Recaps, boosting, face detection

Class 10. 24 Sep 2009

Instructor: Bhiksha Raj

Administrivia: Projects

- Only 3 groups so far
  - Plus two individuals
  - Total of 15 people
- Notify us about your teams by tomorrow
  - Or at least that you are *trying* to form a team
  - Otherwise, on 1st we will assign teams by lots
- Inform us about the project you will be working on
  - Only 4 projects so far

Administrivia: Homeworks

- First homework will be returned to you on 6th
  - Still waiting for the elusive late submissions J
  - Scoring will be completed before that

- Second homework:
  - If you are getting bad results, do not be surprised
  - This is not a great technique
  - A somewhat better technique will be tried for part 2 of the homework
    - Will be put up by Thursday

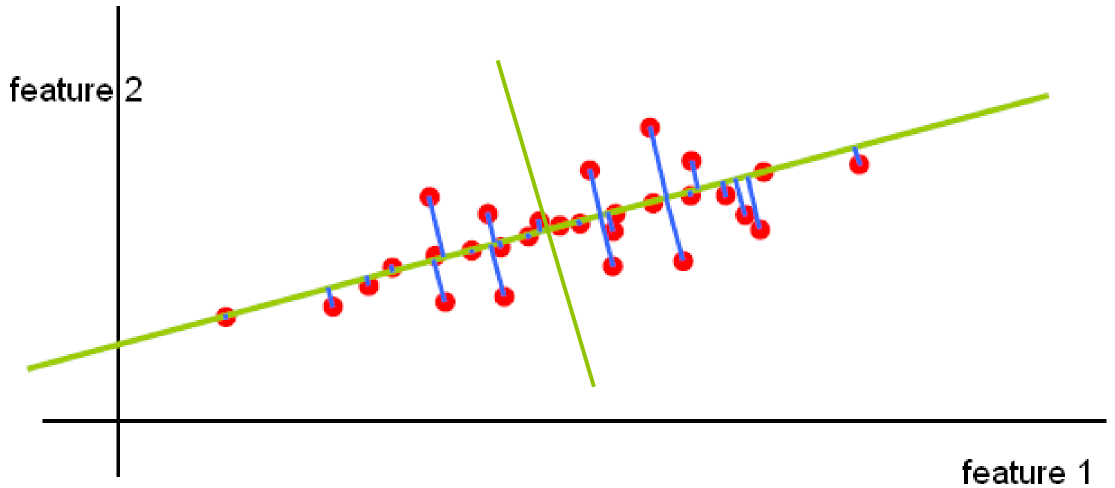
Lecture by Paris Smaragdis

- Thursday.
- Independent Component Analysis and applications to audio

- Seminar by Paris on Friday
  - 3.30 PM, GHC 4303
  - Title “**Making Machines Listen**”
  - Do not miss!
  - Posters can be found in Porter, Wean, Hammerschlag and Roberts

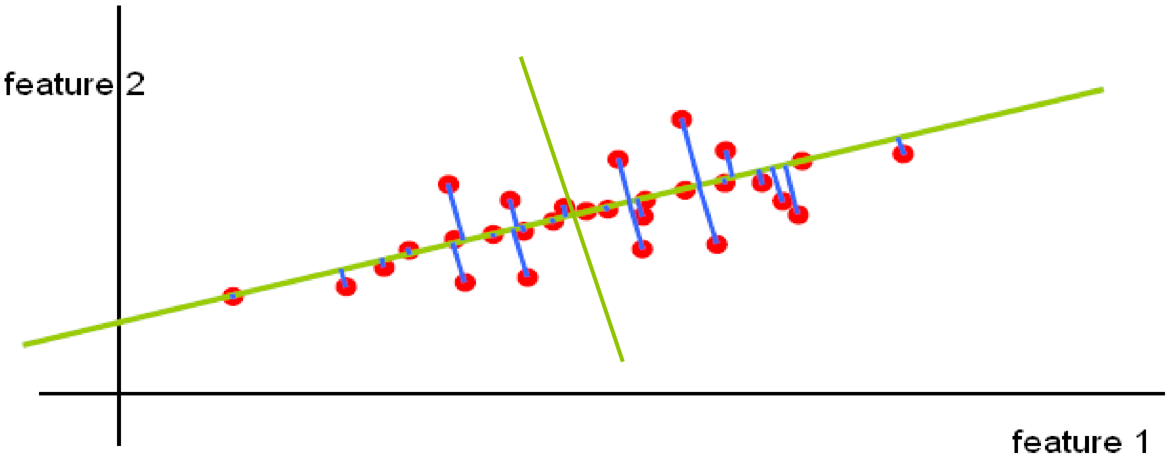
RECAP

Principal Component Analysis



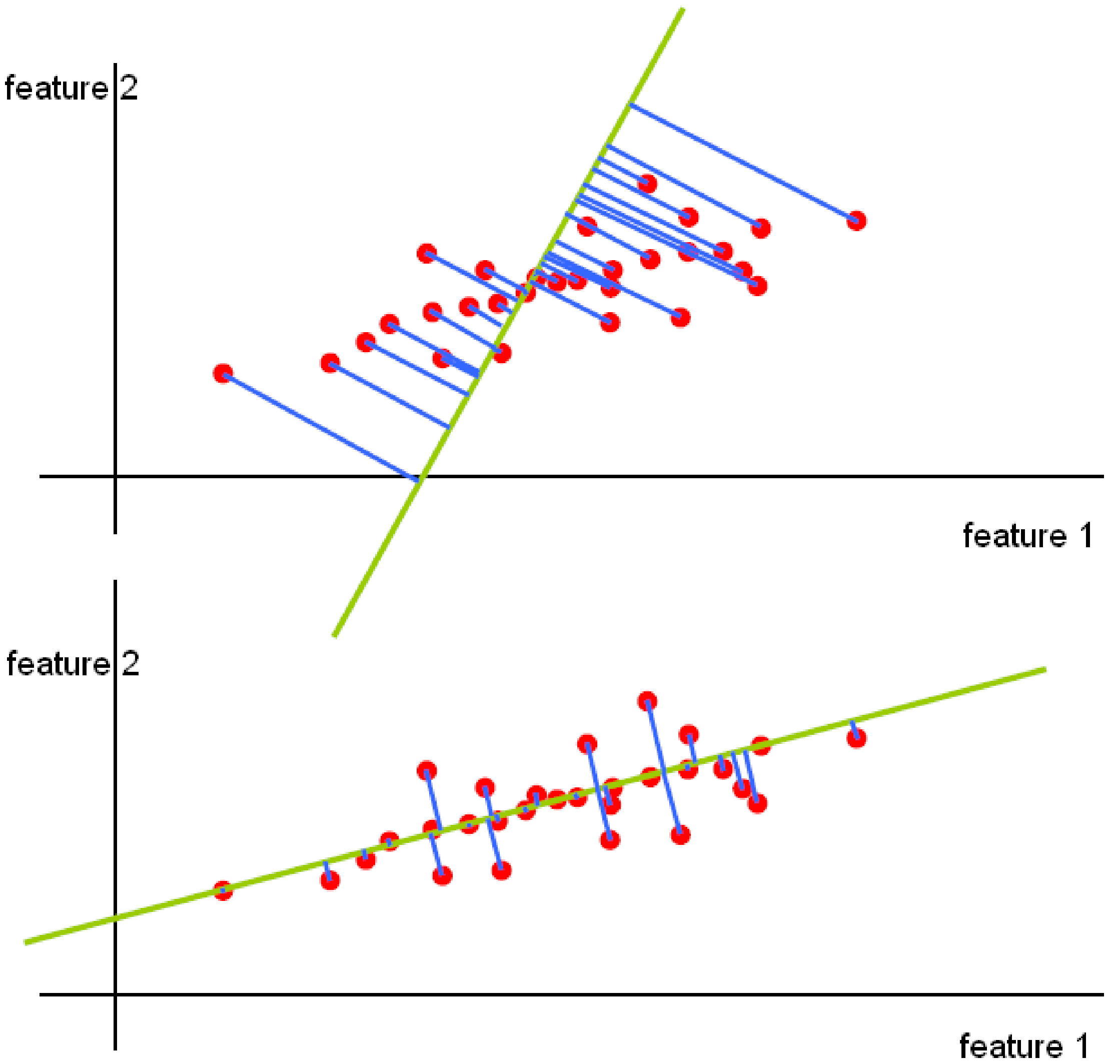
- Computing the "Principal" directions of a data
  - What do they mean
  - Why do we care

Principal Components == Eigen Vectors



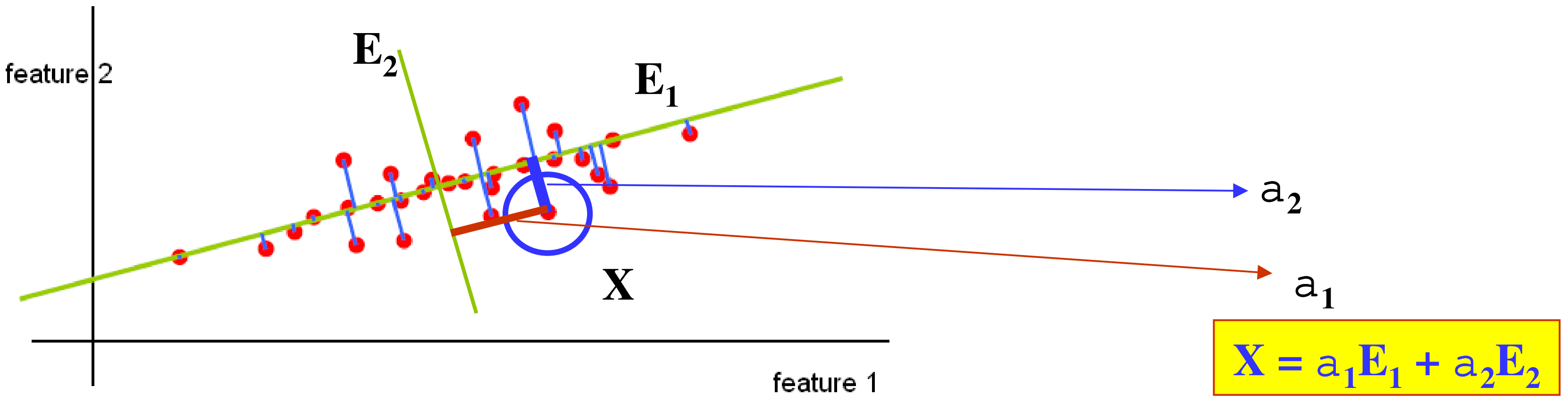
- Principal Component Analysis is the same as Eigen analysis
- The “Principal Components” are the Eigen Vectors
- Again, what are Eigen Vectors?

Principal Component Analysis



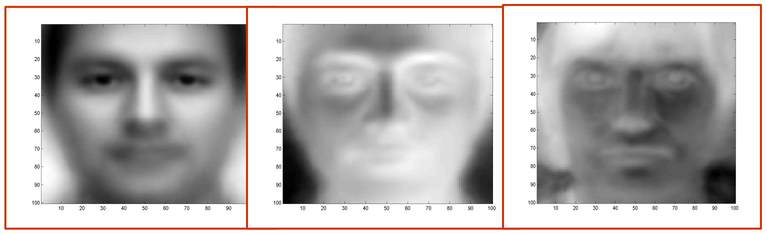
Which line through the mean leads to the smallest reconstruction error (sum of squared lengths of the blue lines) ?

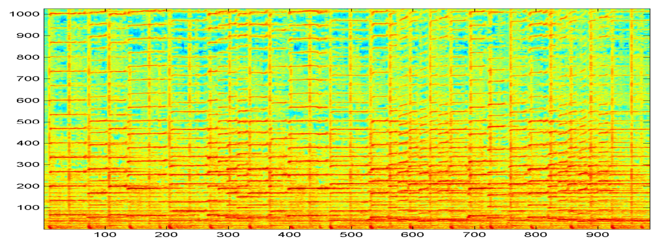
Principal Components

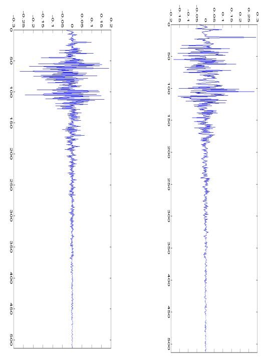


- The first principal component is the *first Eigen* ("typical") vector
  - $X = a_1(X)E_1$
  - The first Eigen face
  - For non-zero-mean data sets, the average of the data
- The second principal component is the second "typical" (or correction) vector
  - $X = a_1(X)E_1 + a_2(X)E_2$

Example of Principal Components

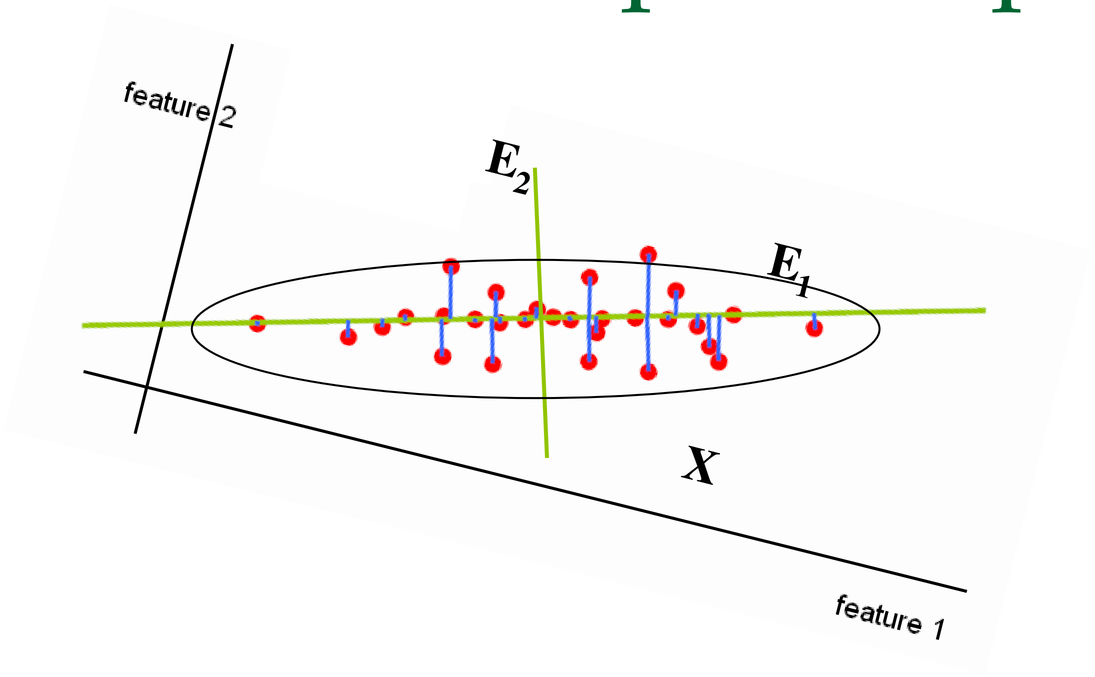






- Faces
  - Principal components: Eigen faces are like faces
- Music
  - Principal components are Eigen vectors
  - Eigen vectors are **NOT** like the notes

