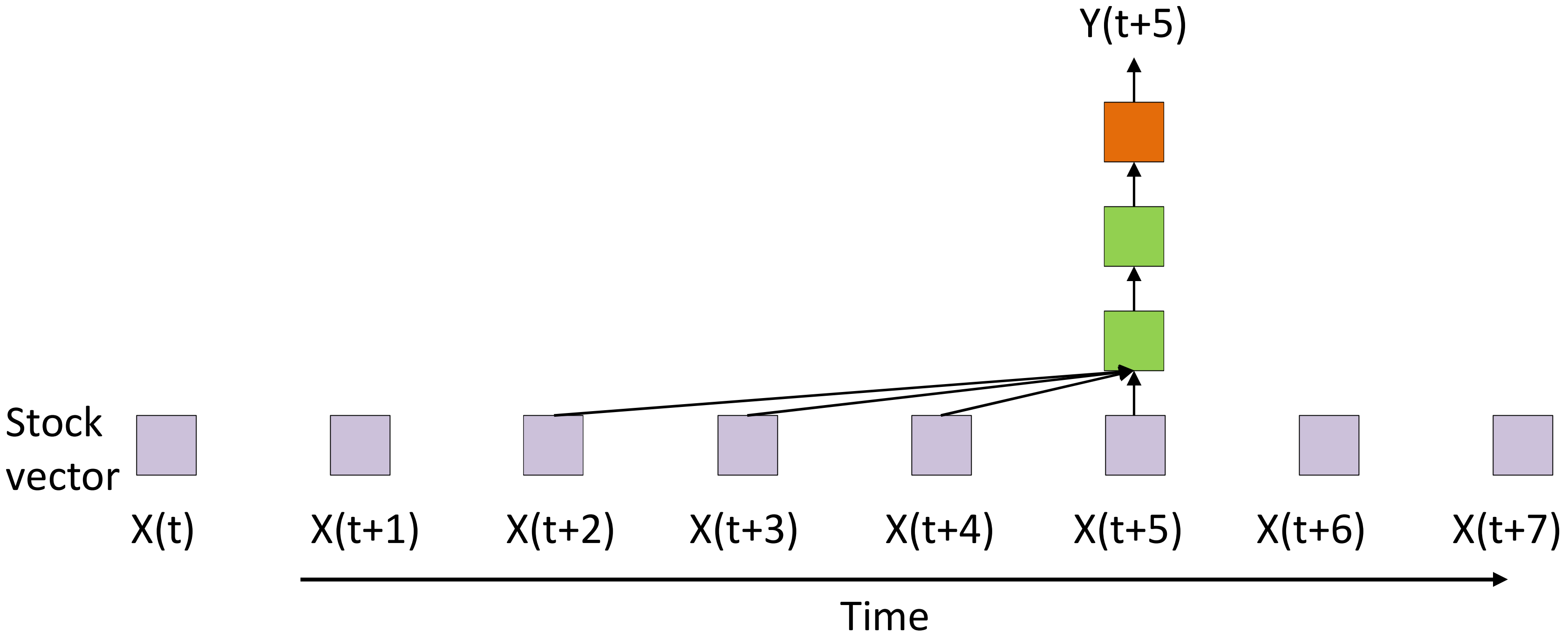

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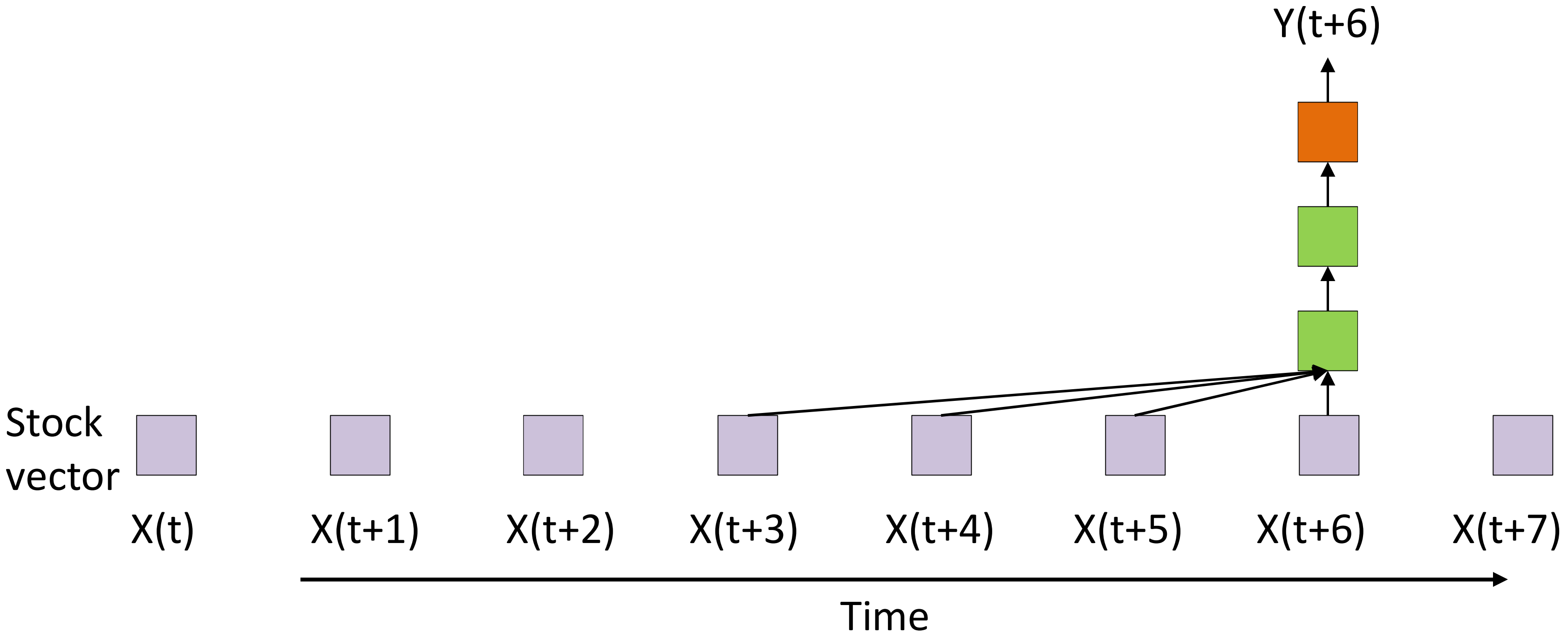

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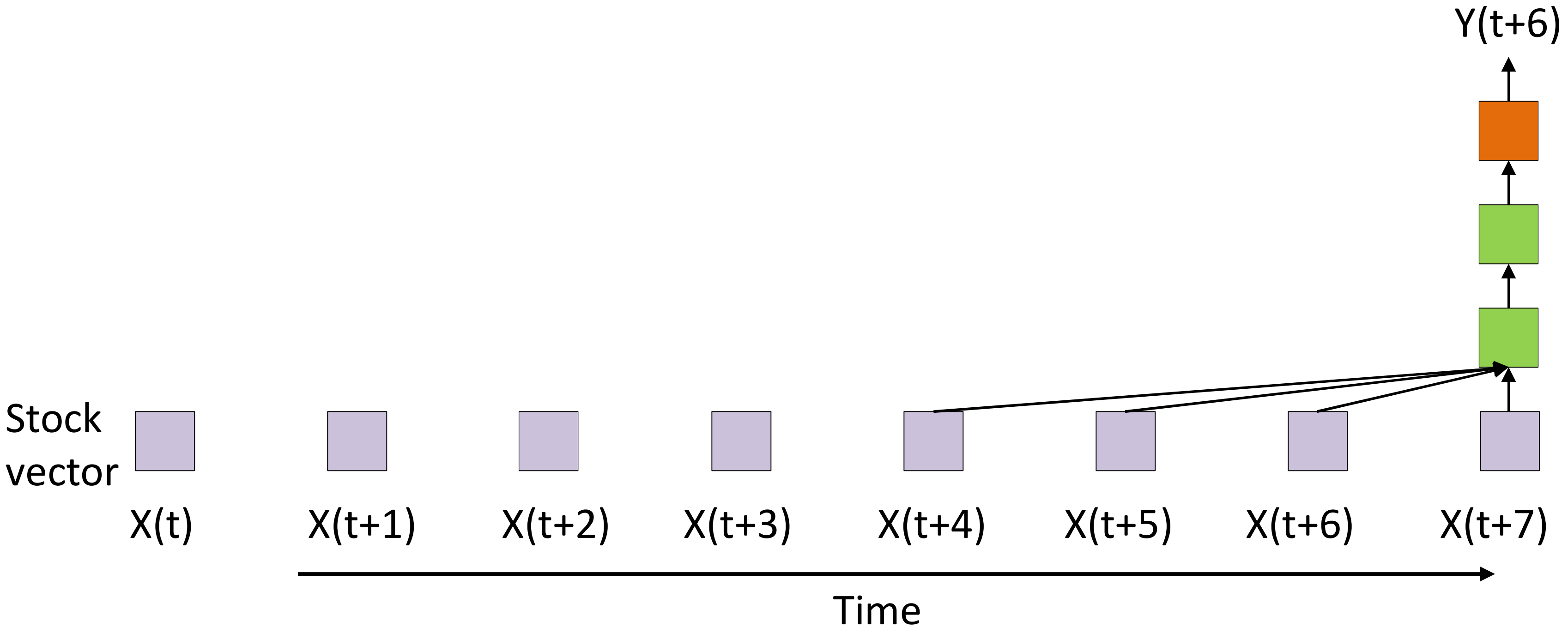

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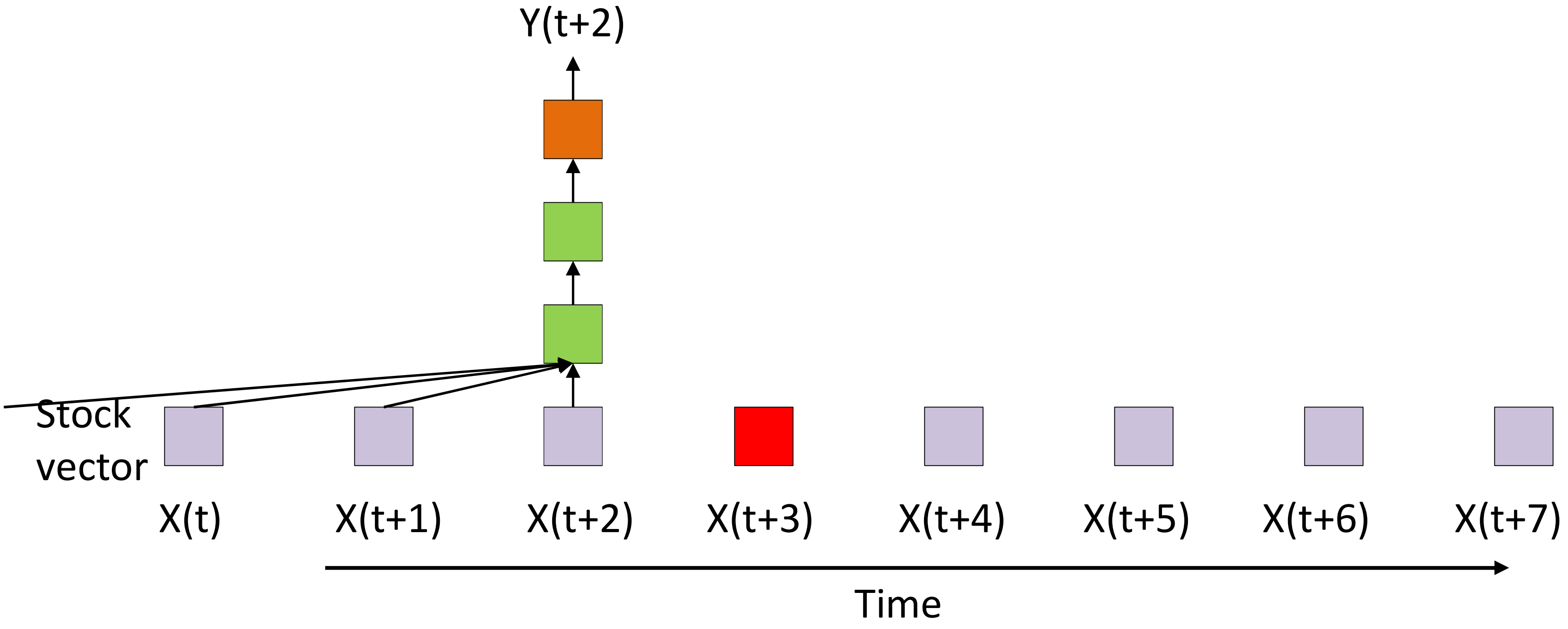
Finite-response model

- This is a *finite response* system
  - Something that happens *today* only affects the output of the system for N days into the future
    - N is the *width* of the system

$$Y_t = f(X_t, X_{t-1}, \dots, X_{t-N})$$

The stock predictor



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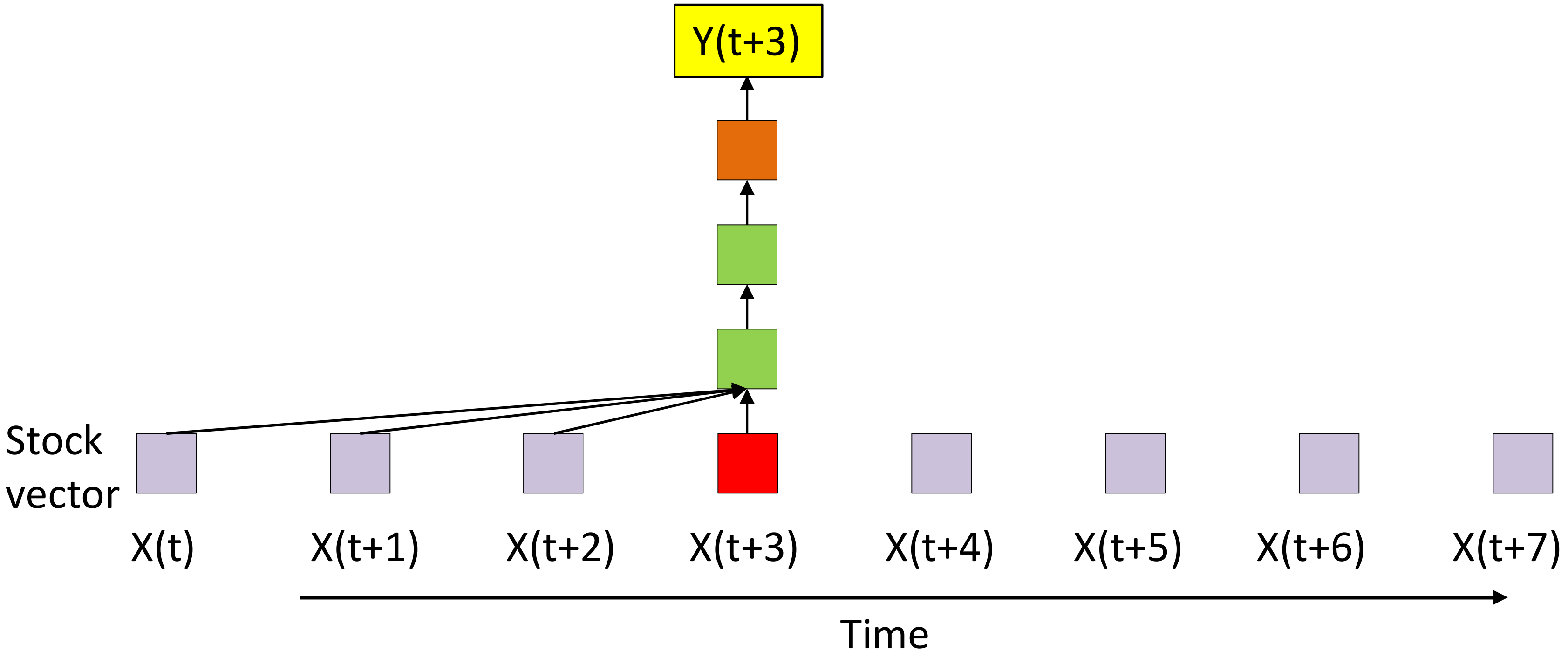

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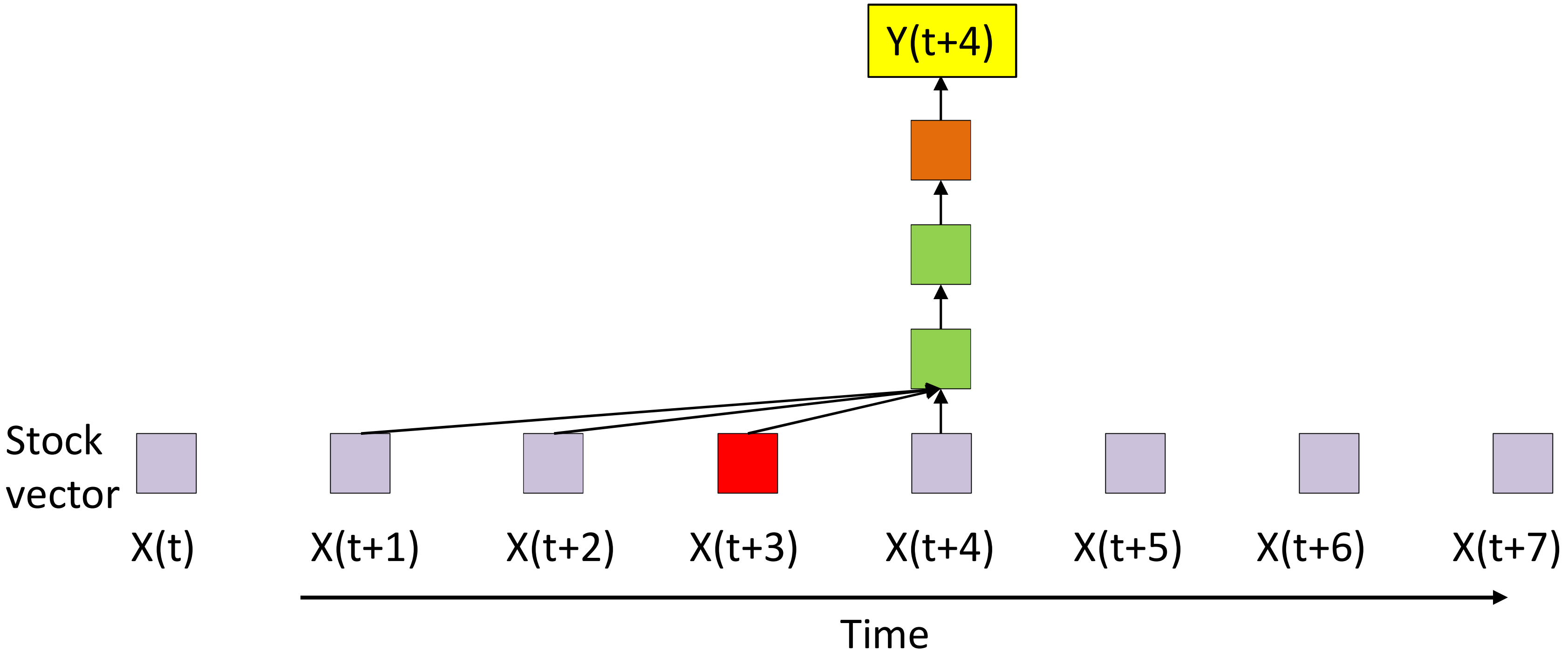

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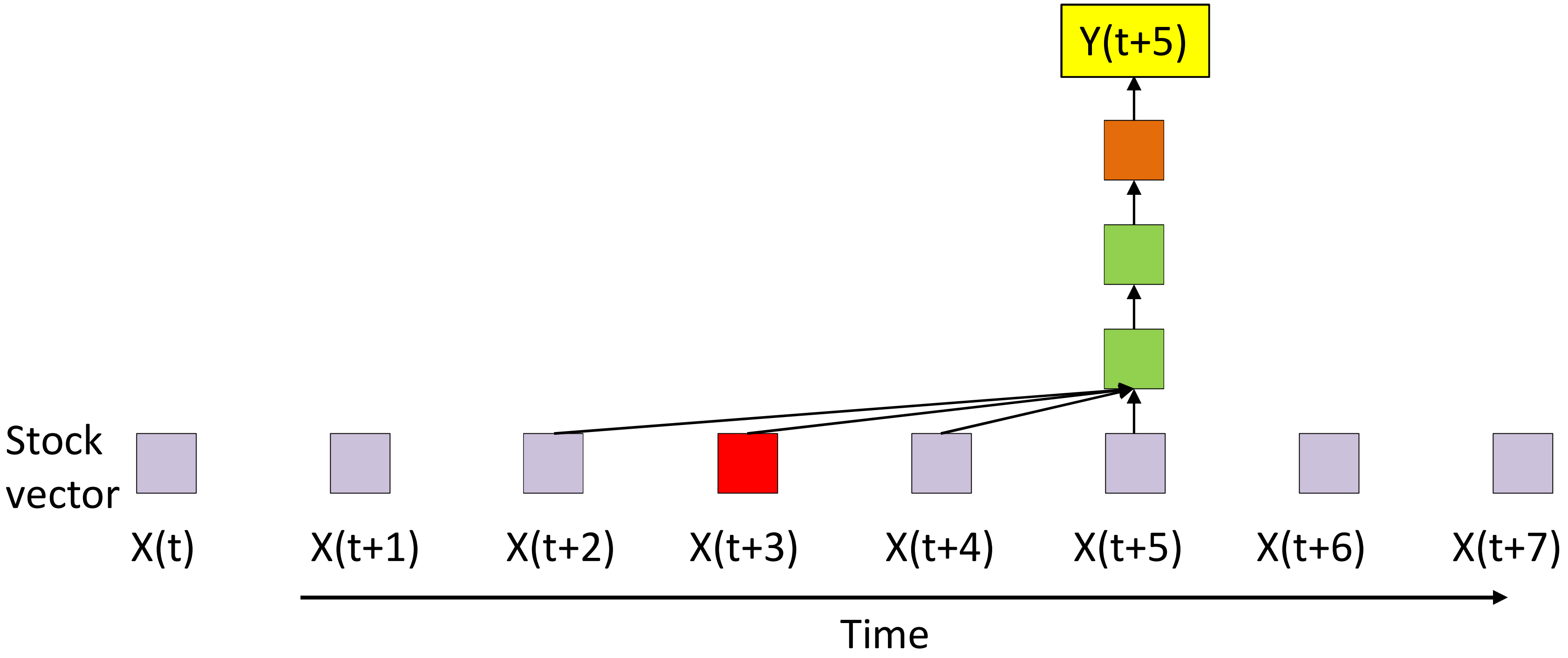

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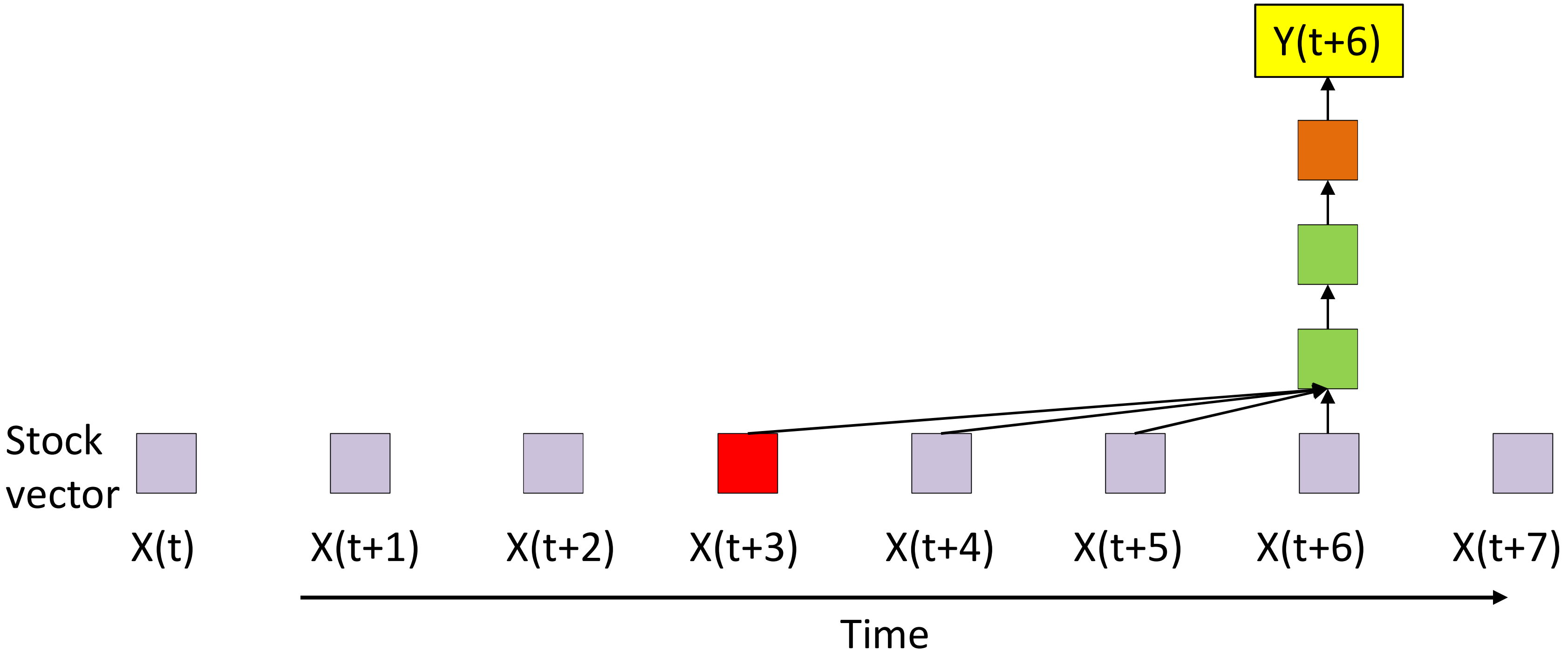

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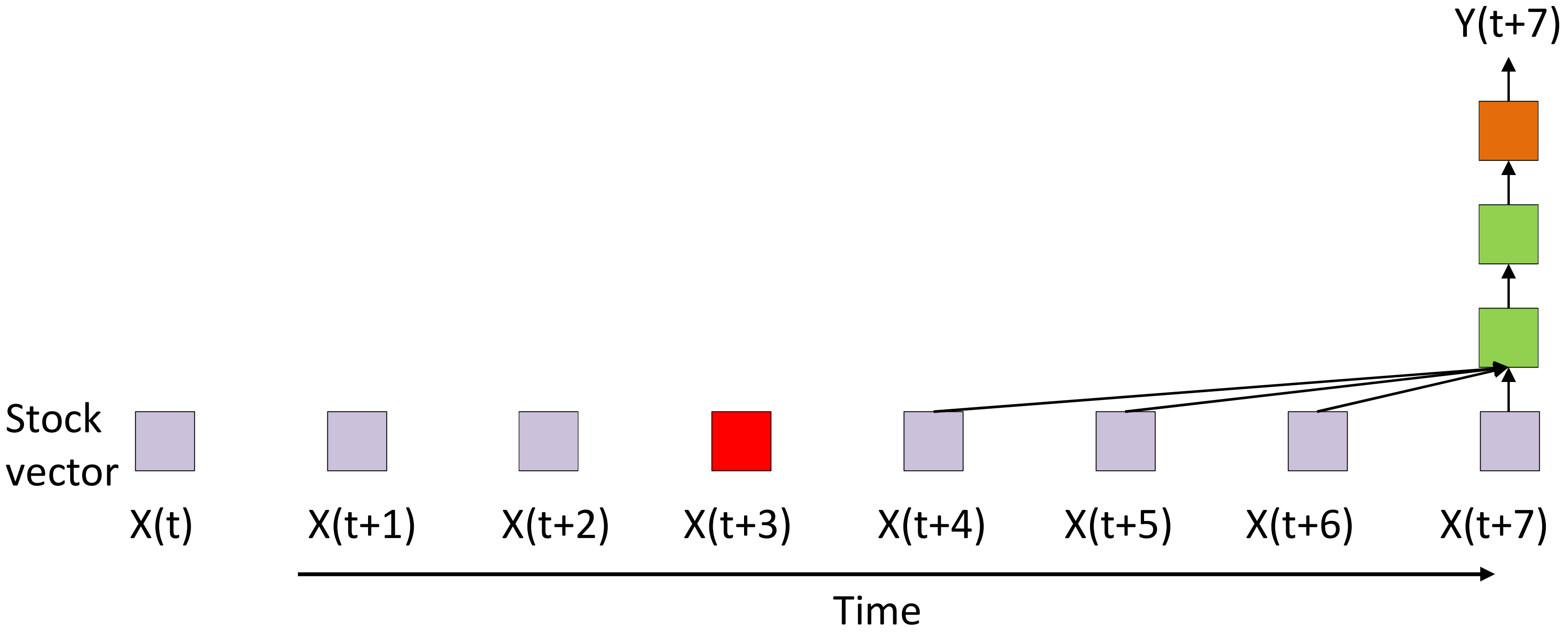

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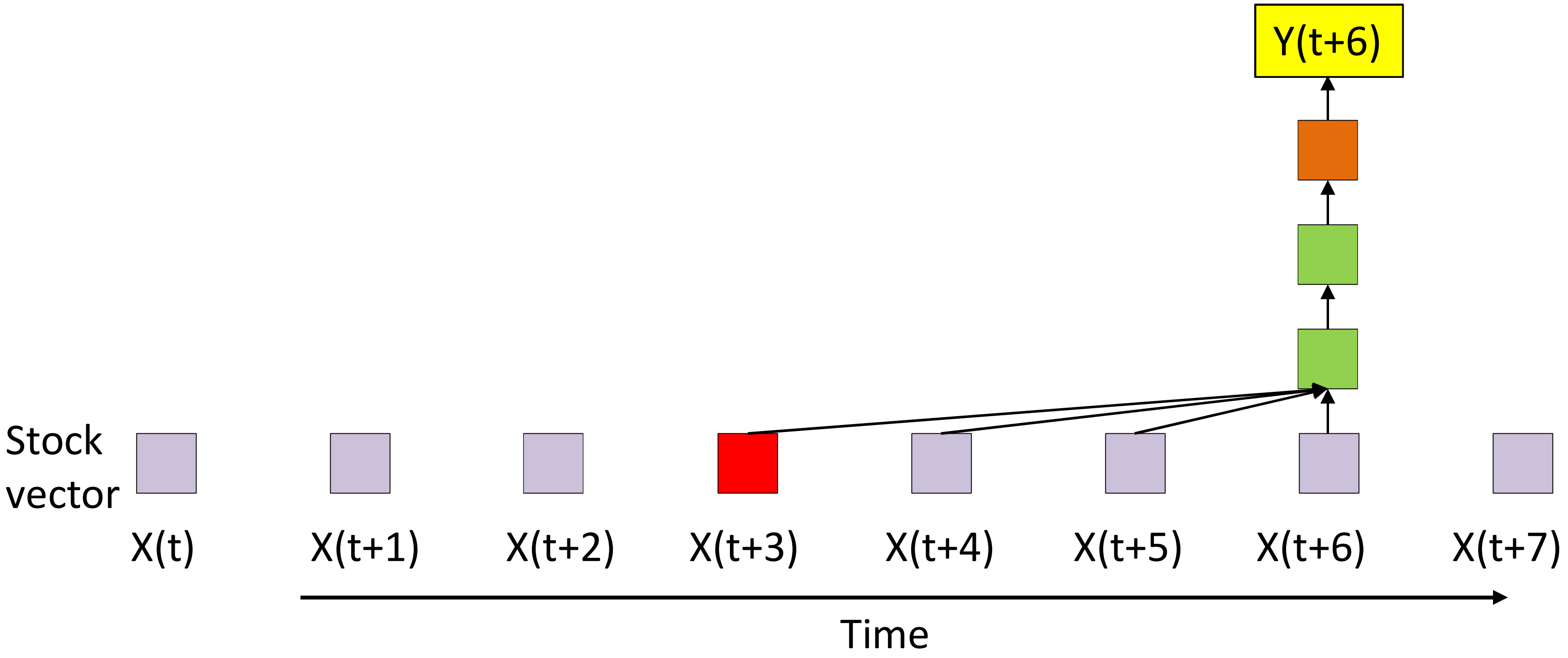
Finite-response model



