# Independent Component Analysis

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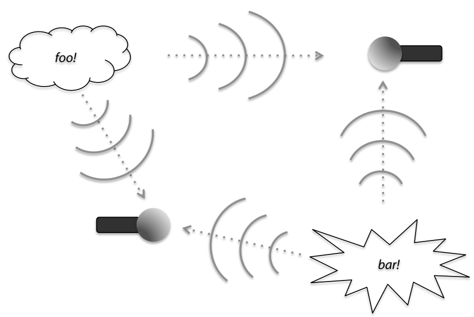
## This lecture's overview

- A motivating example
- The theory
  - Decorrelation
  - Independence vs decorrelation
  - Independent component analysis
- Separating sounds
  - Solving instantaneous mixtures
  - Solving convolutive mixtures
- Data exploration and independence
  - Extracting audio features
  - Extracting multimodal features



## A "simple" audio problem



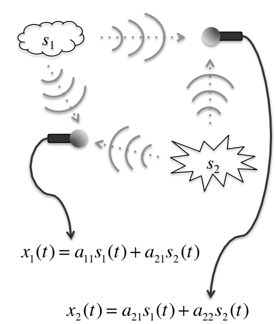



## Formalizing the problem

- Each mic will receive a mix of both sounds
  - Sound waves superimpose linearly
  - We'll ignore propagation delays for now
- The simplified mixing model is:

$$\mathbf{x}(t) = \mathbf{A} \cdot \mathbf{s}(t)$$

- We know $\mathbf{x}(t)$, but nothing else
  - How do we solve this system and find $\mathbf{s}(t)$?



## When can we solve this?

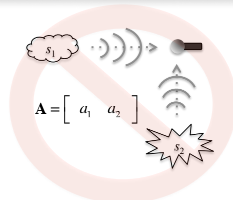
- The mixing equation is:

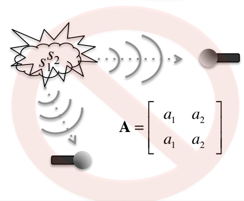
$$\mathbf{x}(t) = \mathbf{A} \cdot \mathbf{s}(t)$$

- Our estimates of $\mathbf{s}(t)$ will be:

$$\hat{\mathbf{s}}(t) = \mathbf{A}^{-1} \cdot \mathbf{x}(t)$$

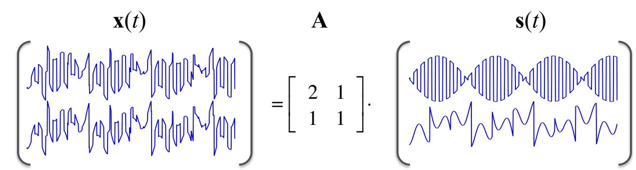
- To recover $\mathbf{s}(t)$, $\mathbf{A}$ must be invertible:
  - We need as many mics as sources
  - The mics/sources must not coincide
  - All sources must be audible
- Otherwise this is a different story …






## A simple example




- A simple invertible problem
  - $\mathbf{s}(t)$ contains two structured waveforms
  - $\mathbf{A}$ is invertible (but we don't know it)
  - $\mathbf{x}(t)$ looks messy, doesn't reveal $\mathbf{s}(t)$ clearly
- How do we solve this? Any ideas?



## What to look for

$$\mathbf{x}(t) = \mathbf{A} \cdot \mathbf{s}(t)$$

- We can only use $\mathbf{x}(t)$
- Is there a property we can take advantage of?
- Yes! We know that different sounds are *"statistically unrelated"*
- The plan: Find a solution that enforces this "*unrelatedness*"



## A first try

- Find $\mathbf{s}(t)$ by minimizing cross-correlation
- Our estimate of $\mathbf{s}(t)$ is computed by:

$$\hat{\mathbf{s}}(t) = \mathbf{W} \cdot \mathbf{x}(t)$$

  - If $\mathbf{W} \approx \mathbf{A}^{-1}$ then we have a good solution
- The goal is that the output becomes uncorrelated:

$$\left\langle \hat{s}_i(t) \cdot \hat{s}_j(t) \right\rangle = 0, \forall i \neq j$$

  - We assume here that our signals are zero mean
- So the overall problem to solve is:

$$\arg\min_{\mathbf{W}} \left\langle \sum_k a_{ik} x_k(t) \cdot \sum_k a_{jk} x_k(t) \right\rangle, \forall i \neq j$$

## How to solve for uncorrelatedness

- Let's use matrices instead of time-series:

$$\mathbf{x}(t) \to \mathbf{X} = \begin{bmatrix} x_1(1), & \cdots, & x_1(N) \\ x_2(1), & \cdots, & x_2(N) \end{bmatrix}, etc \qquad \begin{array}{c} \mathbf{x}(t) = \mathbf{A} \cdot \mathbf{s}(t) \text{ and } \hat{\mathbf{s}}(t) = \mathbf{W} \cdot \mathbf{x}(t) \Rightarrow \\ \Rightarrow \mathbf{X} = \mathbf{A} \cdot \mathbf{S} \text{ and } \hat{\mathbf{S}} = \mathbf{W} \cdot \mathbf{X} \end{array}$$

- The uncorrelatedness translates to:

$$\frac{1}{N}\hat{\mathbf{S}} \cdot \hat{\mathbf{S}}^T = \begin{bmatrix} c_{11} & 0 \\ 0 & c_{22} \end{bmatrix}$$