Properties of Principal Components



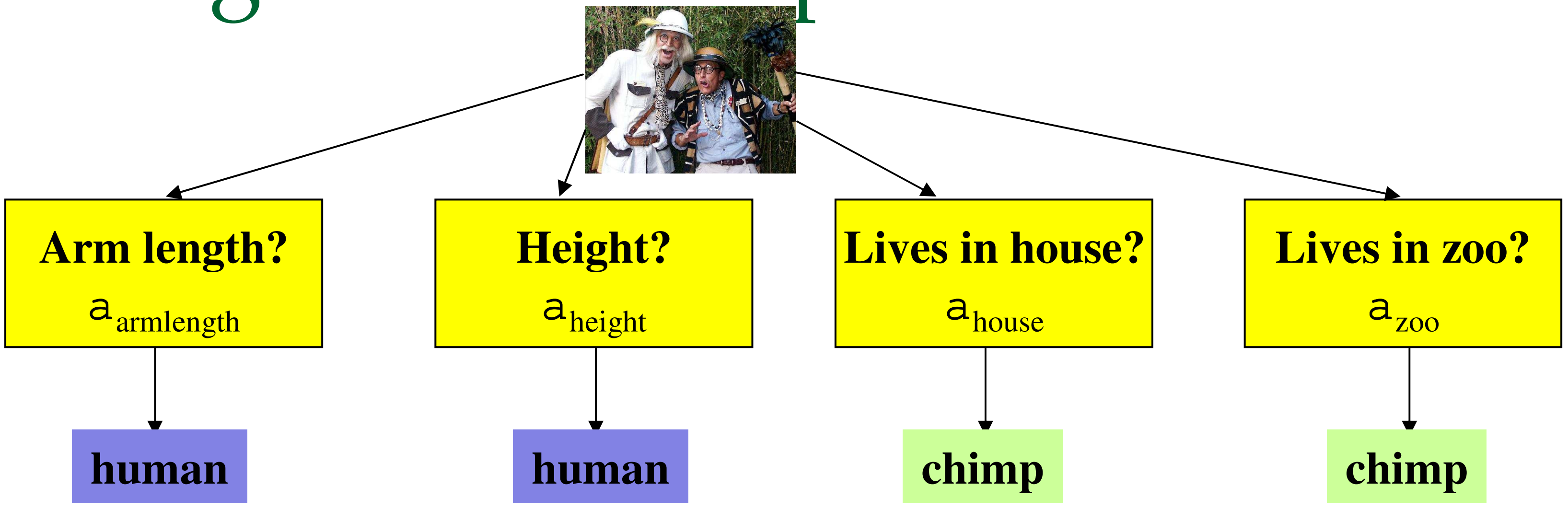
- The first principal component tells us nothing about the *average* value of the second component
- In general, the *k*-th principal component tells us nothing about the *i*-th principal component for *i* not equal to *k*.
- The principal components are ***uncorrelated***
  - The *average* contribution of the second Eigen face to the the collection of faces is the same, regardless of the contribution of the first Eigen face

A Quick Intro to Boosting

Introduction to Boosting

- An *ensemble* method that sequentially combines many simple ***BINARY*** classifiers to construct a final complex classifier
  - Simple classifiers are often called “weak” learners
  - The complex classifiers are called “strong” learners
- Each weak learner focuses on instances where the previous classifier failed
  - Give greater weight to instances that have been incorrectly classified by previous learners
- Restrictions for weak learners
  - Better than 50% correct
- Final classifier is *weighted* sum of weak classifiers

Boosting and the Chimpanzee Problem



- The total confidence in all classifiers that classify the entity as a chimpanzee is

$$Score_{chimp} = \sum_{classifier\ favors\ chimpanzee} a_{\text{classifier}}$$

- The total confidence in all classifiers that classify it as a human is

$$Score_{human} = \sum_{classifier\ favors\ human} a_{\text{classifier}}$$

- If $Score_{chimpanzee} > Score_{human}$ then the our belief that we have a chimpanzee is greater than the belief that we have a human

Boosting: A very simple idea

- One can come up with many rules to classify
  - E.g. Chimpanzee vs. Human classifier:
  - If arms == long, entity is chimpanzee
  - If height > 5’6” entity is human
  - If lives in house == entity is human
  - If lives in zoo == entity is chimpanzee
- Each of them is a reasonable rule, but makes many mistakes
  - Each rule has an intrinsic error rate
- *Combine* the predictions of these rules
  - But not equally
  - Rules that are less accurate should be given lesser weight

Formalizing the Boosting Concept

- Given a set of instances $(x_1, y_1), (x_2, y_2), \ldots (x_N, y_N)$
  - x_i is the set of attributes of the i^{th} instance
  - y_1 is the class for the i^{th} instance
    - y_1 can be +1 or -1 (binary classification only)
- Given a set of classifiers $h_1, h_2, \ldots, h_T$
  - h_i classifies an instance with attributes x as $h_i(x)$
  - $h_i(x)$ is either -1 or +1 (for a binary classifier)

  - y*h(x) is 1 for all correctly classified points and -1 for incorrectly classified points
- Devise a function $f\,(h_1(x), h_2(x), \ldots, h_T(x))$ such that classification based on $f\,()$ is superior to classification by any $h_i(x)$
  - The function is succinctly represented as $f\,(x)$

The Boosting Concept

- A simple combiner function: Voting
  - $f(x) = \Sigma_i h_i(x)$
  - Classifier $H(x) = \text{sign}(f(x)) = \text{sign}(\Sigma_i h_i(x))$
  - Simple majority classifier
    - A simple voting scheme
- A better combiner function: Boosting
  - $f(x) = \Sigma_i \alpha_i h_i(x)$
    - Can be any real number
  - Classifier $H(x) = \text{sign}(f(x)) = \text{sign}(\Sigma_i \alpha_i h_i(x))$
  - A weighted majority classifier
    - The weight α_i for any $h_i(x)$ is a measure of our trust in $h_i(x)$

The ADABoost Algorithm

- Adaboost is ADAPTIVE boosting

- The combined classifier is a *sequence* of weighted classifiers
- We learn classifier weights in an adaptive manner

- Each classifier’s weight optimizes performance on data whose weights are in turn adapted to the accuracy with which they have been classified

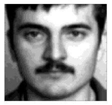
The ADABoost Algorithm

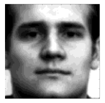
- Initialize $D_1(x_i) = 1/N$
- For $t = 1, \ldots, T$
  - Train a weak classifier h_t using distribution D_t
  - Compute total error on training data
    - $e_t = \text{Sum}\ \{D_t\ (x_i)\ ½(1 - y_i\ h_t(x_i))\}$
  - Set $a_t = ½ \ln ((1 - e_t) / e_t)$
  - For $i = 1 \ldots N$
    - set $D_{t+1}(x_i) = D_t(x_i) \exp(- a_t\, y_i\, h_t(x_i))$
  - Normalize D_{t+1} to make it a distribution
- The final classifier is
  - $H(x) = \text{sign}(S_t\, a_t\, h_t(x))$

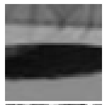
First, some example data

= 0.3 E1 - 0.6 E2

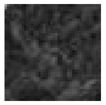
= 0.5 E1 - 0.5 E2

 = 0.7 E1 - 0.1 E2

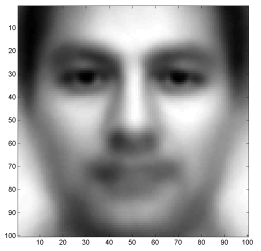
 = 0.6 E1 - 0.4 E2

 = 0.2 E1 + 0.4 E2

= -0.8 E1 - 0.1 E2

 = 0.4 E1 - 0.9 E2

= 0.2 E1 + 0.5 E2

E_1 

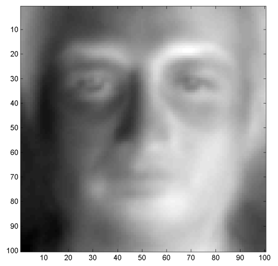
E_2 

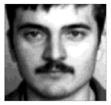
Image = a*E1 + b*E2   a = Image.E1/|Image|

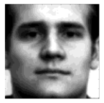
- Face detection with multiple Eigen faces
- Step 0: Derived top 2 Eigen faces from eigen face training data
- Step 1: On a (different) set of examples, express each image as a linear combination of Eigen faces
  - Examples include both faces and non faces
  - Even the non-face images will are explained in terms of the eigen faces

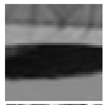
Training Data

= 0.3 E1 - 0.6 E2

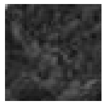
= 0.5 E1 - 0.5 E2

 = 0.7 E1 - 0.1 E2

 = 0.6 E1 - 0.4 E2

 = 0.2 E1 + 0.4 E2

= -0.8 E1 - 0.1 E2

 = 0.4 E1 - 0.9 E2

= 0.2 E1 + 0.5 E2

ID	E1	E2.	Class
A	0.3	-0.6	+1
B	0.5	-0.5	+1
C	0.7	-0.1	+1
D	0.6	-0.4	+1
E	0.2	0.4	-1
F	-0.8	-0.1	-1
G	0.4	-0.9	-1
H	0.2	0.5	-1

Face = +1
Non-face = -1

The ADABoost Algorithm

Initialize $D_1(x_i) = 1/N$

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  - Set $a_t = ½ \ln ((1 – e_t) / e_t)$
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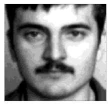
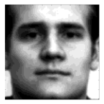
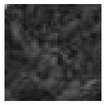
 = 0.7 E1 - 0.1 E2

 = 0.6 E1 - 0.4 E2

= 0.2 E1 + 0.4 E2

= -0.8 E1 - 0.1 E2

 = 0.4 E1 - 0.9 E2

= 0.2 E1 + 0.5 E2

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	-0.1	-1	1/8
G	0.4	-0.9	-1	1/8
H	0.2	0.5	-1	1/8

The ADABoost Algorithm

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    - $e_t = \text{Sum}\{D_t(x_i)\,½(1 - y_i\,h_t(x_i))\}$
  - Set $a_t = ½ \ln(e_t/(1 - e_t))$
  - For $i = 1 \ldots N$
    - set $D_{t+1}(x_i) = D_t(x_i) \exp(-a_t\,y_i\,h_t(x_i))$
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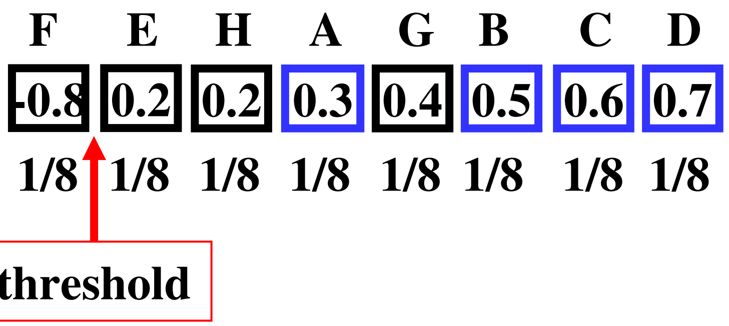
- The final classifier is
  - $H(x) = \text{sign}(S_t\,a_t\,h_t(x))$

The E1 "Stump"



Classifier based on E1:
if (sign*wt(E1) > thresh) > 0)
    face = true

sign = +1 or -1

Sign = +1, error = 3/8
Sign = -1, error = 5/8

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	-0.1	-1	1/8
G	0.4	-0.9	-1	1/8
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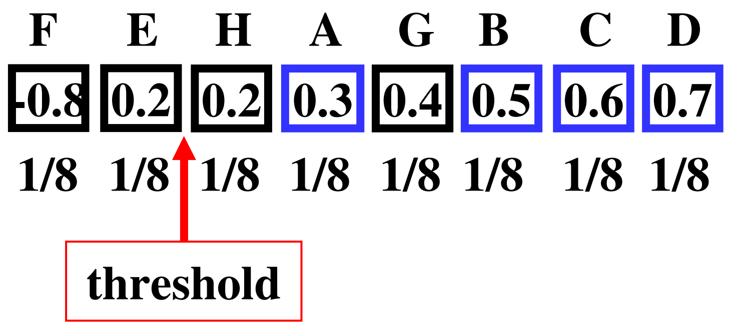
The E1 “Stump”



Classifier based on E1:

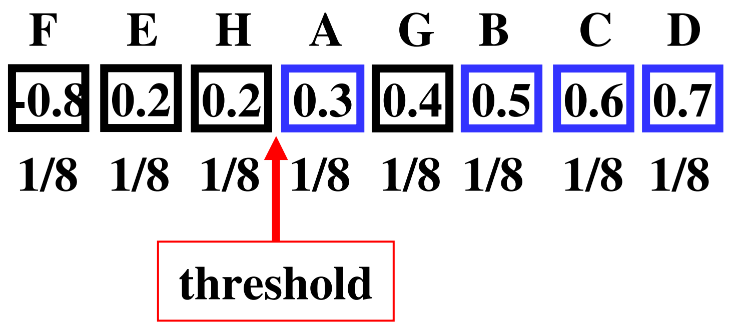
```
if ( sign*wt(E1) > thresh) > 0)
    face = true

sign = +1 or -1
```

Sign = +1, error = 2/8

Sign = -1, error = 6/8

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	-0.1	-1	1/8
G	0.4	-0.9	-1	1/8
H	0.2	0.5	-1	1/8

The E1 "Stump"



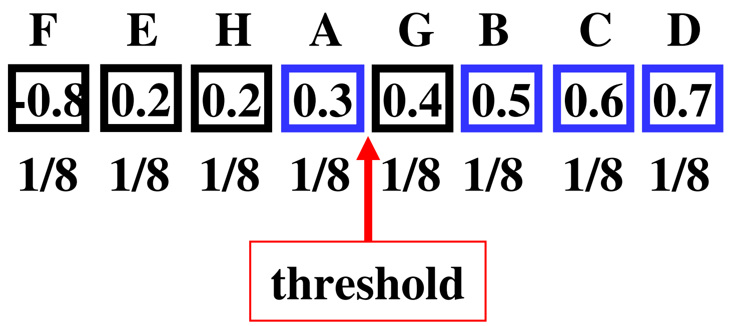
Classifier based on E1:
```
if ( sign*wt(E1) > thresh) > 0)
    face = true

sign = +1 or -1
```