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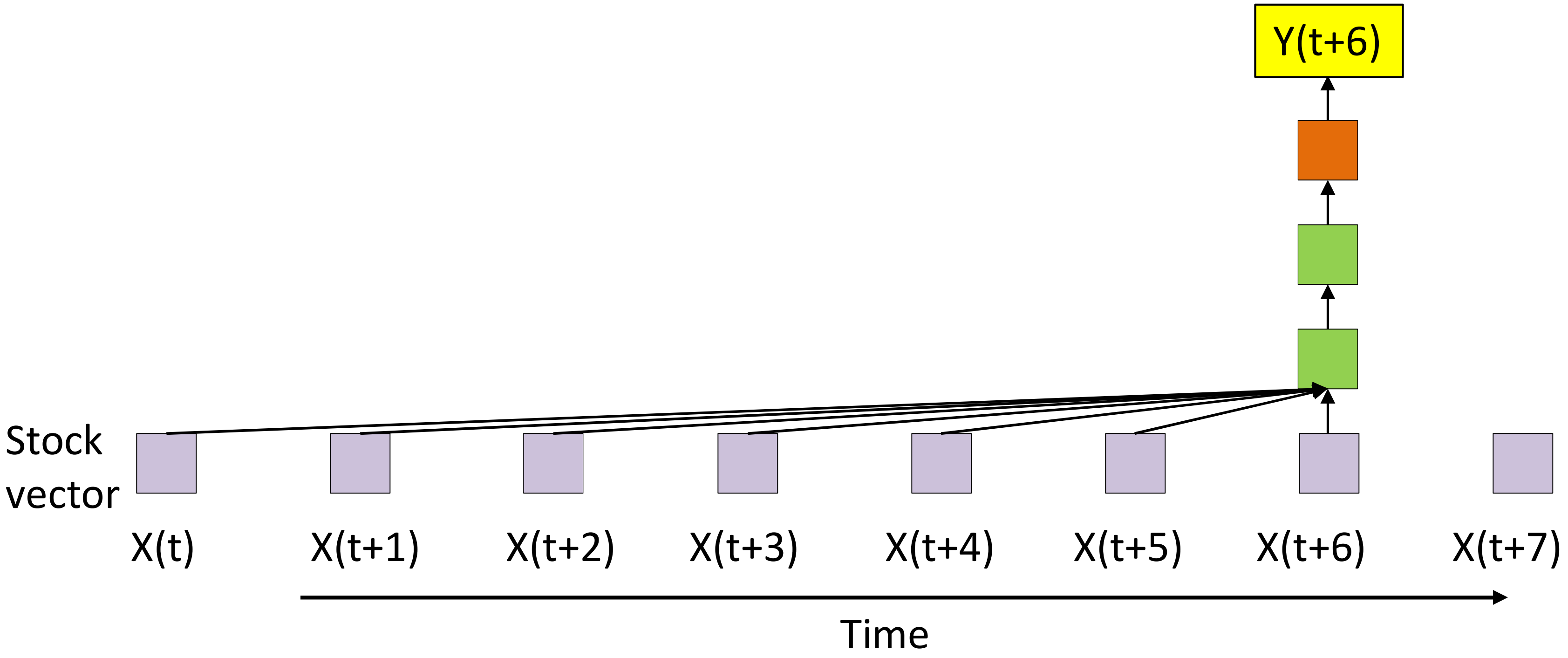
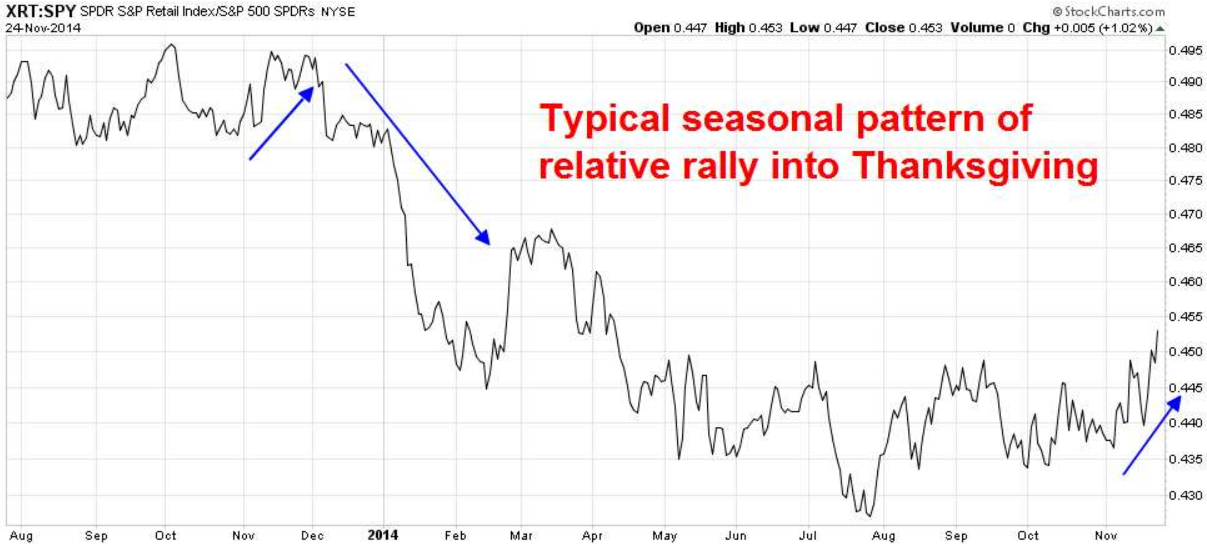
Finite-response



- Problem: Increasing the “history” makes the network more complex
  - No worries, we have the CPU and memory
    - Or do we?

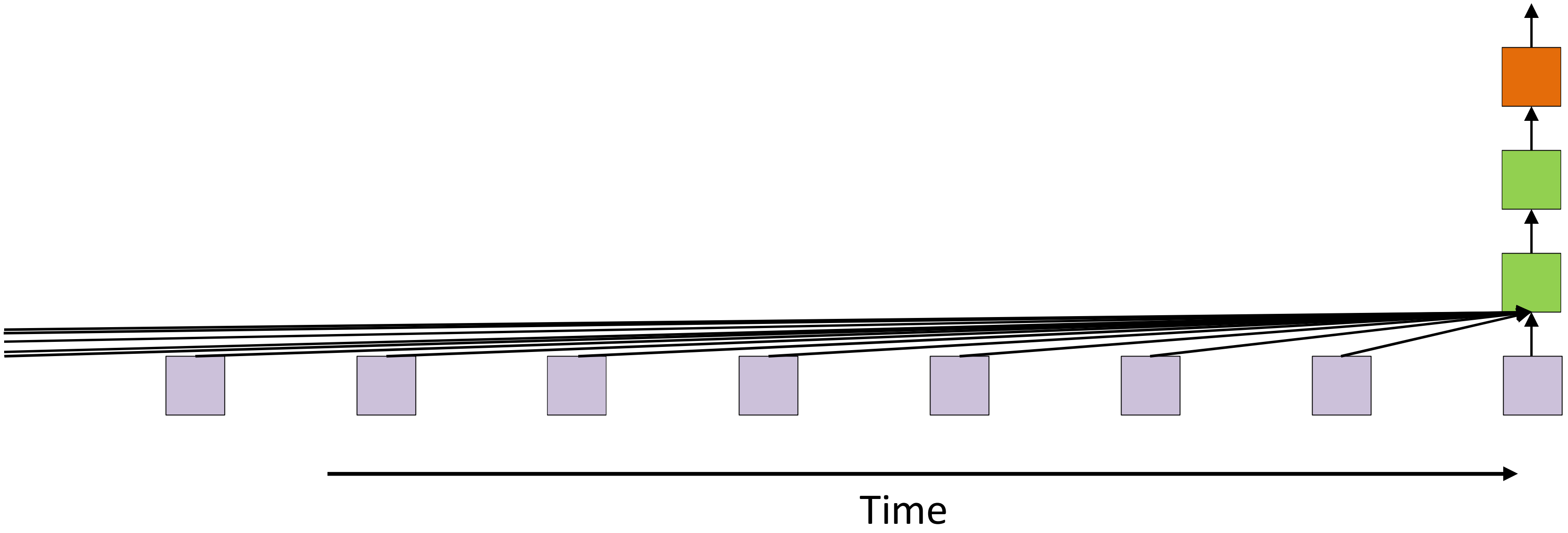
Systems often have long-term dependencies



Typical seasonal pattern of relative rally into Thanksgiving

- Longer-term trends –
  - Weekly trends in the market
  - Monthly trends in the market
  - Annual trends
  - Though longer historic tends to affect us less than more recent events..

We want *infinite* memory



- Required: *Infinite* response systems
  - What happens today can continue to affect the output forever
    - Possibly with weaker and weaker influence

$$Y_t = f(X_t, X_{t-1}, \ldots, X_{t-\infty})$$

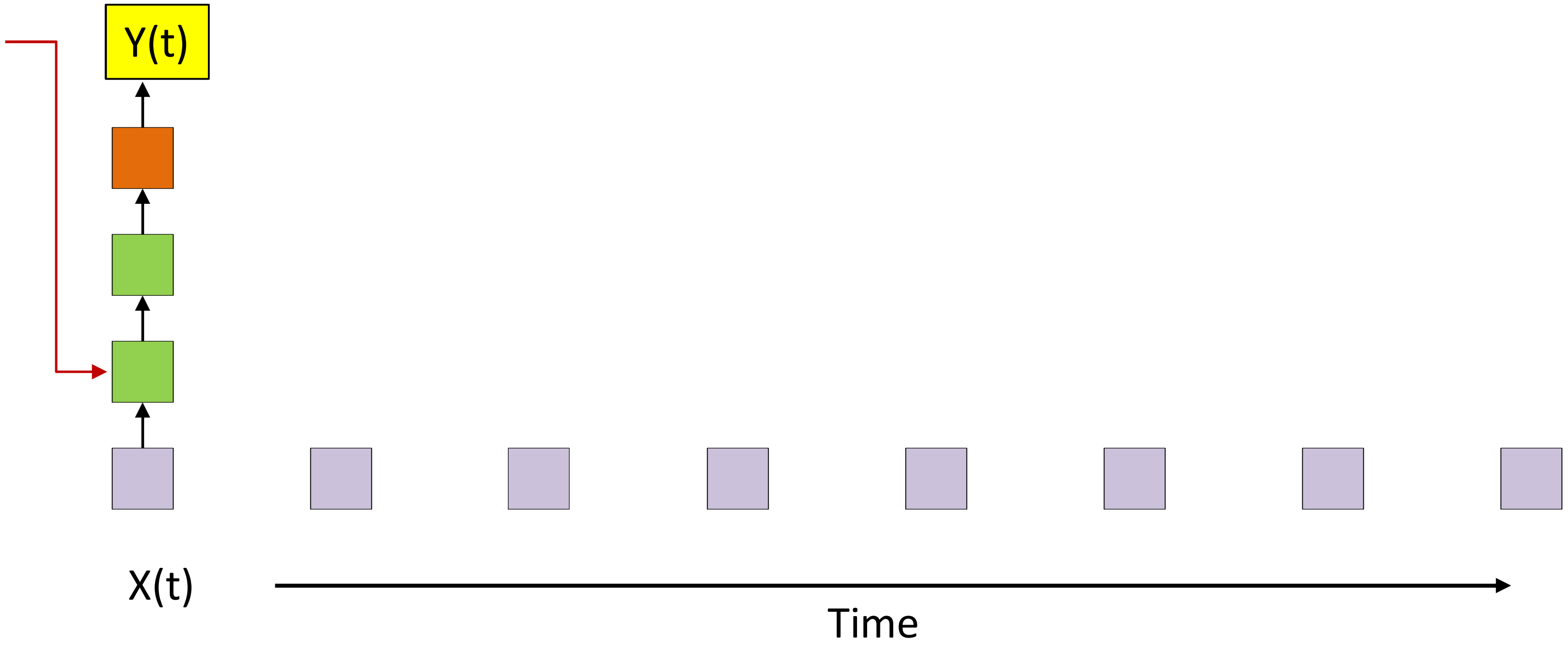
Examples of infinite response systems

$$Y_t = f(X_t, Y_{t-1})$$

- Required: Define initial state: Y_{-1} for $t = 0$
- An input at X_0 at $t = 0$ produces Y_0
- Y_0 produces Y_1 which produces Y_2 and so on until Y_∞ *even if* $X_1 \dots X_\infty$ *are 0*
  - i.e. even if there are no further inputs!

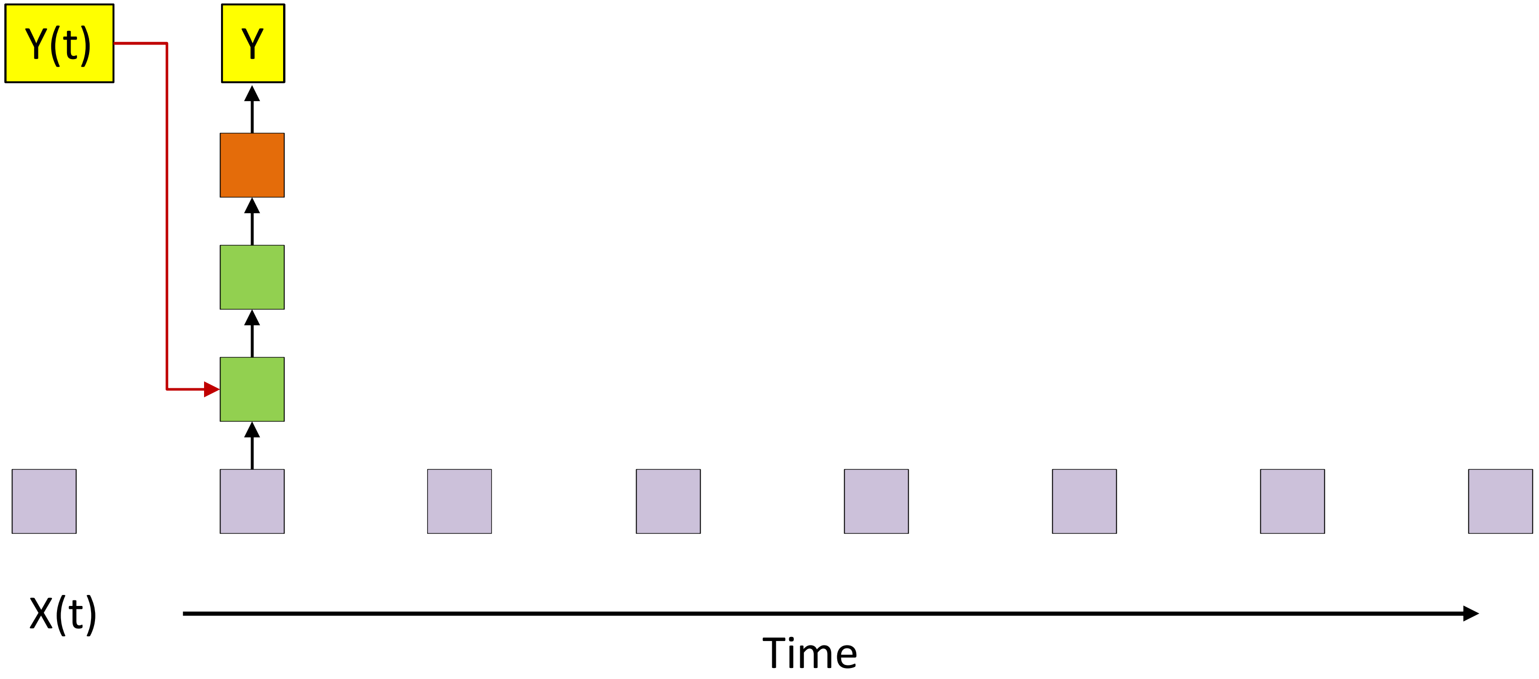
- This is an instance of a NARX network
  - "nonlinear autoregressive network with exogenous inputs"
  - $Y_t = f(X_{0:t}, Y_{0:t-1})$
- *Output* contains information about the entire past

A one-tap NARX network



- A NARX net with recursion from the output

A one-tap NARX network



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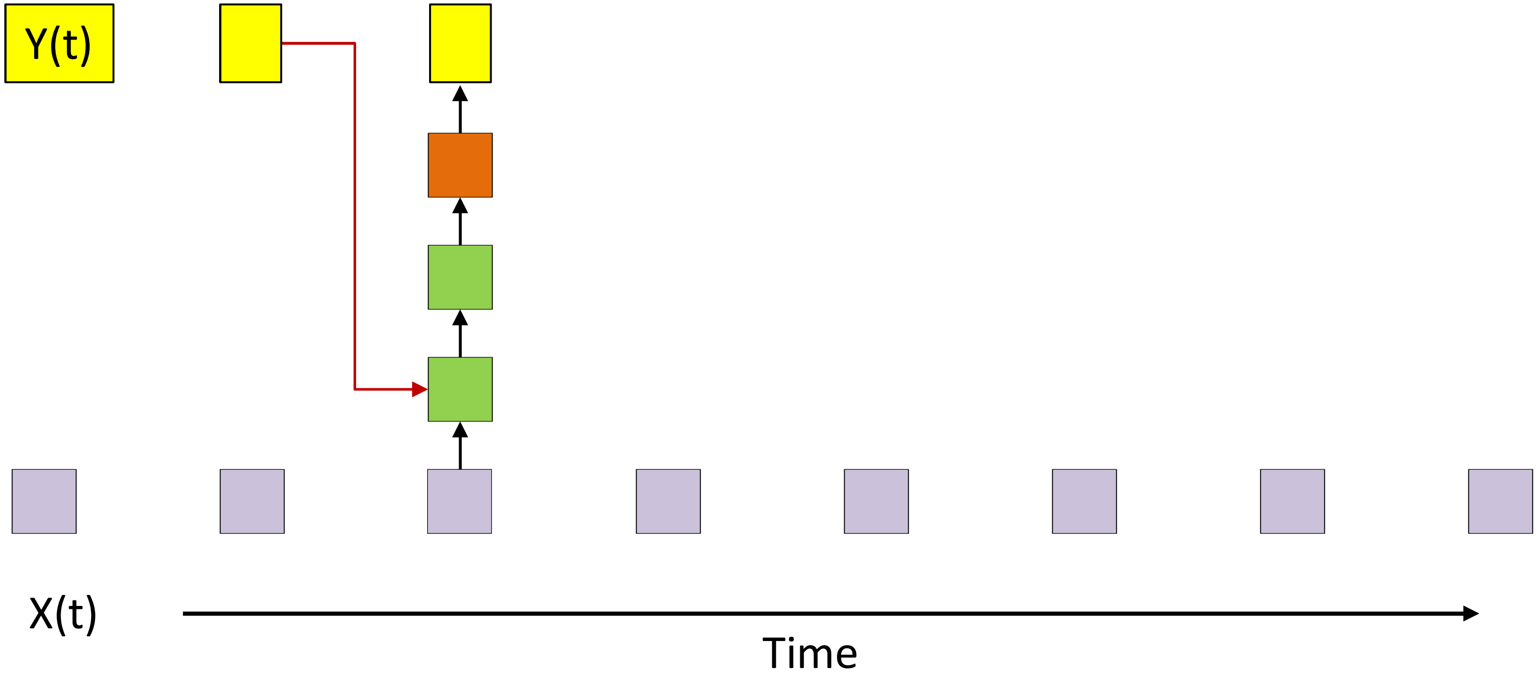

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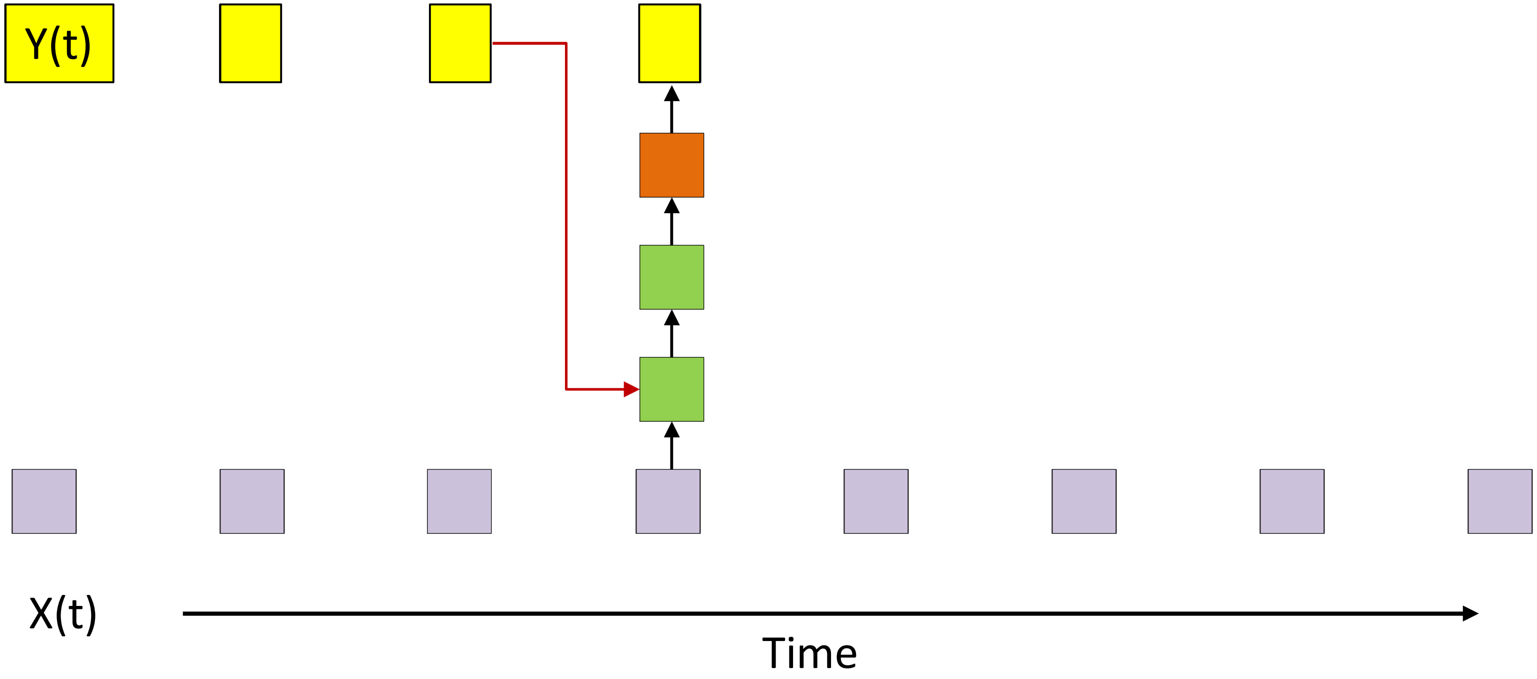

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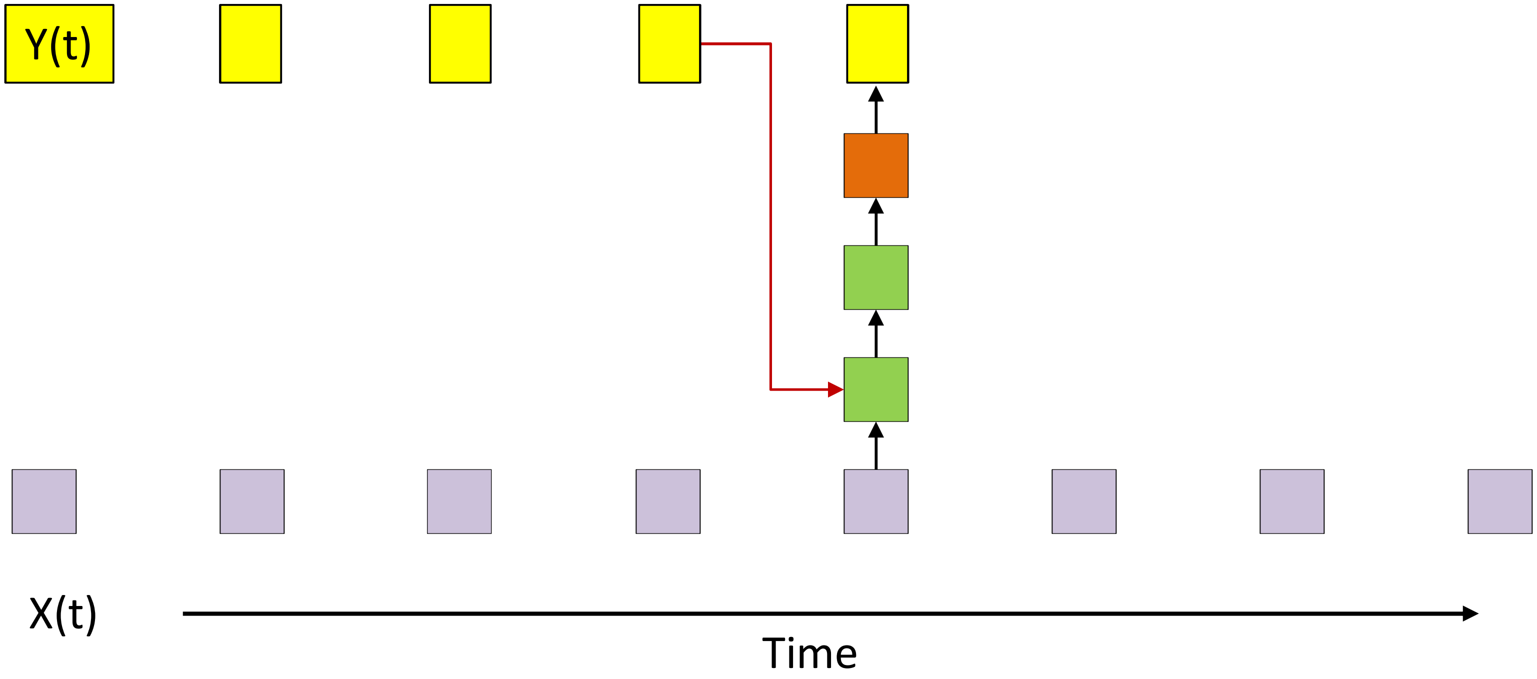

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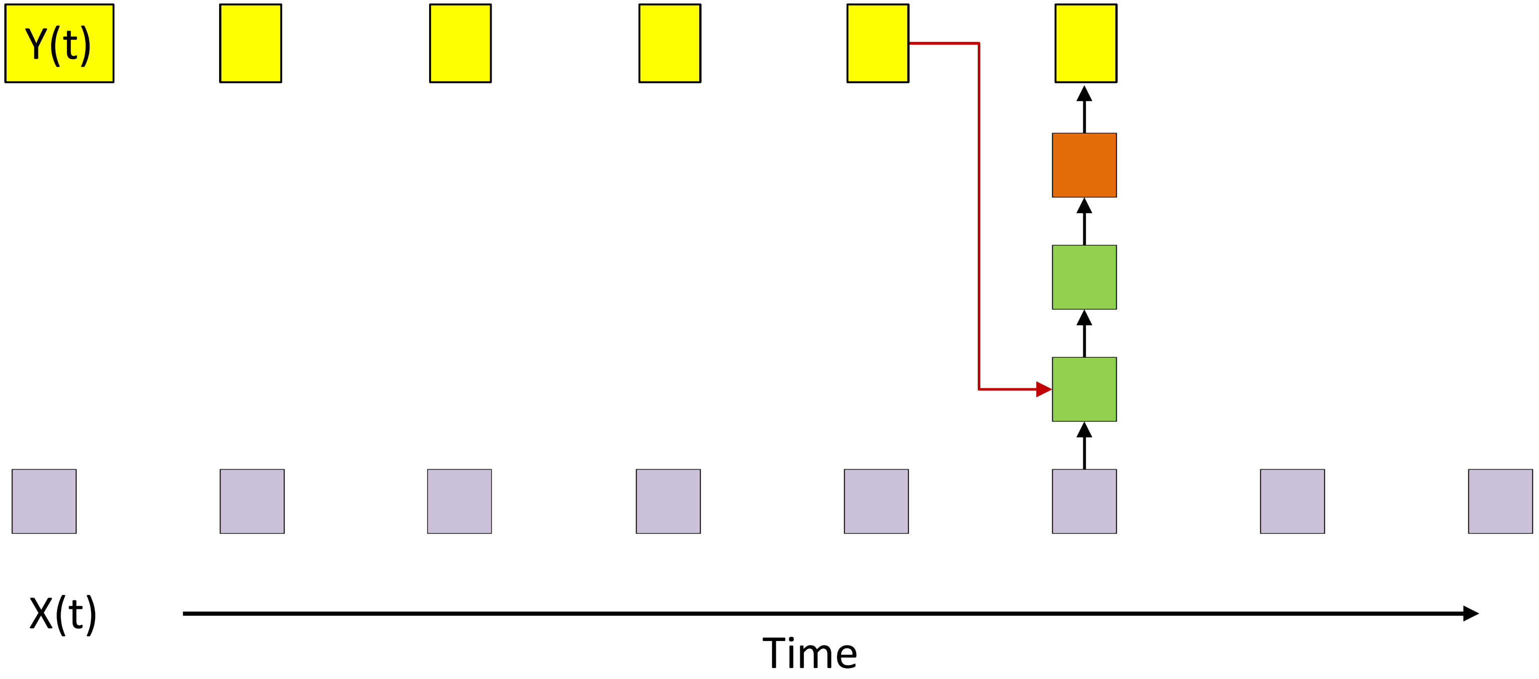

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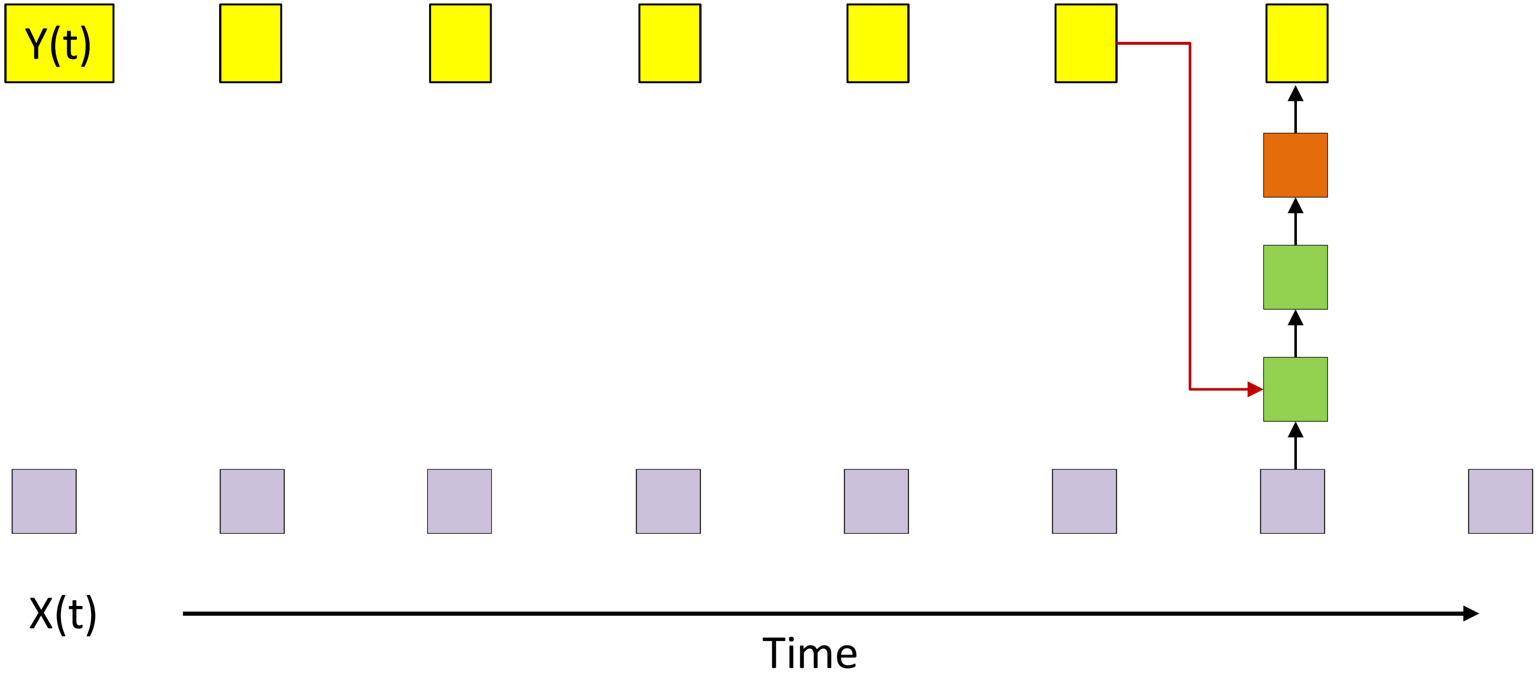