- We then need to diagonalize:

$$\frac{1}{N}\hat{\mathbf{S}} \cdot \hat{\mathbf{S}}^T = \frac{1}{N}(\mathbf{W} \cdot \mathbf{X})(\mathbf{W} \cdot \mathbf{X})^T$$



## How to solve for uncorrelatedness

- This is actually a well known problem in linear algebra
- One solution is Eigenanalysis:

$$\begin{aligned} Cov(\hat{\mathbf{S}}) &= \frac{1}{N}\hat{\mathbf{S}} \cdot \hat{\mathbf{S}}^T \\ &= \frac{1}{N}(\mathbf{W} \cdot \mathbf{X})(\mathbf{W} \cdot \mathbf{X})^T \\ &= \frac{1}{N}\mathbf{W} \cdot \mathbf{X} \cdot \mathbf{X}^T \cdot \mathbf{W}^T \\ &= \mathbf{W} \cdot Cov(\mathbf{X}) \cdot \mathbf{W}^T \end{aligned}$$

- $Cov(\mathbf{X})$ is a symmetric matrix



## How to solve for uncorrelatedness

- For any symmetric matrix $\mathbf{Z}$ we know that:

$$\mathbf{Z} = \mathbf{U} \cdot \mathbf{D} \cdot \mathbf{U}^T = \begin{bmatrix} \uparrow & & \uparrow \\ \mathbf{u}_1 & \cdots & \mathbf{u}_N \\ \downarrow & & \downarrow \end{bmatrix} \cdot \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_N \end{bmatrix} \cdot \begin{bmatrix} \leftarrow & \mathbf{u}_1 & \rightarrow \\ & & \\ \leftarrow & \mathbf{u}_N & \rightarrow \end{bmatrix}$$

- Where $\mathbf{u}_i$ and $d_i$ are $\mathbf{Z}$'s eigenvectors and eigenvalues respectively
- In our case if $[\mathbf{U},\mathbf{D}] = \text{eig}(\ Cov(\ \mathbf{X}))$

$$\begin{aligned} Cov(\hat{\mathbf{S}}) &= \mathbf{W} \cdot Cov(\mathbf{X}) \cdot \mathbf{W}^T \\ &= \mathbf{W} \cdot \mathbf{U} \cdot \mathbf{D} \cdot \mathbf{U}^T \cdot \mathbf{W}^T && \text{let us replace } \mathbf{W} \text{ with } \mathbf{U}^T \\ &= \mathbf{U}^T \cdot \mathbf{U} \cdot \mathbf{D} \cdot \mathbf{U}^T \cdot \mathbf{U} && \mathbf{U} \text{ is orthonormal } (\mathbf{U} \cdot \mathbf{U}^T = \mathbf{I}) \\ &= \mathbf{D} \end{aligned}$$

- This is actually Principal Component Analysis



## Another solution

- We can also solve for a matrix inverse square root:

$$\begin{aligned} Cov(\hat{\mathbf{S}}) &\propto \mathbf{W} \cdot \mathbf{X} \cdot \mathbf{X}^T \cdot \mathbf{W}^T \\ &= (\mathbf{X} \cdot \mathbf{X}^T)^{-1/2} \cdot \mathbf{X} \cdot \mathbf{X}^T \cdot (\mathbf{X}^T \cdot \mathbf{X})^{-1/2} && \text{replace } \mathbf{W} \text{ with } (\mathbf{X} \cdot \mathbf{X}^T)^{-1/2} \\ &= \mathbf{I} \end{aligned}$$

$$(\mathbf{X} \cdot \mathbf{X})^{-1/2} = \mathbf{U} \cdot \begin{bmatrix} d_1^{-1/2} & & \\ & \ddots & \\ & & d_N^{-1/2} \end{bmatrix} \cdot \mathbf{U}^T, \quad \text{where } [\mathbf{U},\mathbf{D}] = eig(\mathbf{X} \cdot \mathbf{X}^T)$$

## Another approach

- What if we want to do this in real-time?
- We can also estimate $\mathbf{W}$ in an online manner:

$$\Delta\mathbf{W} \propto \mu\left(\mathbf{I} - \mathbf{W}\cdot\mathbf{x}(t)\cdot\mathbf{x}(t)^T\cdot\mathbf{W}^T\right)\mathbf{W}$$

- Every time we see a new observation $\mathbf{x}(t)$ we update $\mathbf{W}$
- Using this adaptive approach we can see that:

$$Cov\left(\mathbf{W}\cdot\mathbf{x}(t)\right) = \mathbf{I}, \quad \Delta\mathbf{W} = 0$$

