Advanced Component Analysis Methods for Signal Processing

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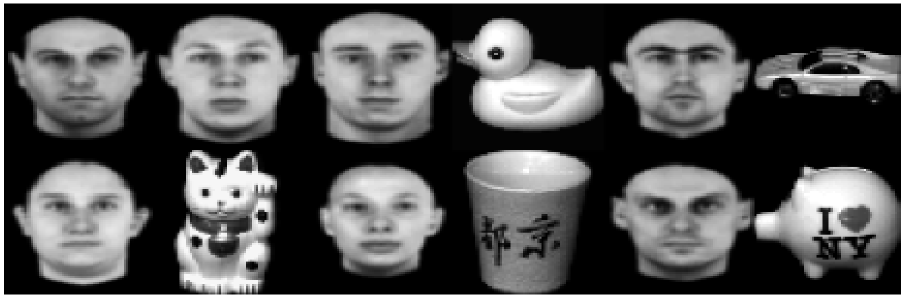
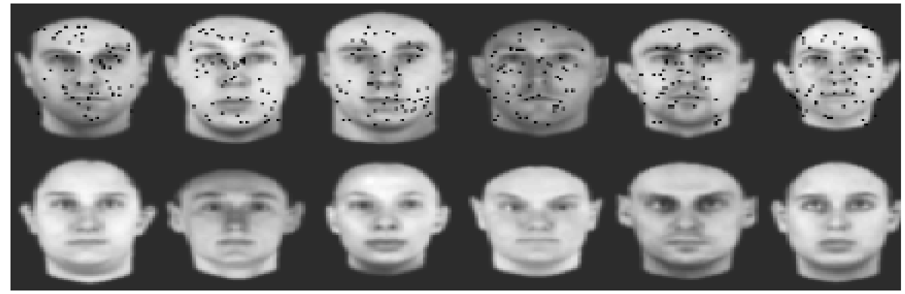
Outline

- Introduction
- Principal component analysis (PCA)
  - Robust Principal Component Analysis (RPCA)
  - PCA with missing data.
  - Parameterized Principal Component Analysis (PaPCA)
- Linear discriminant analysis (LDA)
  - Discriminative cluster analysis (DCA).
- K-means
  - Hierarchical aligned cluster analysis (HACA).
- Canonical correlation analysis (CCA)
  - Canonical time warping (CTW)
  - Dynamic coupled component analysis (DCCA)

1-Robust PCA

- Two types of outliers:

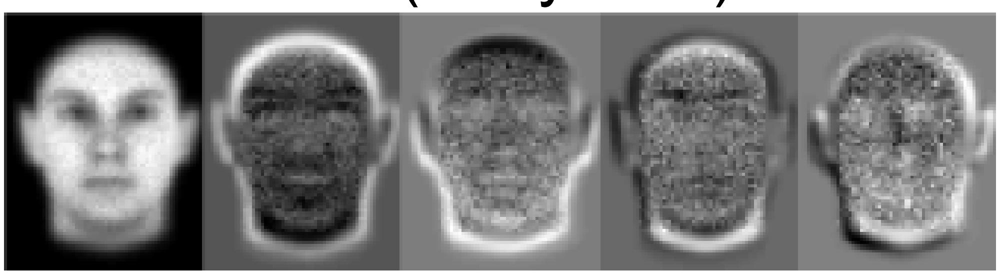


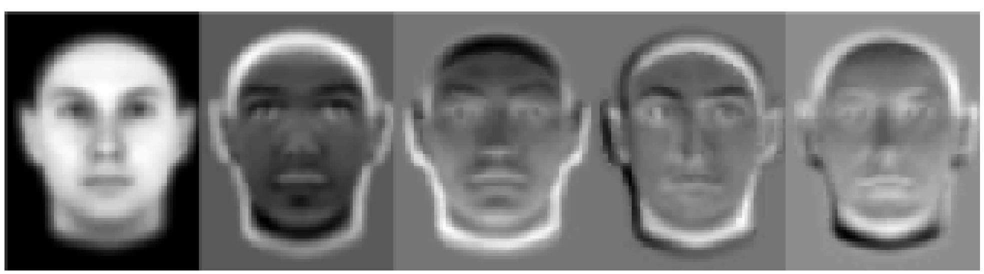


Sample outliers
(Xu & Yuille., 1995)

Intra-sample outliers
(de la Torre & Black, 2001b; Skocaj & Leonardis, 2003)

- Standard PCA solution (noisy data):





Error Function for PCA

- PCA minimizes the following **CONVEX** function.