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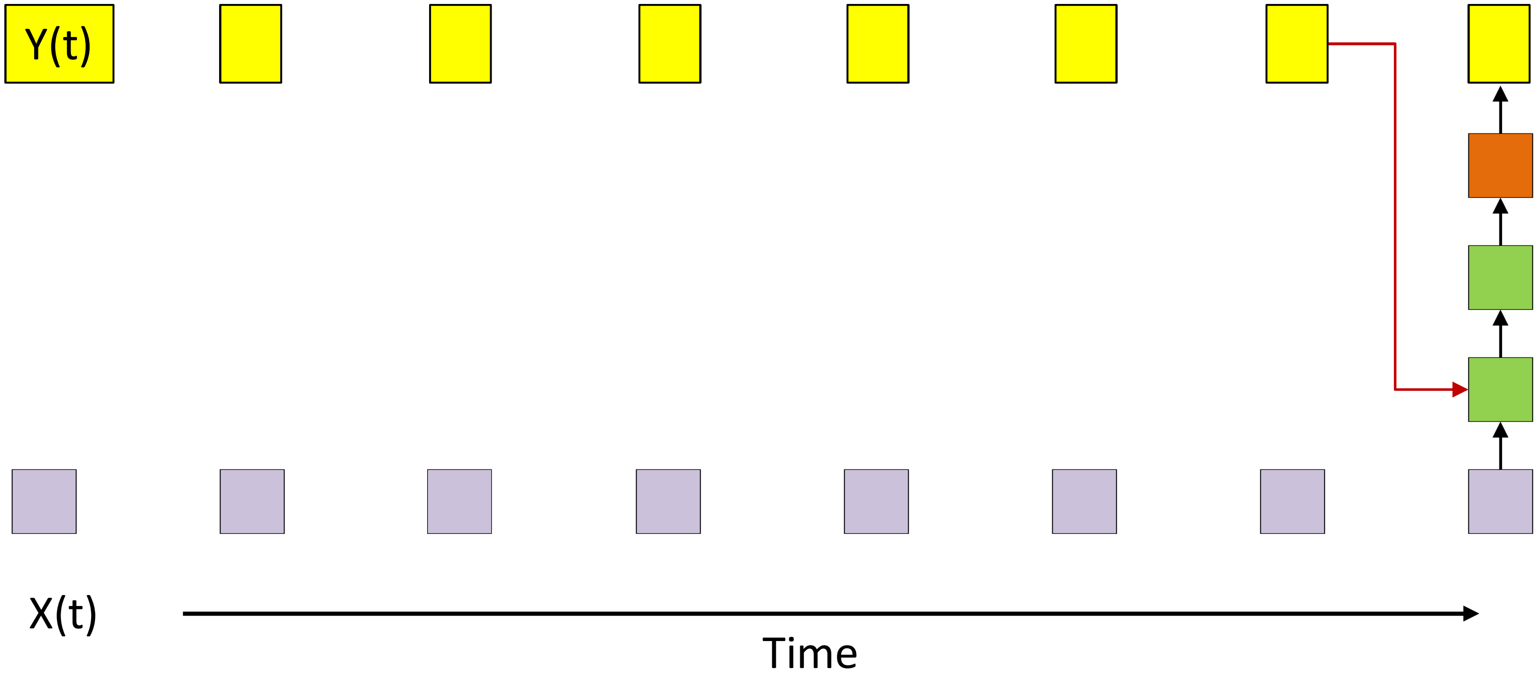
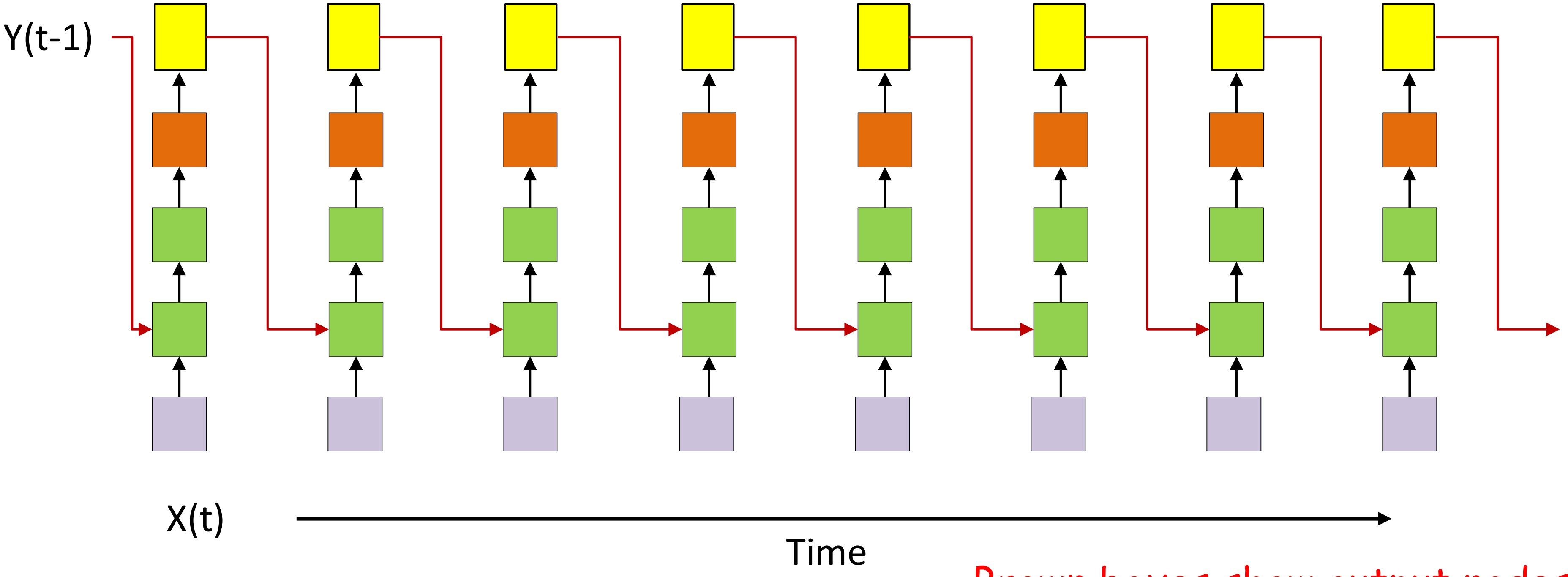


- A NARX net with recursion from the output

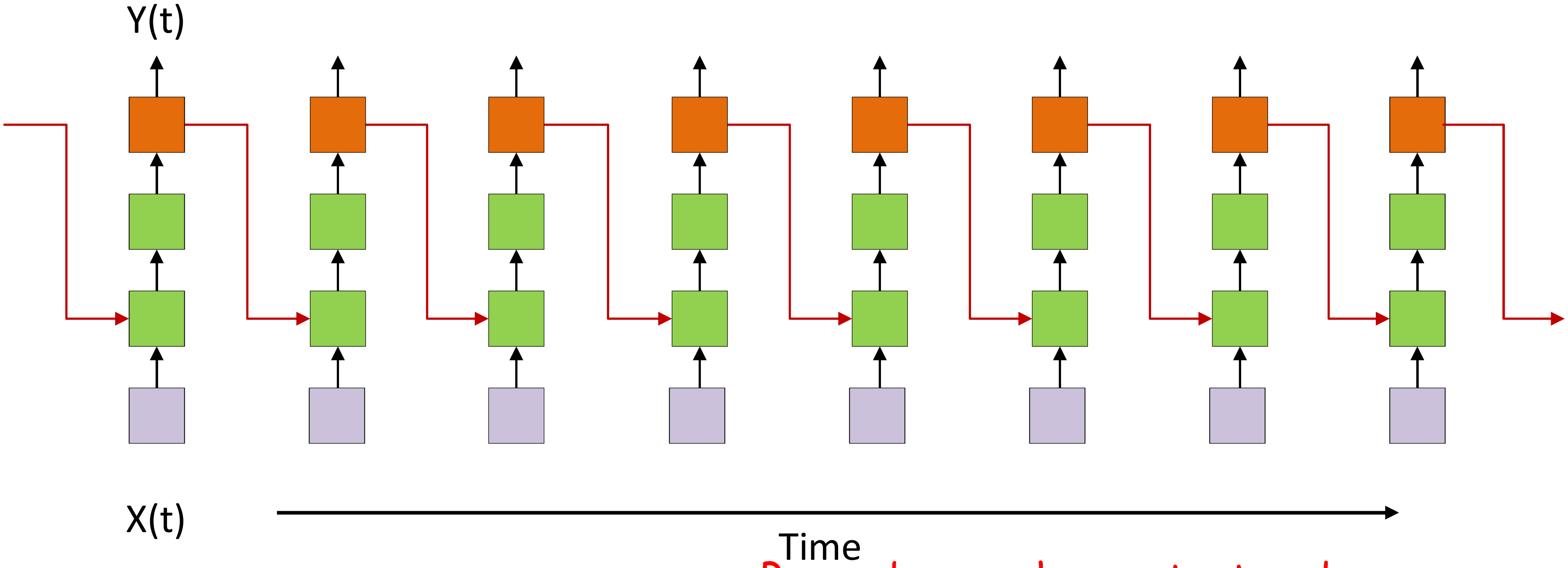
A more complete representation



Brown boxes show output nodes
Yellow boxes are outputs

- A NARX net with recursion from the output
- Showing all computations
- All columns are identical
- *An input at t=0 affects outputs forever*

Same figure redrawn



Brown boxes show output nodes
All outgoing arrows are the same output

- A NARX net with recursion from the output
- Showing all computations
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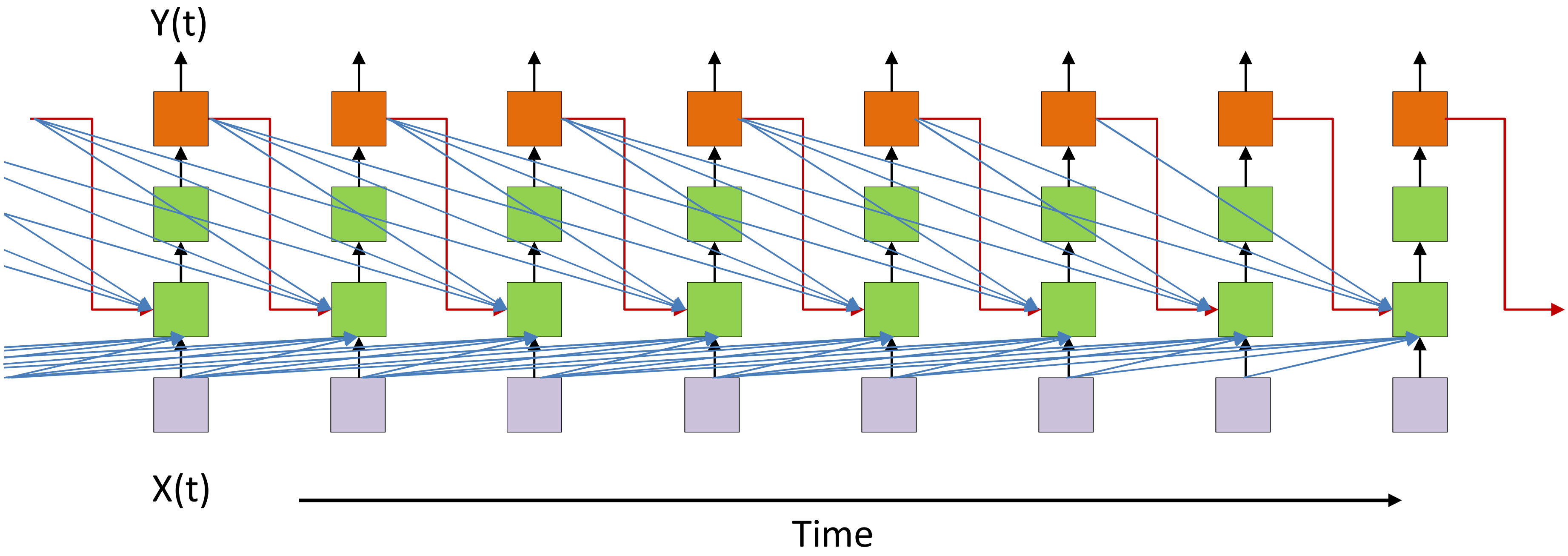
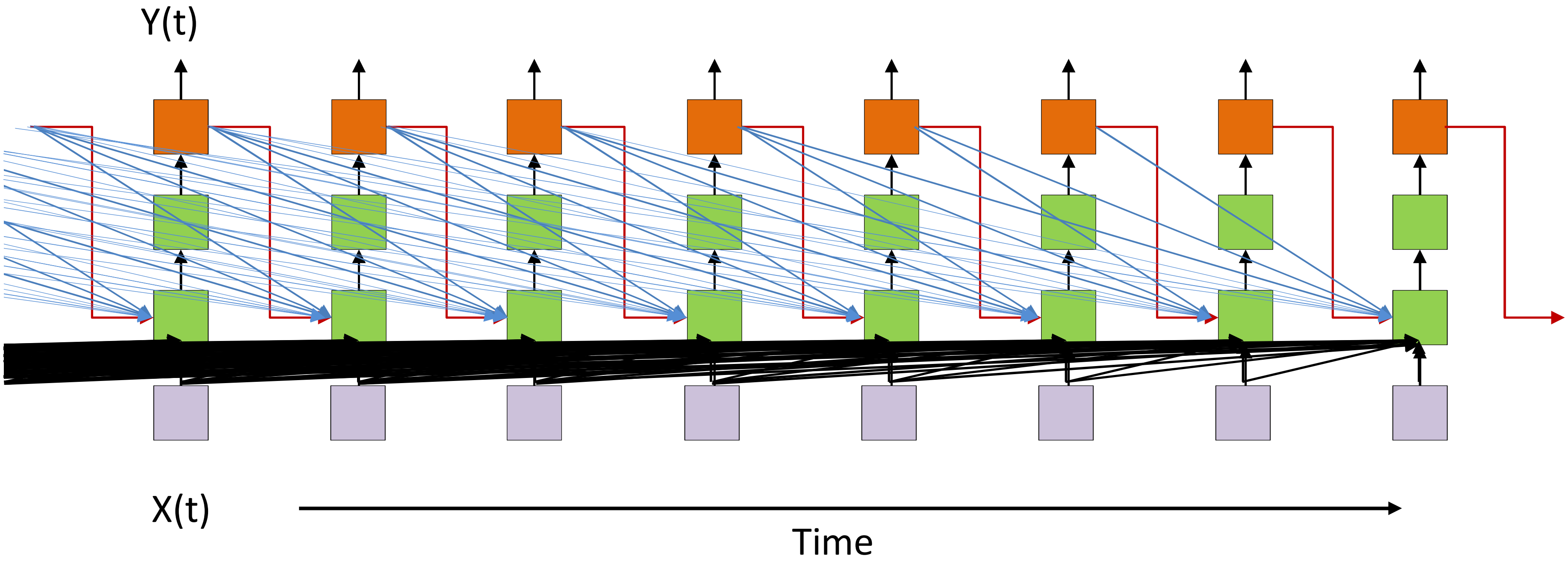
A more generic NARX network



- The output Y_t at time t is computed from the past K outputs $Y_{t-1}, \dots, Y_{t-K}$ and the current and past L inputs $X_t, \dots, X_{t-L}$

A “complete” NARX network



- The output Y_t at time t is computed from *all* past outputs and *all* inputs until time t
  - Not really a practical model

NARX Networks

- Very popular for time-series prediction
  - Weather
  - Stock markets
  - As alternate system models in tracking systems
- Any phenomena with distinct “innovations” that “drive” an output

- Note: here the “memory” of the past is in the output itself, and not in the network

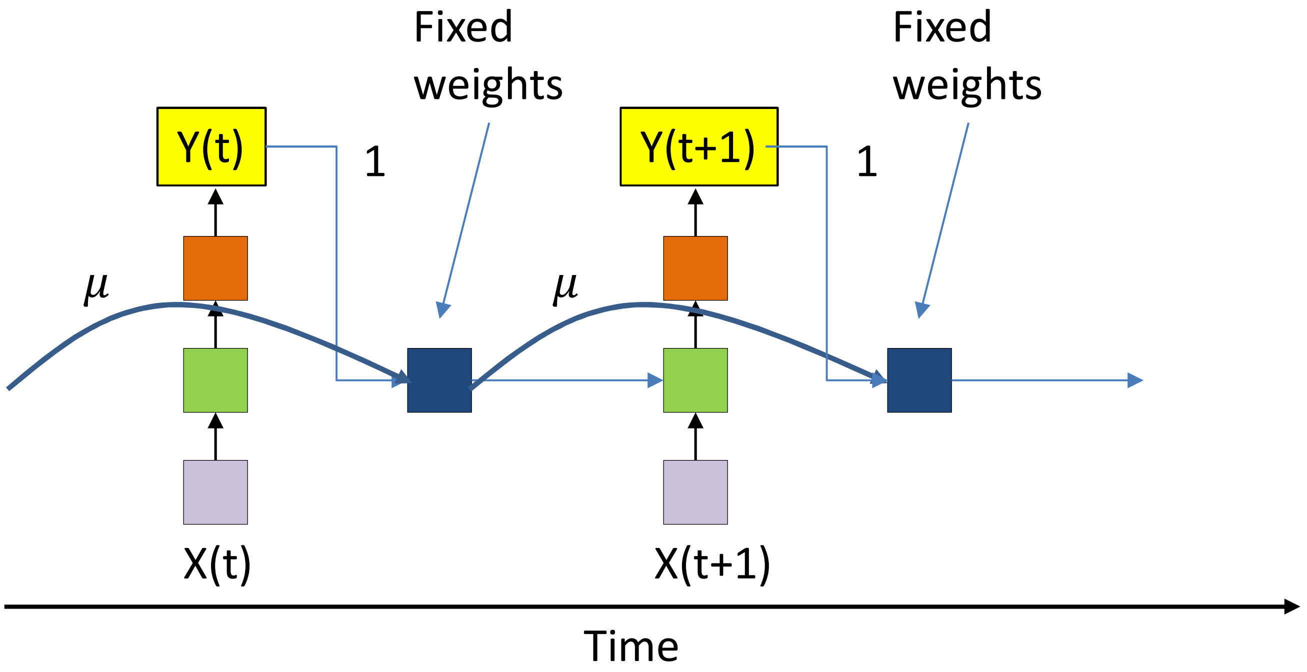
Lets make memory more explicit

- Task is to “remember” the past
- Introduce an explicit *memory* variable whose *job* it is to remember

$$m_t = r(y_{t-1}, h_{t-1}, m_{t-1})$$
$$h_t = f(x_t, m_t)$$
$$y_t = g(h_t\ )$$

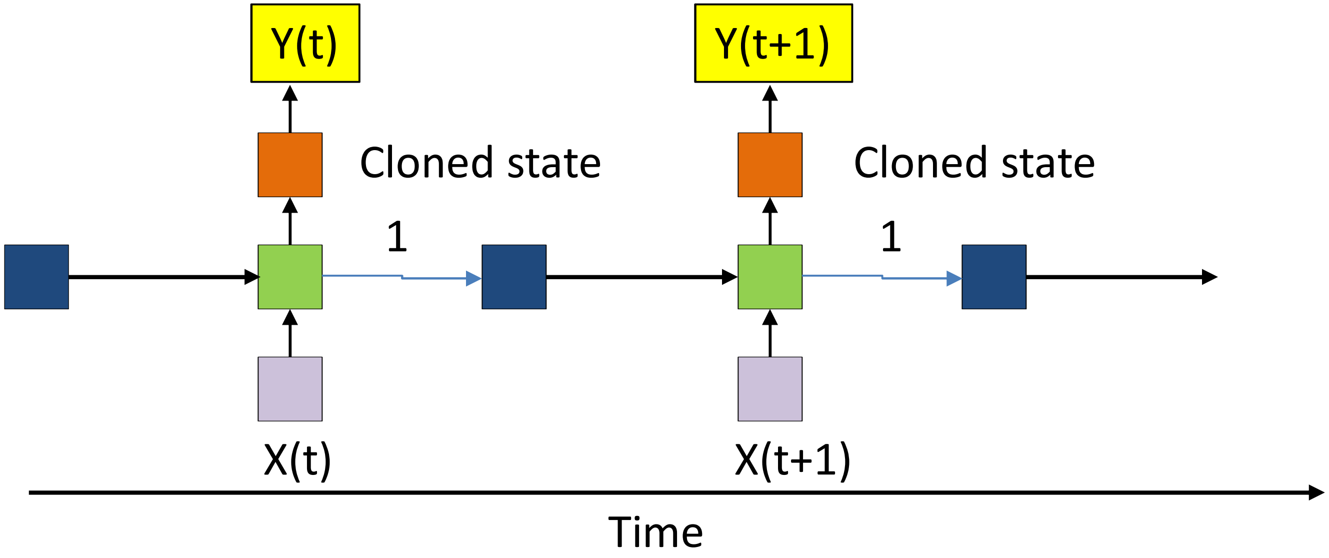
- m_t is a “memory” variable
  - Generally stored in a “memory” unit
  - Used to “remember” the past

Jordan Network



- Memory unit simply retains a running average of past outputs
  - "Serial order: A parallel distributed processing approach", M.I.Jordan, 1986
    - Input is constant (called a "plan")
    - Objective is to train net to produce a specific output, given an input plan
  - Memory has fixed structure; does not "learn" to remember
    - The running average of outputs considers entire past, rather than immediate past

Elman Networks



- Separate memory state from output
  - “Context” units that carry historical state
  - “Finding structure in time”, Jeffrey Elman, Cognitive Science, 1990
    - For the purpose of training, this was approximated as a set of T independent 1-step history nets
- Only the weight *from* the memory unit to the hidden unit is learned
  - But during training no gradient is backpropagated over the “1” link

Story so far

- In time series analysis, models must look at past inputs along with current input
  - Looking at a finite horizon of past inputs gives us a convolutional network
- Looking into the infinite past requires recursion

- NARX networks recurse by feeding back the output to the input
  - May feed back a finite horizon of outputs

- "Simple" recurrent networks:
  - Jordon networks maintain a running average of outputs in a "memory" unit
  - Elman networks store *hidden* unit values for one time instant in a "context" unit
  - "Simple" (or partially recurrent) because during *learning* current error does not actually propagate to the past
    - "Blocked" at the memory units in Jordan networks
    - "Blocked" at the "context" unit in Elman networks

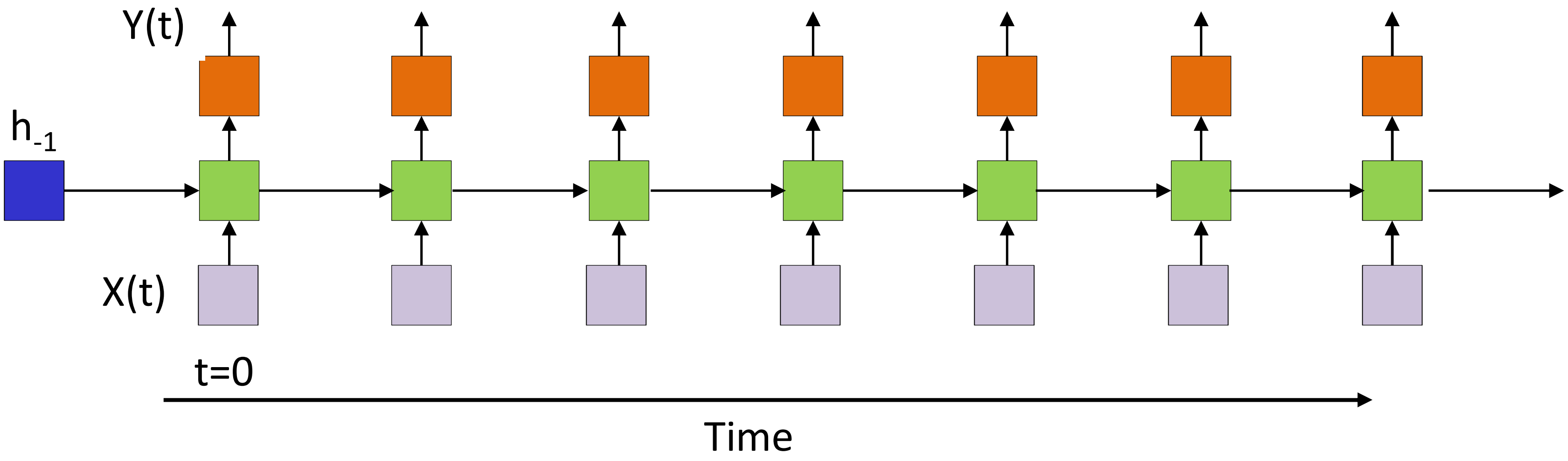
An alternate model for infinite response systems: the state-space model

$$h_t = f(x_t, h_{t-1})$$
$$y_t = g(h_t)$$

- h_t is the *state* of the network
  - Model directly embeds the memory in the state
- Need to define initial state h_{-1}

- This is a *fully recurrent* neural network
  - Or simply a *recurrent neural network*
- *State* summarizes information about the entire past

The simple state-space model



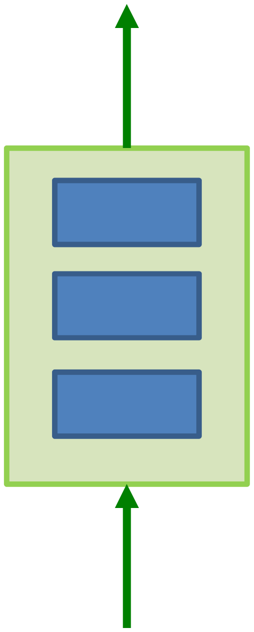
- The state (green) at any time is determined by the input at that time, and the state at the previous time
- *An input at t=0 affects outputs forever*
- Also known as a recurrent neural net

An alternate model for infinite response systems: the state-space model

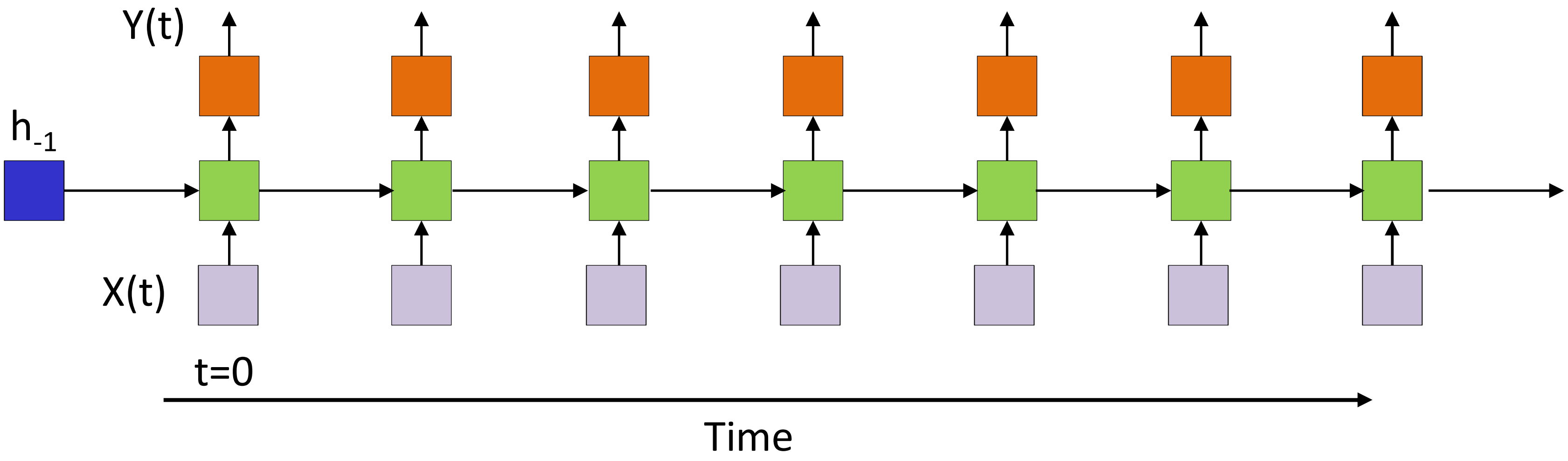
$$h_t = f(x_t, h_{t-1})$$
$$y_t = g(h_t)$$

- h_t is the *state* of the network
- Need to define initial state h_{-1}

- The state an be arbitrarily complex

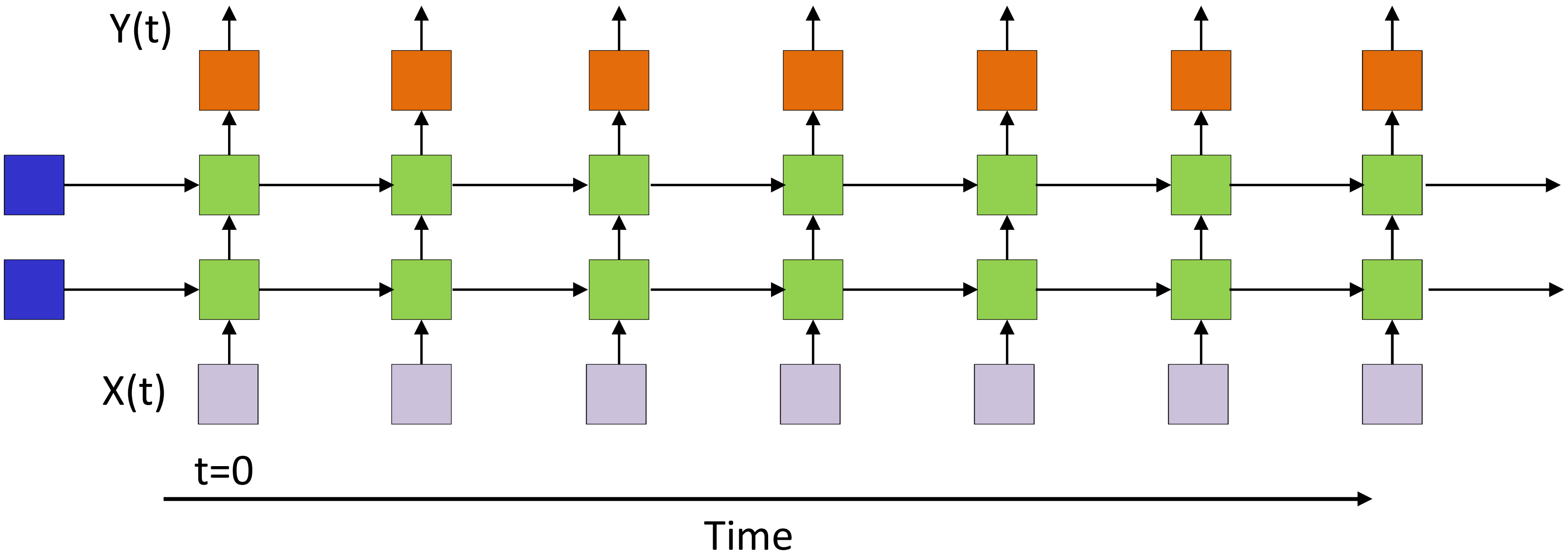


Single hidden layer RNN



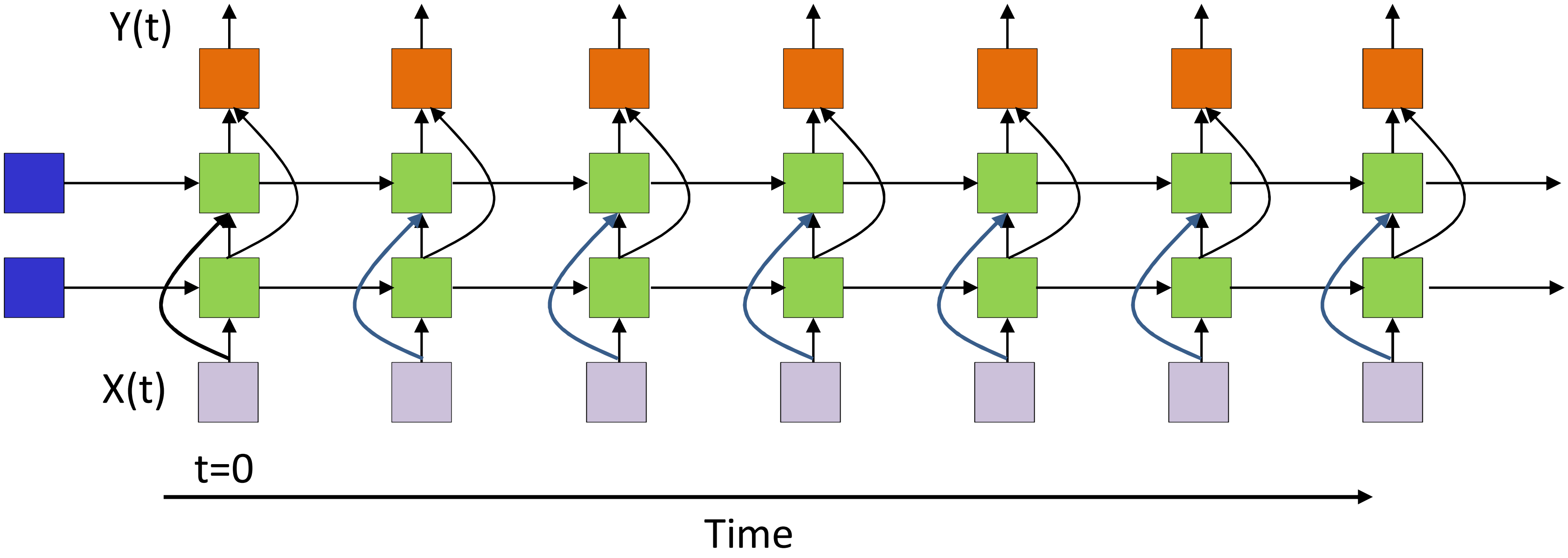
- Recurrent neural network
- All columns are identical
- *An input at t=0 affects outputs forever*