- We'll skip the derivation details for now

## Summary so far

- For a mixture:

$$\mathbf{X} = \mathbf{A}\cdot\mathbf{S}$$

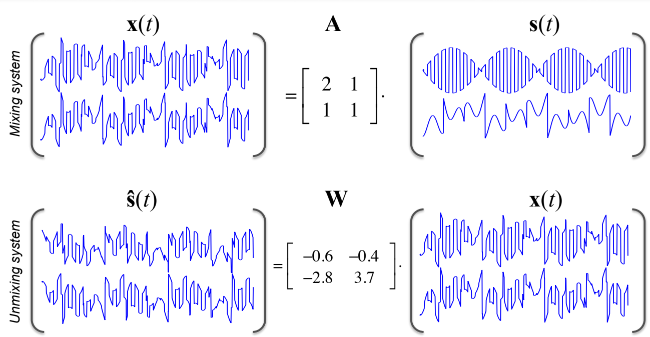
- We can algebraically recover an uncorrelated output using

$$\hat{\mathbf{S}} = \mathbf{W}\cdot\mathbf{X}$$

  - If $\mathbf{W}$ is the eigenvector matrix of $Cov(\mathbf{X})$
  - Or with $\mathbf{W} = Cov(\mathbf{X})^{-1/2}$
- Or we can use an online estimator:

$$\Delta\mathbf{W} \propto \mu\left(\mathbf{I} - \mathbf{W}\cdot\mathbf{x}(t)\cdot\mathbf{x}(t)^T\cdot\mathbf{W}^T\right)\mathbf{W}$$

## So how well does this work?

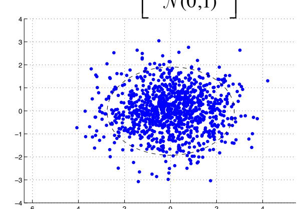


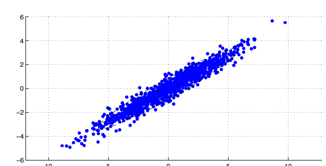

- Well, that was a waste of time …

## What went wrong?

- What does decorrelation mean?
  - That the two things compared are "not related"
- Consider a mixture of two Gaussians

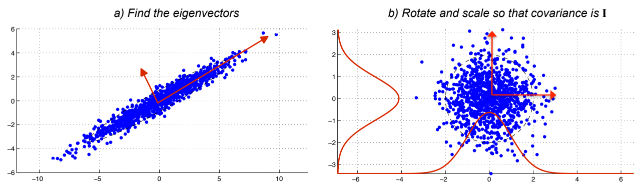
$$\mathbf{s}(t) = \begin{bmatrix} \mathcal{N}(0,2) \\ \mathcal{N}(0,1) \end{bmatrix} \qquad \mathbf{x}(t) = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\cdot\mathbf{s}(t)$$





## Decorrelation

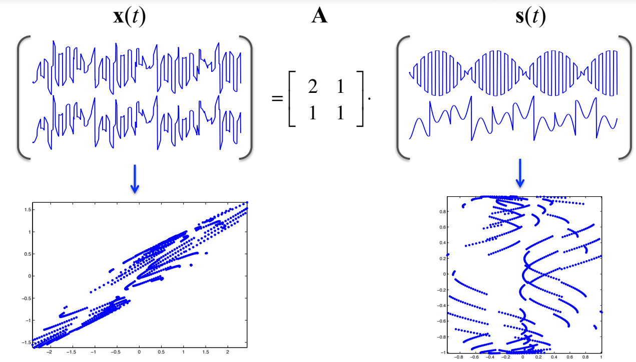
- Now let us do what we derived so far on this signal:




- After we are done the two Gaussians are "*statistically independent*"
  - i.e., $P(s_1, s_2) = P(s_1)P(s_2)$
- We have in effect separated the original signals
  - Save for a scaling ambiguity



## Now lets try this on the original data

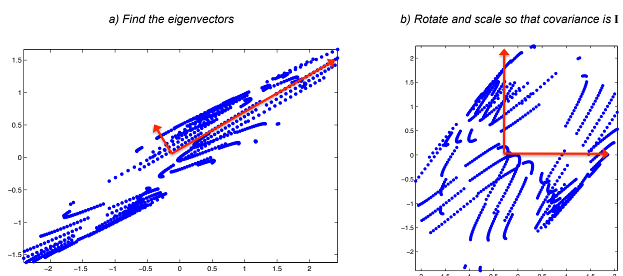



- Stating the obvious: These are not very Gaussian signals!!



## Doing PCA doesn't give us the right solution

- The result is not what we want
  - We are off by a rotation
- This idea doesn't seem to work for non-Gaussian signals






## So what's wrong?

- For Gaussian data decorrelation means independence
  - Gaussians have up to second order statistics (1st is mean, 2nd is variance)
  - By minimizing the 2nd-order cross-statistics we achieve independence
  - These statistics can be expressed by the 2nd-order cumulants:

$$cum(x_i, x_j) = \langle x_i x_j \rangle$$

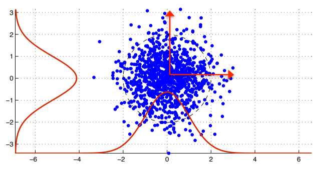
  - Which happen to be the diagonals of the covariance matrix
- But real-world data are seldom Gaussian
  - Non-Gaussian data have higher orders which are not taken care of with PCA
  - We can measure their dependence using higher order cumulants:

$$3^{rd} \text{ order:} \quad cum(x_i, x_j, x_k) = \langle x_i x_j x_k \rangle$$

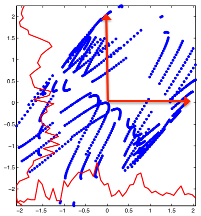
$$4^{th} \text{ order:} \quad cum(x_i, x_j, x_k, x_l) = \langle x_i x_j x_k x_l \rangle - \langle x_i x_j \rangle \langle x_k x_l \rangle - \langle x_i x_k \rangle \langle x_j x_l \rangle - \langle x_i x_l \rangle \langle x_k x_j \rangle$$