  (Eckardt & Young, 1936; Gabriel & Zamir, 1979; Baldi & Hornik, 1989; Shum et al., 1995; de la Torre & Black, 2003a)

$$E_1(\mathbf{B},\mathbf{C}) = \sum_{i=1}^{n} \left\| \mathbf{d}_i - \mathbf{B}\mathbf{c}_i \right\|_2^2 = \left\| \mathbf{D} - \mathbf{B}\mathbf{C} \right\|_F$$

- Not unique solution: $\mathbf{B}\mathbf{R}\mathbf{R}^{-1}\mathbf{C} = \mathbf{B}\mathbf{C}$ $\qquad \mathbf{R} \in \Re^{k \times k}$
- To obtain same PCA solution **R** has to satisfy:

$$\hat{\mathbf{B}} = \mathbf{B}\mathbf{R} \qquad \hat{\mathbf{C}} = \mathbf{R}^{-1}\mathbf{C}$$

$$\hat{\mathbf{B}}^T\hat{\mathbf{B}} = \mathbf{I} \qquad \hat{\mathbf{C}}\hat{\mathbf{C}}^T = \Lambda$$

- **R** is computed as a generalized k×k eigenvalue problem.

  (de la Torre, 2006)

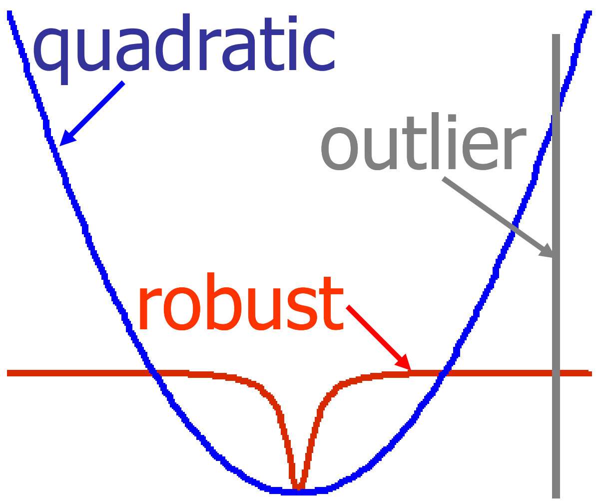
$$\left(\mathbf{C}\mathbf{C}^T\right)^{-1}\mathbf{R} = \mathbf{B}^T\mathbf{B}\mathbf{R}\Lambda^{-1}$$

Robust PCA

- Using robust statistics:
  (de la Torre & Black, 2001b; de la Torre & Black, 2003a)

Pixel residual

$$E_{rpca}(\mathbf{B},\mathbf{C},\boldsymbol{\mu}) = \sum_{i=1}^{n}\sum_{p=1}^{d}\rho\Big(d_{pi} - \mu_p - \sum_{j=1}^{k} b_{pj}c_{ji}, \sigma_p\Big)$$



mean

Basis (**B**) &
Coefficients(**c**)

Numerical Problems

- No closed form solution in terms of an eigen-equation.
- Deflation approaches do not hold.

$$\mathbf{A}' = \mathbf{A} - \lambda_1 \mathbf{u}_1 \mathbf{u}_1^{\mathrm{T}}$$

First eigenvector with highest eigenvalue.

$$\mathbf{A}'' = \mathbf{A}' - \lambda_2 \mathbf{u}_2 \mathbf{u}_2^{\mathrm{T}}$$

Second eigenvector with highest eigenvalue.

$$\cdots$$

- In the robust case all the basis have to be computed simultaneously (including the mean).