Sign = +1, error = 1/8

Sign = -1, error = 7/8

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	-0.1	-1	1/8
G	0.4	-0.9	-1	1/8
H	0.2	0.5	-1	1/8

The E1 "Stump"



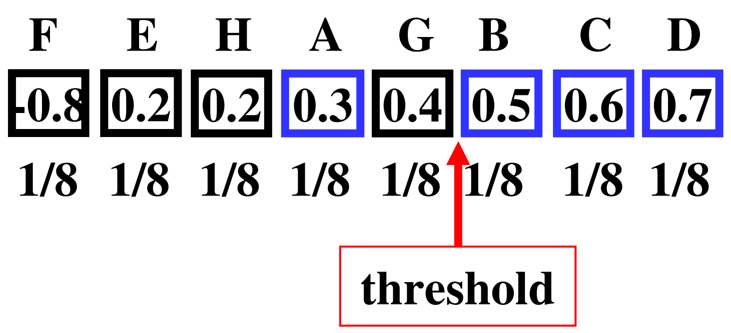
Classifier based on E1:
if (sign*wt(E1) > thresh) > 0)
    face = true

sign = +1 or -1

Sign = +1, error = 2/8
Sign = -1, error = 6/8

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	-0.1	-1	1/8
G	0.4	-0.9	-1	1/8
H	0.2	0.5	-1	1/8

The E1 “Stump”



Classifier based on E1:

```
if ( sign*wt(E1) > thresh) > 0)
    face = true
```

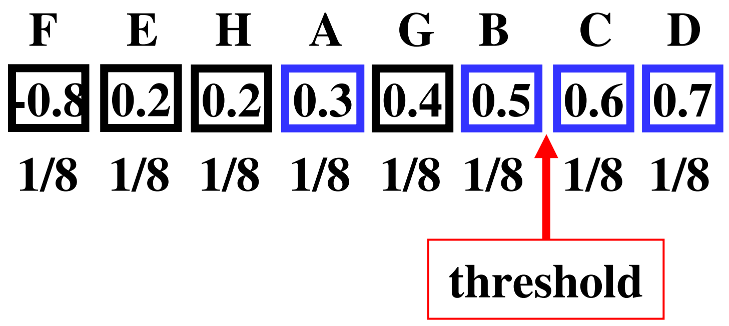
sign = +1 or -1

Sign = +1, error = 1/8

Sign = -1, error = 7/8

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	-0.1	-1	1/8
G	0.4	-0.9	-1	1/8
H	0.2	0.5	-1	1/8

The E1 “Stump”



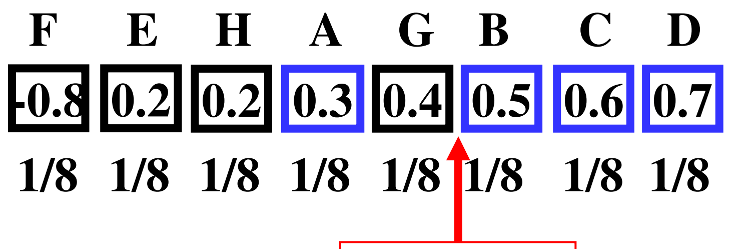
```
Classifier based on E1:
if ( sign*wt(E1) > thresh) > 0)
    face = true

sign = +1 or -1
```

Sign = +1, error = 2/8

Sign = -1, error = 6/8

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	-0.1	-1	1/8
G	0.4	-0.9	-1	1/8
H	0.2	0.5	-1	1/8

The Best E1 "Stump"



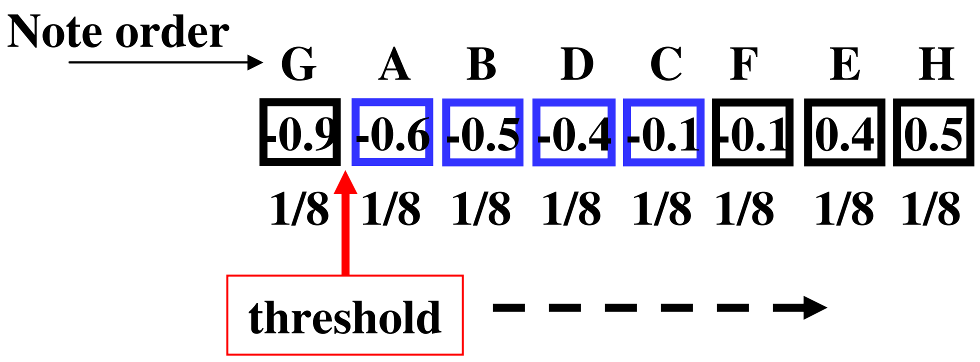
Sign = +1, error = 1/8

Classifier based on E1:
```
if ( sign*wt(E1) > thresh) > 0)
    face = true
```

Sign = +1
Threshold = 0.45

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	-0.1	-1	1/8
G	0.4	-0.9	-1	1/8
H	0.2	0.5	-1	1/8

The E2"Stump"

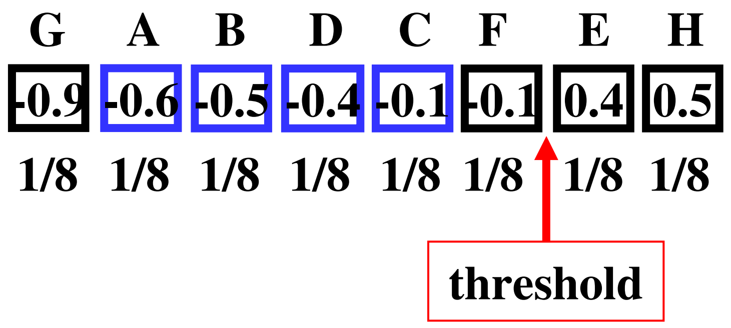


```
Classifier based on E2:
if ( sign*wt(E2) > thresh) > 0)
     face = true

sign = +1 or -1
```

Sign = +1, error = 3/8
Sign = -1, error = 5/8

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	-0.1	-1	1/8
G	0.4	-0.9	-1	1/8
H	0.2	0.5	-1	1/8

The Best E2"Stump"



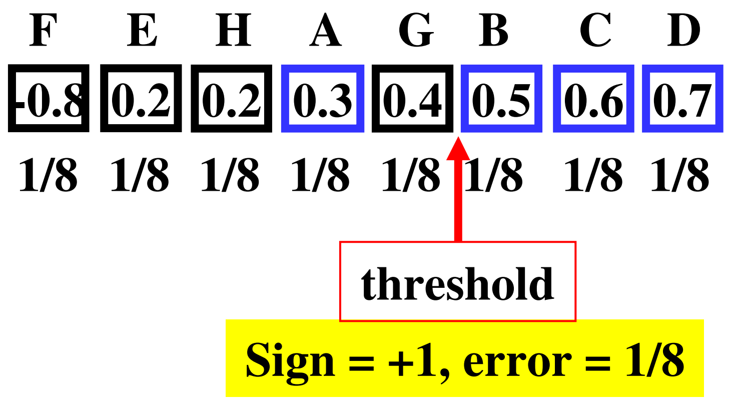
Classifier based on E2:

```
if ( sign*wt(E2) > thresh) > 0)
    face = true

sign = -1
Threshold = 0.15
```

Sign = -1, error = 2/8

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	-0.1	-1	1/8
G	0.4	-0.9	-1	1/8
H	0.2	0.5	-1	1/8

The Best "Stump"



The Best overall classifier based on a single feature is based on E1

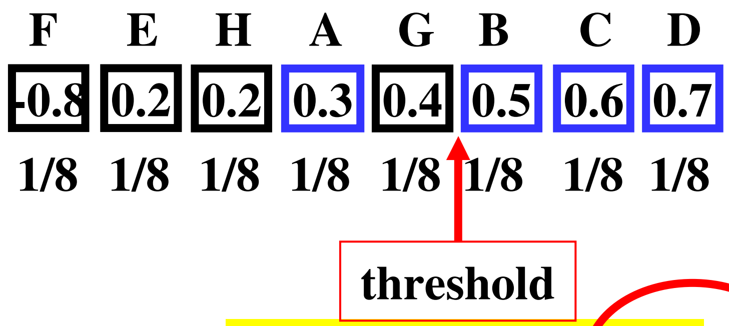
If (wt(E1) > 0.45) Face

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	0.1	-1	1/8
G	0.4	-0.9	-1	1/8
H	0.2	0.5	-1	1/8

The ADABoost Algorithm

- Initialize $D_1(x_i) = 1/N$
- For $t = 1, \ldots, T$
  - Train a weak classifier h_t using distribution D_t
  - Compute total error on training data
    - $e_t = \text{Sum}\ \{D_t\ (x_i)\ ½(1 – y_i\ h_t(x_i))\}$
  - Set $a_t = ½ \ln\ (e_t/(1 – e_t))$
  - For $i = 1 \ldots N$
    - set $D_{t+1}(x_i) = D_t(x_i) \exp(- a_t\ y_i\ h_t(x_i))$
  - Normalize D_{t+1} to make it a distribution
- The final classifier is
  - $H(x) = \text{sign}(S_t\ a_t\ h_t(x))$

The Best Error



The Error of the classifier is the sum of the weights of the misclassified instances

Sign = +1, error = 1/8

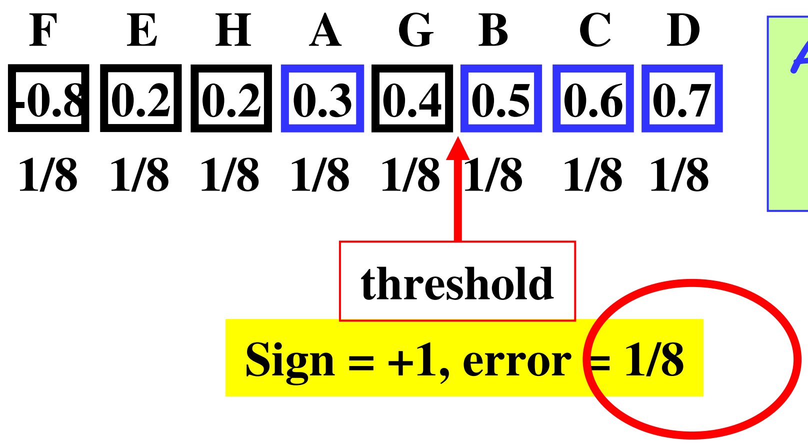
ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	1/8
B	0.5	-0.5	+1	1/8
C	0.7	-0.1	+1	1/8
D	0.6	-0.4	+1	1/8
E	0.2	0.4	-1	1/8
F	-0.8	0.1	-1	1/8
G	0.4	-0.9	-1	1/8
H	0.2	0.5	-1	1/8

NOTE: THE ERROR IS THE SUM OF THE WEIGHTS OF MISCLASSIFIED INSTANCES

The ADABoost Algorithm

- Initialize $D_1(x_i) = 1/N$
- For $t = 1, \ldots, T$
  - Train a weak classifier h_t using distribution D_t
  - Compute total error on training data
    - $e_t = \text{Sum}\ \{D_t\ (x_i)\ ½(1 - y_i\ h_t(x_i))\}$
  - Set $a_t = ½ \ln ((1 - e_t) / e_t)$
  - For $i = 1 \ldots N$
    - set $D_{t+1}(x_i) = D_t(x_i) \exp(- a_t\ y_i\ h_t(x_i))$
  - Normalize D_{t+1} to make it a distribution
- The final classifier is
  - $H(x) = \text{sign}(S_t\ a_t\ h_t(x))$

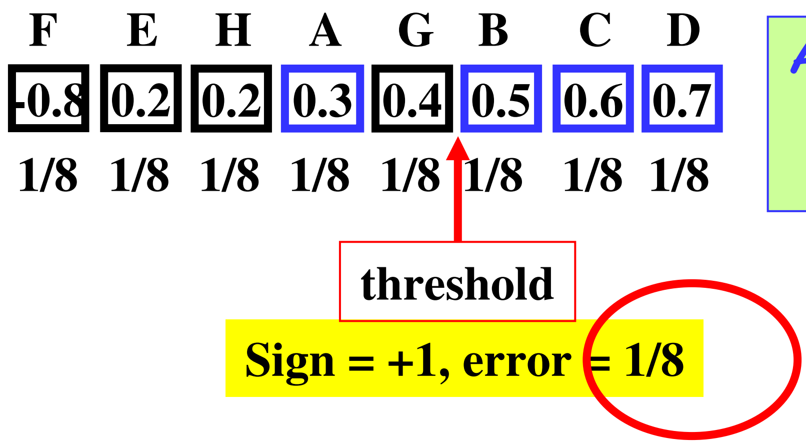
Computing Alpha



Alpha = 0.5ln((1-1/8) / (1/8))

= 0.5 ln(7) = 0.97

The Boosted Classifier Thus Far



Sign = +1, error = 1/8

Alpha = 0.5ln((1-1/8) / (1/8))

= 0.5 ln(7) = 0.97

h1(X) = wt(E1) > 0.45 ? +1 : -1

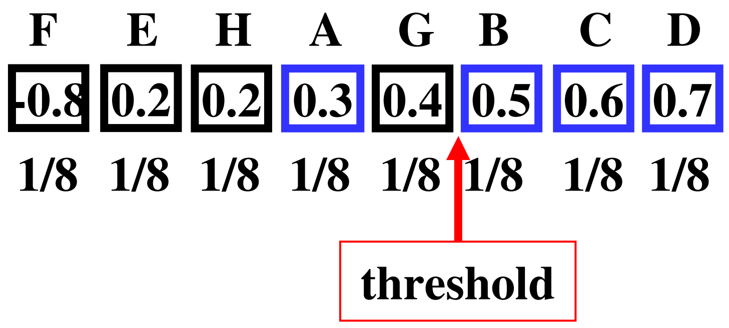
H(X) = sign(0.97 * h1(X))

It's the same as h1(x)