## Cumulants for Gaussian vs non-Gaussian case

*Gaussian case*



*Non-Gaussian case*



*Cross-cumulants tend to zero*

| | |
|---|---|
| $cum(x_i, x_j)$ | $1e^{-13}$ |
| $cum(x_i, x_j, x_k)$ | 0.0008, 0.0004 |
| $cum(x_i, x_j, x_k, x_l)$ | -0.003, 0.0005, 0.0007 |

*Only 2nd order cross-cumulants tend to zero*

| | |
|---|---|
| $cum(x_i, x_j)$ | $-4e^{-14}$ |
| $cum(x_i, x_j, x_k)$ | -0.08, -0.1 |
| $cum(x_i, x_j, x_k, x_l)$ | 0.42, -0.3, 0.16 |



## The real problem to solve

- For statistical independence we need to minimize <u>all</u> cross-cumulants
  - In practice up to 4th order is enough
- For 2nd order we minimized the off-diagonal covariance elements

$$\begin{bmatrix} cum(x_1,x_1) & cum(x_1,x_2) \\ cum(x_2,x_1) & cum(x_2,x_2) \end{bmatrix}$$

- For 4th order we will do the same for a *tensor*

$$Q_{i,j,k,l} = cum(x_i, x_j, x_k, x_l)$$

- The process is similar to PCA, but in more dimensions
  - We now find *"eigenmatrices"* instead of eigenvectors
- Algorithms like JADE and FOBI solve this problem
  - Can you see a potential problem though?



## An alternative approach

- Tensorial methods can be very very computationally intensive
- How about an on-line method instead?
- Independence can also be coined as *"non-linear decorrelation"*
  - $x$ and $y$ are independent if and only if:

$$\langle f(x)g(y)\rangle = \langle f(x)\rangle\langle g(y)\rangle$$

  - For all continuous functions $f$ and $g$
  - This is a non-linear extension of 2nd order independence where $f(x) = g(x) = x$
- We can try solving for that then



## Online ICA

- Conceptually this is very similar to online decorrelation
  - For decorrelation:

$$\Delta\mathbf{W} \propto \mu\left(\mathbf{I} - \mathbf{W}\cdot\mathbf{x}(t)\cdot\mathbf{x}(t)^T\cdot\mathbf{W}^T\right)\mathbf{W}$$

  - For non-linear decorrelation:

$$\Delta\mathbf{W} \propto \mu\left(\mathbf{I} - f\left(\mathbf{W}\cdot\mathbf{x}(t)\right)\cdot g\left(\mathbf{W}\cdot\mathbf{x}(t)\right)^T\right)\mathbf{W}$$

- This adaptation method is known as the Cichocki-Unbehauen update
  - But we can obtain it using many different ways
- But how do we pick the non-linearities?
  - Depends on the prior we have on the sources

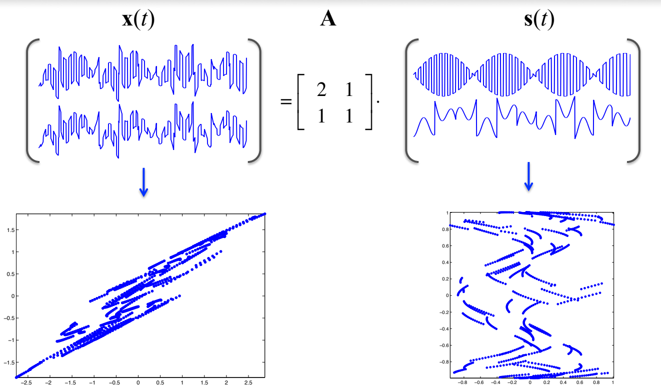
$$f(x_i) = \begin{cases} x + \tanh(x), & \text{for super-Gaussians} \\ x - \tanh(x), & \text{for sub-Gaussians} \end{cases}$$

## Other popular approaches

- Minimum Mutual Information
  - Minimize the mutual information of the output
  - Creates maximally statistically independent outputs
- Infomax
  - Maximize the entropy of the output or Mutual Information of input/output
- Non-Gaussianity
  - Adding signals tends towards Gaussianity (Central Limit Theorem)
  - Find the maximally non-Gaussian outputs undoes the mixing
- Maximum Likelihood
  - Less straightforward at first, but elegant nevertheless
- Geometric methods
  - Trying to “eyeball” the proper way to rotate