Multiple recurrent layer RNN



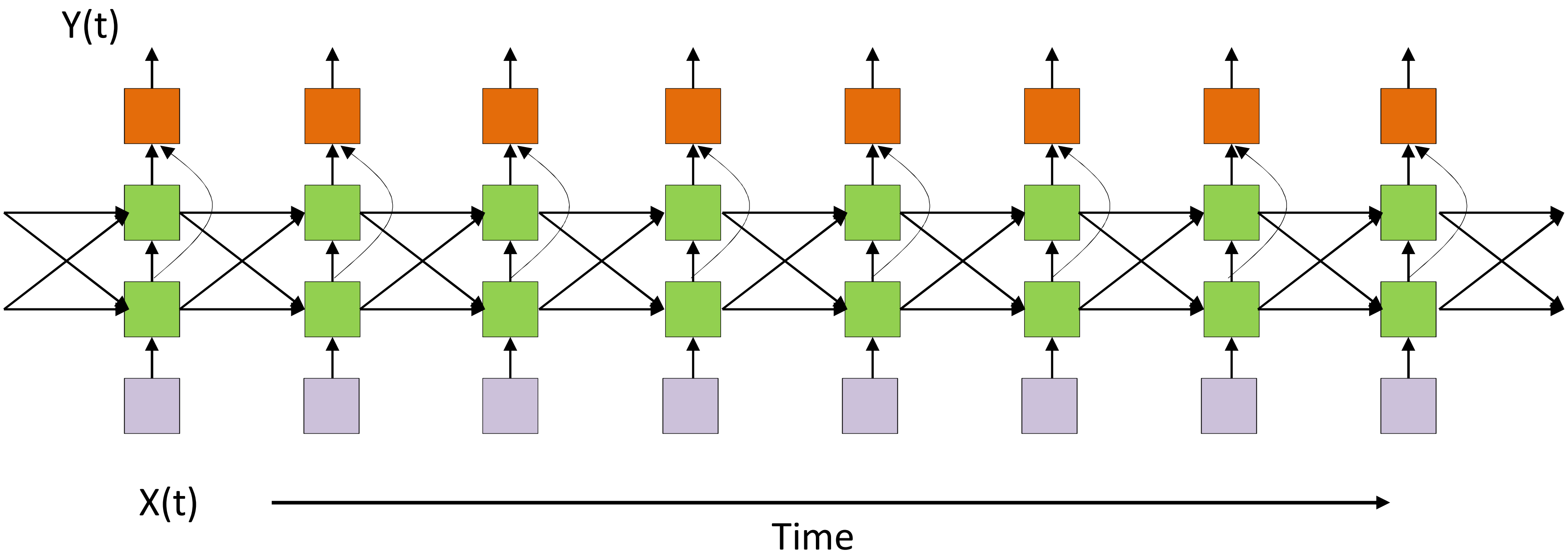
- Recurrent neural network
- All columns are identical
- *An input at t=0 affects outputs forever*

Multiple recurrent layer RNN



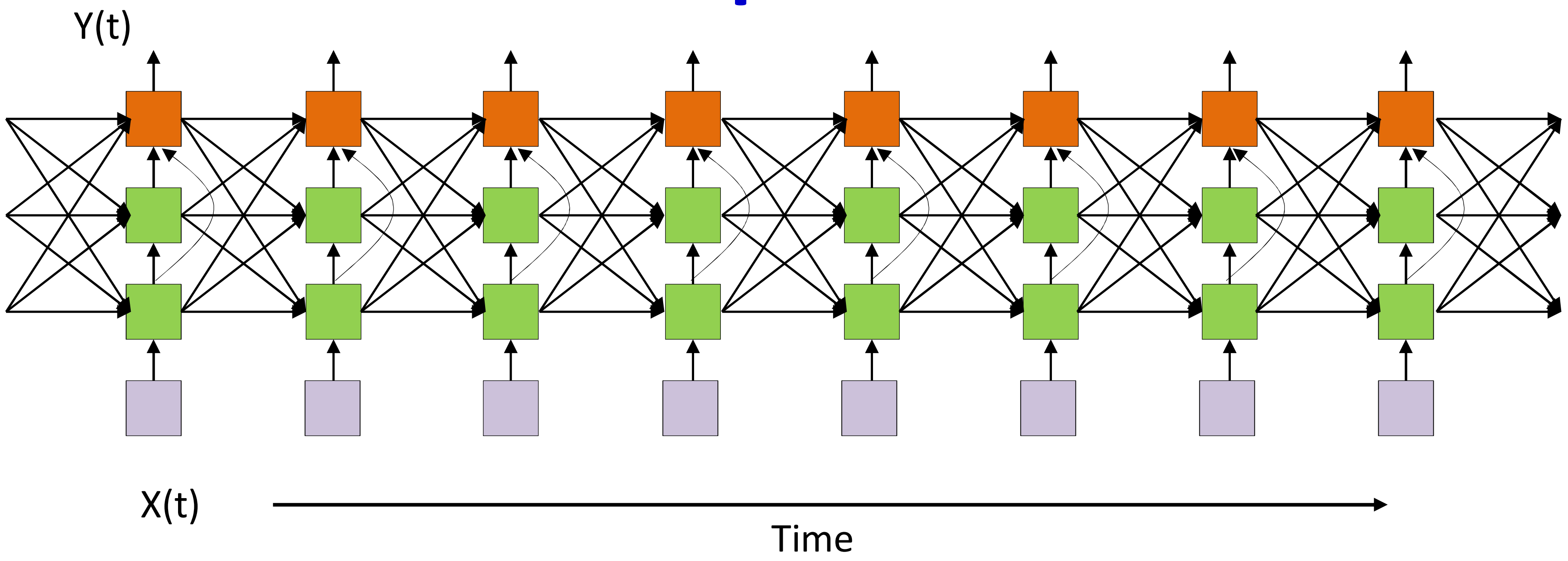
- We can also have skips..

A more complex state



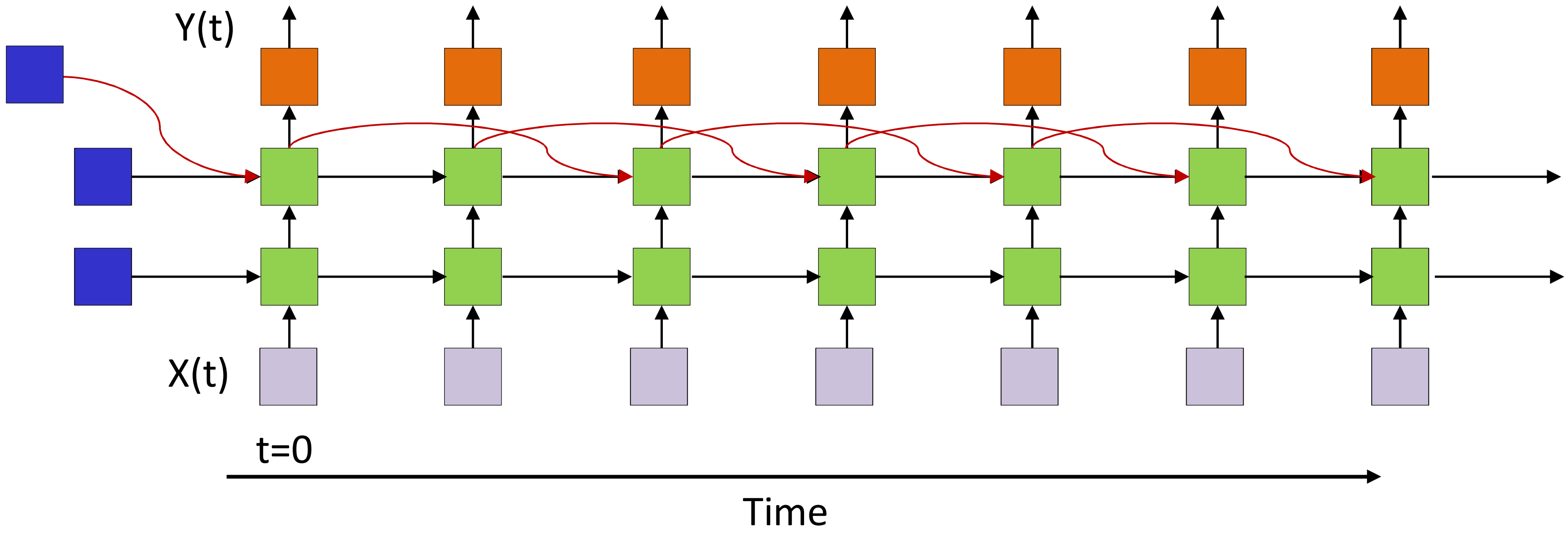
- All columns are identical
- *An input at t=0 affects outputs forever*

Or the network may be even more complicated



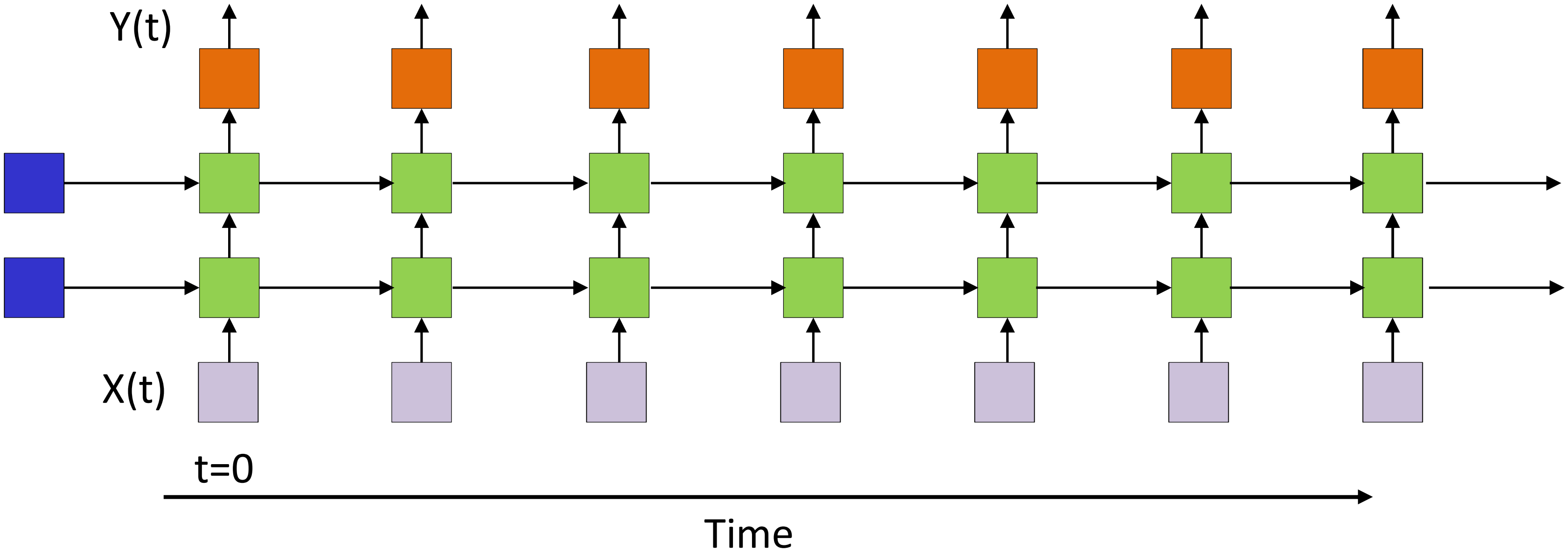
- Shades of NARX
- All columns are identical
- *An input at t=0 affects outputs forever*

Generalization with other recurrences



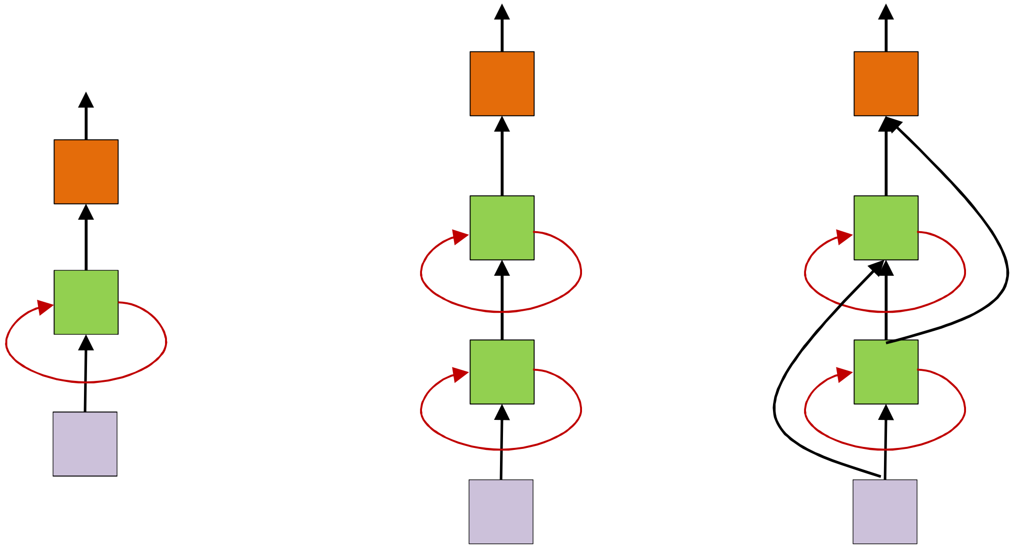
- All columns (including incoming edges) are identical

The simplest structures are most popular



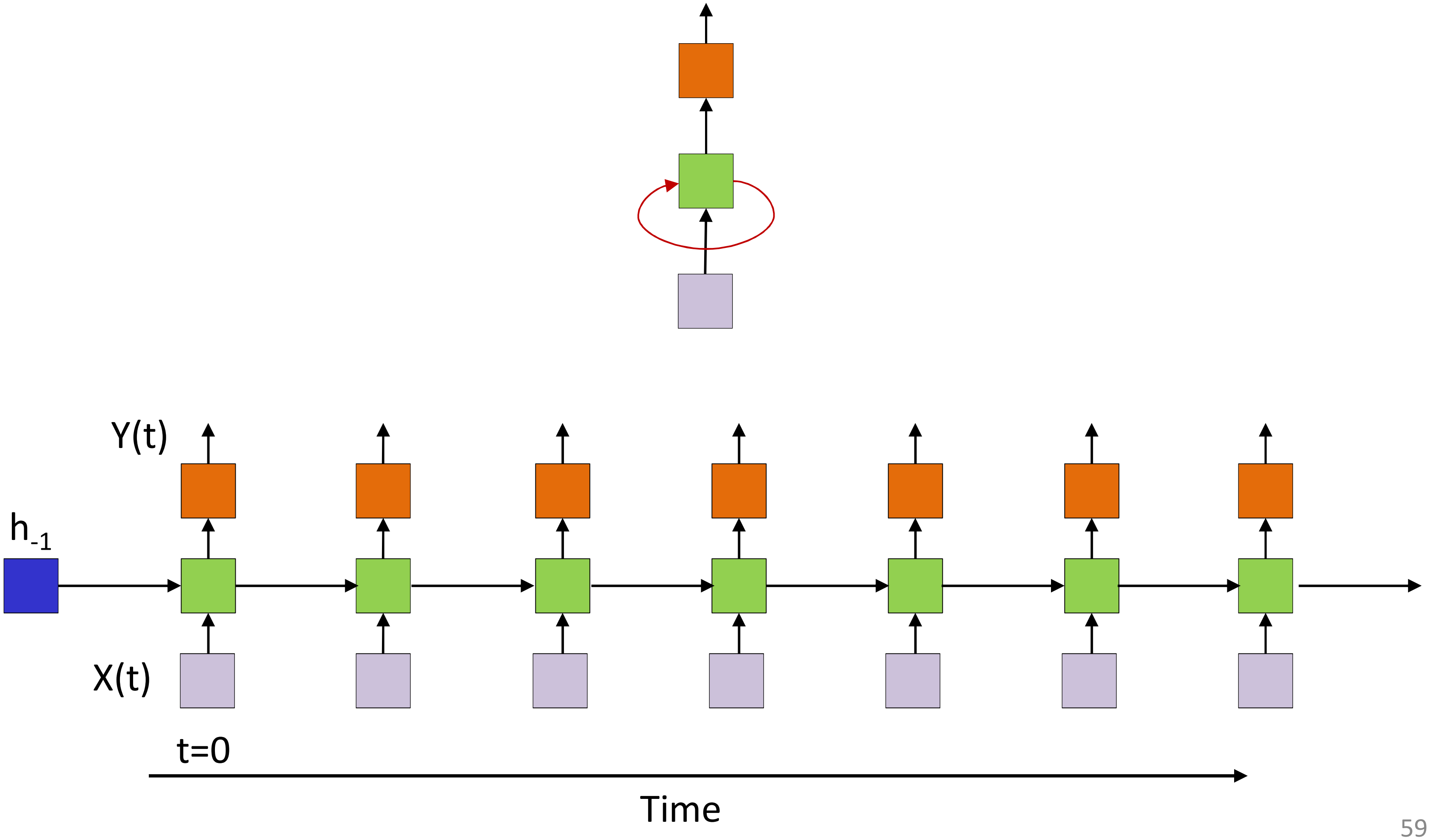
- Recurrent neural network
- All columns are identical
- *An input at t=0 affects outputs forever*

A Recurrent Neural Network

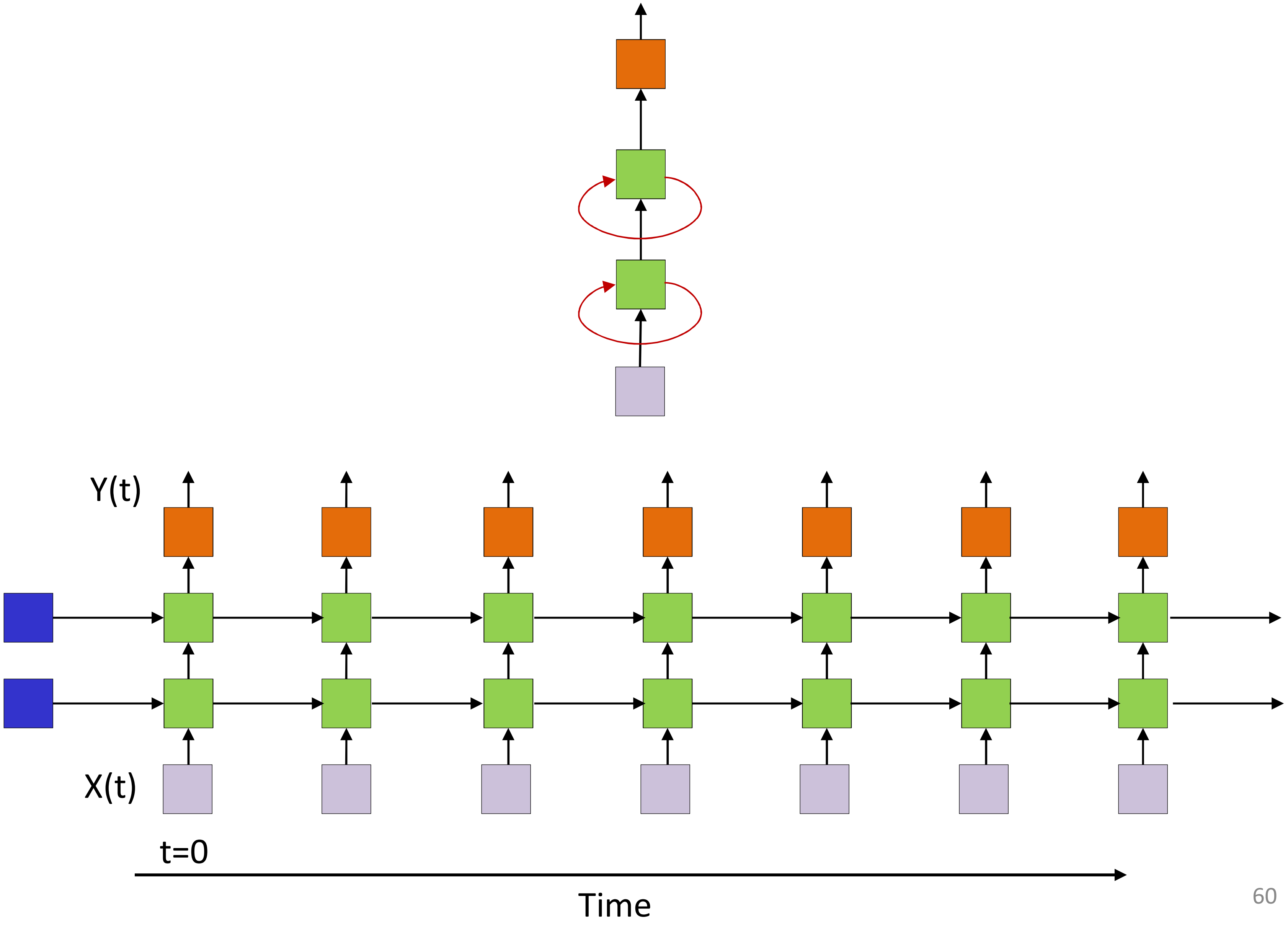


- Simplified models often drawn
- The loops imply recurrence

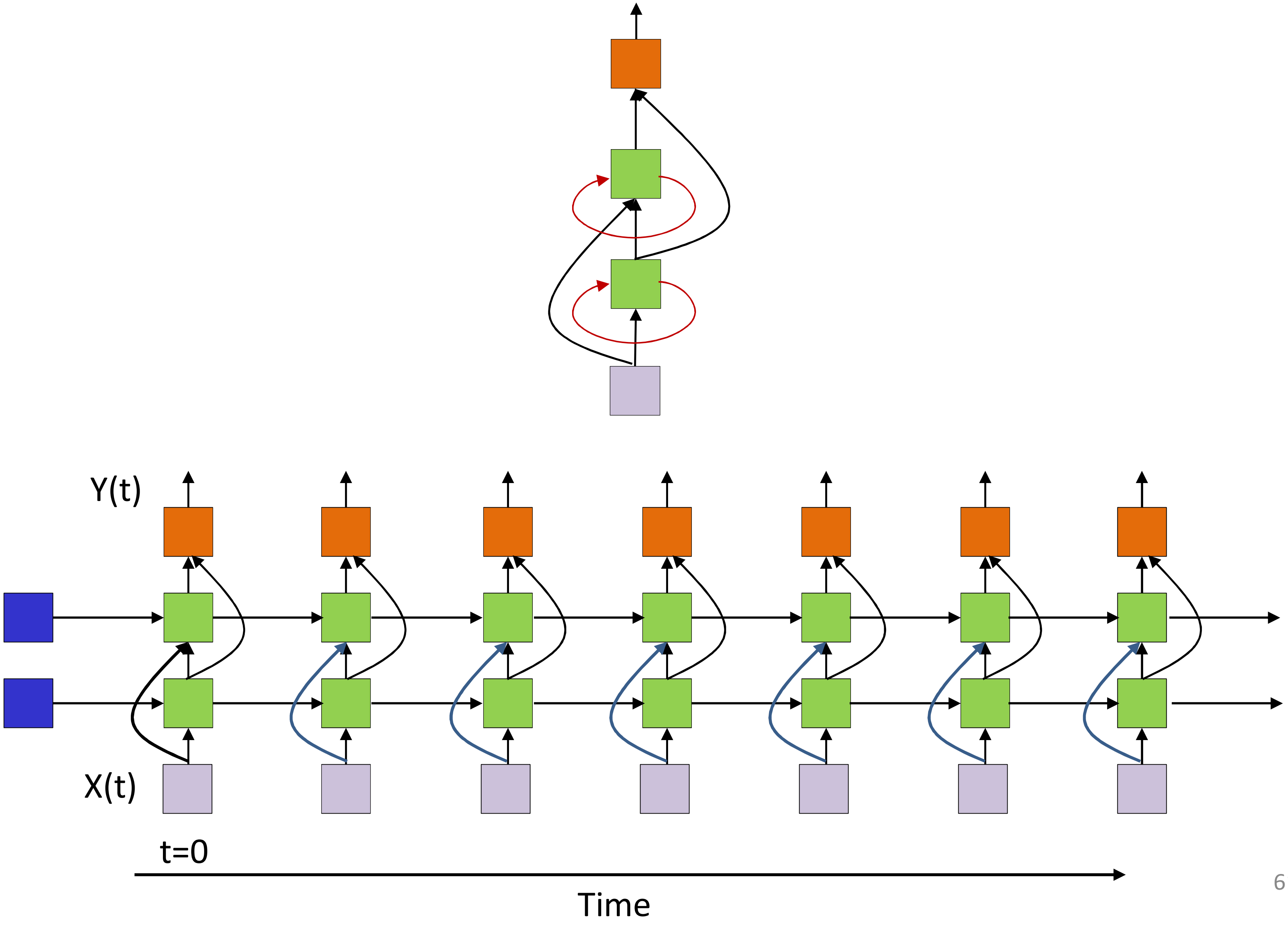
The detailed version of the simplified representation



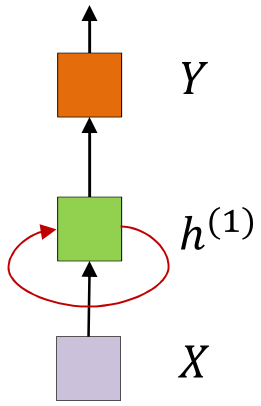
Multiple recurrent layer RNN



Multiple recurrent layer RNN



Equations



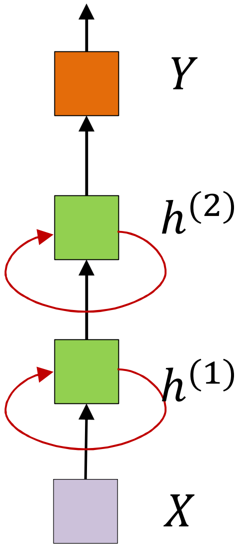
$$h_i^{(1)}(-1) = part\ of\ network\ parameters$$

$$h_i^{(1)}(t) = f_1\left(\sum_j w_{ji}^{(0)} X_j(t) + \sum_j w_{ji}^{(11)} h_i^{(1)}(t-1) + b_i^{(1)}\right)$$

$$Y(t) = f_2\left(\sum_j w_{jk}^{(1)} h_j^{(1)}(t) + b_k^{(1)}, k = 1..M\right)$$

- Note superscript in indexing, which indicates layer of network from which inputs are obtained
- Assuming vector function at output, e.g. softmax
- The *state* node activation, $f_1()$ is typically $\tanh()$
- Every neuron also has a *bias* input

Equations



$$h_i^{(1)}(-1) = part\ of\ network\ parameters$$

$$h_i^{(2)}(-1) = part\ of\ network\ parameters$$

$$h_i^{(1)}(t) = f_1\left(\sum_j w_{ji}^{(0)} X_j(t) + \sum_j w_{ji}^{(11)} h_i^{(1)}(t-1) + b_i^{(1)}\right)$$

$$h_i^{(2)}(t) = f_2\left(\sum_j w_{ji}^{(1)} h_j^{(1)}(t) + \sum_j w_{ji}^{(22)} h_i^{(2)}(t-1) + b_i^{(2)}\right)$$

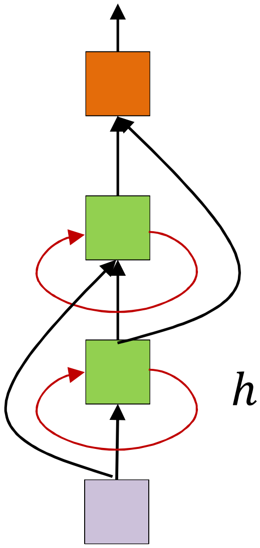
$$Y(t) = f_3\left(\sum_j w_{jk}^{(2)} h_j^{(2)}(t) + b_k^{(3)}, k = 1..M\right)$$

- Assuming vector function at output, e.g. softmax $f_3()$
- The *state* node activations, $f_k()$ are typically tanh()
- Every neuron also has a *bias* input