The ADABoost Algorithm

- Initialize $D_1(x_i) = 1/N$
- For $t = 1, \ldots, T$
  - Train a weak classifier h_t using distribution D_t
  - Compute total error on training data
    - $\varepsilon_t = \text{Average}\{\frac{1}{2}(1 - y_i\, h_t(x_i))\}$
  - Set $\alpha_t = \frac{1}{2} \ln((1 - \varepsilon_t) / \varepsilon_t)$
  - For $i = 1 \ldots N$
    - set $D_{t+1}(x_i) = D_t(x_i) \exp(-\alpha_t\, y_i\, h_t(x_i))$
  - Normalize D_{t+1} to make it a distribution
- The final classifier is
  - $H(x) = \text{sign}(\Sigma_t\, \alpha_t\, h_t(x))$

The Best Error



F	E	H	A	G	B	C	D
-0.8	0.2	0.2	0.3	0.4	0.5	0.6	0.7
1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

threshold

$$D_{t+1}(x_i) = D_t(x_i)\ \exp(-\ a_t\, y_i\, h_t\,(x_i))$$

$$\exp(a_t) = \exp(0.97) = 2.63$$
$$\exp(-a_t) = \exp(-0.97) = 0.38$$

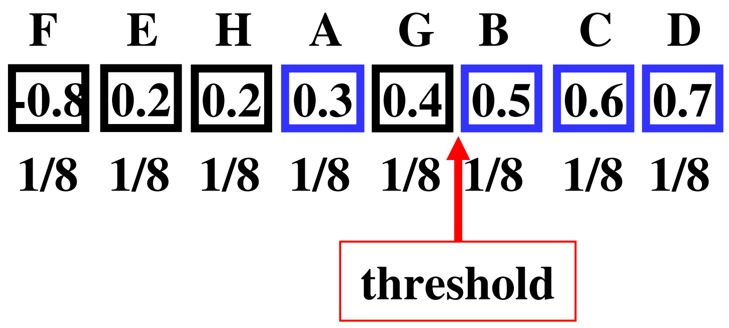
ID	E1	E2.	Class	Weight	Weight
A	0.3	-0.6	+1	1/8 * 2.63	0.33
B	0.5	-0.5	+1	1/8 * 0.38	0.05
C	0.7	-0.1	+1	1/8 * 0.38	0.05
D	0.6	-0.4	+1	1/8 * 0.38	0.05
E	0.2	0.4	-1	1/8 * 0.38	0.05
F	-0.8	0.1	-1	1/8 * 0.38	0.05
G	0.4	-0.9	-1	1/8 * 0.38	0.05
H	0.2	0.5	-1	1/8 * 0.38	0.05

Multiply the correctly classified instances by 0.38
Multiply incorrectly classified instances by 2.63

The ADABoost Algorithm

- Initialize $D_1(x_i) = 1/N$
- For $t = 1, \ldots, T$
  - Train a weak classifier h_t using distribution D_t
  - Compute total error on training data
    - $\varepsilon_t = \text{Average}\ \{½\ (1 – y_i\ h_t(x_i))\}$
  - Set $\alpha_t = ½ \ln ((1 – \varepsilon_t) / \varepsilon_t)$
  - For $i = 1\ldots N$
    - set $D_{t+1}(x_i) = D_t(x_i) \exp(- \alpha_t\ y_i\ h_t(x_i))$
  - Normalize D_{t+1} to make it a distribution
- The final classifier is
  - $H(x) = \text{sign}(\Sigma_t\ \alpha_t\ h_t(x))$

The Best Error

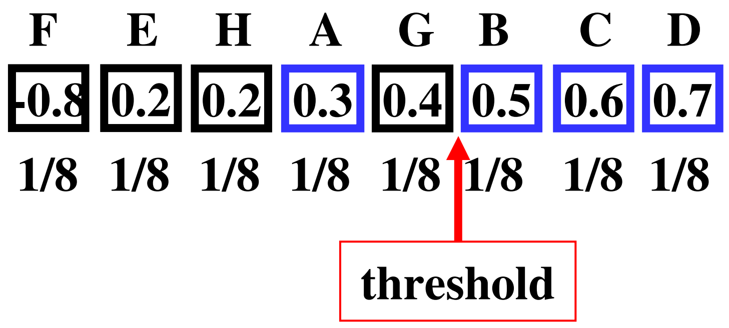


$D' = D / sum(D)$

ID	E1	E2.	Class	Weight	Weight	Weight
A	0.3	-0.6	+1	1/8 * 2.63	0.33	0.48
B	0.5	-0.5	+1	1/8 * 0.38	0.05	0.074
C	0.7	-0.1	+1	1/8 * 0.38	0.05	0.074
D	0.6	-0.4	+1	1/8 * 0.38	0.05	0.074
E	0.2	0.4	-1	1/8 * 0.38	0.05	0.074
F	-0.8	0.1	-1	1/8 * 0.38	0.05	0.074
G	0.4	-0.9	-1	1/8 * 0.38	0.05	0.074
H	0.2	0.5	-1	1/8 * 0.38	0.05	0.074

Multiply the correctly classified instances by 0.38
Multiply incorrectly classified instances by 2.63
Normalize to sum to 1.0

The Best Error



$D' = D / sum(D)$

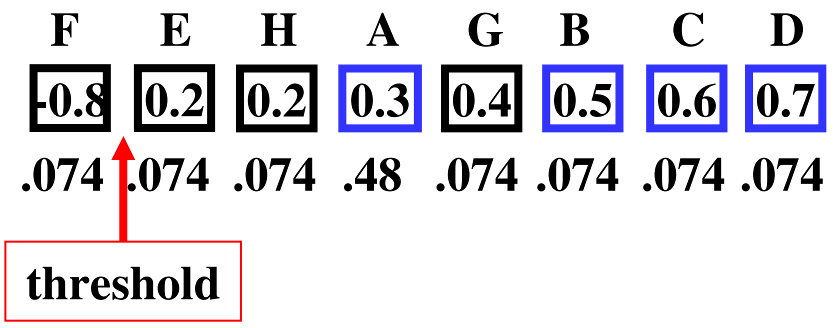
ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	0.48
B	0.5	-0.5	+1	0.074
C	0.7	-0.1	+1	0.074
D	0.6	-0.4	+1	0.074
E	0.2	0.4	-1	0.074
F	-0.8	0.1	-1	0.074
G	0.4	-0.9	-1	0.074
H	0.2	0.5	-1	0.074

Multiply the correctly classified instances by 0.38
Multiply incorrectly classified instances by 2.63
Normalize to sum to 1.0

The ADABoost Algorithm

- Initialize $D_1(x_i) = 1/N$
- For $t = 1, \ldots, T$
  - Train a weak classifier h_t using distribution D_t
  - Compute total error on training data
    - $\varepsilon_t = \text{Average}\,\{½\,(1 - y_i\, h_t(x_i))\}$
  - Set $\alpha_t = ½ \ln(\varepsilon_t/(1 - \varepsilon_t))$
  - For $i = 1 \ldots N$
    - set $D_{t+1}(x_i) = D_t(x_i) \exp(-\alpha_t\, y_i\, h_t(x_i))$
  - Normalize D_{t+1} to make it a distribution
- The final classifier is
  - $H(x) = \text{sign}(\Sigma_t\, \alpha_t\, h_t(x))$

E1 classifier

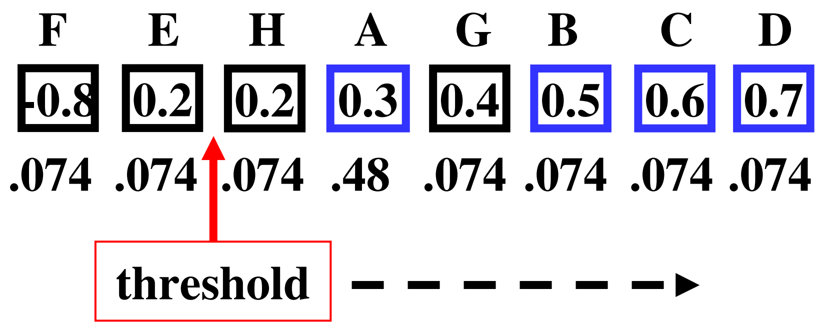


```
Classifier based on E1:
if ( sign*wt(E1) > thresh) > 0)
     face = true

sign = +1 or -1
```

Sign = +1, error = 0.222
Sign = -1, error = 0.778

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	0.48
B	0.5	-0.5	+1	0.074
C	0.7	-0.1	+1	0.074
D	0.6	-0.4	+1	0.074
E	0.2	0.4	-1	0.074
F	-0.8	0.1	-1	0.074
G	0.4	-0.9	-1	0.074
H	0.2	0.5	-1	0.074

E1 classifier



Classifier based on E1:

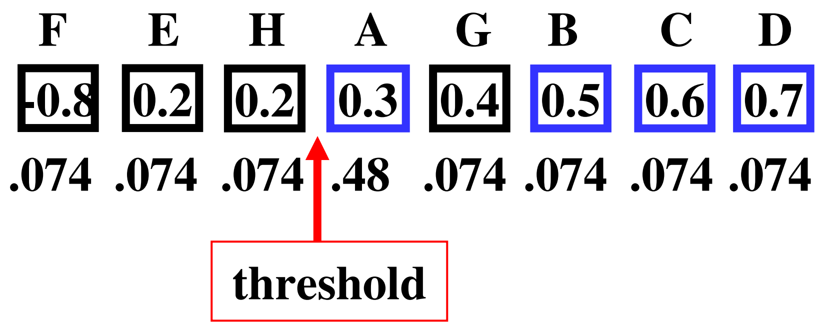
```
if ( sign*wt(E1) > thresh) > 0)
    face = true

sign = +1 or -1
```

Sign = +1, error = 0.148

Sign = -1, error = 0.852

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	0.48
B	0.5	-0.5	+1	0.074
C	0.7	-0.1	+1	0.074
D	0.6	-0.4	+1	0.074
E	0.2	0.4	-1	0.074
F	-0.8	0.1	-1	0.074
G	0.4	-0.9	-1	0.074
H	0.2	0.5	-1	0.074

The Best E1 classifier

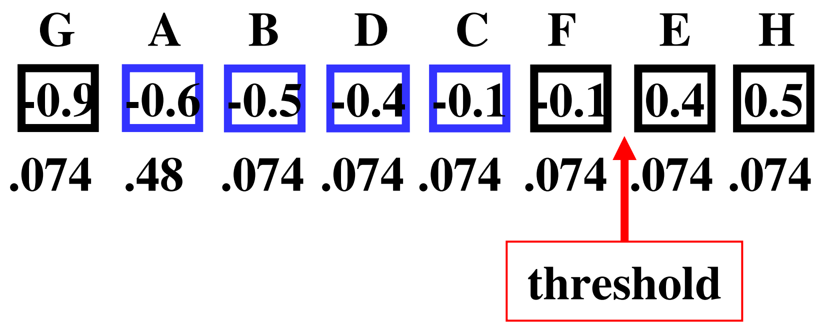


```
Classifier based on E1:
if ( sign*wt(E1) > thresh) > 0)
     face = true

sign = +1 or -1
```

Sign = +1, error = 0.074

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	0.48
B	0.5	-0.5	+1	0.074
C	0.7	-0.1	+1	0.074
D	0.6	-0.4	+1	0.074
E	0.2	0.4	-1	0.074
F	-0.8	0.1	-1	0.074
G	0.4	-0.9	-1	0.074
H	0.2	0.5	-1	0.074

The Best E2 classifier



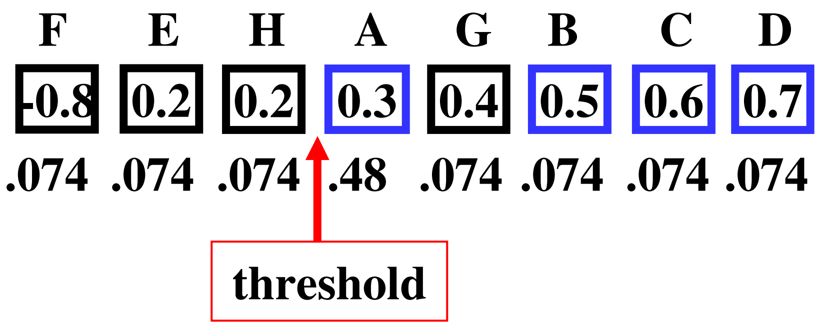
Classifier based on E2:

```
if ( sign*wt(E2) > thresh) > 0)
    face = true

sign = +1 or -1
```

Sign = -1, error = 0.148

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	0.48
B	0.5	-0.5	+1	0.074
C	0.7	-0.1	+1	0.074
D	0.6	-0.4	+1	0.074
E	0.2	0.4	-1	0.074
F	-0.8	-0.1	-1	0.074
G	0.4	-0.9	-1	0.074
H	0.2	0.5	-1	0.074

The Best Classifier



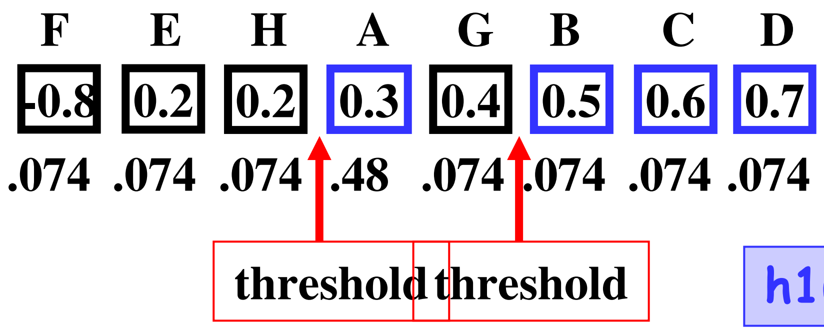
Sign = +1, error = 0.074

Classifier based on E1:
if (wt(E1) > 0.45) face = true

Alpha = 0.5ln((1-0.074) / 0.074)
= 1.26

ID	E1	E2.	Class	Weight
A	0.3	-0.6	+1	0.48
B	0.5	-0.5	+1	0.074
C	0.7	-0.1	+1	0.074
D	0.6	-0.4	+1	0.074
E	0.2	0.4	-1	0.074
F	-0.8	0.1	-1	0.074
G	0.4	-0.9	-1	0.074
H	0.2	0.5	-1	0.074

The Boosted Classifier Thus Far

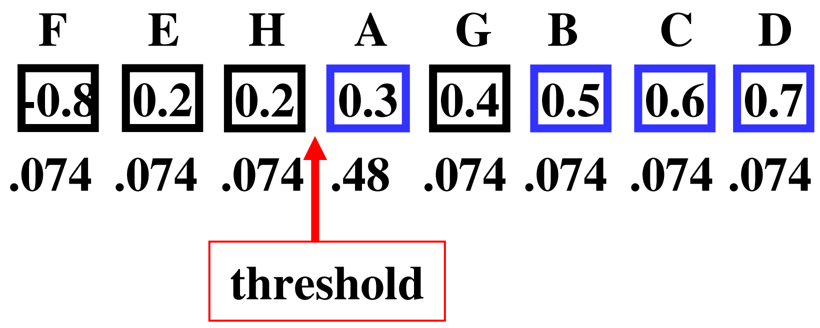


h1(X) = wt(E1) > 0.45 ? +1 : -1

h2(X) = wt(E1) > 0.25 ? +1 : -1

H(X) = sign(0.97 * h1(X) + 1.26 * h2(X))