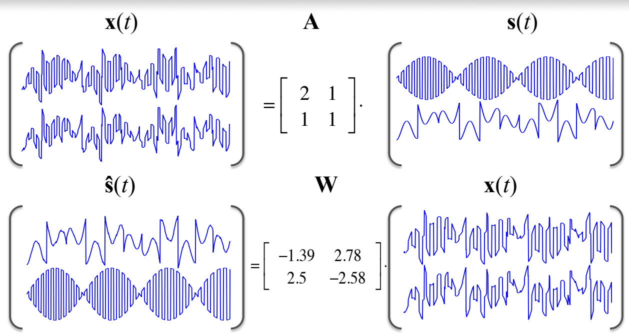


## Trying this on our dataset





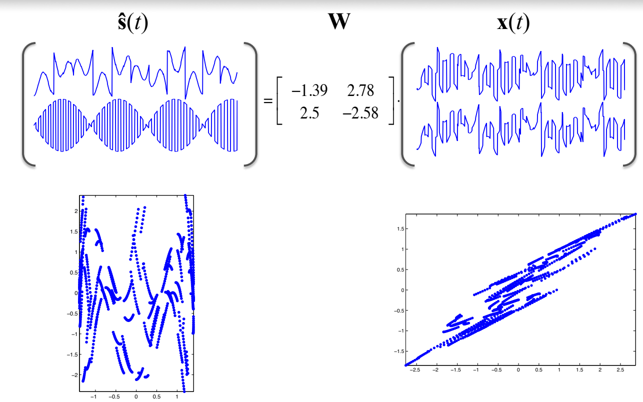

## Trying this on our dataset




- We actually separated the mixture!



## Trying this on our dataset

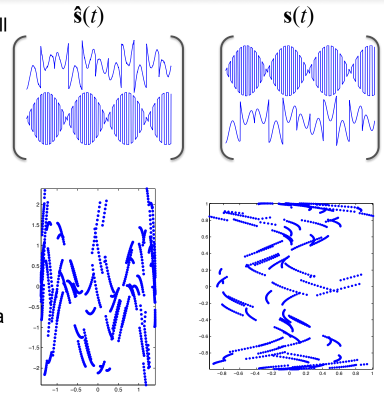


## But something is amiss ..

- There some things that ICA will not resolve
- Scale
  - Statistical independence is invariant of scale (and sign)
- Order of inputs
  - Order of inputs is irrelevant when talking about independence
- ICA will actually recover:

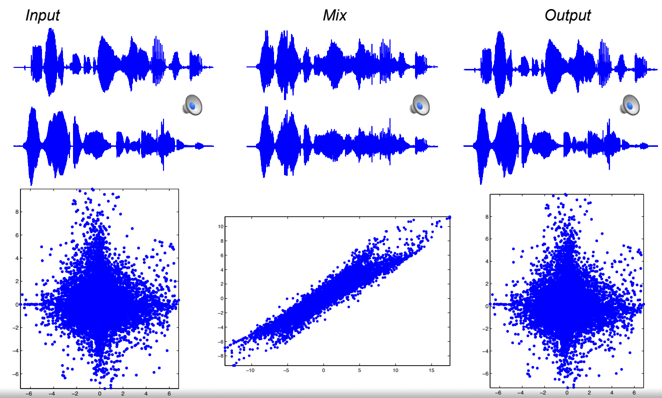
$$\hat{\mathbf{s}}(t) = \mathbf{D} \cdot \mathbf{P} \cdot \mathbf{s}(t)$$

- Where $\mathbf{D}$ is diagonal and $\mathbf{P}$ is a permutation matrix





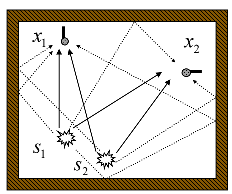

## This works really well for audio mixtures!






## Problems with instantaneous mixing

- Sounds don't really mix instantaneously
- There are multiple effects
  - Room reflections
  - Sensor response
  - Propagation delays
  - Propagation and reflection filtering
- Most can be seen as filters
- We need a *convolutive mixing* model




*Estimated sources using the instantaneous model on convolutive mix*



## Convolutive mixing

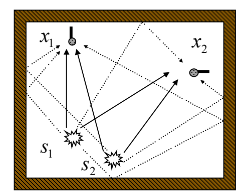
- Instead of instantaneous mixing:

$$x_i(t) = \sum_{j=1} a_{ij} s_j(t)$$

- We now have *convolutive* mixing:

$$x_i(t) = \sum_j \sum_k a_{ij}(k) s_j(t-k)$$

- The mixing filters $a_{ij}(k)$ encapsulate all the mixing effects in this model
- But how do we do ICA now?
  - This is an ugly equation!



## FIR matrix algebra

- Matrices with FIR filters as elements

$$\underline{\mathbf{A}} = \begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix}$$

$$\underline{a}_{ij} = \begin{bmatrix} a_{ij}(0) & \cdots & a_{ij}(k-1) \end{bmatrix}$$

- FIR matrix multiplication performs convolution and accumulation

$$\underline{\mathbf{A}} \cdot \underline{\mathbf{b}} = \begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix} \cdot \begin{bmatrix} \underline{b}_1 \\ \underline{b}_2 \end{bmatrix} = \begin{bmatrix} \underline{a}_{11} * \underline{b}_1 + \underline{a}_{12} * \underline{b}_2 \\ \underline{a}_{21} * \underline{b}_1 + \underline{a}_{22} * \underline{b}_2 \end{bmatrix}$$

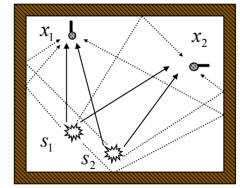


## Back to convolutive mixing

- Now we can rewrite convolutive mixing as:

$$x_i(t) = \sum_j \sum_k a_{ij}(k) s_j(t-k) \Rightarrow$$

$$\Rightarrow \mathbf{x}(t) = \underline{\mathbf{A}} \cdot \mathbf{s}(t) = \begin{bmatrix} a_{11} * s_1(t) + a_{12} * s_2(t) \\ a_{21} * s_1(t) + a_{22} * s_2(t) \end{bmatrix}$$