How to Optimize it?

$$E_{rpca}(\mathbf{B},\mathbf{C},\boldsymbol{\mu}) = \sum_{i=1}^{n}\sum_{p=1}^{d}\rho(d_{pi} - \mu_p - \sum_{j=1}^{k} b_{pj}c_{ji}, \sigma_p)$$

- Normalized Gradient descent

$$\mathbf{B}^{n+1} = \mathbf{B}^{n} - [\mathbf{H}_b]^{-1} \circ \frac{\partial E_{rpca}}{\partial \mathbf{B}} \qquad \mathbf{H}_{\mathbf{b}} = \max \; diag\left(\frac{\partial^2 E_{rpca}}{\partial \mathbf{b}_i \partial \mathbf{b}_i^T}\right)$$

$$\mathbf{C}^{n+1} = \mathbf{C}^{n} - [\mathbf{H}_c]^{-1} \circ \frac{\partial E_{rpca}}{\partial \mathbf{C}} \qquad \mathbf{H}_{\mathbf{c}} = \max \; diag\left(\frac{\partial^2 E_{rpca}}{\partial \mathbf{c}_i \partial \mathbf{c}_i^T}\right)$$

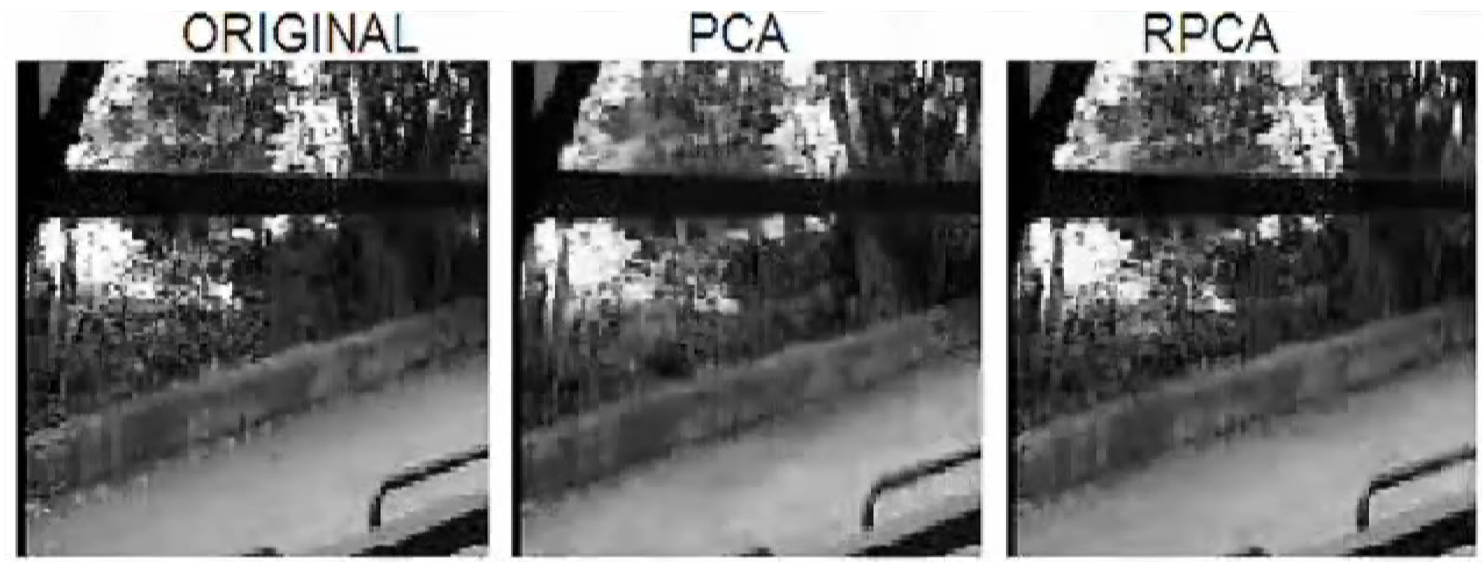
- Deterministic annealing methods to avoid local minima.