Reweighting the Data



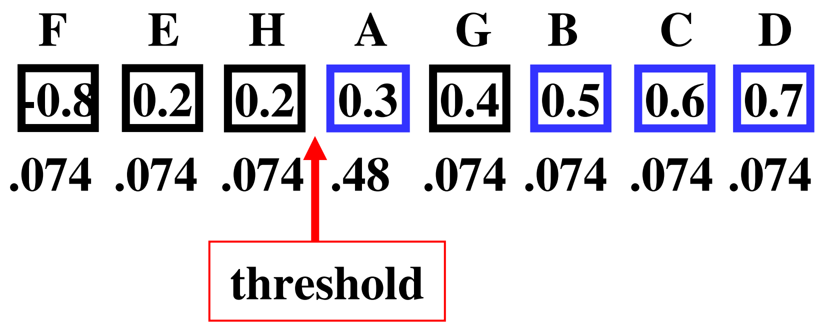
Sign = +1, error = 0.074

Exp(alpha) = exp(2.36) = 10
Exp(-alpha) = exp(-2.36) = 0.1

ID	E1	E2.	Class	Weight	
A	0.3	-0.6	+1	0.48*0.1	0.06
B	0.5	-0.5	+1	0.074*0.1	0.01
C	0.7	-0.1	+1	0.074*0.1	0.01
D	0.6	-0.4	+1	0.074*0.1	0.01
E	0.2	0.4	-1	0.074*0.1	0.01
F	-0.8	0.1	-1	0.074*0.1	0.01
G	0.4	-0.9	-1	0.074*10	0.86
H	0.2	0.5	-1	0.074*0.1	0.01

RENORMALIZE

Reweighting the Data



Sign = +1, error = 0.074

NOTE: THE WEIGHT OF "G" WHICH WAS MISCLASSIFIED BY THE SECOND CLASSIFIER IS NOW SUDDENLY HIGH

ID	E1	E2.	Class	Weight	
A	0.3	-0.6	+1	0.48*0.1	0.06
B	0.5	-0.5	+1	0.074*0.1	0.01
C	0.7	-0.1	+1	0.074*0.1	0.01
D	0.6	-0.4	+1	0.074*0.1	0.01
E	0.2	0.4	-1	0.074*0.1	0.01
F	-0.8	0.1	-1	0.074*0.1	0.01
G	0.4	-0.9	-1	0.074*10	0.86
H	0.2	0.5	-1	0.074*0.1	0.01

RENORMALIZE

AdaBoost

- In this example both of our first two classifiers were based on E1
    - Additional classifiers may switch to E2
- In general, the reweighting of the data will result in a different feature being picked for each classifier

- This also automatically gives us a *feature selection* strategy
    - In this data the wt(E1) is the most important feature

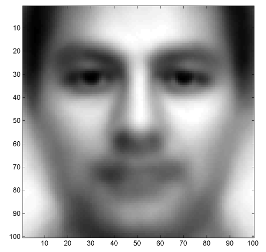
AdaBoost

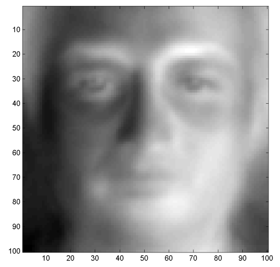
- NOT required to go with the best classifier so far
- For instance, for our second classifier, we might use the best E2 classifier, even though its worse than the E1 classifier
  - So long as its right more than 50% of the time

- We can *continue* to add classifiers even after we get 100% classification of the training data
  - Because the weights of the data keep changing
  - Adding new classifiers beyond this point is often a good thing to do

ADA Boost

= 0.4 E1 - 0.4 E2

$\mathbf{E_1}$ 

$\mathbf{E_2}$ 

- The final classifier is
  - $H(x) = \text{sign}(\Sigma_t\, \alpha_t\, h_t(x))$
- The output is 1 if the total weight of all weak learners that classify *x* as 1 is greater than the total weight of all weak learners that classify it as -1

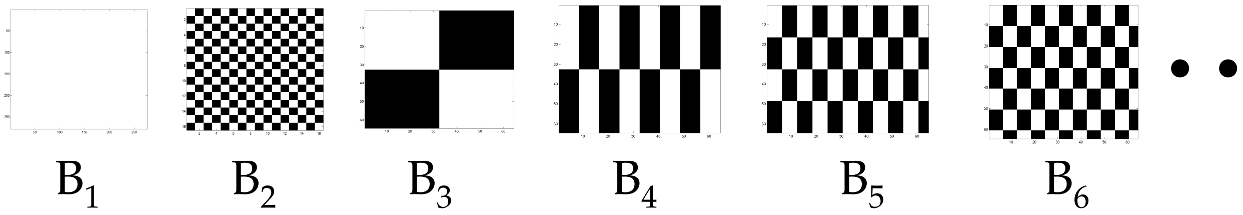
Boosting and Face Detection

- Boosting forms the basis of the most common technique for face detection today: The Viola-Jones algorithm.

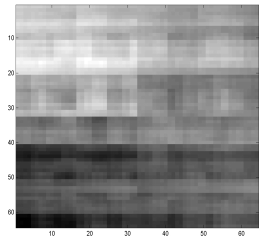
The problem of face detection

- Defining Features
  - Should we be searching for noses, eyes, eyebrows etc.?
    - Nice, but expensive
  - Or something simpler
- Selecting Features
  - Of all the possible features we can think of, which ones make sense
- Classification: Combining evidence
  - How does one combine the evidence from the different features?

Features: The Viola Jones Method



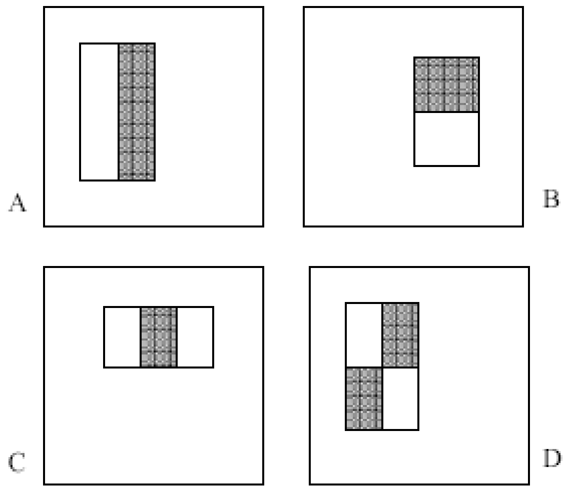
$$\text{Image} \approx w_1 B_1 + w_2 B_2 + w_3 B_3 + \ldots$$

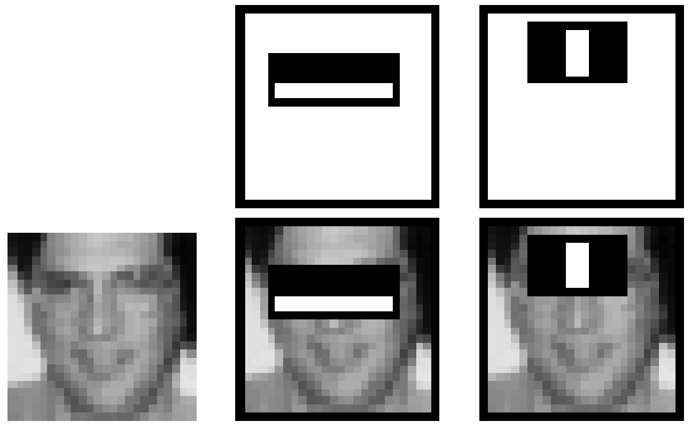


- Integral Features!!
  - Like the Checkerboard
- The same principle as we used to decompose images in terms of checkerboards:
  - The image of any object has changes at various scales
  - These can be represented coarsely by a checkerboard pattern
- The checkerboard patterns must however now be *localized*
  - Stay within the region of the face

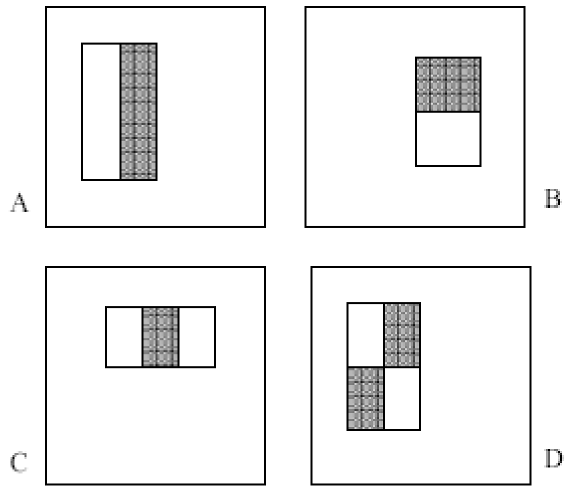
Features

- Checkerboard Patterns to represent facial features
  - The white areas are subtracted from the black ones.
  - Each checkerboard explains a *localized* portion of the image
- Four types of checkerboard patterns (only)



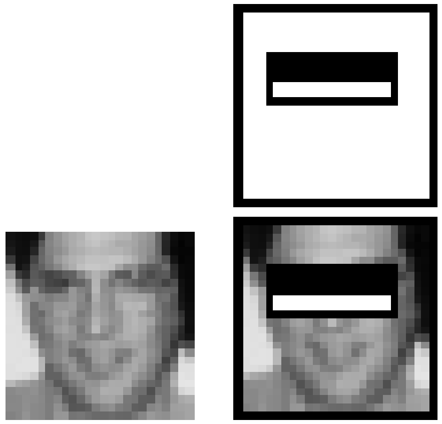


“Integral” features



- Each checkerboard has the following characteristics
  - Length
  - Width
  - Type
    - Specifies the number and arrangement of bands
- The four checkerboards above are the four used by Viola and Jones

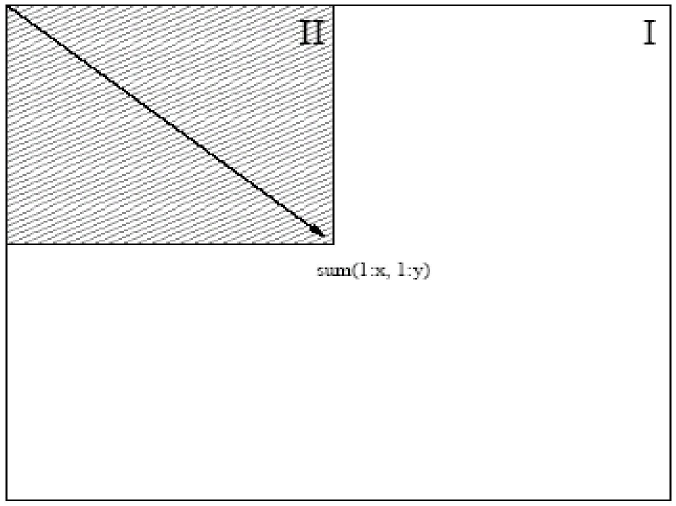
Explaining a portion of the face with a checker..



- How much is the difference in average intensity of the image in the black and white regions
  - Sum(pixel values in white region) – Sum(pixel values in black region)
- This is actually the dot product of the region of the face covered by the rectangle and the checkered pattern itself
  - White = 1, Black = -1

Integral images

- Summed area tables



- For each pixel store the sum of ALL pixels to the left of and above it.

Fast Computation of Pixel Sums

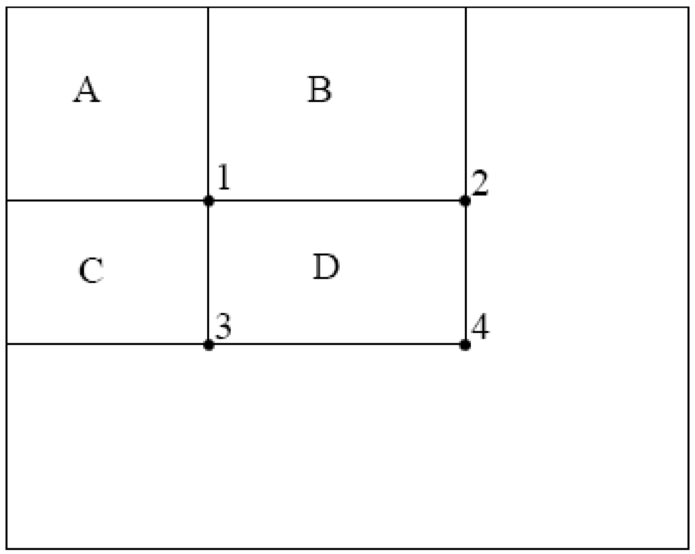
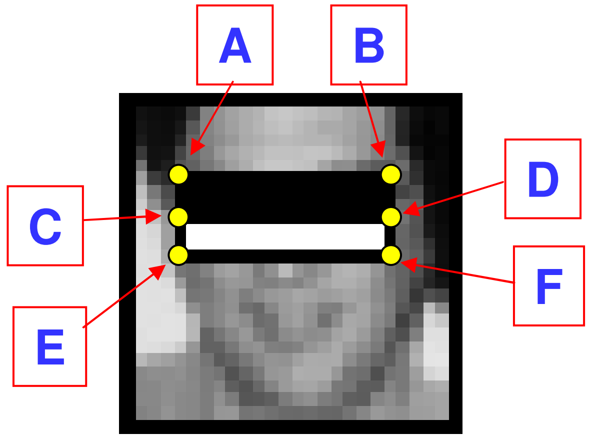


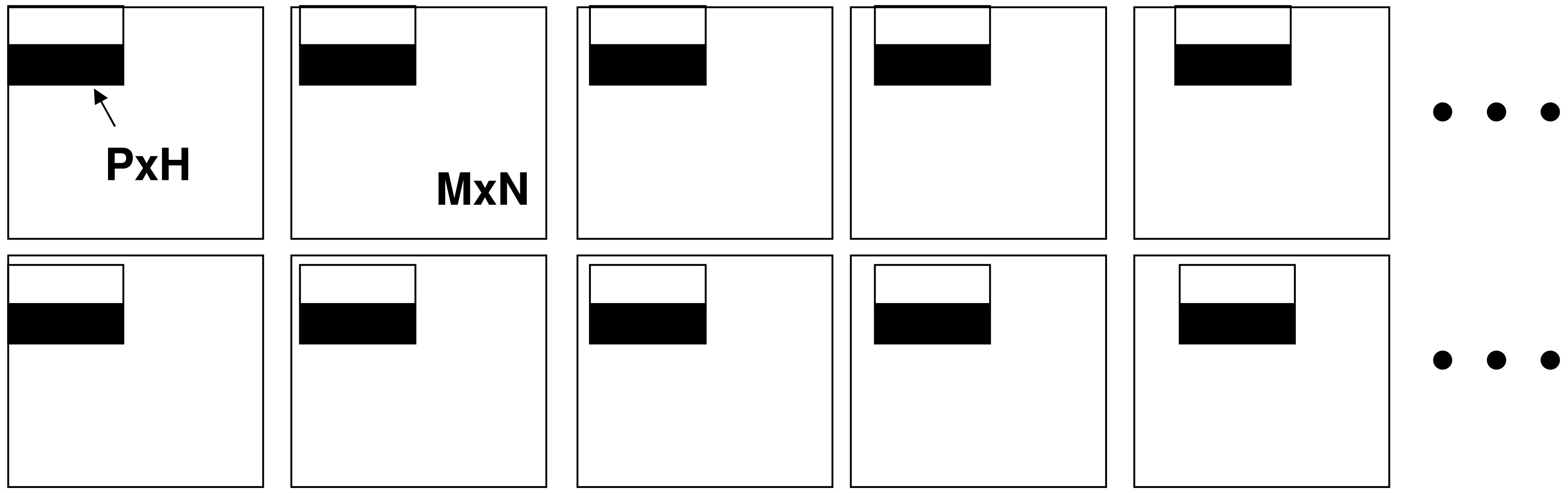
Figure 3: The sum of the pixels within rectangle D can be computed with four array references. The value of the integral image at location 1 is the sum of the pixels in rectangle A. The value at location 2 is $A + B$, at location 3 is $A + C$, and at location 4 is $A + B + C + D$. The sum within D can be computed as $4 + 1 - (2 + 3)$.