- Tidier formulation!
- We can use the FIR matrix abstraction to solve this problem now



## An easy way to solve convolutive mixing

- Straightforward translation of instantaneous learning rules using FIR matrices:

$$\Delta \underline{\mathbf{W}} \propto \left( \mathbf{I} + f(\underline{\mathbf{W}} \cdot \underline{\mathbf{x}}) \cdot (\underline{\mathbf{W}} \cdot \underline{\mathbf{x}})^T \right) \cdot \underline{\mathbf{W}}$$

- Not so easy with algebraic approaches!
- Multiple other (and more rigorous/better behaved) approaches have been developed



## Complications with this approach

- Required convolutions are expensive
  - Real-room filters are long
  - Their FIR inverses are very long
  - FIR products can become very time consuming
- Convergence is hard to achieve
  - Huge parameter space
  - Tightly interwoven parameter relationships
- A slow optimization nightmare!

## FIR matrix algebra, part II

- FIR matrices have frequency domain counterparts:

$$\underline{\mathbf{A}} = \begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix} \xrightarrow{\text{frequency domain}} \hat{\underline{\mathbf{A}}} = \begin{bmatrix} \hat{\underline{a}}_{11} & \hat{\underline{a}}_{12} \\ \hat{\underline{a}}_{21} & \hat{\underline{a}}_{22} \end{bmatrix}$$

$$\hat{\underline{a}}_{ij} = \mathrm{DFT}[\underline{a}_{ij}]$$

- And their products are simpler:

$$\hat{\underline{\mathbf{A}}} \cdot \hat{\underline{\mathbf{b}}} = \begin{bmatrix} \hat{\underline{a}}_{11} \cdot \hat{\underline{b}}_1 + \hat{\underline{a}}_{12} \cdot \hat{\underline{b}}_2 \\ \hat{\underline{a}}_{21} \cdot \hat{\underline{b}}_1 + \hat{\underline{a}}_{22} \cdot \hat{\underline{b}}_2 \end{bmatrix}$$

$$\hat{\underline{a}} \cdot \hat{\underline{b}} = \begin{bmatrix} a(0) \cdot b(0) & \cdots & a(k-1) \cdot b(k-1) \end{bmatrix}$$



## Yet another convolutive mixing formulation

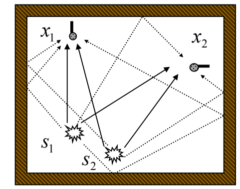
- We can now model the process in the frequency domain:

$$\hat{\underline{\mathbf{X}}} = \hat{\underline{\mathbf{A}}} \cdot \hat{\underline{\mathbf{S}}}$$

- For every frequency we have:

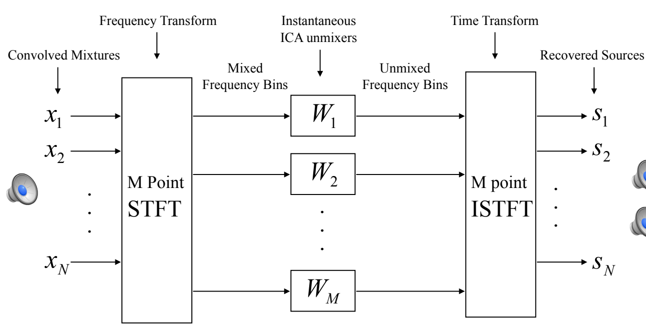
$$\mathbf{X}_f(t) = \mathbf{A}_f \cdot \mathbf{S}_f(t), \ \forall f, t$$

- Hey, that's instantaneous mixing!
  - We can solve that!






## Overall flowgraph






## Some complications …

- Permutation issues
  - We don't know which source will end up in each narrowband output …
  - Resulting output can have separated narrowband elements from both sounds!

*Extracted source with permutation*

- Scaling issues
  - Narrowband outputs can be scaled arbitrarily
  - This results in spectrally colored outputs

*Original source* *Colored source*

## Scaling issue

- One simple fix is to normalize the separating matrices

$$\mathbf{W}_f^{norm} = \mathbf{W}_f^{orig} \cdot \left|\mathbf{W}_f^{orig}\right|^{\frac{1}{N}}$$

  - Results into more reasonable scaling

- More sophisticated approaches exist but this is not a major problem
- Some spectral coloration is however unavoidable

*Original source* *Colored source* *Corrected source*



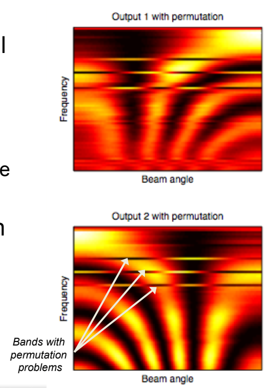
## Some solutions for permutation problems

- Continuity of unmixing matrices
  - Adjacent unmixing matrices tend to be a little similar, we can permute/bias them accordingly
    - Doesn't work that great
- Smoothness of spectral output
  - Narrowband components from each source tend to modulate the same way
  - Permute unmixing matrices to ensure adjacent narrowband output are similarly modulated
    - Works fine
- The above can fail miserably for more than two sources!
  - Combinatorial explosion!