Equations

$$h_i^{(1)}(-1) = part\ of\ network\ parameters$$

$$h_i^{(2)}(-1) = part\ of\ network\ parameters$$

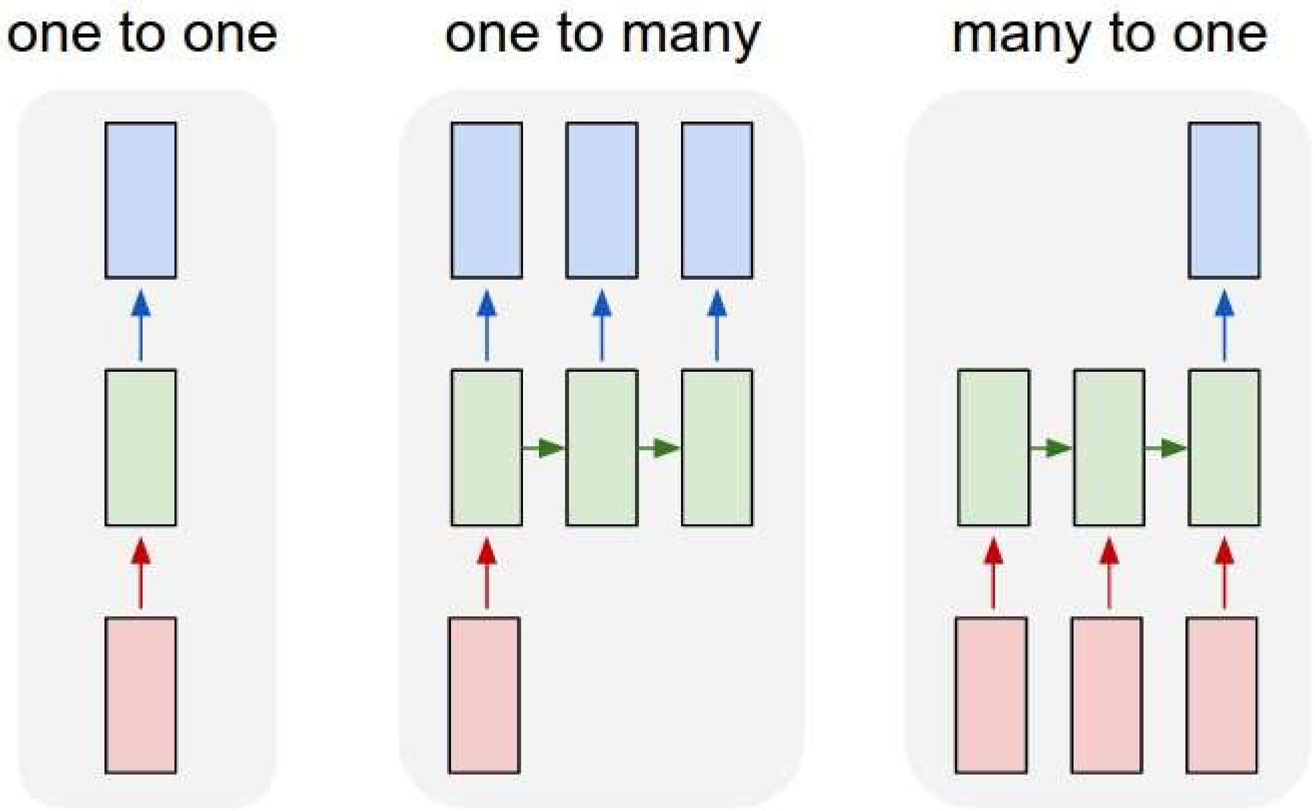


$$h_i^{(1)}(t) = f_1\left(\sum_j w_{ji}^{(0,1)} X_j(t) + \sum_i w_{ii}^{(1,1)} h_i^{(1)}(t-1) + b_i^{(1)}\right)$$

$$h_i^{(2)}(t) = f_2\left(\sum_j w_{ji}^{(1,2)} h_j^{(1)}(t) + \sum_j w_{ji}^{(0,2)} X_j(t) + \sum_i w_{ii}^{(2,2)} h_i^{(2)}(t-1) + b_i^{(2)}\right)$$

$$Y_i(t) = f_3\left(\sum_j w_{jk}^{(2)} h_j^{(2)}(t) + \sum_j w_{jk}^{(1,3)} h_j^{(1)}(t) + b_k^{(3)}, k = 1..M\right)$$

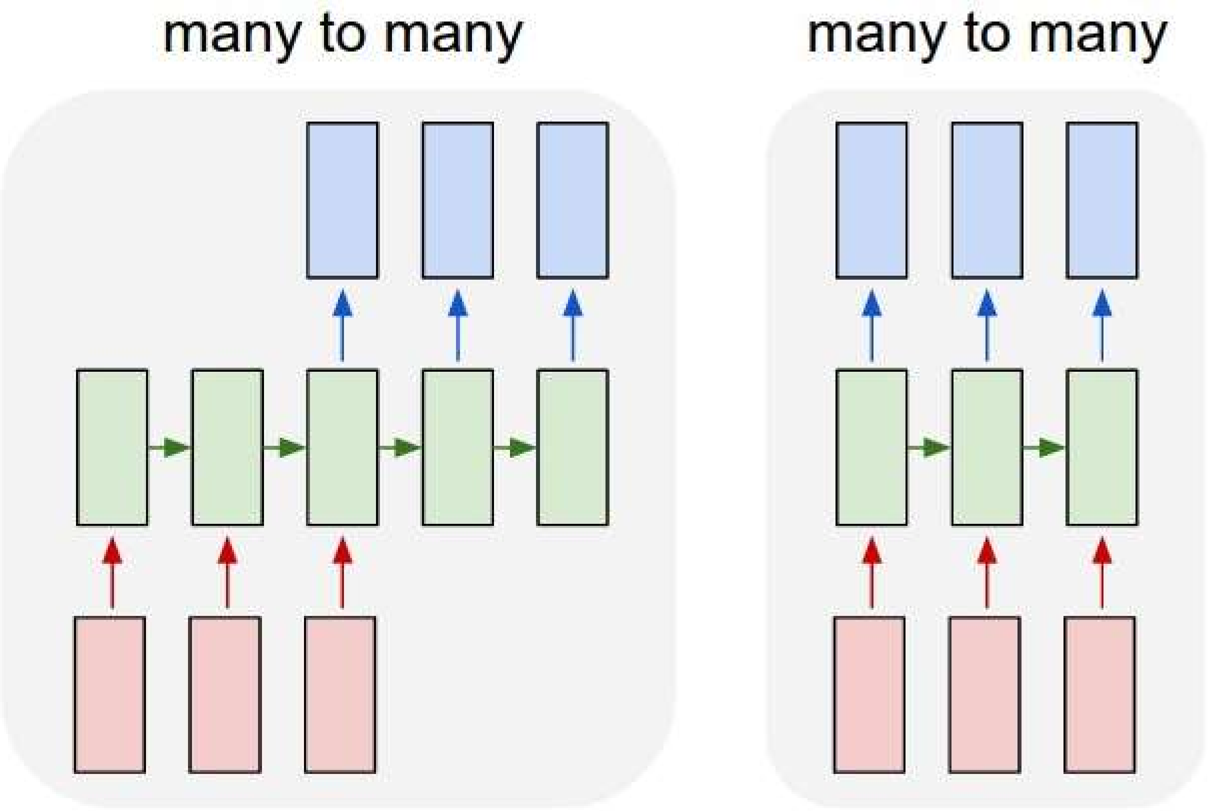
Variants on recurrent nets



Images from Karpathy

- 1: Conventional MLP
- 2: Sequence *generation*, e.g. image to caption
- 3: Sequence based *prediction or classification,* e.g. Speech recognition, text classification

Variants



Images from Karpathy

- 1: *Delayed* sequence to sequence
- 2: Sequence to sequence, e.g. stock problem, label prediction
- Etc…

Story so far

- Time series analysis must consider past inputs along with current input
- Looking into the infinite past requires recursion

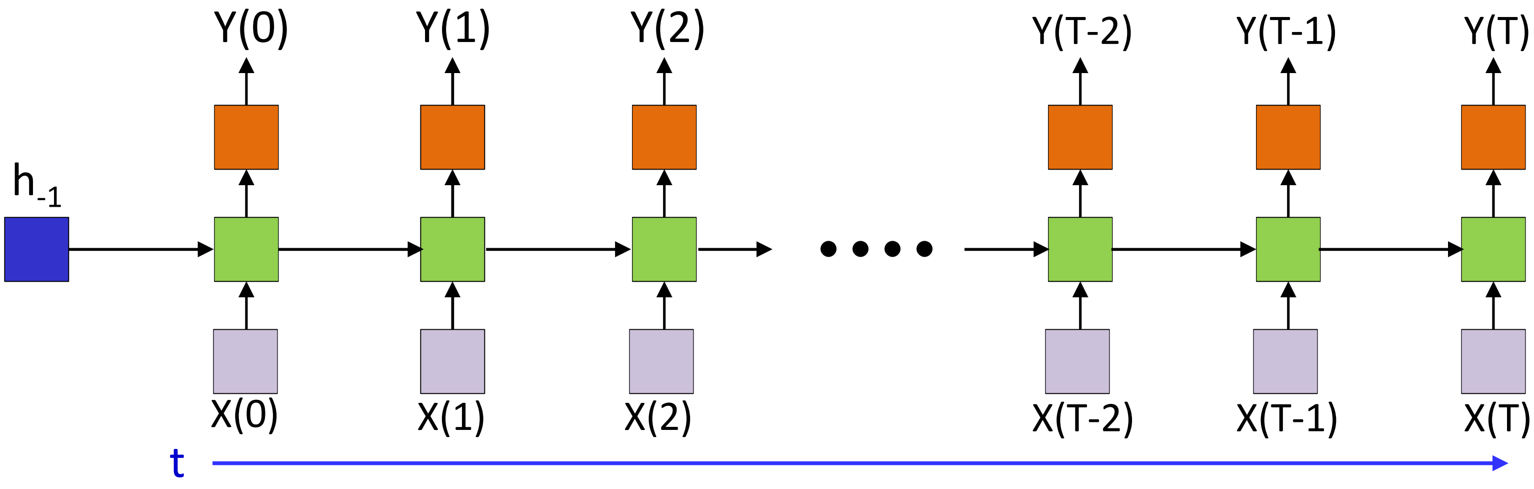
- NARX networks achieve this by feeding back the output to the input

- “Simple” recurrent networks maintain separate “memory” or “context” units to retain some information about the past
  - But during learning the current error does not influence the past

- State-space models retain information about the past through recurrent hidden states
  - These are “fully recurrent” networks
  - The initial values of the hidden states are generally learnable parameters as well

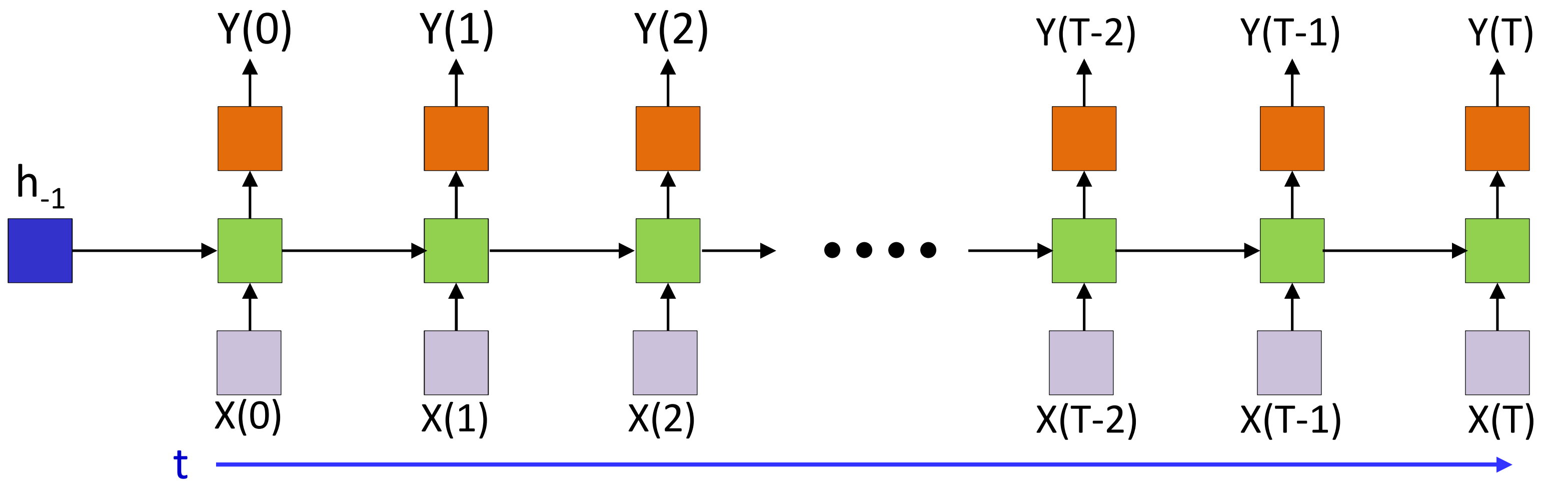
- State-space models enable current error to update parameters in the past

How do we *train* the network



- Back propagation through time (BPTT)
- Given a collection of *sequence* inputs
  - $(\mathbf{X}_i, \mathbf{D}_i)$, where
  - $\mathbf{X}_i = X_{i,0}, \dots, X_{i,T}$
  - $\mathbf{D}_i = D_{i,0}, \dots, D_{i,T}$
- Train network parameters to minimize the error between the output of the network $\mathbf{Y}_i = Y_{i,0}, \dots, Y_{i,T}$ and the desired outputs
  - This is the most generic setting. In other settings we just “remove” some of the input or output entries

Training: Forward pass

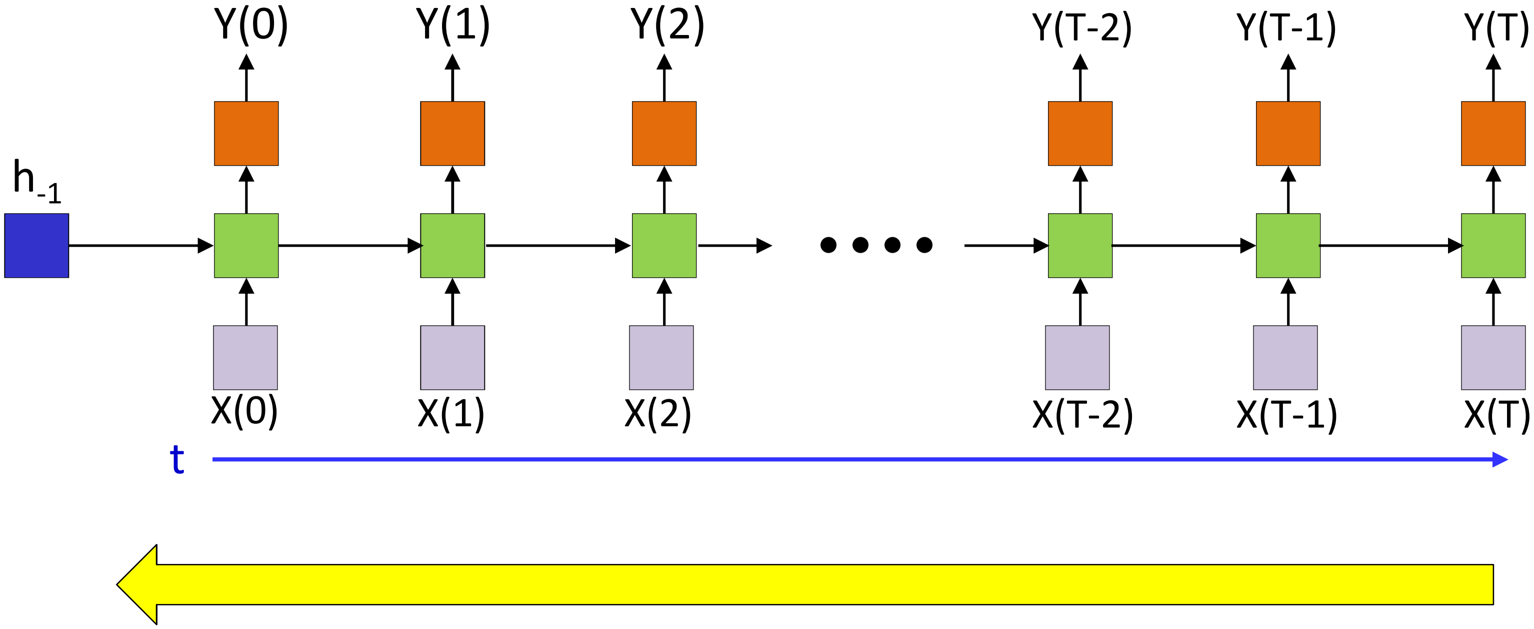


- For each training input:
- Forward pass: pass the entire data sequence through the network, generate outputs

Recurrent Neural Net
Assuming time-synchronous output

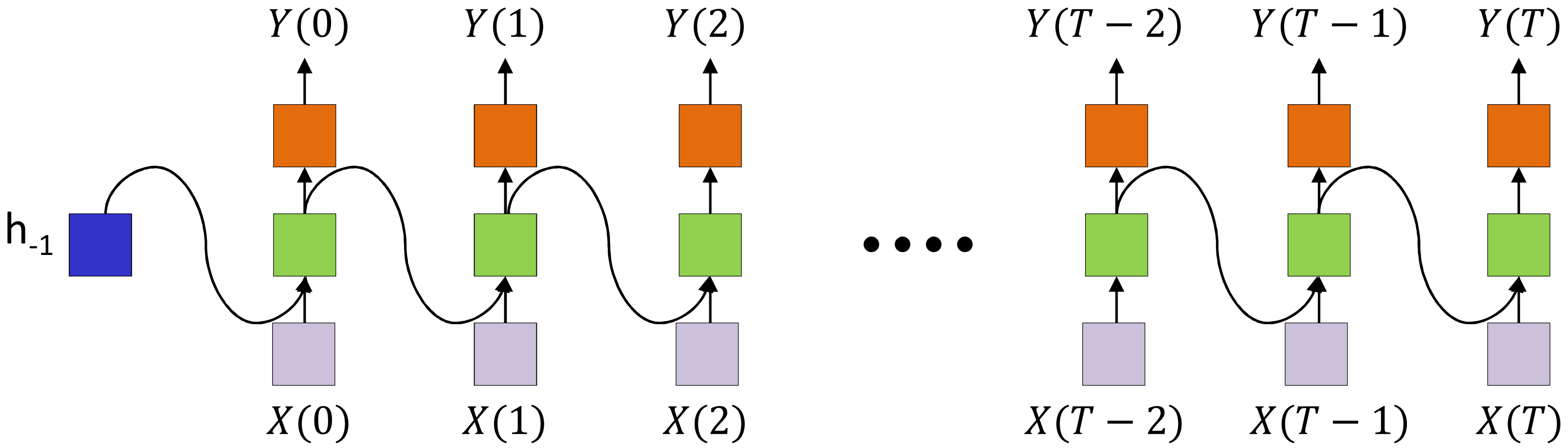
```
# Assuming h(-1,*) is known
# Assuming L hidden-state layers and an output layer
# $W_c$(*) and $W_r$(*) are matrics, b(*) are vectors
# $W_c$ are weights for inputs from current time
# $W_r$ is recurrent weight applied to the previous time
# $W_o$ are output layre weights

for t = 0:T-1  # Including both ends of the index
    h(t,0) = x(t) # Vectors. Initialize h(0) to input
    for l = 1:L  # hidden layers operate at time t
        z(t,l) = $W_c$(l)h(t,l-1) + $W_r$(l)h(t-1,l) + b(l)
        h(t,l) = tanh(z(t,l)) # Assuming tanh activ.
    $z_o$(t) = $W_o$h(t,L) + $b_o$
    Y(t) = softmax( $z_o$(t) )
```

Training: Computing gradients



- For each training input:
- Backward pass: Compute gradients via backpropagation
  - *Back Propagation Through Time*

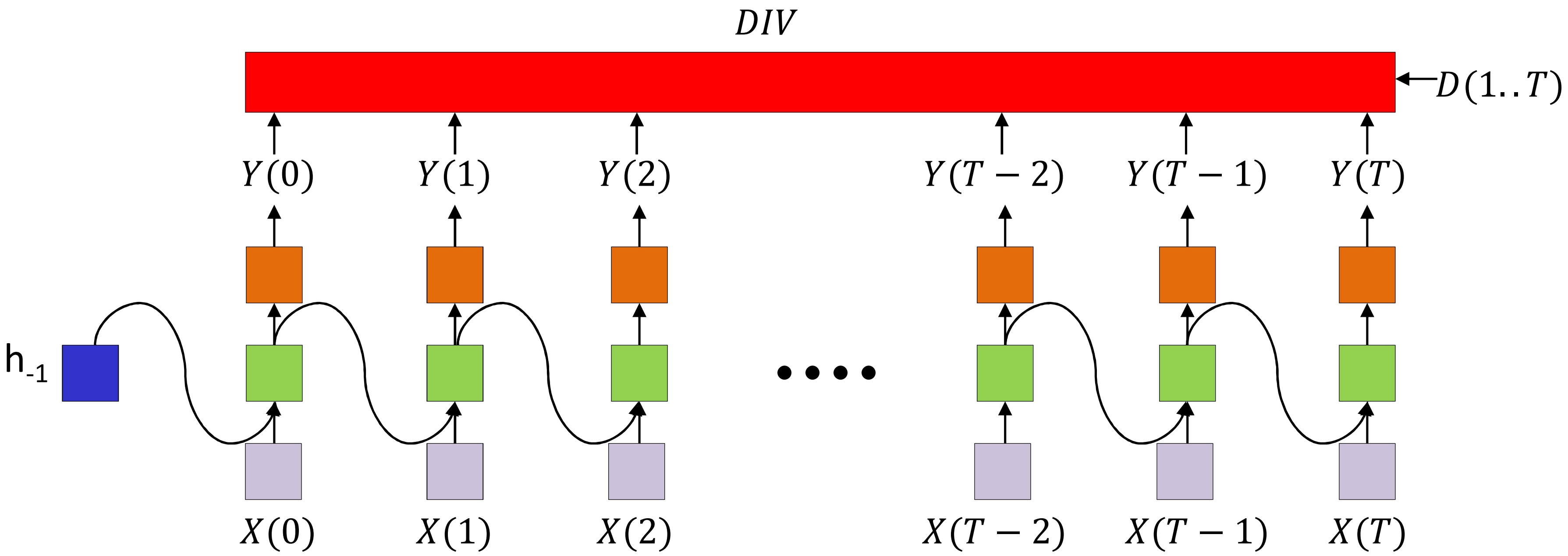
Back Propagation Through Time



Will only focus on *one* training instance

All subscripts represent *components* and not training instance index

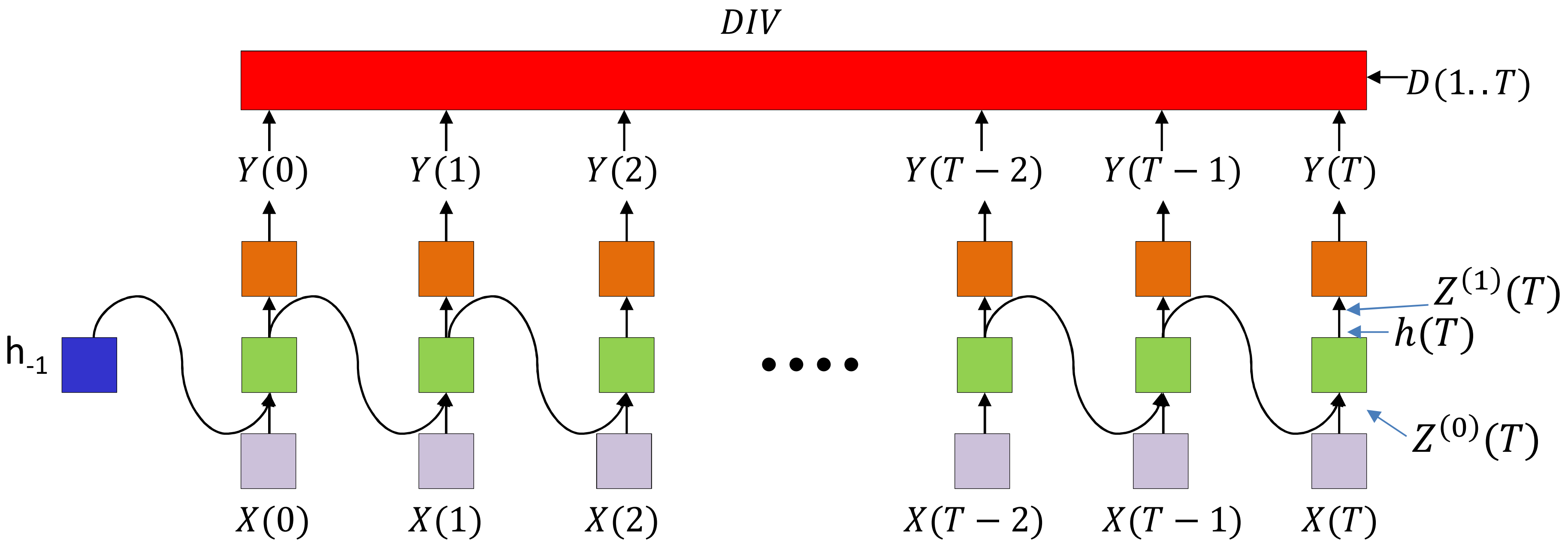
Back Propagation Through Time



- The divergence computed is between the *sequence of outputs* by the network and the *desired sequence of outputs*
  - DIV is a scalar function of a series of vectors!

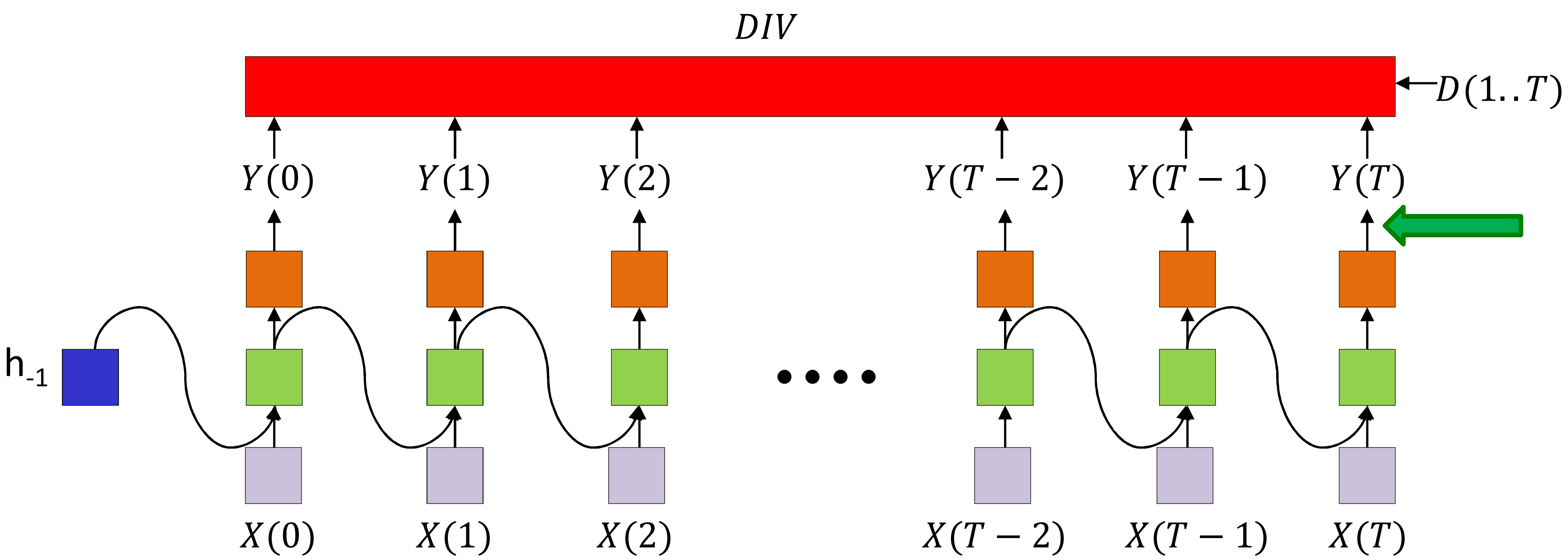
- This is *not* just the sum of the divergences at individual times
  - Unless we explicitly define it that way

Notation



- $Y(t)$ is the output at time t
  - $Y_i(t)$ is the ith output
- $Z^{(1)}(t)$ is the pre-activation value of the neurons at the output layer at time t
- $h(t)$ is the output of the hidden layer at time t
  - Assuming only one hidden layer in this example
- $Z^{(0)}(t)$ is the pre-activation value of the hidden layer at time t

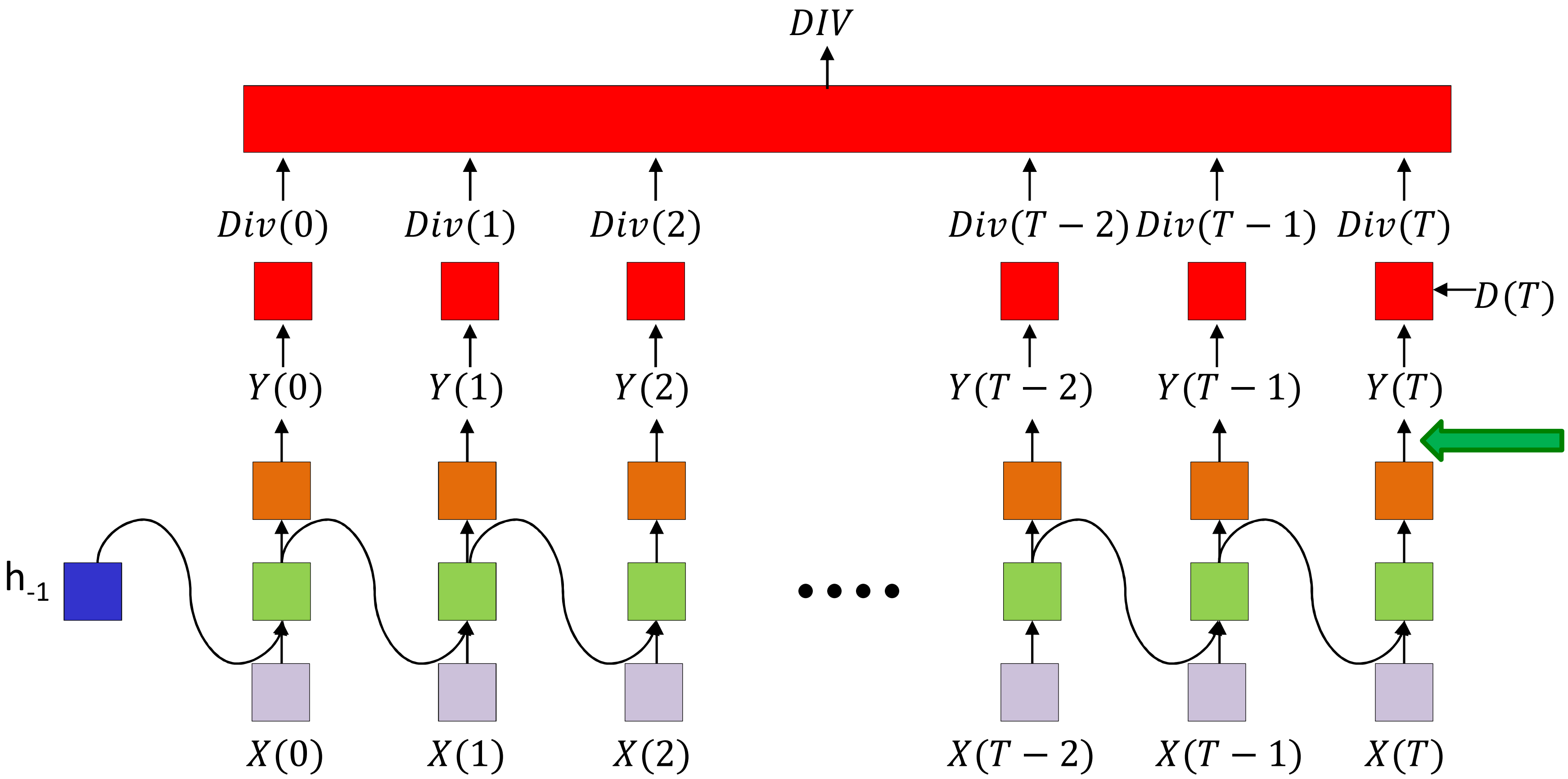
Back Propagation Through Time



First step of backprop: Compute $\frac{dDIV}{dY_i(T)}$ for all i

Note: DIV is a function of *all* outputs Y(0) … Y(T)

In general we will be required to compute $\frac{dDIV}{dY_i(t)}$ for all i and t as we will see. This can be a source of significant difficulty in many scenarios.