A Fast Way to Compute the Feature



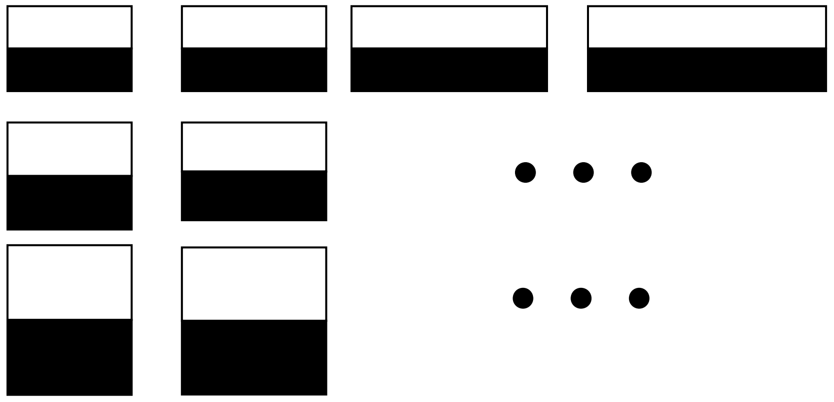
- Store pixel table for every pixel in the image
  - The sum of all pixel values to the left of and above the pixel
- Let A, B, C, D, E, F be the pixel table values at the locations shown
  - Total pixel value of black area = D + A – B – C
  - Total pixel value of white area = F + C – D – E
  - Feature value = (F + C – D – E) – (D + A – B – C)

How many features?



- Each checker board of width P and height H can start at
  - (0,0), (0,1),(0,2), … (0, N-P)
  - (1,0), (1,1),(1,2), … (1, N-P)
  - ..
  - (M-H,0), (M-H,1), (M-H,2), … (M-H, N-P)
- (M-H)*(N-P) possible starting locations
  - Each is a unique checker feature
    - E.g. at one location it may measure the forehead, at another the chin

How many features

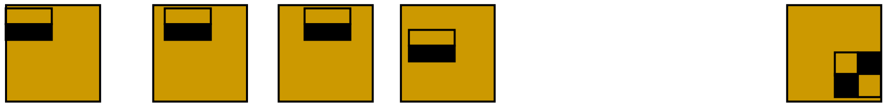


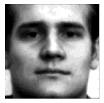
- Each feature can have many sizes
  - Width from (min) to (max) pixels
  - Height from (min ht) to (max ht) pixels
- At each size, there can be many starting locations
  - Total number of possible checkerboards of one type:
    No. of possible sizes x No. of possible locations
- There are four types of checkerboards
  - Total no. of possible checkerboards: VERY VERY LARGE!

Learning: No. of features

- Analysis performed on images of 24x24 pixels only
  - Reduces the no. of possible features to about 180000
- Restrict checkerboard size
  - Minimum of 8 pixels wide
  - Minimum of 8 pixels high
    - Other limits, e.g. 4 pixels may be used too
  - Reduces no. of checkerboards to about 50000

No. of features



	F1	F2	F3	F4	…..	F180000
	7	9	2	-1	…..	12
	-11	3	19	17	…..	2

- Each possible checkerboard gives us one feature
- A total of up to 180000 features derived from a 24x24 image!
- Every 24x24 image is now represented by a set of 180000 numbers
  - This is the set of features we will use for classifying if it is a face or not!

The Classifier

- The Viola-Jones algorithm uses a simple Boosting based classifier
- Each "weak learner" is a simple threshold
- At each stage find the best feature to classify the data with
    - I.e the feature that gives us the best classification of all the training data
        - Training data includes many examples of faces and non-face images
    - The classification rule is of the kind
        - If feature > threshold, face (or if feature < threshold, face)
        - The optimal value of "threshold" must also be determined.

The Weak Learner

- Training (for each weak learner):
  - For each feature f (of all 180000 features)
    - Find a threshold q(f) and polarity *p*(f) (*p*(f) *= -1 or p*(f) *= 1*) such that (f > *p*(f) *q(f)) performs the best classification of faces
      - Lowest overall error in classifying all training data
        - Error counted over *weighted* samples
    - Let the optimal overall error for *f* be error(*f*)
  - Find the feature f' such that error(f') is lowest
  - The weak learner is the test (f' > *p*(f')*q(f')) => face

- Note that the procedure for learning weak learners also identifies the most useful features for face recognition

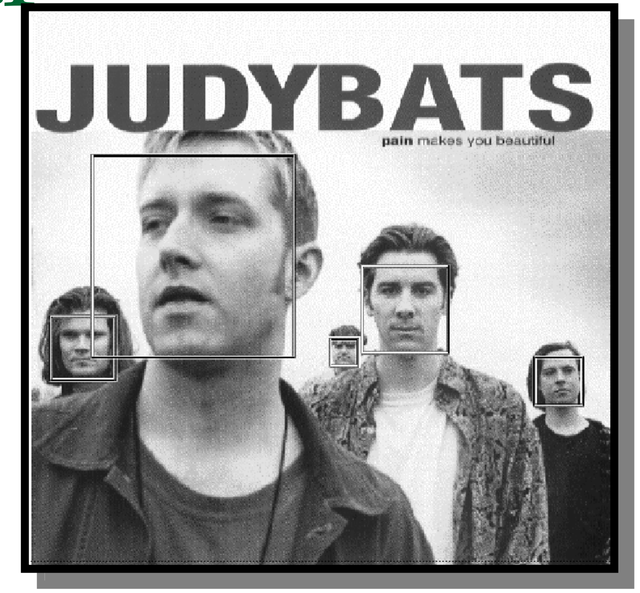
The Viola Jones Classifier

- A boosted threshold-based classifier
- First weak learner: Find the best feature, and its optimal threshold
  - Second weak learner: Find the best feature, for the weighted training data, and its threshold (weighting from one weak learner)
    - Third weak learner: Find the best feature for the reweighted data and its optimal threshold (weighting from two weak learners)
      - Fourth weak learner: Find the best feature for the reweighted data and its optimal threhsold (weighting from three weak learners)
        - ..

To Train

- Collect a large number of histogram equalized facial images
  - Resize all of them to 24x24
  - These are our “face” training set
- Collect a much much much larger set of 24x24 non-face images of all kinds
  - Each of them is histogram equalized
  - These are our “non-face” training set
- Train a boosted classifier

The Viola Jones Classifier



- During tests:
  - Given any new 24x24 image
    - $H(f) = \text{Sign}(S_f\ a_f\ (f > p_f\ q(f)))$
    - Only a small number of features (f < 100) typically used
- Problems:
  - Only classifies 24 x 24 images entirely as faces or non-faces
    - Typical pictures are much larger
    - They may contain many faces
    - Faces in pictures can be much larger or smaller
  - Not accurate enough

Multiple faces in the picture

- Scan the image
  - Classify each 24x24 rectangle from the photo
  - All rectangles that get classified as having a face indicate the location of a face
- For an NxM picture, we will perform (N-24)*(M-24) classifications
- If overlapping 24x24 rectangles are found to have faces, merge them

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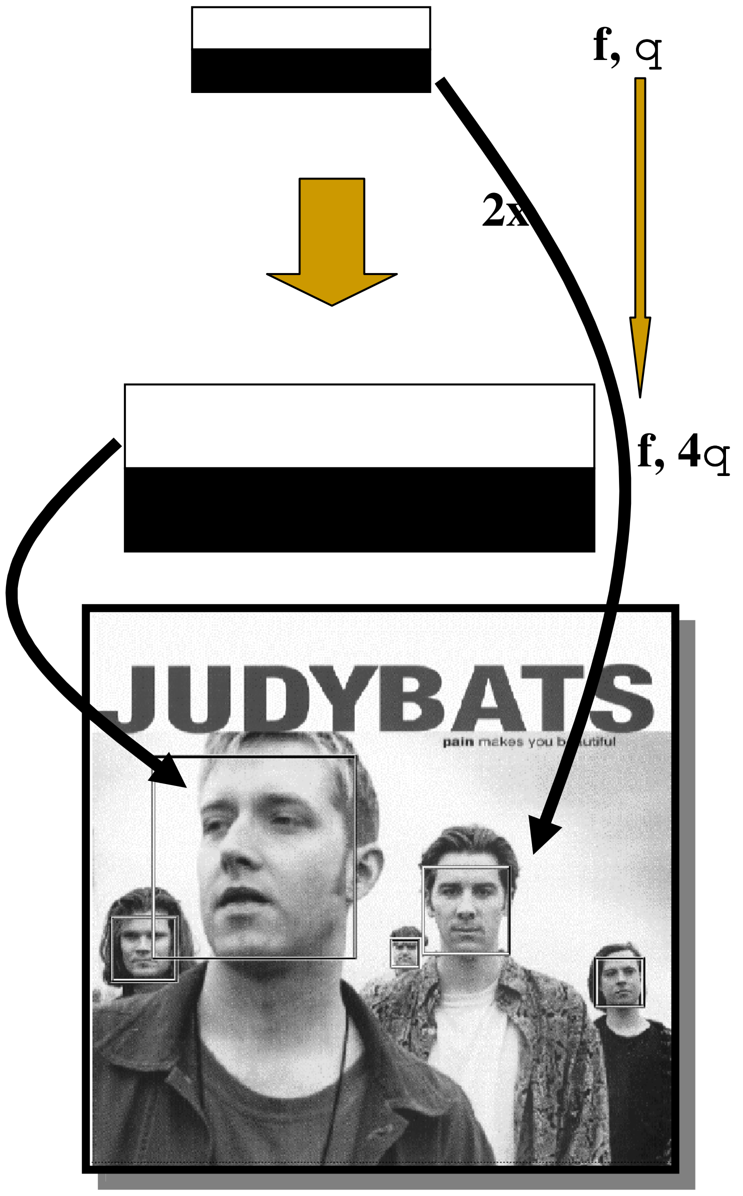
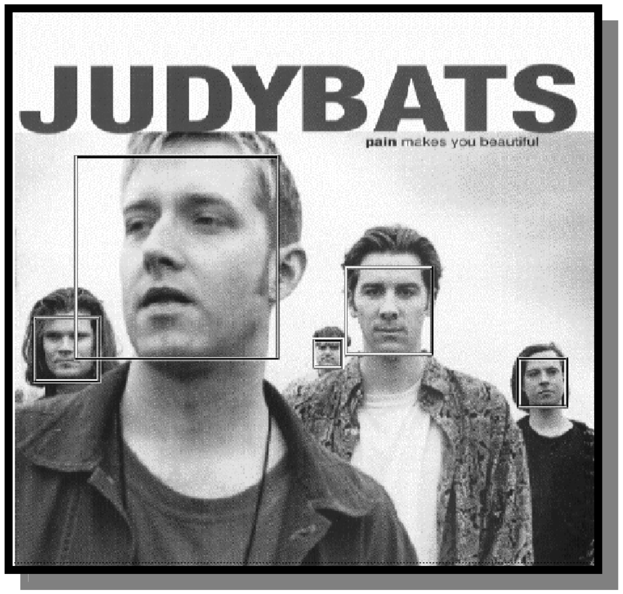
Face size solution

- We already have a classifier
  - That uses weak learners
- *Scale each classifier*
  - Every weak learner
  - Scale its size up by factor α. Scale the threshold up to $\alpha^2\theta$.
  - Do this for many scaling factors



Overall solution



- Scan the picture with classifiers of size 24x24
- Scale the classifier to 26x26 and scan
- Scale to 28x28 and scan etc.

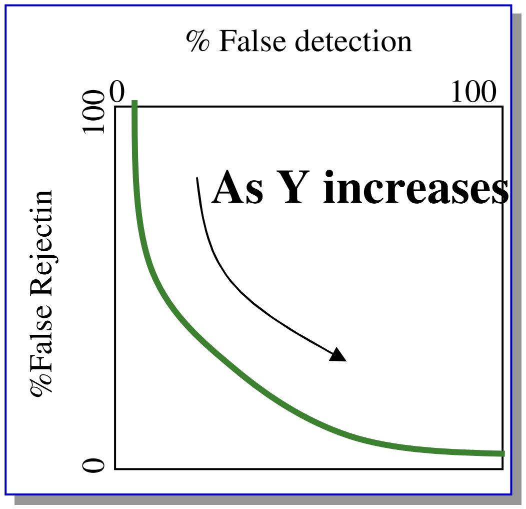
- Faces of different sizes will be found at different scales

False Rejection vs. False detection

- False Rejection: There's a face in the image, but the classifier misses it
  - Rejects the hypothesis that there's a face
- False detection: Recognizes a face when there is none.