## Beamforming and ICA

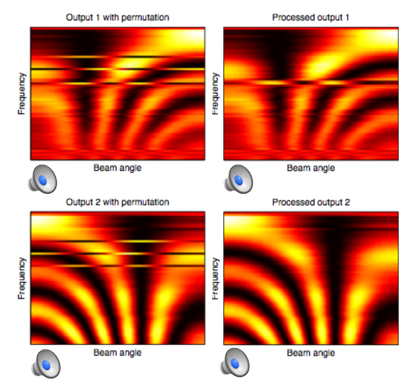
- If we know the placement of the sensors we can obtain the spatial response of the ICA solution
- ICA places nulls to cancel out interfering sources
  - Just as in the instantaneous case we cancel out sources
- We can visualize the permutation problem now
  - Out of place bands






## Using beamforming to resolve permutations

- Spatial information can be used to resolve permutations
  - Find permutations that preserve zeros or smooth out the responses
- Works fine, although it can be flaky if the array response is not that clean



## The N-input N-output problem

- ICA, in either formulation inverts a square matrix (whether scalar, or FIR)
  - This implies that we have the same number of input as outputs
  - E.g. in a street with 30 noise sources we need at least 30 mics!
- Solutions exist for $M$ ins - $N$ outs where $M > N$
- If $N > M$ we can only beamform
  - In some cases extra sources can be treated as noise
- This can be restrictive in some situations



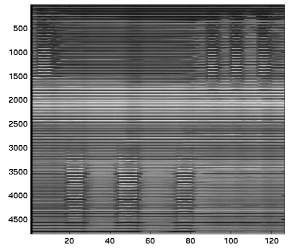
## Separation recap

- Orthogonality is not independence!!
  - Not all signals are Gaussian which is a usual assumption
- We can model instantaneous mixtures with ICA and get good results
  - ICA algorithms can optimize a variety of objectives, but ultimately result in statistical independence between the outputs
  - Same model is useful for all sorts of mixing situations
- Convolutive mixtures are more challenging but solvable
  - There's more ambiguity, and a closer link to signal processing approaches



## ICA for data exploration

- ICA is also great for data exploration
  - If PCA is, then ICA should be, right?
- With data of large dimensionalities we want to find structure
- PCA can reduce the dimensionality
  - And clean up the data structure a bit
- But ICA can find much more intuitive projections

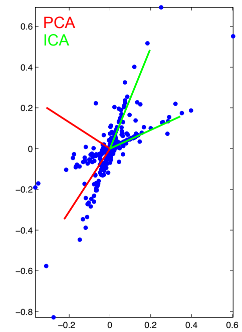





## Example cases of PCA vs ICA

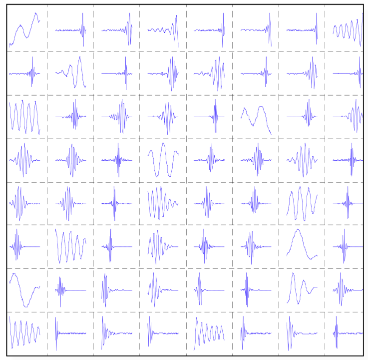
- Motivation for using ICA vs PCA
- PCA will indicate orthogonal directions of maximal variance
  - This is great for Gaussian data
  - Also great if we are into LS models
- Real-world is not Gaussian though
- ICA finds directions that are more "revealing"

*Non-Gaussian data*



## Finding useful transforms with ICA

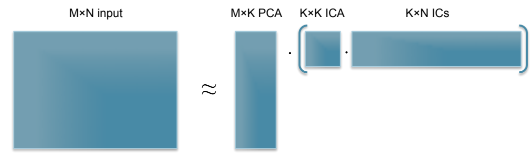
- Audio preprocessing example
- Take a lot of audio snippets and concatenate them in a big matrix, do component analysis
- PCA results in the DCT bases
  - Do you see why?
- ICA returns time/freq localized sinusoids which is a better way to analyze sounds
- Ditto for images
  - ICA returns localizes edge filters





## Enhancing PCA with ICA

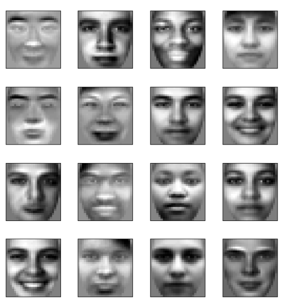
- ICA cannot perform dimensionality reduction
  - The goal is to find independent components, hence there is no sense of order
- PCA does a great job at reducing dimensionality
  - Keeps the elements that carry most of the input's energy
- It turns out that PCA is a great preprocessor for ICA
  - There is no guarantee that the PCA subspace will be appropriate for the independent components but for most practical purposes this doesn't make a big difference



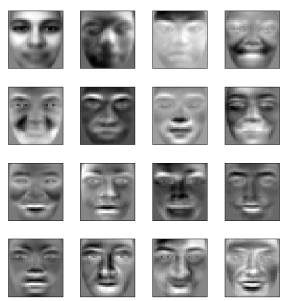



## Example case: ICA-faces vs. Eigenfaces

ICA-faces



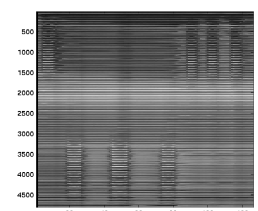
Eigenfaces





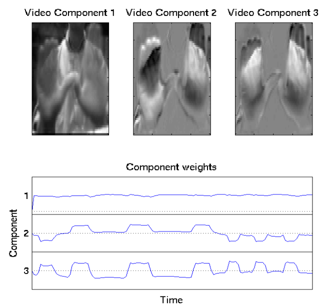
## A Video Example

- The movie is a series of frames
  - Each frame is a data point
  - 126, 80×60 pixel frames
  - Data $\mathbf{X}$ will be 4800×126
- Using PCA/ICA
  - $\mathbf{X} = \mathbf{W} \times \mathbf{H}$
  - $\mathbf{W}$ will contain visual components
  - $\mathbf{H}$ will contain their time weights