Special case, when the overall divergence is a simple combination of local divergences at each time:

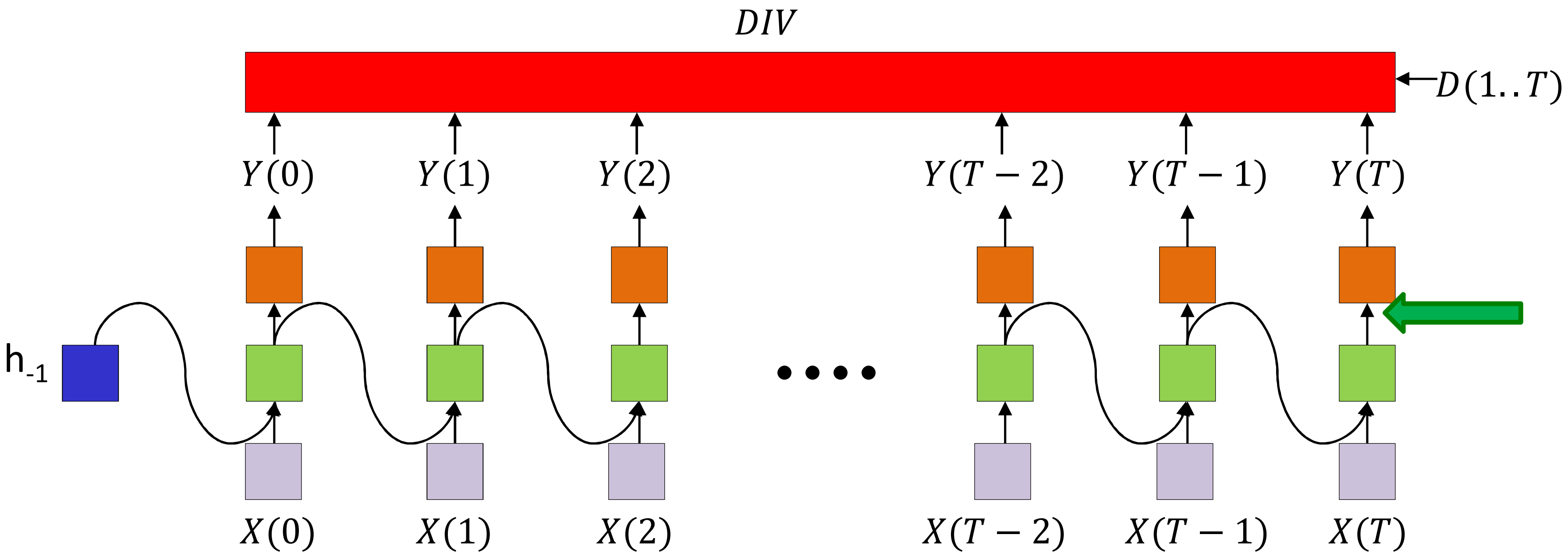
Must compute

$$\frac{dDIV}{dY_i(t)} \text{ for all i for all T}$$

Will usually get

$$\frac{dDIV}{dY_i(t)} = \frac{dDiv(t)}{dY_i(t)}$$

Back Propagation Through Time



First step of backprop: Compute $\frac{dDIV}{dY_i(T)}$ for all i

$$\nabla_{Z^{(1)}(T)} DIV = \nabla_{Y(T)} DIV \nabla_{Z^{(1)}(T)} Y(T)$$

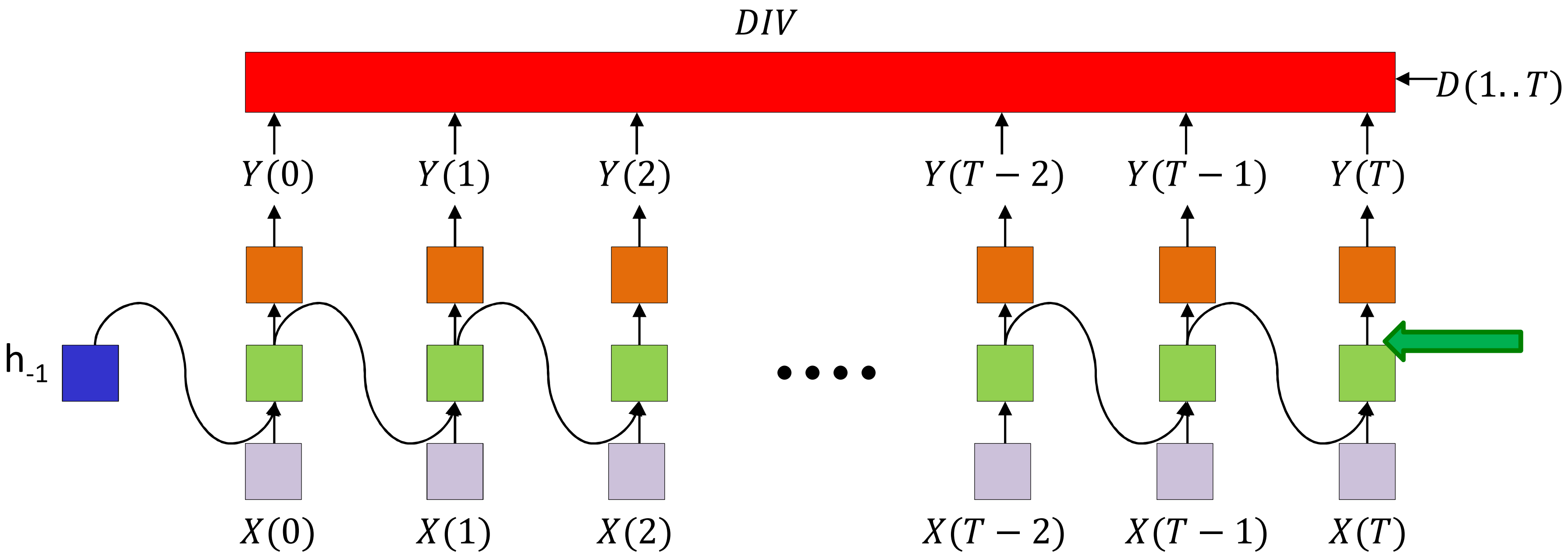
Vector output activation

$$\frac{dDIV}{dZ_i^{(1)}(T)} = \frac{dDIV}{dY_i(T)} \frac{dY_i(T)}{dZ_i^{(1)}(T)}$$

OR

$$\frac{dDIV}{dZ_i(T)} = \sum_j \frac{dDIV}{dY_j(T)} \frac{dY_j(T)}{dZ_j^{(1)}(T)}$$

Back Propagation Through Time



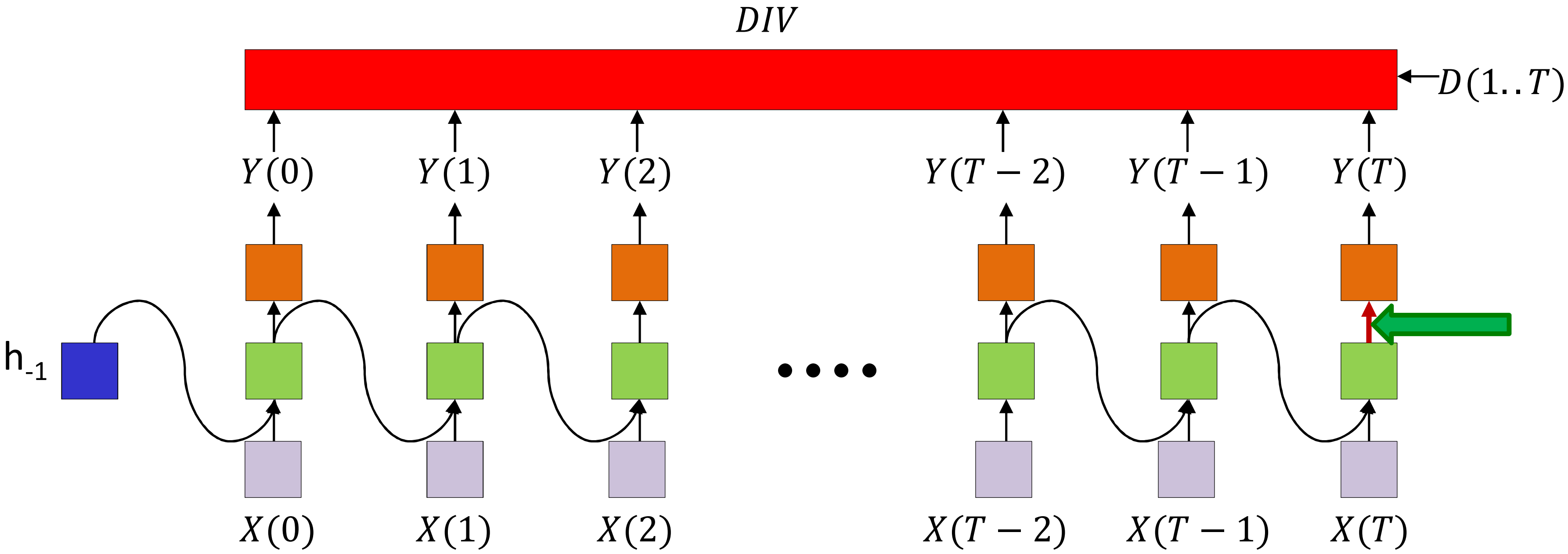
$$\frac{dDIV}{dY_i(T)} \text{ for all i}$$

$$\frac{dDIV}{dZ_i^{(1)}(T)} = \frac{dDiv(T)}{dY_i(T)} \frac{dY_i(T)}{dZ_i^{(1)}(T)}$$

$$\frac{dDIV}{dh_i(T)} = \sum_j \frac{dDIV}{dZ_j^{(1)}(T)} \frac{dZ_j^{(1)}(T)}{dh_i(T)} = \sum_j w_{ij}^{(1)} \frac{dDIV}{dZ_j^{(1)}(T)}$$

$$\nabla_{h(T)}DIV = \nabla_{Z^{(1)}(T)}DIV\; W^{(1)}$$

Back Propagation Through Time



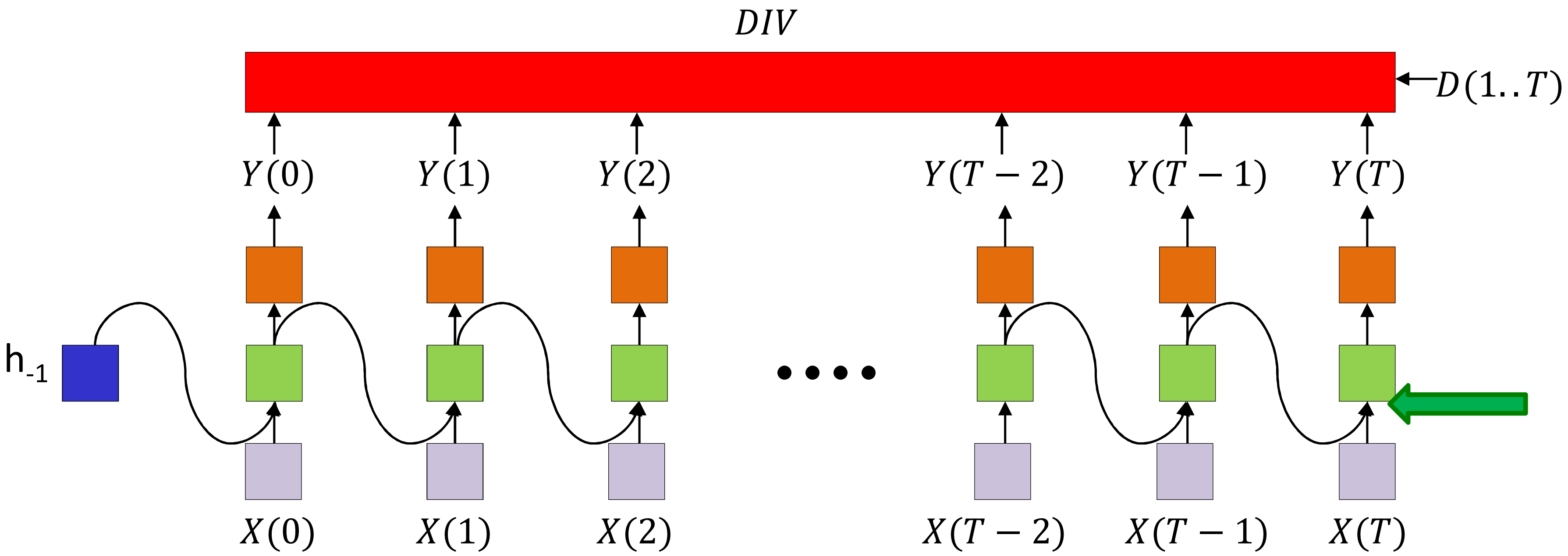
$$\frac{dDIV}{dZ_i^{(1)}(T)} = \frac{dDiv(T)}{dY_i(T)}\frac{dY_i(T)}{dZ_i^{(1)}(T)}$$

$$\frac{dDIV}{dh_i(T)} = \sum_j w_{ij}^{(1)} \frac{dDIV}{dZ_j^{(1)}(T)}$$

$$\frac{dDIV}{dw_{ij}^{(1)}} = \frac{dDIV}{dZ_j^{(1)}(T)} h_i(T)$$

$$\nabla_{W^{(1)}} DIV = h(T) \nabla_{Z^{(1)}(T)} DIV$$

Back Propagation Through Time



$$\nabla_{Z^{(0)}(T)} DIV = \nabla_{h(T)} DIV \, \nabla_{Z^{(0)}(T)} h(T)$$

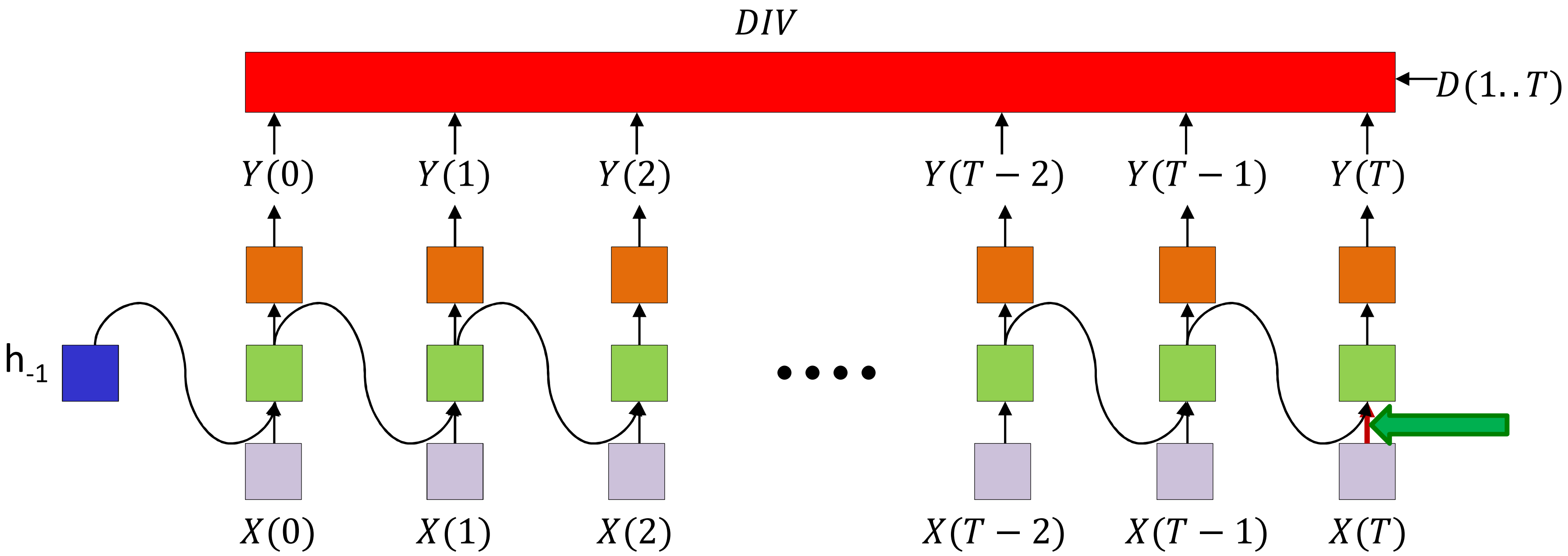
$$\frac{dDIV}{dZ_i^{(0)}(T)} = \frac{dDIV}{dh_i(T)} \frac{dh_i(T)}{dZ_i^{(0)}(T)}$$

$$\frac{dDIV}{dZ_i^{(1)}(T)} = \frac{dDIV}{dY_i(T)} \frac{dY_i(T)}{dZ_i^{(1)}(T)}$$

$$\frac{dDIV}{dh_i(T)} = \sum_j w_{ij}^{(1)} \frac{dDIV}{dZ_j^{(1)}(T)}$$

$$\frac{dDIV}{dw_{ij}^{(1)}} = \frac{dDIV}{dZ_j^{(1)}(T)} h_i(T)$$

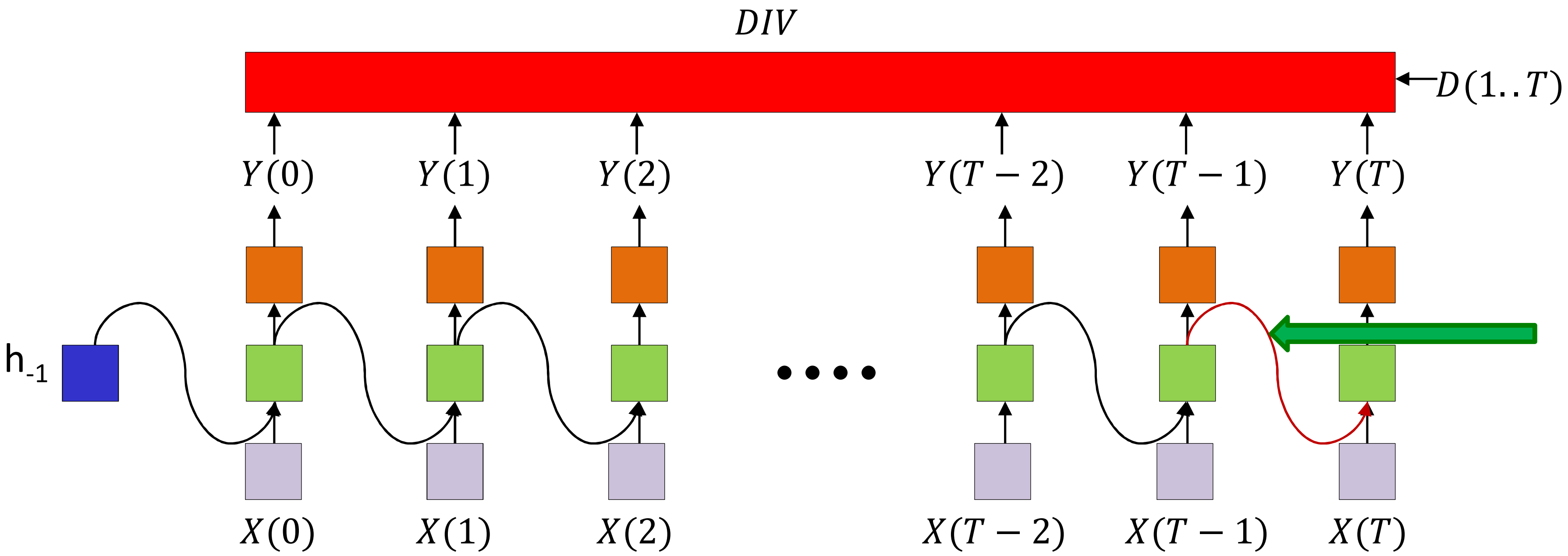
Back Propagation Through Time



$$\frac{dDIV}{dw_{ij}^{(0)}} = \frac{dDIV}{dZ_j^{(0)}(T)} X_i(T)$$

$$\nabla_{W^{(0)}} DIV = X(T) \nabla_{Z^{(0)}(T)} DIV$$

Back Propagation Through Time

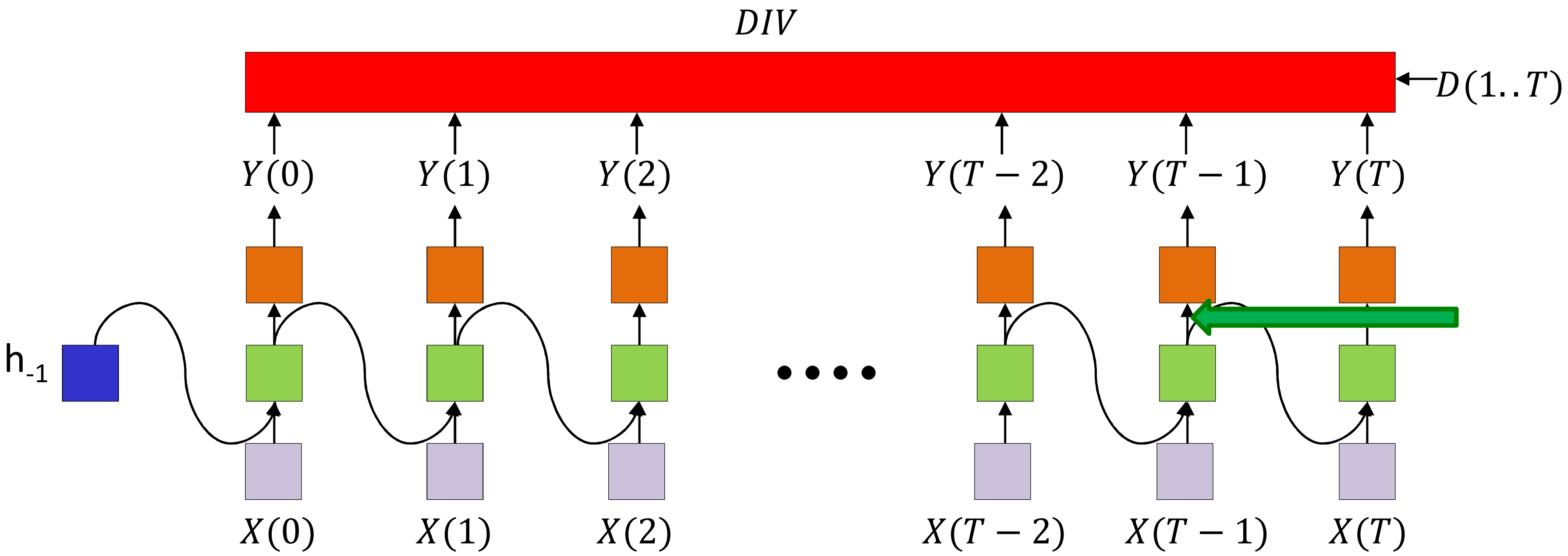


$$\nabla_{W^{(11)}} DIV = h(T-1)\nabla_{Z^{(0)}(T)} DIV$$

$$\frac{dDIV}{dw_{ij}^{(0)}} = \frac{dDIV}{dZ_j^{(0)}(T)} X_i(T)$$

$$\frac{dDIV}{dw_{ij}^{(11)}} = \frac{dDIV}{dZ_j^{(0)}(T)} h_i(T-1)$$

Back Propagation Through Time



$$\nabla_{Z^{(1)}(T-1)}DIV = \nabla_{Y(T-1)}DIV \ \nabla_{Z^{(1)}(T)}Y(T-1)$$

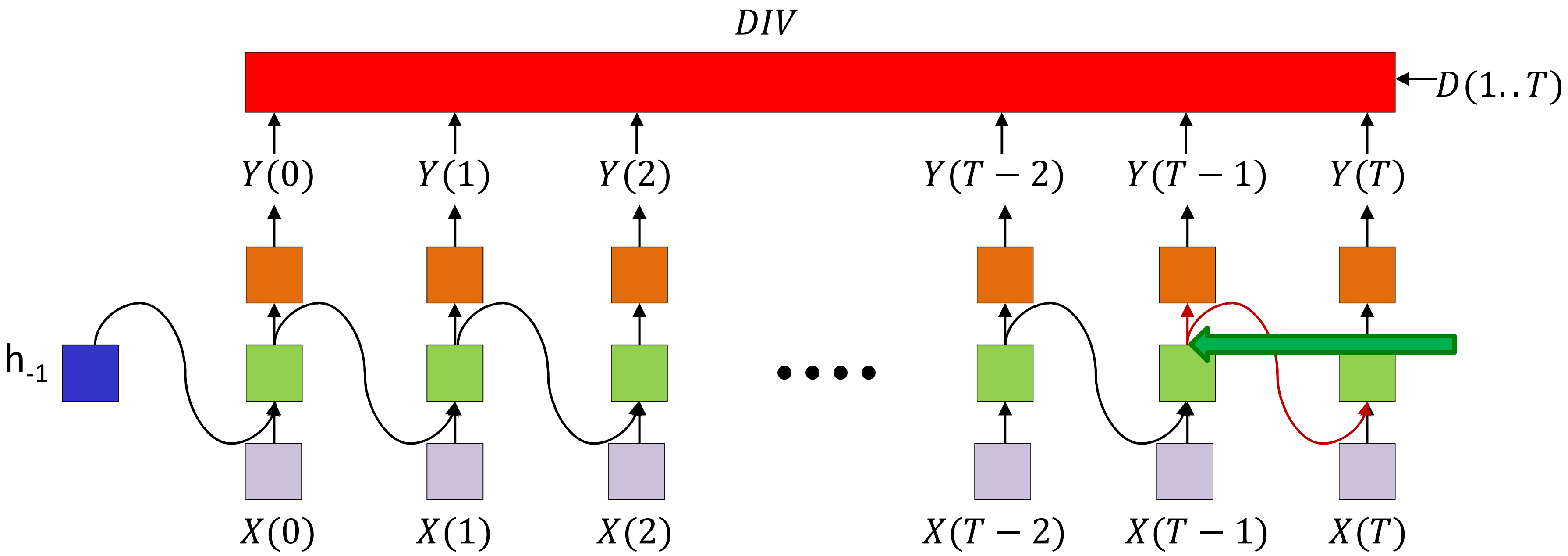
Vector output activation

$$\frac{dDIV}{dZ_i^{(1)}(T-1)} = \frac{dDIV}{dY_i(T-1)}\frac{dY_i(T-1)}{dZ_i^{(1)}(T-1)}$$

OR

$$\frac{dDIV}{dZ_i^{(1)}(T-1)} = \sum_j \frac{dDIV}{dY_j(T-1)}\frac{dY_j(T-1)}{dZ_i^{(1)}(T-1)}$$

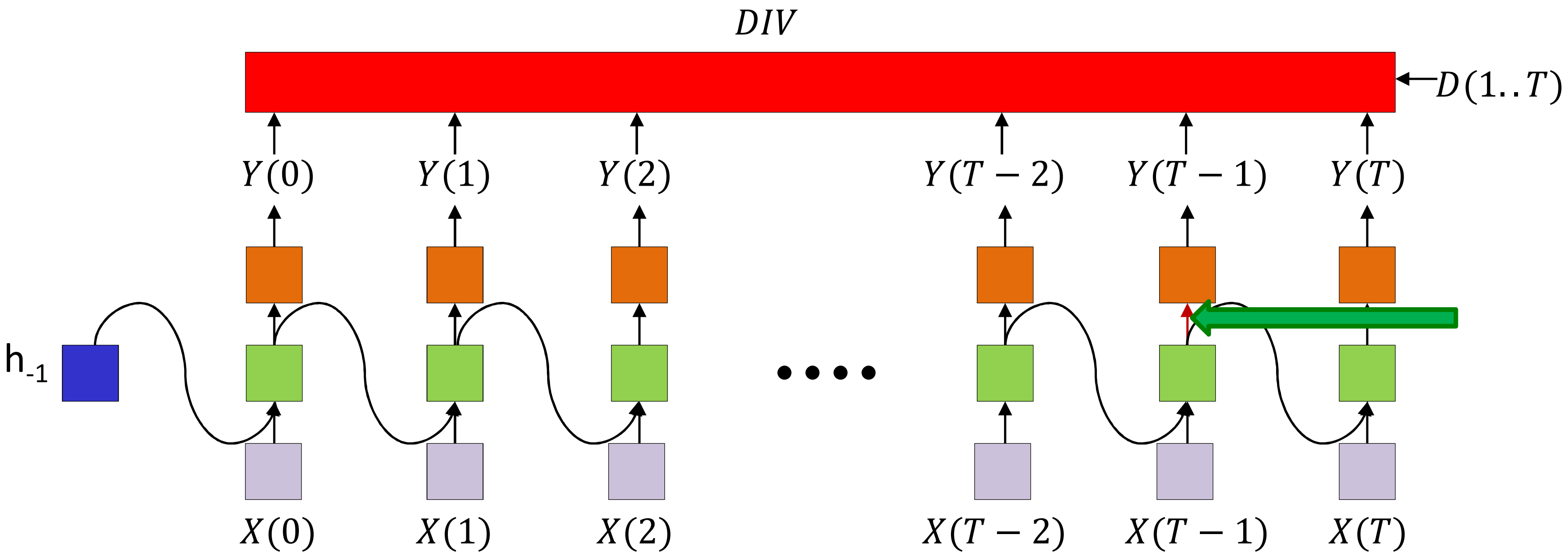
Back Propagation Through Time



$$\frac{dDIV}{dh_i(T-1)} = \sum_j w_{ij}^{(1)} \frac{dDIV}{dZ_j^{(1)}(T-1)} + \sum_j w_{ij}^{(11)} \frac{dDIV}{dZ_j^{(0)}(T)}$$

$$\nabla_{h(T-1)}DIV = \nabla_{Z^{(1)}(T-1)}DIV\ W^{(1)} + \nabla_{Z^{(0)}(T)}DIV\ W^{(11)}$$

Back Propagation Through Time



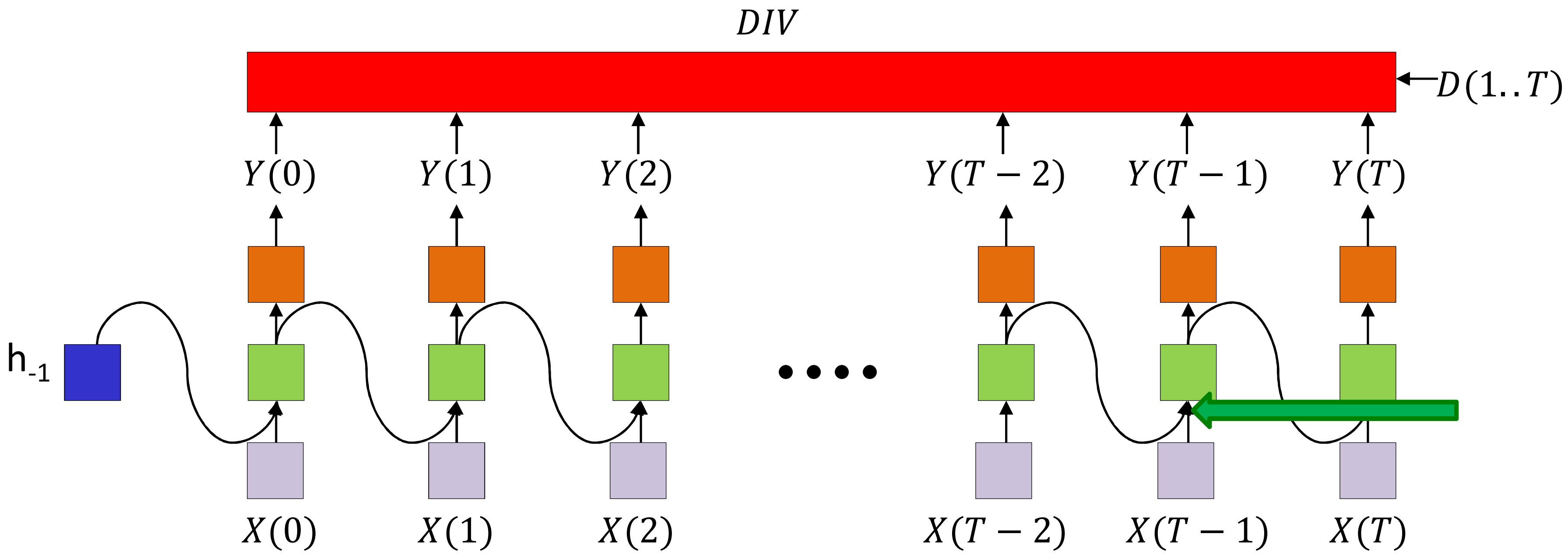
$$\frac{dDIV}{dh_i(T-1)} = \sum_j w_{ij}^{(1)} \frac{dDIV}{dZ_j^{(1)}(T-1)} + \sum_j w_{ij}^{(11)} \frac{dDIV}{dZ_j^{(0)}(T)}$$