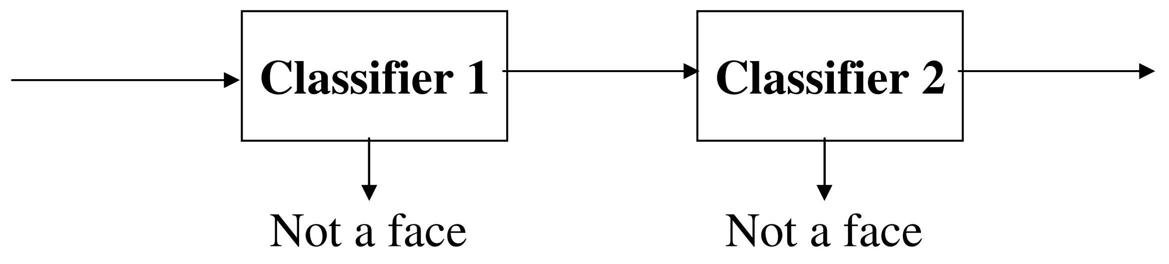
- Classifier:
  - Standard boosted classifier: $H(x) = \text{sign}(\Sigma_t \alpha_t h_t(x))$
  - Modified classifier $H(x) = \text{sign}(\Sigma_t \alpha_t h_t(x) + Y)$
    - Y is a bias that we apply to the classifier.
    - If Y is large, then we assume the presence of a face even when we are not sure
  - By increasing Y, we can reduce false rejection, while increasing false detection
    - Many instances for which $\Sigma_t \alpha_t h_t(x)$ is negative get classified as faces

ROC



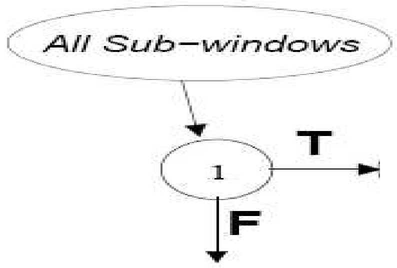
- Ideally false rejection will be 0%, false detection will also be 0%
- As Y increases, we reject faces less and less
  - But accept increasing amounts of garbage as faces
- Can set Y so that we rarely miss a face

Problem: Not accurate enough, too slow



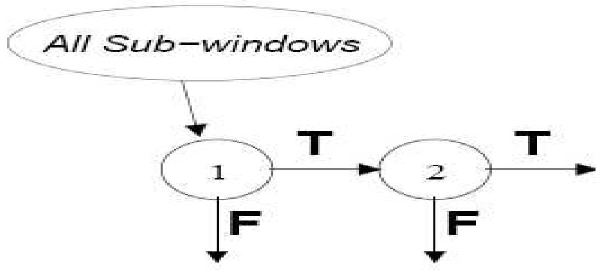
- If we set Y high enough, we will never miss a face
  - But will classify a lot of junk as faces
- Solution: Classify the output of the first classifier with a second classifier
  - And so on.

Cascaded Classifiers



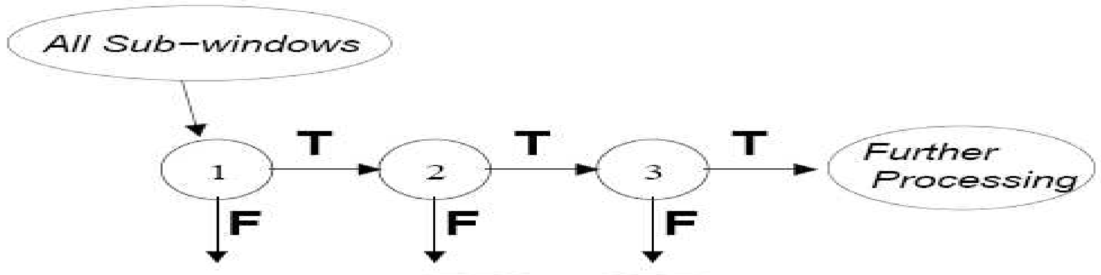
- Build the first classifier to have near-zero false rejection rate
  - But will reject a large number of non-face images

Cascaded Classifiers



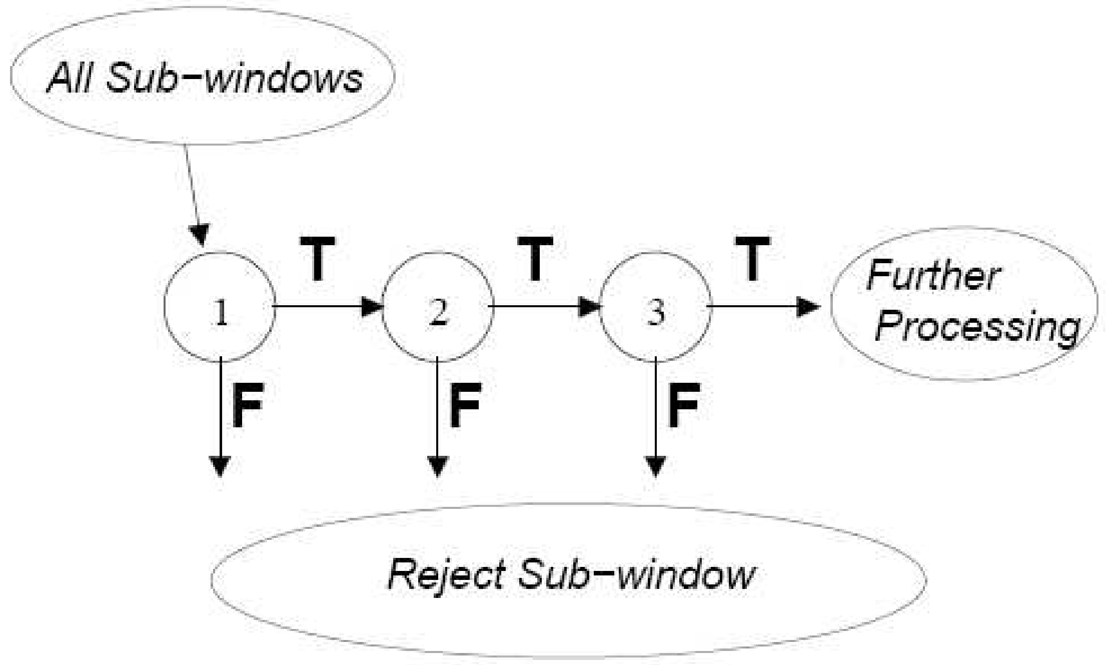
- Build the first classifier to have near-zero false rejection rate
  - But will reject a large number of non-face images
- Filter all training data with this classifier
- Build a second classifier on the data that have been passed by the first classifier, to have near-zero false rejection rate
  - This classifier will be different from the first one
    - Different data set

Cascaded Classifiers

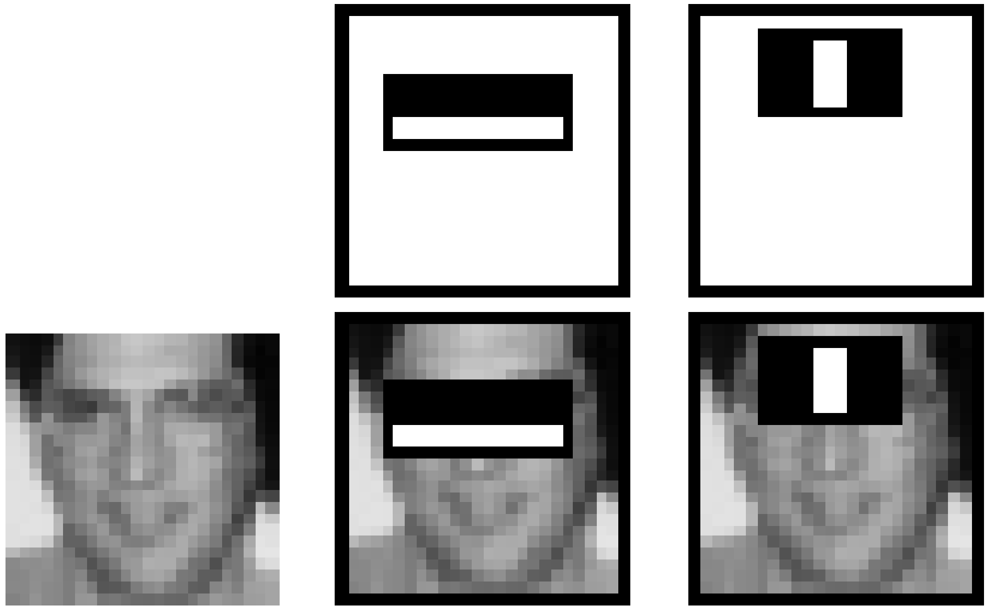


- Build the first classifier to have near-zero false rejection rate
  - But will reject a large number of non-face images
- Filter all training data with this classifier
- Build a second classifier on the data that have been passed by the first classifier, to have near-zero false rejection rate
  - This classifier will be different from the first one
    - Different data set
- Filter all training data with the cascade of the first two classifiers
- Build a third classifier on data passed by the cascade..
  - And so on..

Final Cascade of Classifiers



Useful Features Learned by Boosting



Detection in Real Images

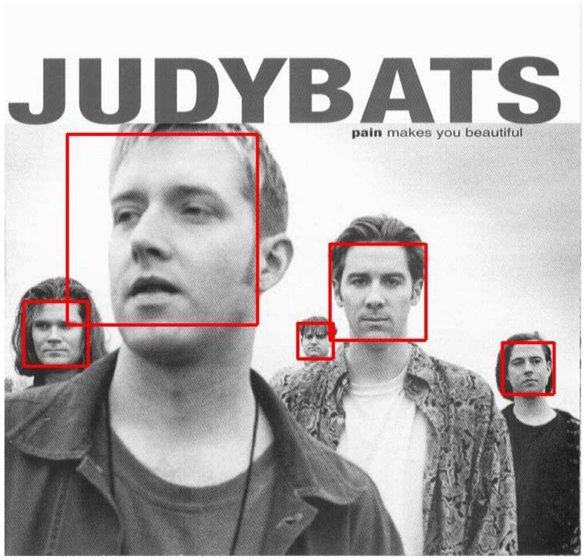
- Basic classifier operates on 24 x 24 subwindows
- Scaling:
  - Scale the detector (rather than the images)
  - Features can easily be evaluated at any scale
  - Scale by factors of 1.25
- Location:
  - Move detector around the image (e.g., 1 pixel increments)
- Final Detections
  - A real face may result in multiple nearby detections
  - Postprocess detected subwindows to combine overlapping detections into a single detection

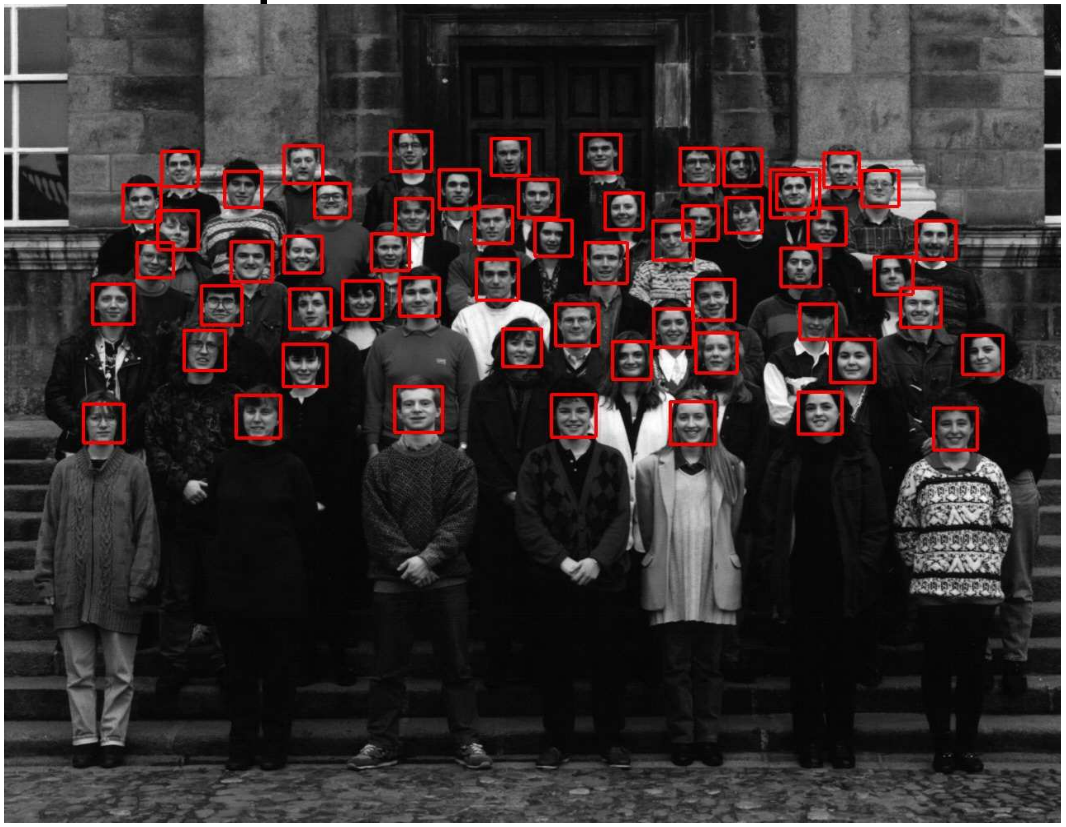
Training

- In paper, 24x24 images of faces and non faces (positive and negative examples).