(Blake & Zisserman, 1987)

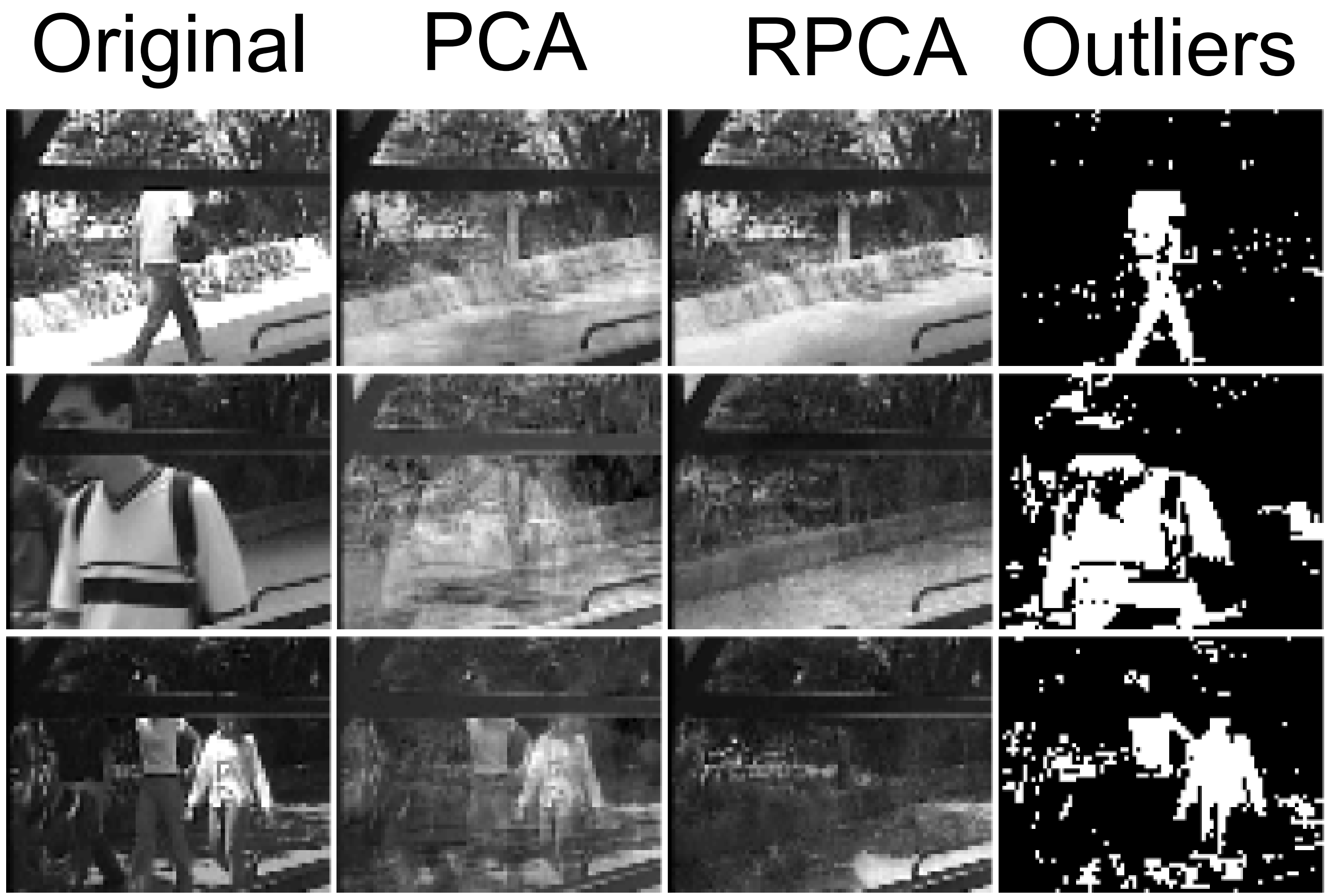
Example

Statistical outlier

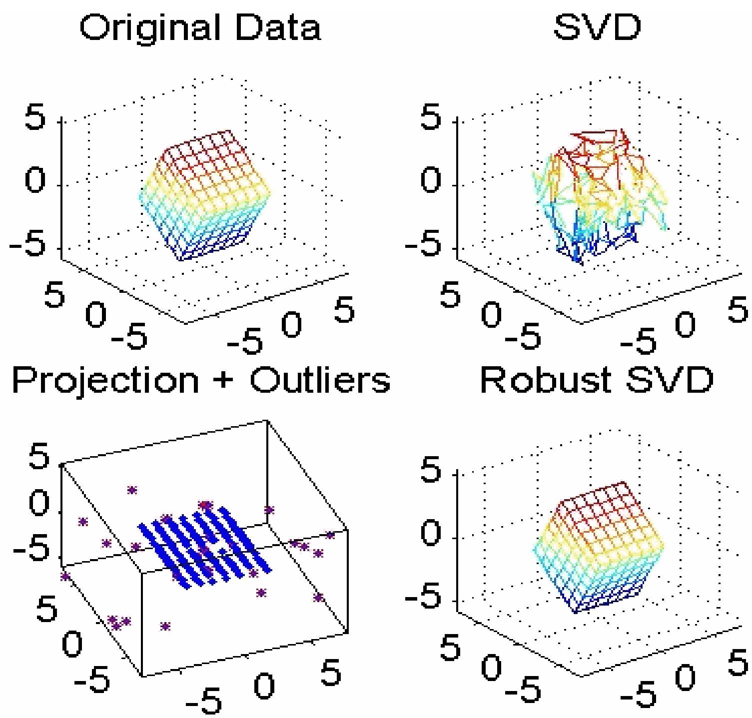
- Small region
- Short amount of time



Robust PCA



Structure from Motion



More work on Robust PCA

- Robust estimation of coefficients (Black & Jepson, 1998; Leonardis & Bischof, 2000; Ke & Kanade, 2004)
- Robust estimation of basis and coefficients (Gabriel & Odoro, 1984; Croux & Filzmoser., 1981; Skocaj et al., 2002;Skocaj & Leonardis, 2003; de la Torre & Black, 2001b; de la Torre & Black, 2003a)
- Other Robust PCA techniques (sample outliers) (Campbell, 1980; Ruymagaart, 1981; Xu & Yuille., 1995)

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2- PCA with Uncertainty and Missing Data

- Adding uncertainty $E_2(\mathbf{B},\mathbf{C}) = \|\mathbf{W} \circ (\mathbf{D} - \mathbf{BC})\|_F = \sum_{i=1}^{d}\sum_{j=1}^{n} w_{ij}(d_{ij} - \sum_{s=1}^{k} b_{is}c_{sj})^2$

- If weights are separable $\mathbf{W} = \mathbf{w}_r \mathbf{w}_c^T$ closed-form solution.

$$\mathbf{w}^c = \begin{pmatrix} w_1^c & w_2^c & \cdots & w_n^c \end{pmatrix}$$