Note the addition

$$\frac{dDIV}{dw_{ij}^{(1)}} += \frac{dDIV}{dZ_j^{(1)}(T-1)} h_i(T-1)$$

$$\nabla_{W^{(1)}} DIV += h(T-1)\nabla_{Z^{(1)}(T-1)} DIV$$

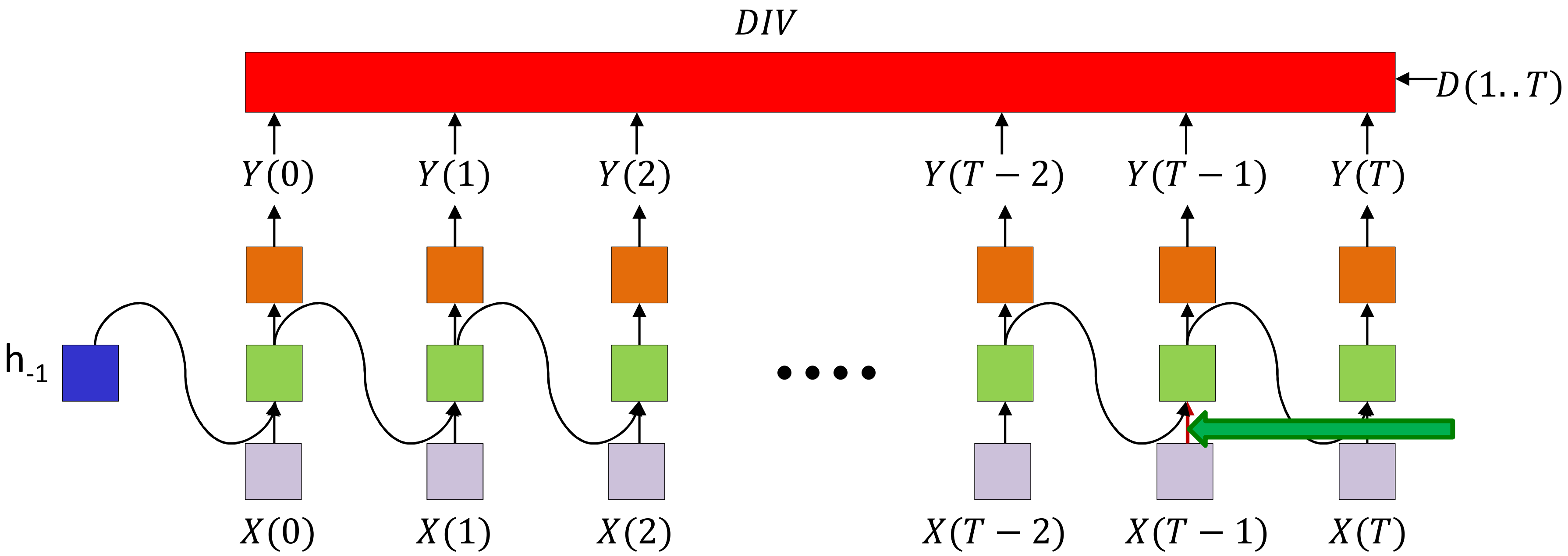
Back Propagation Through Time



$$\boxed{\frac{dDIV}{dZ_i^{(0)}(T-1)} = \frac{dDIV}{dh_i(T-1)} \frac{dh_i(T-1)}{dZ_i^{(0)}(T-1)}}$$

$$\boxed{\nabla_{Z^{(0)}\ (T-1)} DIV = \nabla_{h(T-1)} DIV\ \nabla_{Z^{(0)}(T-1)} h(T-1)}$$

Back Propagation Through Time



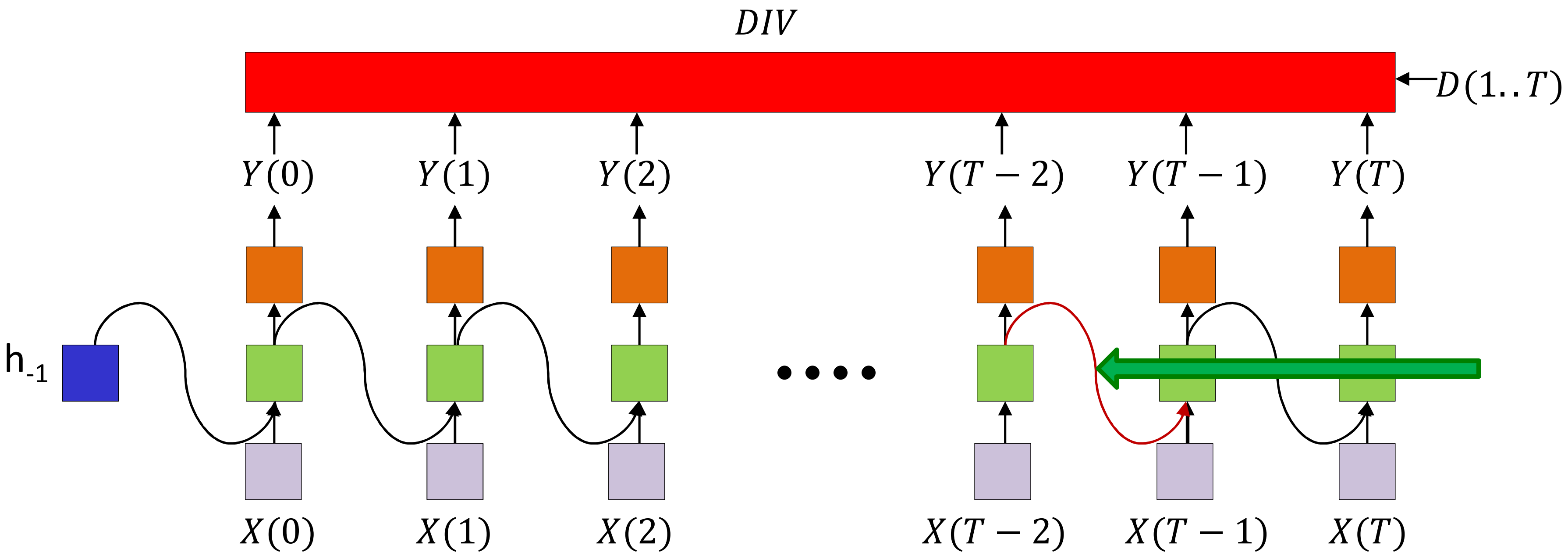
$$\frac{dDIV}{dZ_i^{(0)}(T-1)} = \frac{dDIV}{dh_i(T-1)} \frac{dh_i(T-1)}{dZ_i^{(0)}(T-1)}$$

$$\frac{dDIV}{dw_{ij}^{(0)}} += \frac{dDIV}{dZ_j^{(0)}(T-1)} X_i(T-1)$$

Note the addition

$$\nabla_{W^{(0)}} DIV += X(T-1) \nabla_{Z^{(0)}(T-1)} DIV$$

Back Propagation Through Time



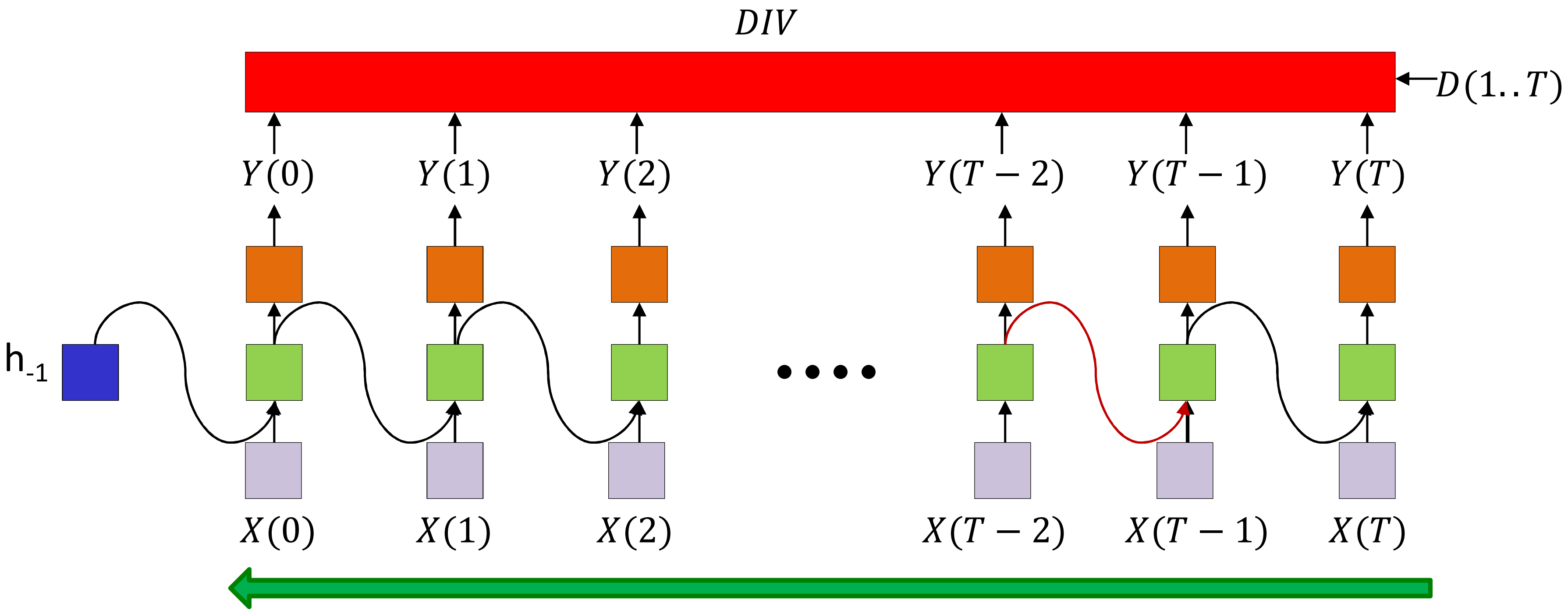
$$\frac{dDIV}{dw_{ij}^{(0)}} += \frac{dDIV}{dZ_j^{(0)}(T-1)} X_i(T-1)$$

Note the addition

$$\frac{dDIV}{dw_{ij}^{(11)}} += \frac{dDIV}{dZ_j^{(0)}(T-1)} h_i(T-2)$$

$$\nabla_{W^{(11)}} DIV += h(T-2) \nabla_{Z^{(0)}(T-1)} DIV$$

Back Propagation Through Time

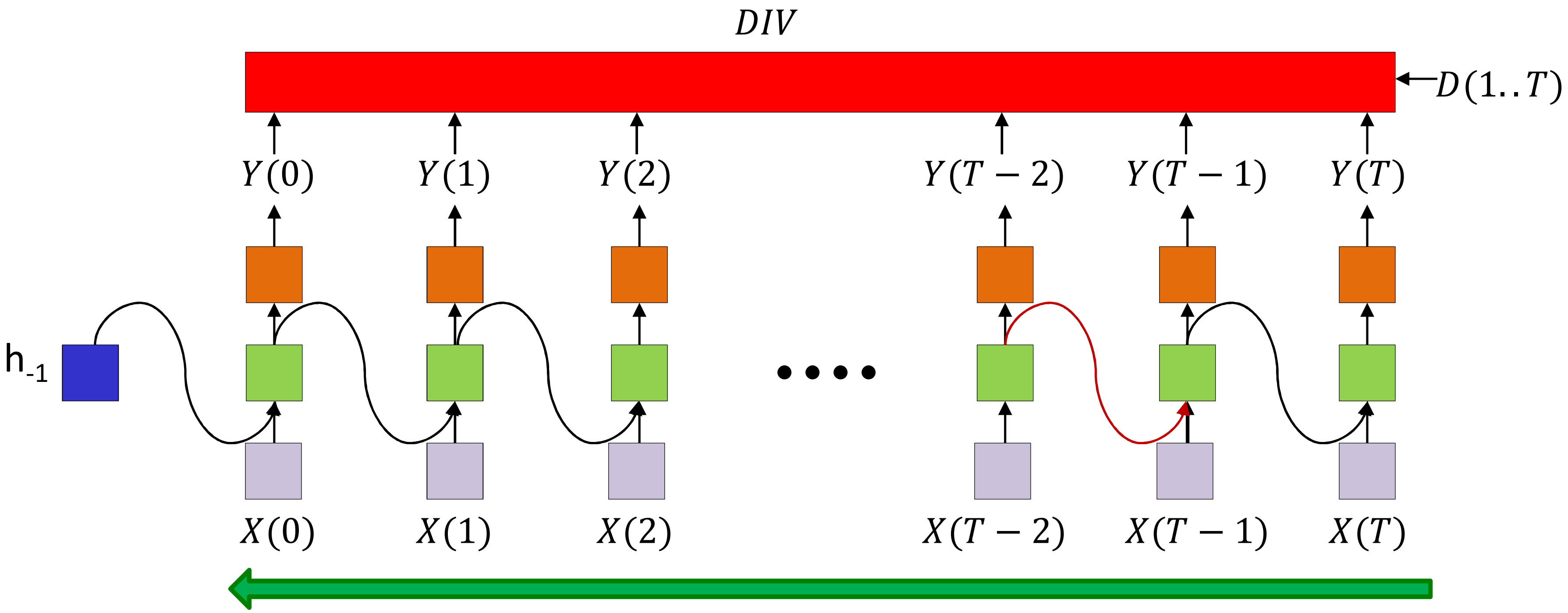


Continue computing derivatives going backward through time until..

$$\frac{dDIV}{dh_{-1}} = \sum_j w_{ij}^{(11)} \frac{dDIV}{dZ_j^{(1)}(0)}$$

$$\nabla_{h_{-1}} DIV = \nabla_{Z^{(1)}(0)} DIV W^{(11)}$$

Back Propagation Through Time

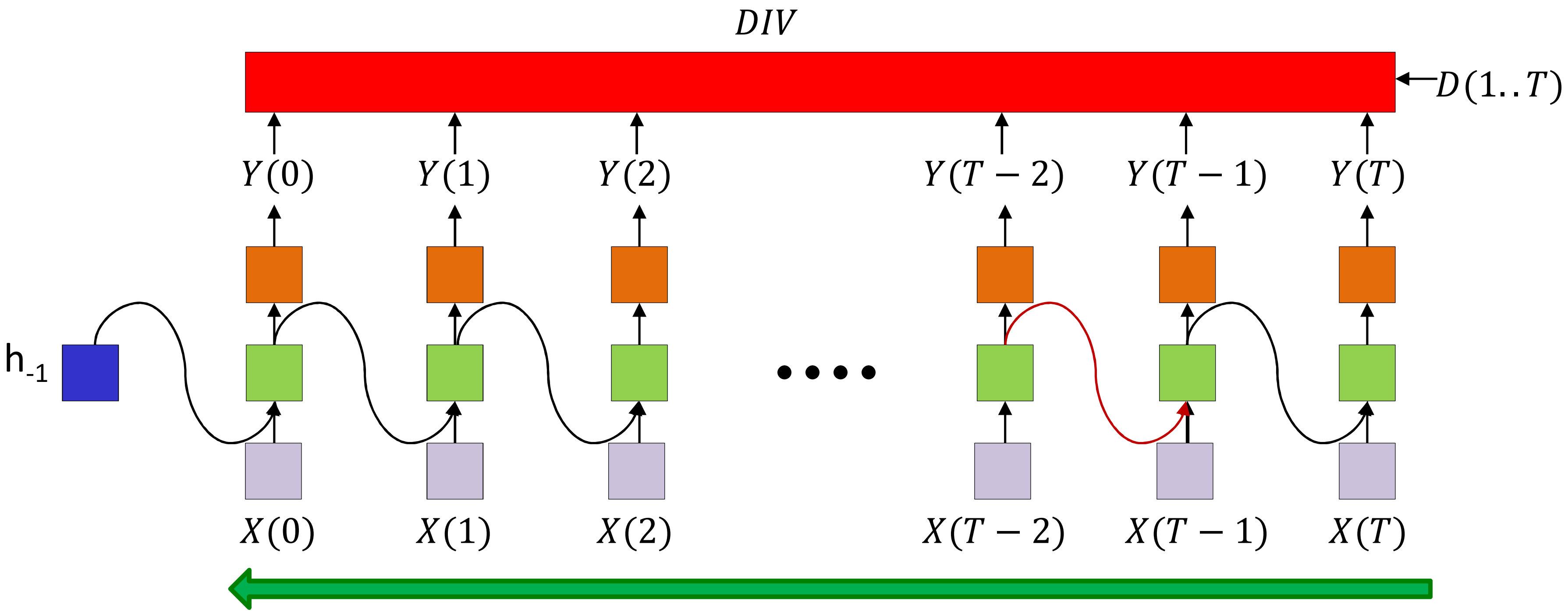


$$\boxed{\frac{dDIV}{dh_i^{(k)}(t)} = \sum_j w_{i,j}^{(k)} \frac{dDIV}{dZ_j^{(k+1)}(t)} + \sum_j w_{i,j}^{(k,k)} \frac{dDIV}{dZ_j^{(k)}(t+1)}}$$

Not showing derivatives
at output neurons

$$\boxed{\frac{dDIV}{dZ_i^{(k)}(t)} = \frac{dDIV}{dh_i^{(k)}(t)} f_k'\left(Z_i^{(k)}(t)\right)}$$

Back Propagation Through Time



$$\frac{dDIV}{dh_{-1}} = \sum_j w_{ij}^{(11)} \frac{dDIV}{dZ_j^{(1)}(0)}$$

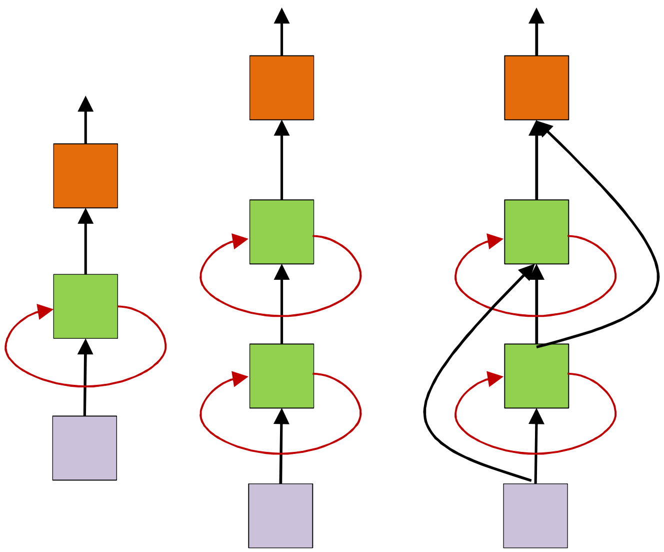
$$\frac{dDIV}{dw_{ij}^{(0)}} = \sum_t \frac{dDIV}{dZ_j^{(0)}(t)} X_i(t)$$

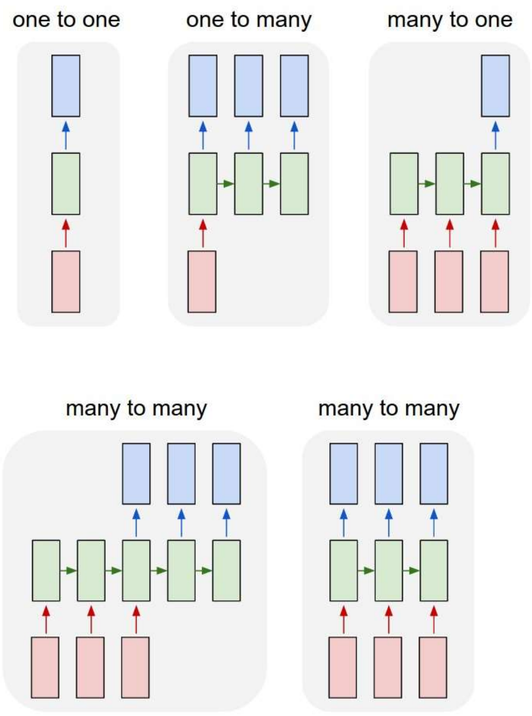
$$\frac{dDIV}{dw_{ij}^{(11)}} = \sum_t \frac{dDIV}{dZ_j^{(0)}(t)} h_i(t-1)$$

BPTT

```
# Assuming forward pass has been completed
# Jacobian(x,y) is the jacobian of x w.r.t. y
# Assuming dY(t) = gradient(div,Y(t)) available for all t
# Assuming all dz, dh, dW and db are initialized to 0

for t = T-1:downto:0  # Backward through time
    $dz_o(t)$ = dY(t)Jacobian(Y(t),$z_o(t)$)
    $dW_o$ += h(t,L)$dz_o(t)$
    db(L) += $dz_o(t)$
    dh(t,L) += $dz_o(t)W_o$

    for l = L:1  # Reverse through layers
        dz(t,l) = dh(t,l)Jacobian(h(t,l),z(t,l))
        dh(t,l-1) += dz(t,l) $W_c(l)$
        dh(t-1,l) += dz(t,l) $W_r(l)$

        $dW_c(l)$ += h(t,l-1)dz(t,l)
        $dW_r(l)$ += h(t-1,l)dz(t,l)
        db(l) += dz(t,l)
```

BPTT

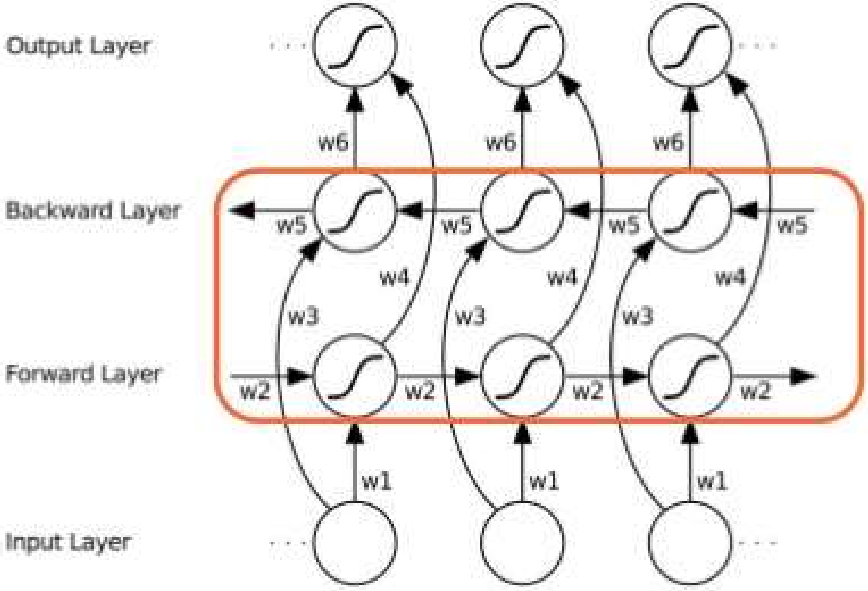




- Can be generalized to any architecture

Extensions to the RNN: *Bidirectional RNN*

Bidirectional RNN (BRNN)



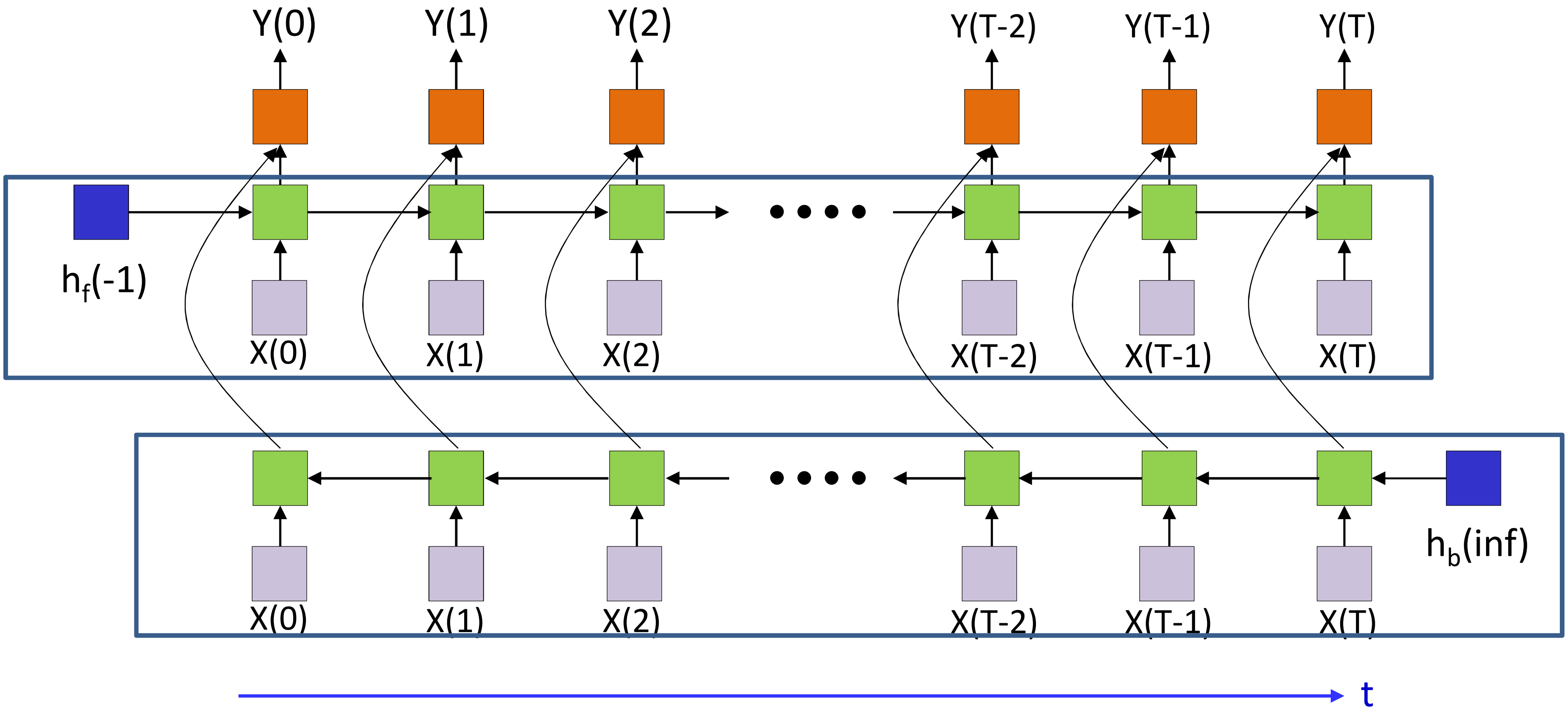
Must learn weights w2, w3, w4 & w5; in addition to w1 & w6.

Proposed by Schuster and Paliwal 1997

Alex Graves, "Supervised Sequence Labelling with Recurrent Neural Networks"

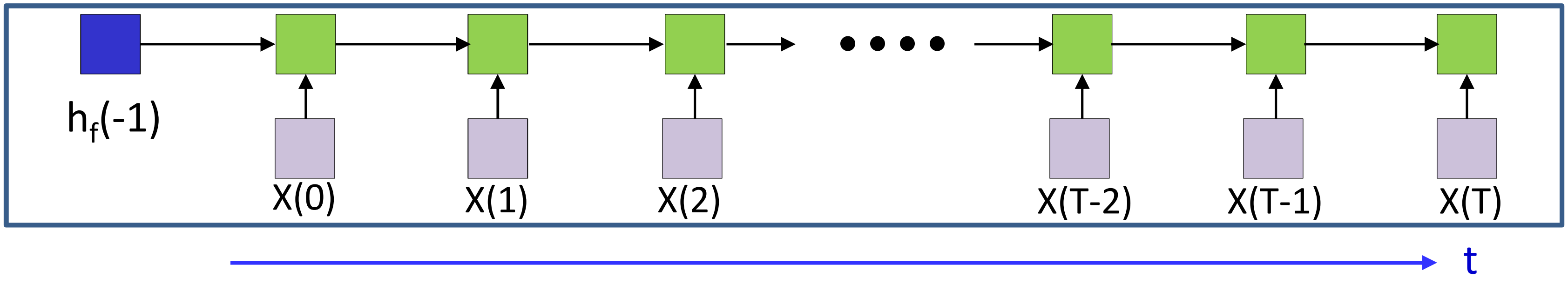
- RNN with both forward and backward recursion
  - Explicitly models the fact that just as the future can be predicted from the past, the past can be deduced from the future

Bidirectional RNN



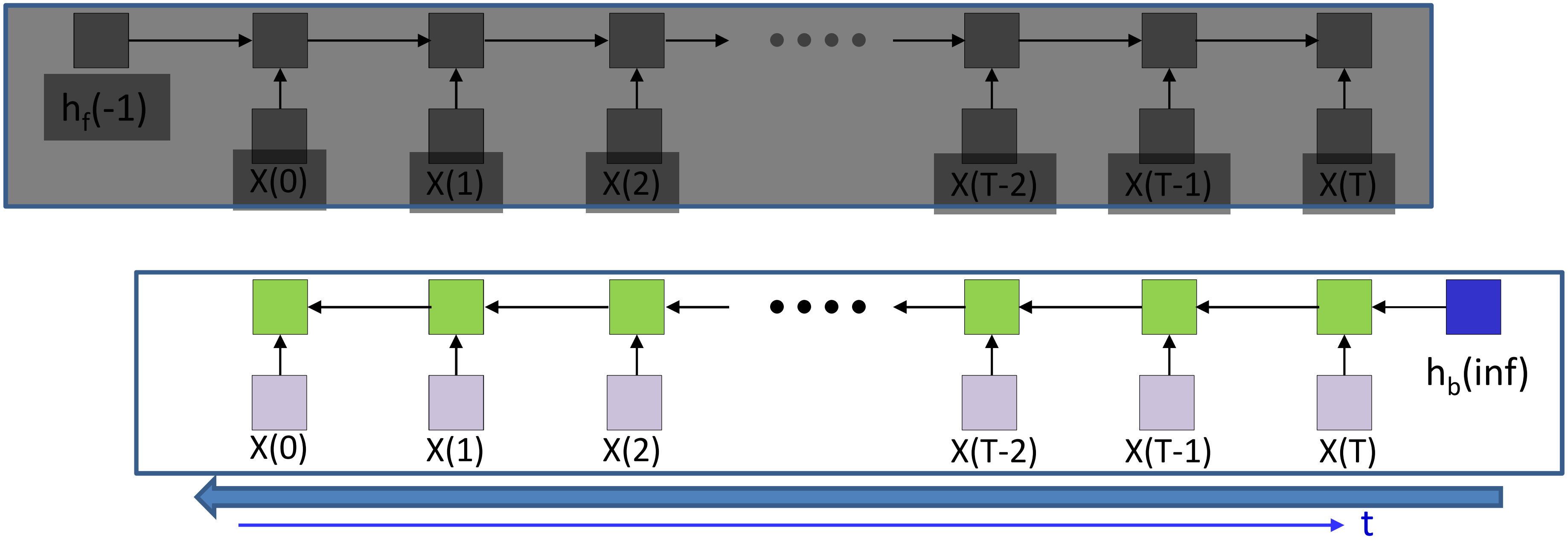
- A forward net process the data from t=0 to t=T
- A backward net processes it backward from t=T down to t=0

Bidirectional RNN: Processing an input string



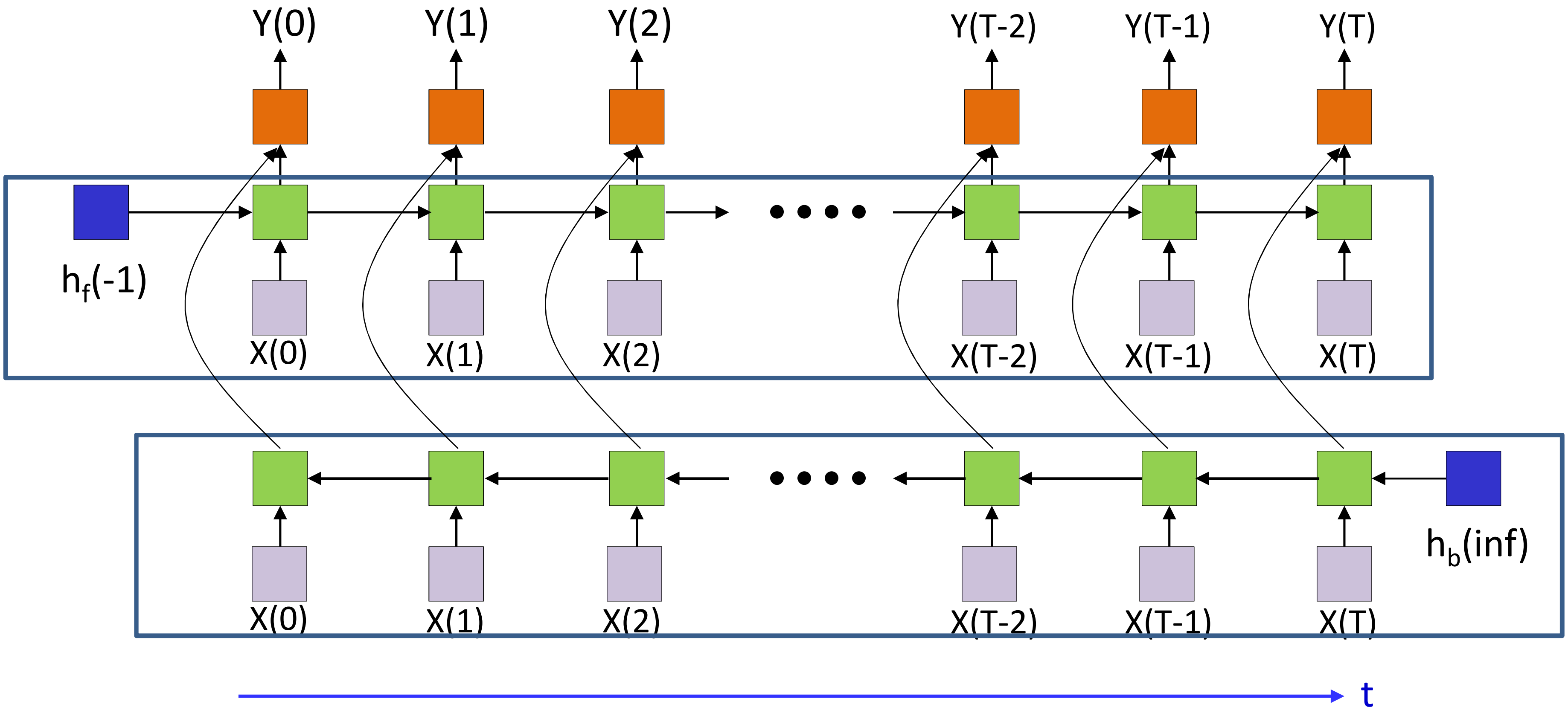
- The forward net process the data from t=0 to t=T
  - Only computing the hidden states, initially