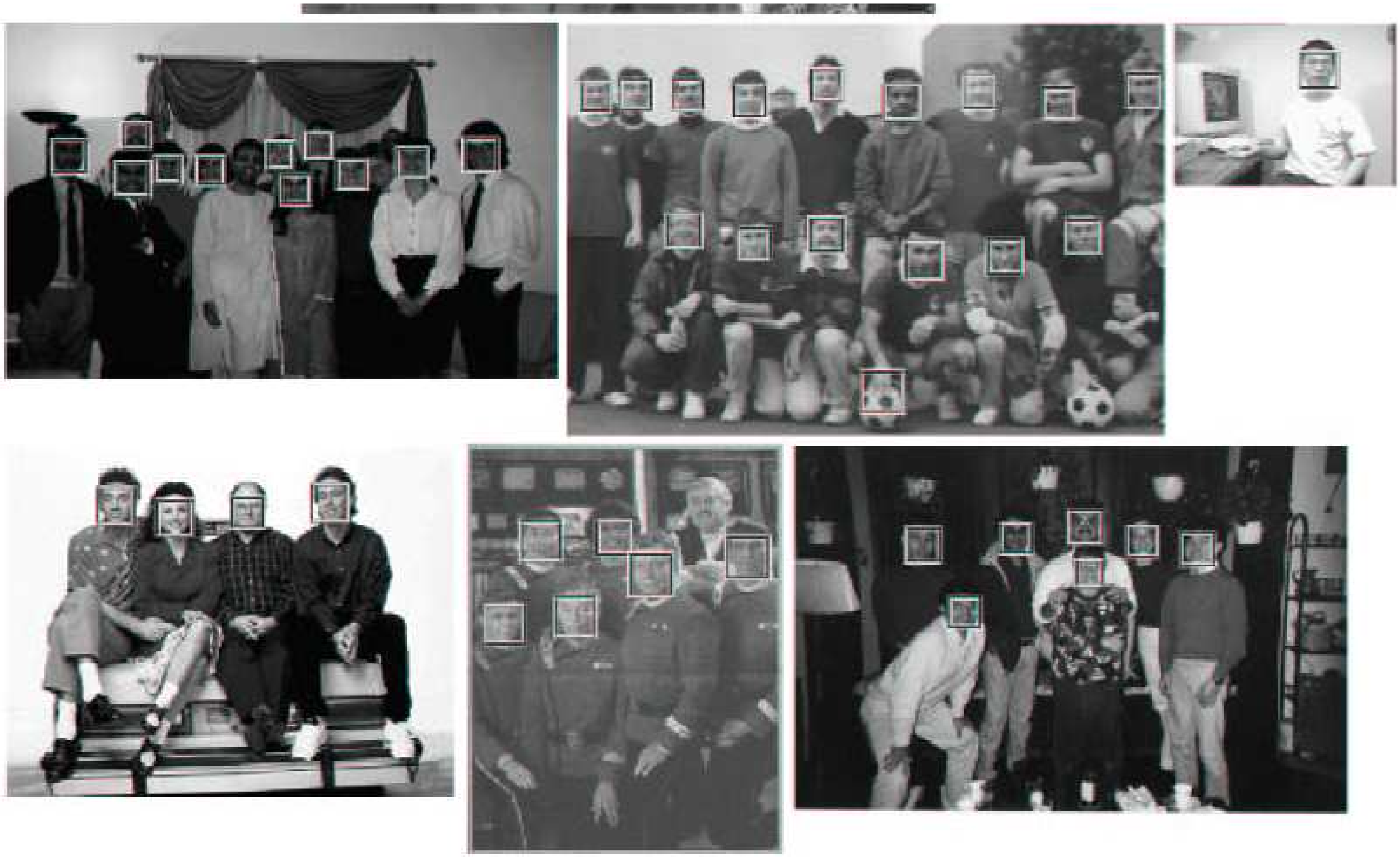
Sample results using the Viola-Jones Detector

- Notice detection at multiple scales





More Detection Examples

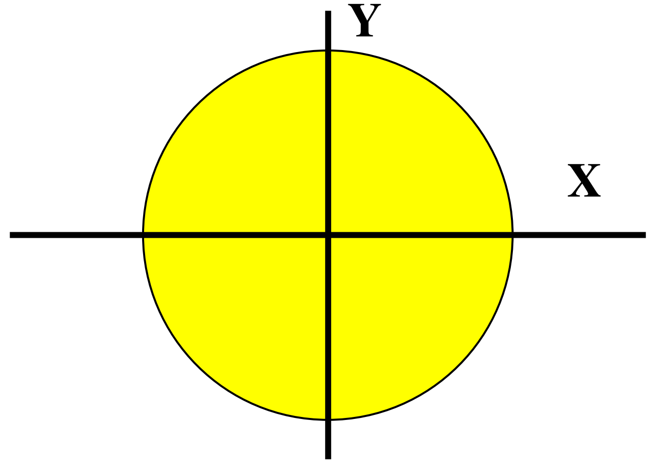


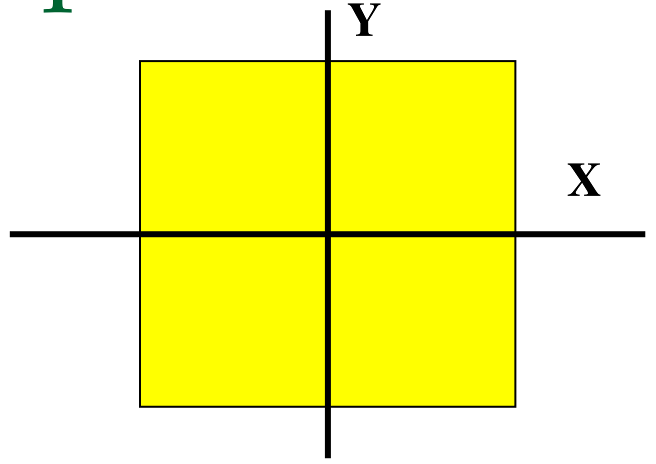
Practical implementation

- Details discussed in Viola-Jones paper
- Training time = weeks (with 5k faces and 9.5k non-faces)
- Final detector has 38 layers in the cascade, 6060 features
- 700 Mhz processor:
  - Can process a 384 x 288 image in 0.067 seconds (in 2003 when paper was written)

- MORE RECAPS

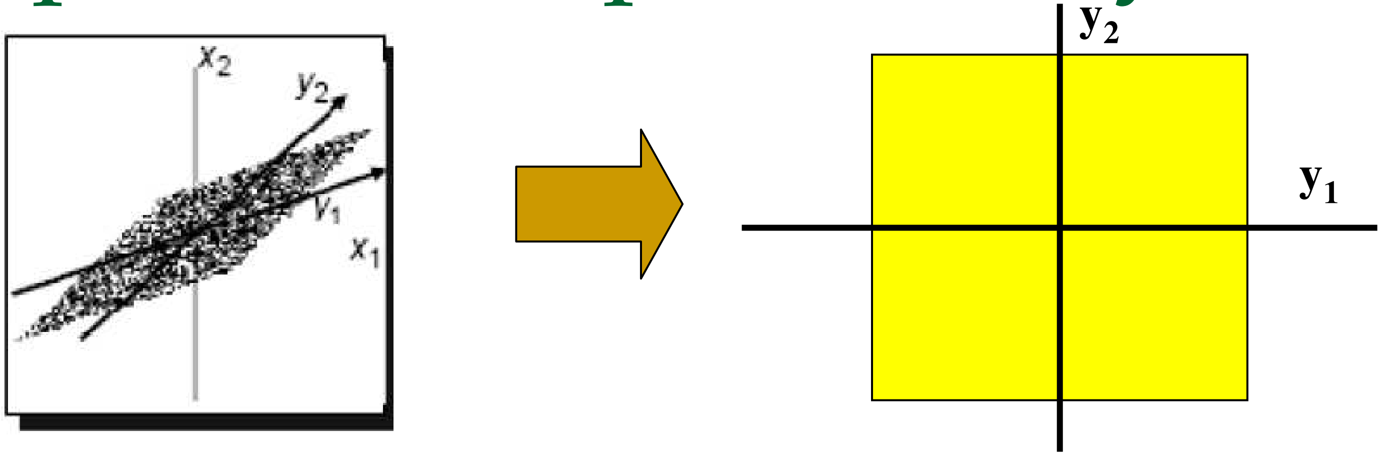
Uncorrelated vs. Independence





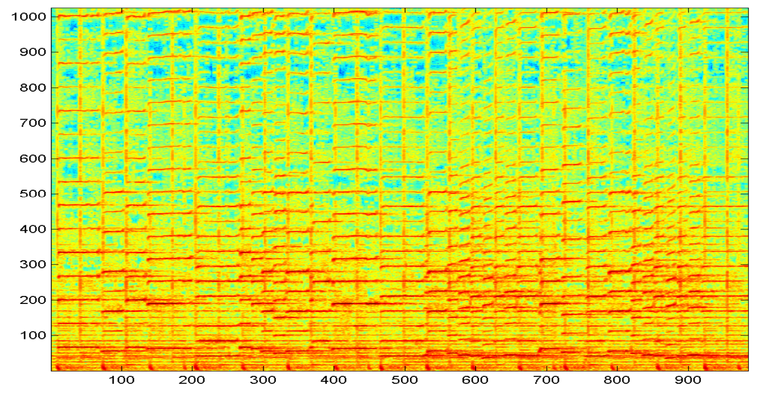
- Left panel: What does the value of X tell you about the average value of Y?
  - But what does X tell you about the distribution of Y?
- Right panel: What does the value of X tell you about the average value of Y?
  - What about the distribution?
  - X and Y are independent!

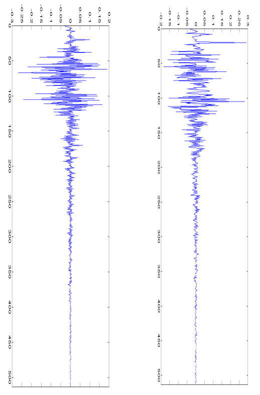
Independent component analysis



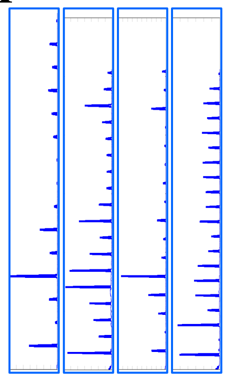
- Pick “basis” vectors such that projections along one tell you *nothing* about projections along another
  - Not merely such that they do not tell you anything about the average value
- These represent “independent” factors that compose the data
  - E.g. knowing where one note occurs in music tells you nothing about where another note occurs
  - These are independent factors

Non-negative Matrix Factorization



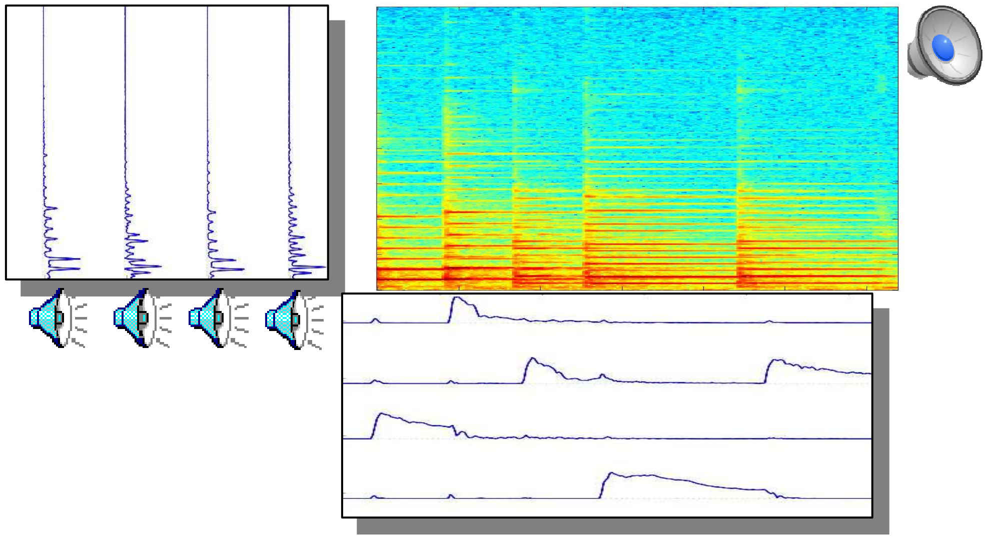


What we need to explain the music



- Some times components only add
  - Notes in a piece of music are purely additive
  - Playing one note will not cancel out another that is simultaneously played
- PCA / Eigen analysis result in bases that combine both additively and subtractively
  - E.g. for the piece of music above, the first eigen vector includes frequencies that are not in the first note. They must be subtracted out by subsequent eigen vectors

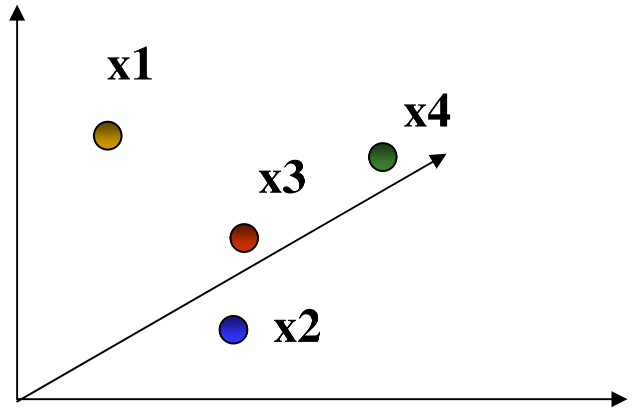
Non-negative Matrix Factorization



- NMF will give you *purely additive* bases
  - Bases will be non-negative
  - They will only add and never subtract
- For the music above this *automatically* discovers the notes

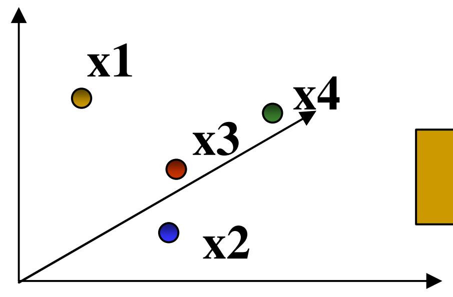
Multi-Dimensional Scaling

	x1	x2	x3	x4
x1	0	1.9	7	6.2
x2		0	11	5.3
x3			0	4.9
x4				0

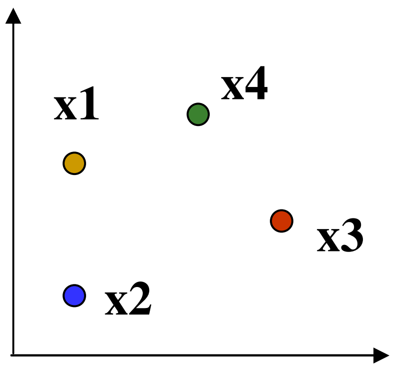


- Given only the distances between data, how do you find their locations in some N-dimensional space
  - The distances may be from anything
    - KL distances, counts, etc.

MDS for dimensionality reduction



	x1	x2	x3	x4
x1	0	1.9	7	6.2
x2		0	11	5.3
x3			0	4.9
x4				0



- Given vectors with very large dimensionality
  - E.g. spectral vectors: 1025 components (frequencies)
  - Images: 10000 components (pixels)
- Compute for each vector Y a new low-dimensional vector Y’ such that the distances between vectors is preserved
  - Compute distances between all vector pairs
  - Employ MDS to get new low-dimensional vectors
    - E.g. 100 dimensions instead of 10000

Additional Topics

- Covered later as required