## PCA Results

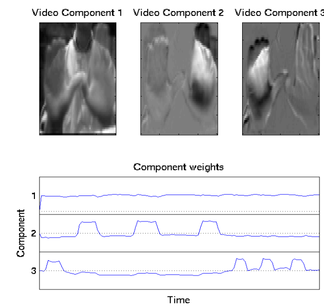
- Nothing special about the visual components
- They are orthogonal pictures
  - Does this mean anything? (not really …)
  - Some segmentation of constant vs. moving parts
- Some highlighting of the action in the weights






## ICA Results

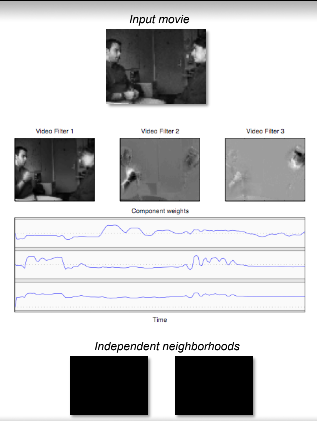
- Much more interesting visual components
- They are *independent*
  - Unrelated elements (left/ right hands, background) are now highlighted
  - We have some decomposition by parts
- Components weights are now describing the scene






## A Video Example

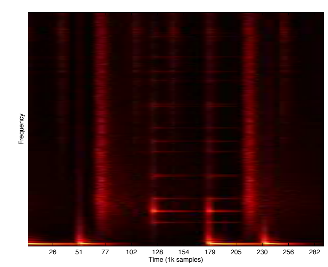
- The movie is a series of frames
  - Each frame is a data point
  - 315, 80×60 pixel frames
  - Data $\mathbf{X}$ will be 4800×315
- Using PCA/ICA
  - $\mathbf{X} = \mathbf{W} \times \mathbf{H}$
  - $\mathbf{W}$ will contain visual components
  - $\mathbf{H}$ will contain their time weights





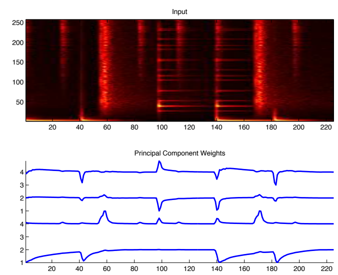

## What about the soundtrack?

- We can also analyze audio in a similar way
- We do a frequency transform and get an audio spectrogram $\mathbf{X}$
  - $\mathbf{X}$ is frequencies × time
  - Distinct audio elements can be seen in $\mathbf{X}$
- Unlike before we have only one input this time



## PCA on Audio

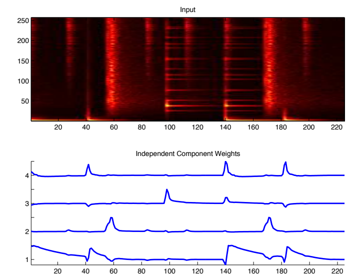
- Umm … it sucks!
- Orthogonality doesn't mean much for audio components
  - Results are mathematically optimal, perceptually useless






## ICA on Audio

- A definite improvement
- Independence helps pick up somewhat more meaningful sound objects
  - Not too clean results, but the intentions are clear
  - Misses some details



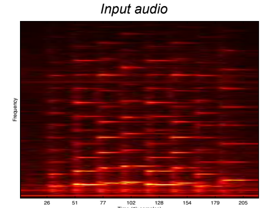



## Audio Visual Components?

- We can can even take in both audio and video data and try to find structure
- Sometimes there is a very strong correlation between auditory and visual elements
- We should be able to discover that automatically

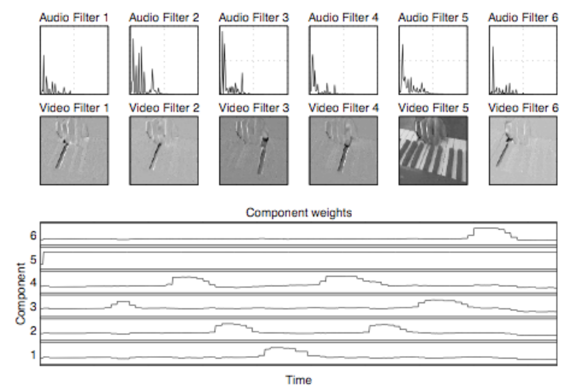
*Input video*

*Input audio*





## Audio/Visual Components



## Which allows us to play with output

- And of course once we have such a nice description we can resynthesize at will

*Resynthesis*



## Recap

- A motivating example
- The theory
  - Decorrelation
  - Independence vs decorrelation
  - Independent component analysis
- Separating sounds
  - Solving instantaneous mixtures
  - Solving convolutive mixtures
- Data exploration and independence
  - Extracting audio features
  - Extracting multimodal features