Bidirectional RNN: Processing an input string



- The backward nets processes the input data in *reverse time,* end to beginning
  - Initially only the hidden state values are computed
    - Clearly, this is not an online process and requires the *entire* input data
  - Note: *This is not the backward pass of backprop.*

Bidirectional RNN: Processing an input string



- The computed states of both networks are used to compute the final output at each time

Bidirectional RNN
Assuming time-synchronous output

```
# Subscript f represents forward net, b is backward net
# Assuming $h_f(-1,*)$ and $h_b(inf,*)$ are known

#forward pass
for t = 0:T-1 # Going forward in time
    $h_f(t,0) = x(t)$ # Vectors. Initialize h(0) to input
    for l = 1:$L_f$  # $L_f$ is depth of forward network hidden layers
        $z_f(t,l) = W_{fc}(l)h_f(t,l-1) + W_{fr}(l)h_f(t-1,l) + b_f(l)$
        $h_f(t,l) = tanh(z_f(t,l))$ # Assuming tanh activ.

#backward
h(T,:,:) = h(inf,:,:) # Just the initial value
for t = T-1:downto:0 # Going backward in time
    $h_b(t,0) = x(t)$ # Vectors. Initialize h(0) to input
    for l = 1:$L_b$ # $L_b$ is depth of backward network hidden layers
        $z_b(t,l) = W_{bc}(l)h_b(t,l-1) + W_{br}(l)h(t+1,l) + b_b(l)$
        $h_b(t,l) = tanh(z_b(t,l))$ # Assuming tanh activ.

for t = 0:T-1  # The output combines forward and backward
     $z_o(t) = W_{fo}h_f(t,L_f) +  W_{bo}h_b(t,L_b) + b_o$
     Y(t) = softmax( $z_o(t)$ )
```

Bidirectional RNN: Simplified code

- Code can be made modular and simplified for better interpretability…

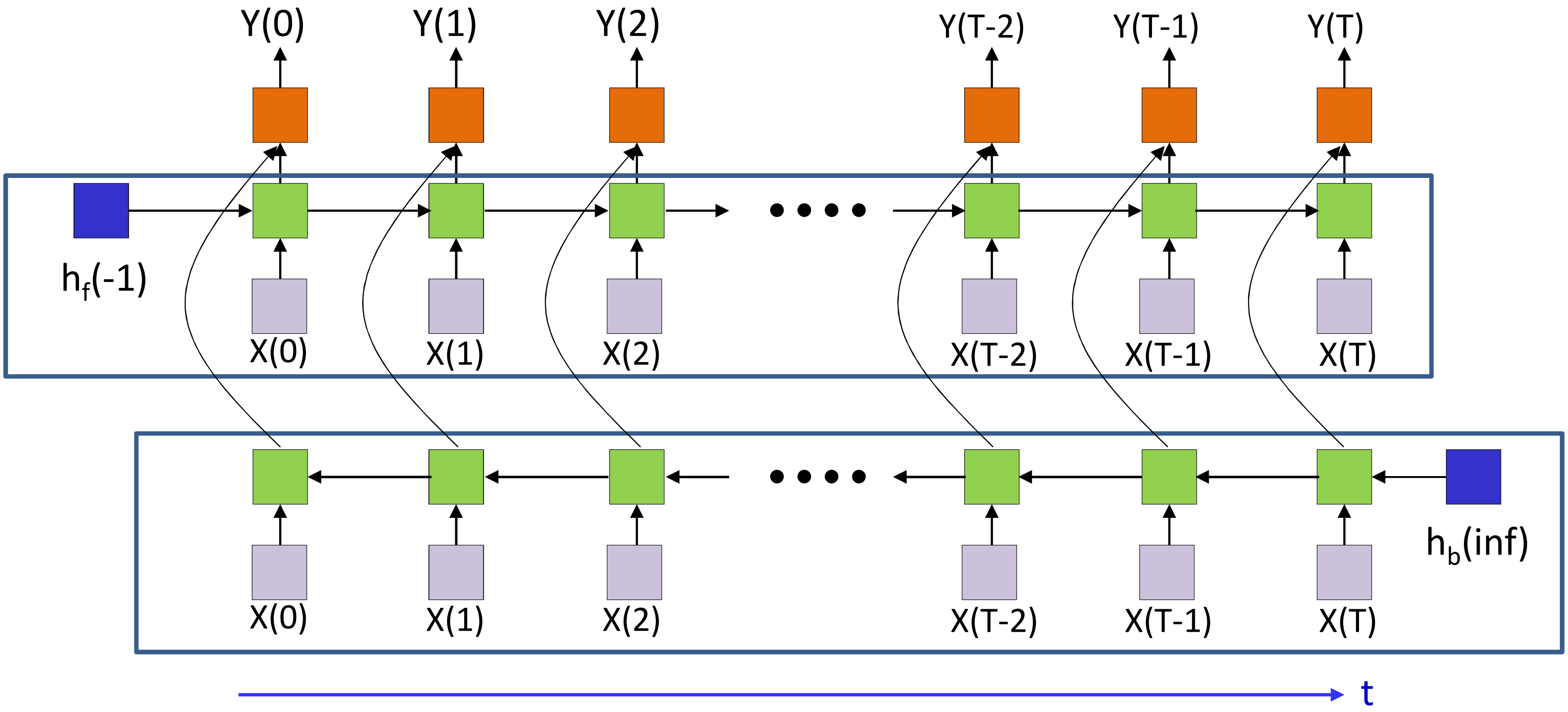
First: Define basic RNN with only hidden units

```
# Inputs:
#    L : Number of hidden layers
#    $W_c$,$W_r$,b: current weights,  recurrent weights, biases
#    hinit:  initial value of h(representing h(-1,*))
#    x: input vector sequence
#    T: Length of input vector sequence
# Output:
#    h, z: sequence of pre-and post activation hidden
#          representations from all layers of the RNN

function [h,z] = RNN_forward(L, $W_c$, $W_r$, b, hinit, x, T)
    h(-1,:) = hinit  # hinit is the initial value for all layers
    for t = 0:T-1 # Going forward in time
        h(t,0) = x(t) # Vectors. Initialize h(0) to input
        for l = 1:L
            z(t,l) = $W_c$(l)h(t,l-1) + $W_r$(l)h(t-1,l) + b(l)
            h(t,l) = tanh(z(t,l)) # Assuming tanh activ.
    return h,z
```

Bidirectional RNN
Assuming time-synchronous output

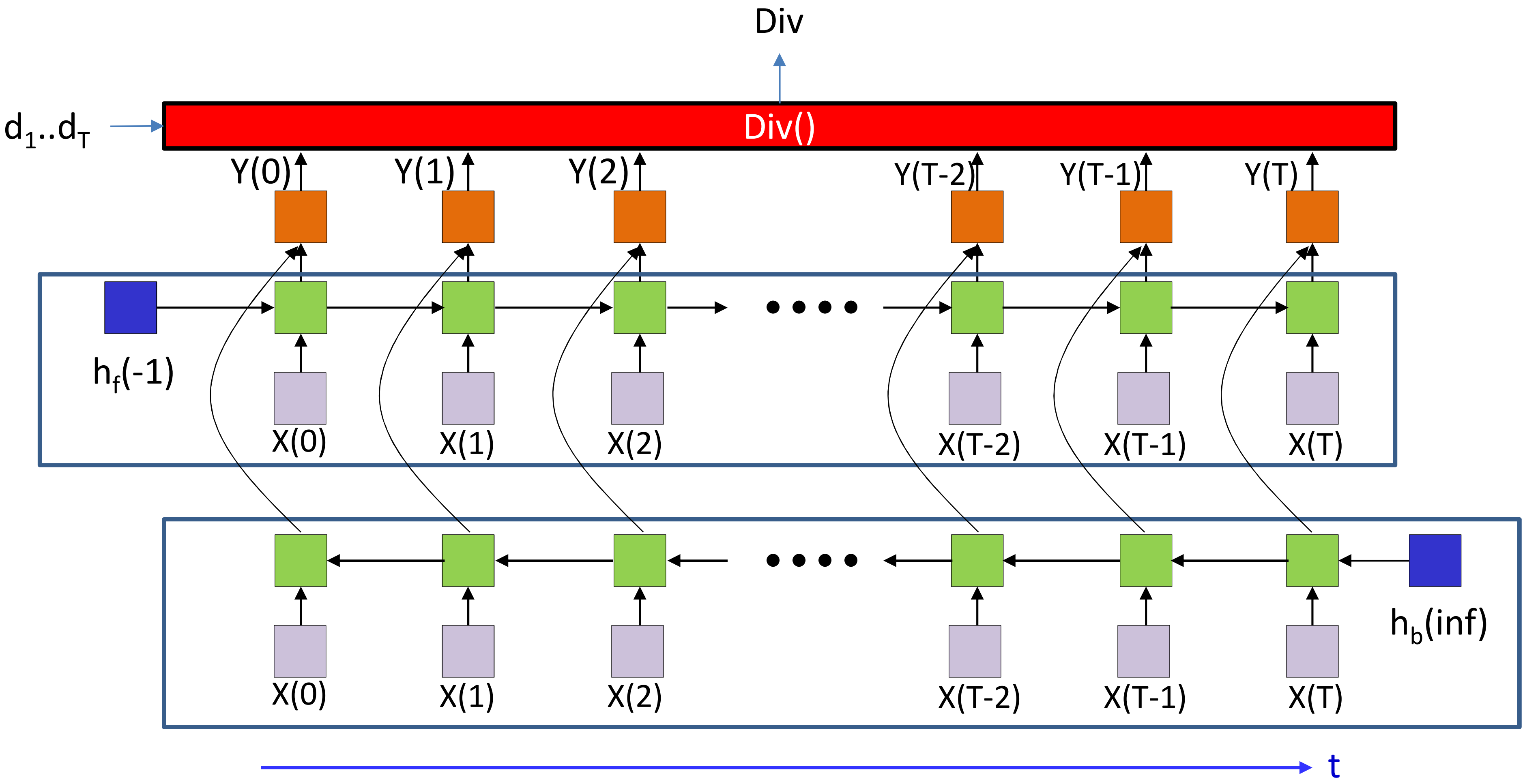
```
# Subscript f represents forward net, b is backward net
# Assuming $h_f(-1,*)$ and $h_b(inf,*)$ are known

#forward pass
[$h_f$, $z_f$] = RNN_forward($L_f$, $W_{fc}$, $W_{fr}$, $b_f$, h(-1,:), x, T)

#backward pass
$x_{rev}$ = fliplr(x)   # Flip it in time
[$h_{brev}$, $z_{brev}$] = RNN_forward($L_b$, $W_{bc}$, $W_{br}$, $b_b$, h(inf,:), $x_{rev}$, T)
$h_b$ = fliplr($h_{brev}$)   # Flip back to straighten time
$z_b$ = fliplr($z_{brev}$)

#combine the two for the output
for t = 0:T-1   # The output combines forward and backward
    $z_o(t) = W_{fo}h_f(t,L_f) + W_{bo}h_b(t,L_b) + b_o$
    Y(t) = softmax( $z_o(t)$ )
```

Backpropagation in BRNNs



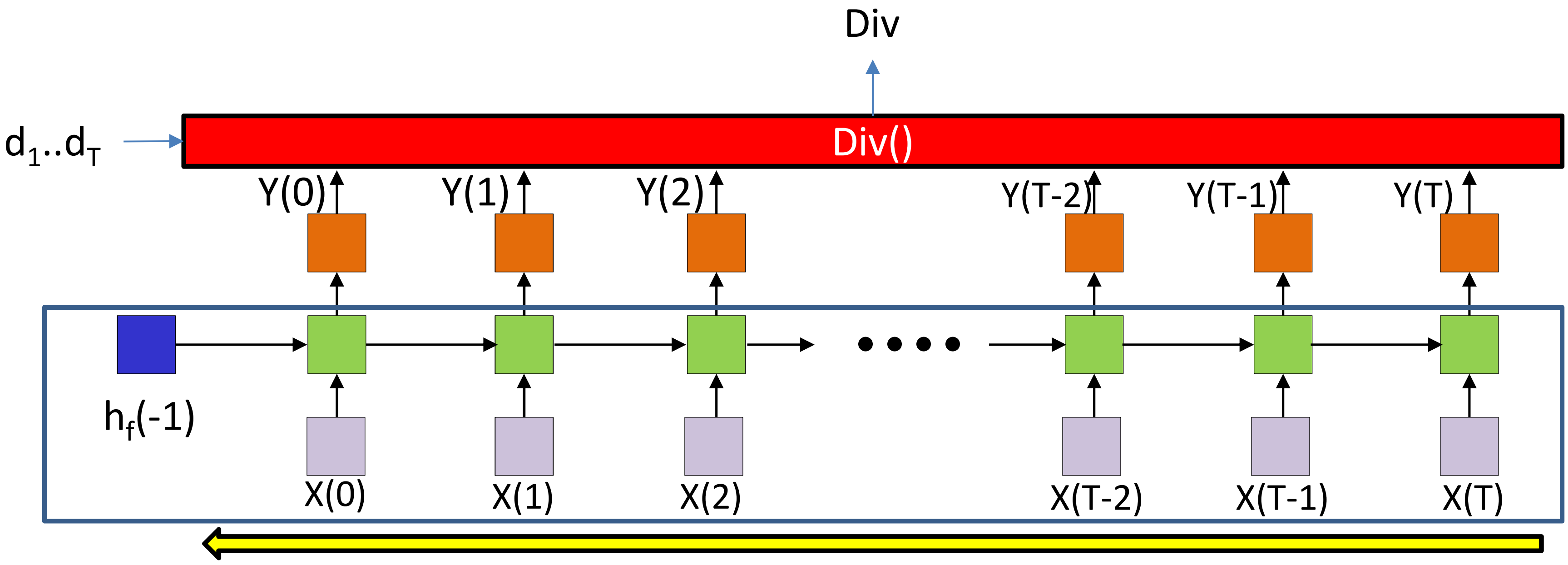
- Forward pass: Compute both forward and backward networks and final output

Backpropagation in BRNNs



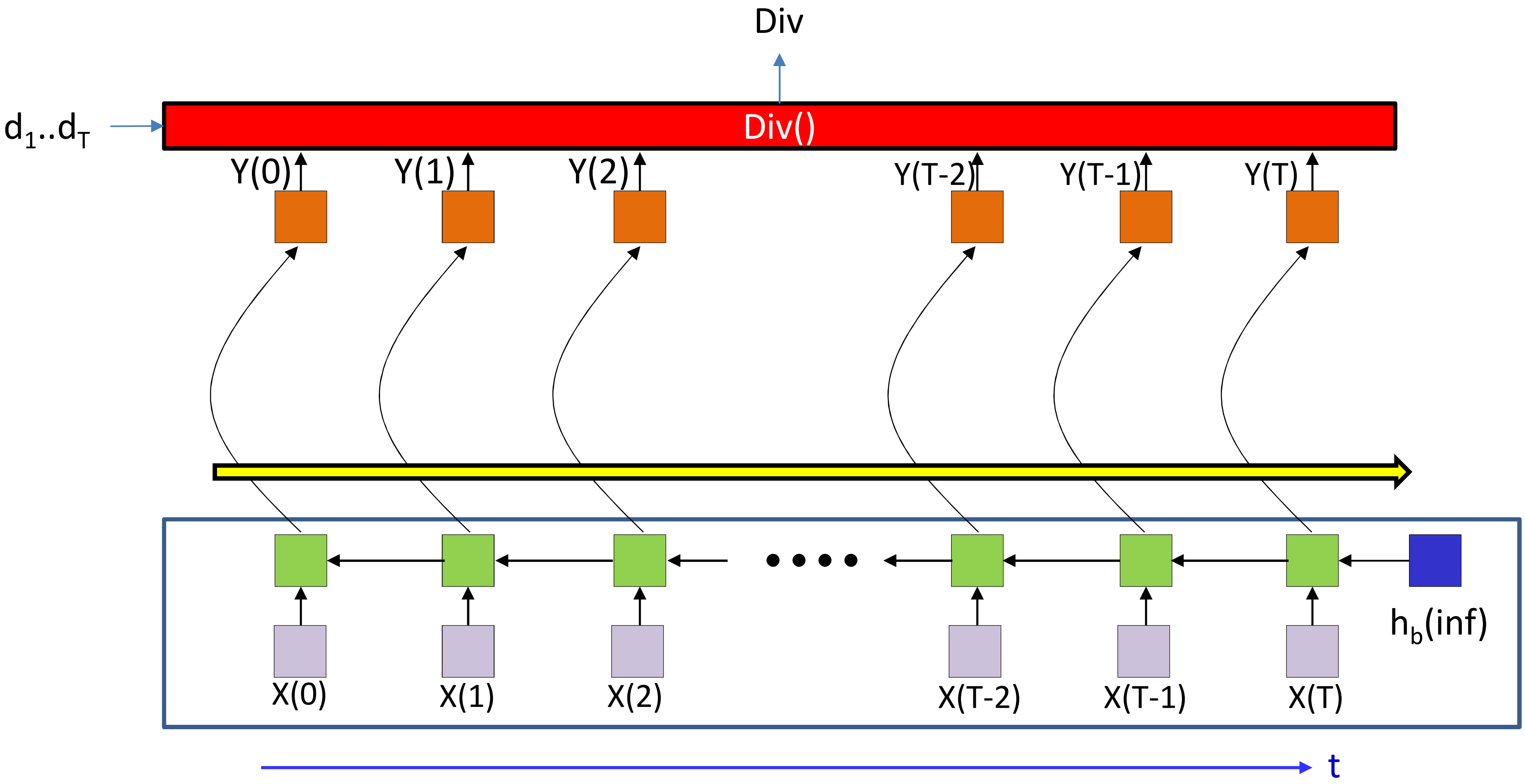
- Backward pass: Define a divergence from the desired output

Backpropagation in BRNNs



- Backward pass: Define a divergence from the desired output
- Separately perform back propagation on both nets
  - **From t=T down to t=0 for the forward net**

Backpropagation in BRNNs



- Backward pass: Define a divergence from the desired output
- Separately perform back propagation on both nets
  - From t=T down to t=0 for the forward net
  - **From t=0 up to t=T for the backward net**

Backpropagation: Pseudocode

- As before we will use a 2-step code:
  - A basic backprop routine that we will call
  - Two calls to the routine within a higher-level wrapper

First: backprop through a recurrent net

```
# Inputs:
#     (In addition to inputs used by L : Number of hidden layers
#    $dh_{top}$:  derivatives ddiv/$dh_*$(t,L) at each time (* may be f or b)
#    h, z:  h and z values returned by the forward pass
#    T: Length of input vector sequence
# Output:
#    $dW_c$, $dW_b$, db $dh_{init}$: derivatives w.r.t current and recurrent weights,
#                           biases, and initial h.
# Assuming all dz, dh, $dW_c$, $dW_r$ and db are initialized to 0

function [$dW_c$,$dW_r$,db,$dh_{init}$] = RNN_bptt(L, $W_c$, $W_r$, b, hinit, x, T, $dh_{top}$, h, z)
    dh = zeros
    for t = T-1:downto:0  # Backward through time
        dh(t,L) += $dh_{top}$(t)
        for l = L:1  # Reverse through layers
            dz(t,l) = dh(t,l)Jacobian(h(t,l),z(t,l))
            dh(t,l-1) += dz(t,l) $W_c$(l)
            dh(t-1,l) += dz(t,l) $W_r$(l)

            $dW_c$(l) += h(t,l-1)dz(t,l)
            $dW_r$(l) += h(t-1,l)dz(t,l)
            db(l) += dz(t,l)

    return dWc, dWr, db, dh(-1)  # dh(-1) is actually dh(-1,1:L,:)
```

Bi-RNN gradient computatoin
Assuming time-synchronous output

```
# Subscript f represents forward net, b is backward net
# First compute derivatives that directly relate to dY(t) for all t,
# then pass the derivatives into RNN_bptt to compute forward and backward
# parameter derivatives

for t = 0:T-1  # The output combines forward and backward
    $dz_o(t)$ = dY(t)Jacobian(Y(t),$z_o(t)$)
    $dh_{fo}(t)$ = $dz_o(t)W_{fo}$
    $dh_{bo}(t)$ = $dz_o(t)W_{bo}$
    $db_o$  += $dz_o(t)$
    $dW_{fo}$ += $h_f(t,L)dz_o(t)$
    $dW_{bo}$ += $h_b(t,L)dz_o(t)$

#forward net
[$dW_{fc}$,$dW_{fr}$,$db_f$,$dh_f(-1)$] = RNN_bptt(L, $W_{fc}$, $W_{fr}$, $b_f$, $h_f(-1)$, x, T, $dh_{fo}$, $h_f$, $z_f$)
#backward net
$x_{rev}$ = fliplr(x)  # Flip it in time
[$dW_{bc}$,$dW_{br}$,$db_b$,$dh_b$(inf)] = RNN_bptt(L, $W_{bc}$, $W_{br}$, $b_b$, $h_b$(inf), $x_{rev}$, T, $dh_{bo}$, $h_b$, $z_b$)
```

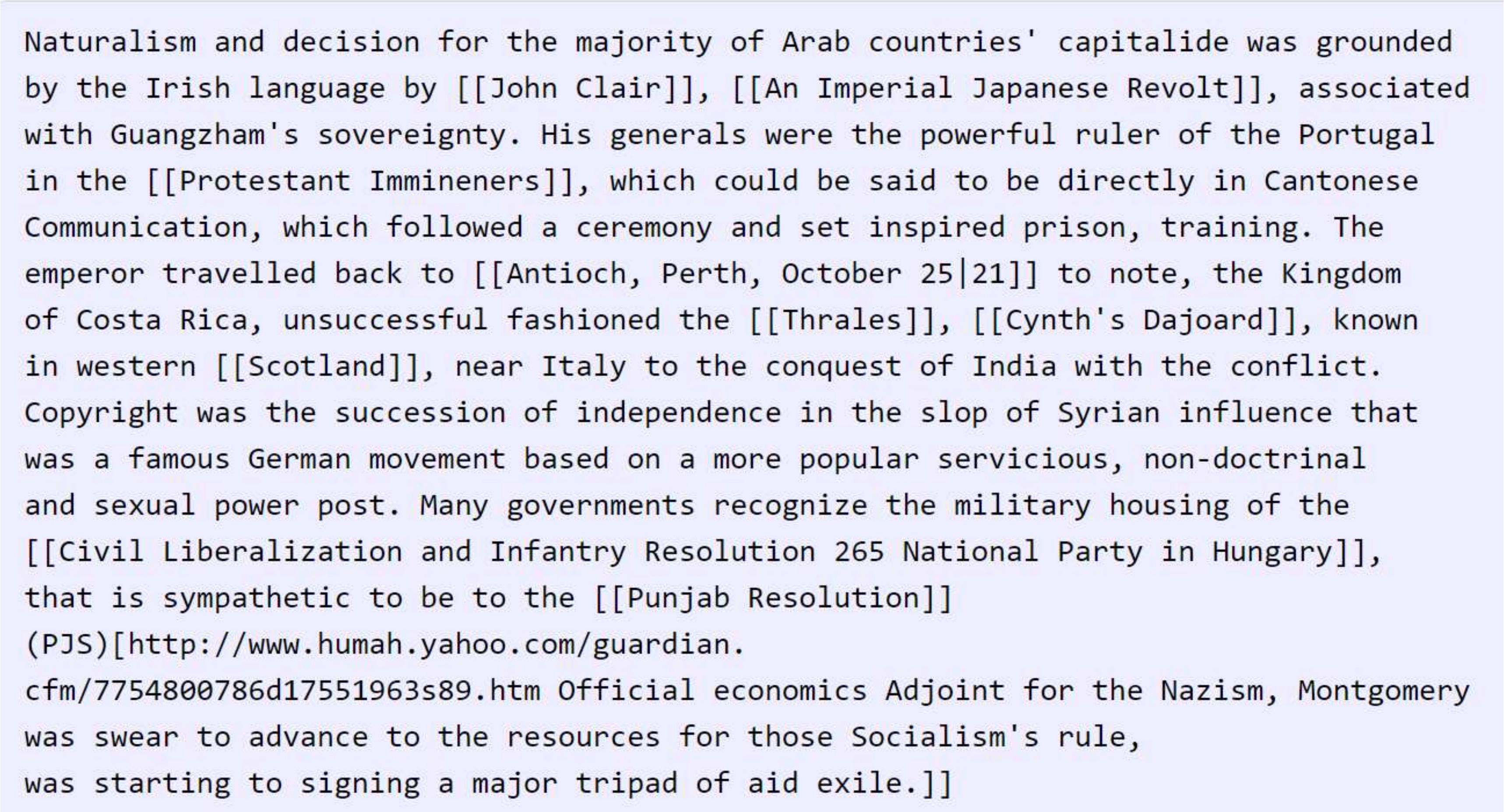
Story so far

- Time series analysis must consider past inputs along with current input
- Recurrent networks look into the infinite past through a state-space framework
  - Hidden states that recurse on themselves
- Training recurrent networks requires
  - Defining a divergence between the actual and desired output *sequences*
  - Backpropagating gradients over the entire chain of recursion
    - Backpropagation through time
  - Pooling gradients with respect to individual parameters over time
- Bidirectional networks analyze data both ways, begin→end and end→beginning to make predictions
  - In these networks, backprop must follow the chain of recursion (and gradient pooling) separately in the forward and reverse nets

RNNs..

- Excellent models for time-series analysis tasks
  - Time-series prediction
  - Time-series classification
  - Sequence prediction..

So how did this happen



```
Naturalism and decision for the majority of Arab countries' capitalide was grounded
by the Irish language by [[John Clair]], [[An Imperial Japanese Revolt]], associated
with Guangzham's sovereignty. His generals were the powerful ruler of the Portugal
in the [[Protestant Immineners]], which could be said to be directly in Cantonese
Communication, which followed a ceremony and set inspired prison, training. The
emperor travelled back to [[Antioch, Perth, October 25|21]] to note, the Kingdom
of Costa Rica, unsuccessful fashioned the [[Thrales]], [[Cynth's Dajoard]], known
in western [[Scotland]], near Italy to the conquest of India with the conflict.
Copyright was the succession of independence in the slop of Syrian influence that
was a famous German movement based on a more popular servicious, non-doctrinal
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cfm/7754800786d17551963s89.htm Official economics Adjoint for the Nazism, Montgomery
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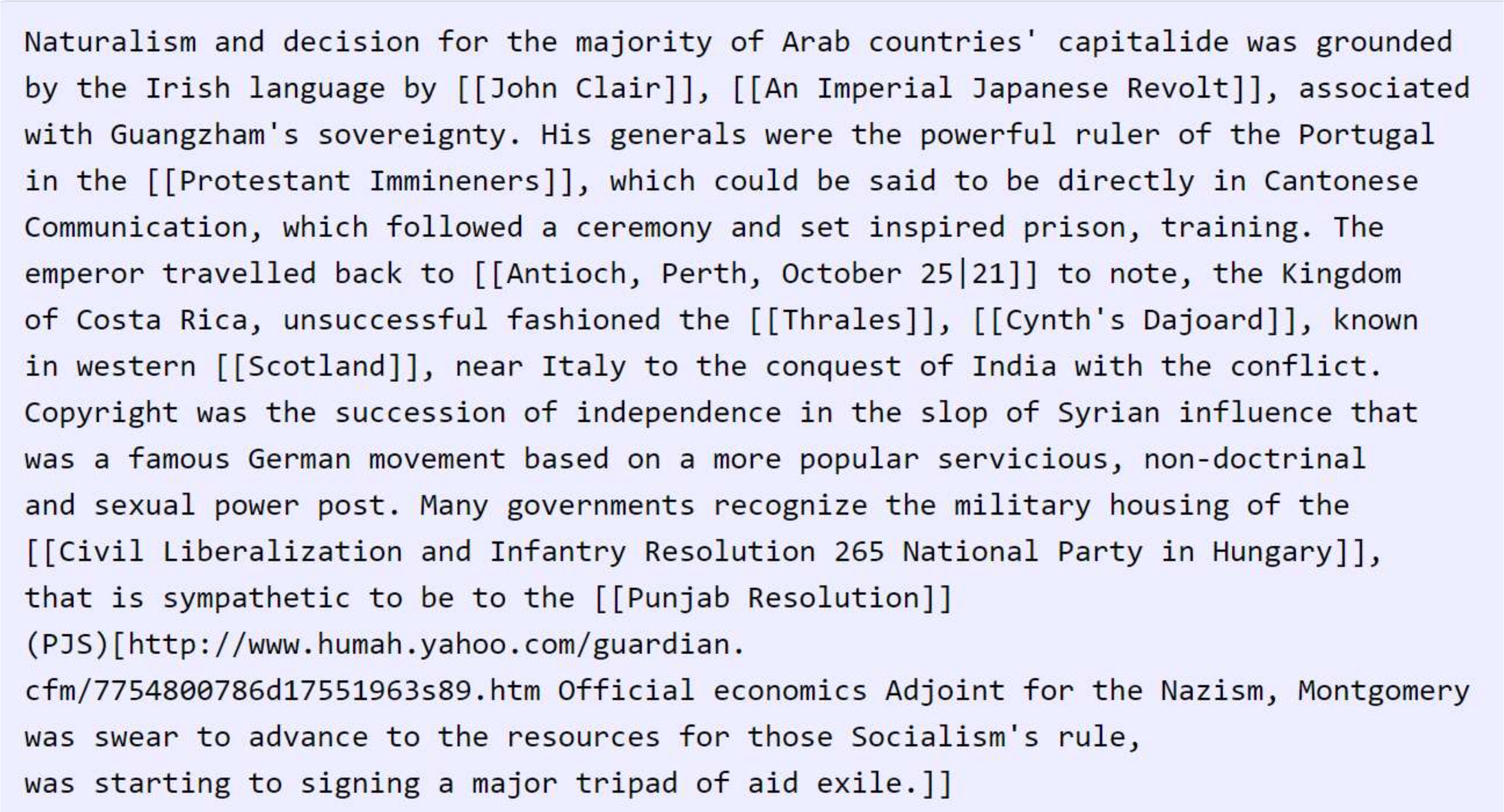

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```

More on this later..

RNNs..

- Excellent models for time-series analysis tasks
  - Time-series prediction
  - Time-series classification
  - Sequence prediction..
  - They can even simplify some problems that are difficult for MLPs
    - Next class..