$$\mathbf{w}^r = \begin{pmatrix} w_1^r \\ w_2^r \\ \cdots \\ w_d^r \end{pmatrix} \qquad \begin{pmatrix} d_{11} & \cdots & \cdots & d_{1n} \\ d_{21} & \ddots & & d_{2n} \\ \vdots & & \ddots & \vdots \\ d_{d1} & \cdots & \cdots & d_{dn} \end{pmatrix} = \mathbf{D}$$

$\mathbf{W} \in \Re^{d\times n}$

$w_{ij} \geq 0$

$\circ$ *Hadamard product*

- Generalized SVD

(Greenacre, 1984; Irani & Anandan, 2000;)

General Case

- For arbitrary weights no closed-form solution.

$$E_2(\mathbf{B},\mathbf{C}) = \left\| \mathbf{W} \circ (\mathbf{D} - \mathbf{BC}) \right\|_F = \sum_{i=1}^{n} (\mathbf{d}_i - \mathbf{Bc}_i)^T diag(\mathbf{w}_i)(\mathbf{d}_i - \mathbf{Bc}_i) =$$

$$\sum_{p=1}^{d} (\mathbf{d}^p - \mathbf{C}^T\mathbf{b}^p)^T diag(\mathbf{w}^p)(\mathbf{d}^p - \mathbf{C}^T\mathbf{b}^p)$$ (Torre & Black, 2003a)

- Alternated least squares algorithms
  – Slow convergence, easy implementation.
- Damped Newton Algorithm
  – Fast convergence. (Buchanan & Fitzgibbon., 2005)

$$E_2(\mathbf{B},\mathbf{C}) = \left\| \mathbf{W} \circ (\mathbf{D} - \mathbf{BC}) \right\|_F + \lambda_1 \|\mathbf{B}\|_F + \lambda_2 \|\mathbf{C}\|_F$$

$$\mathbf{v} = \begin{bmatrix} vec(\mathbf{B}) \\ vec(\mathbf{C}) \end{bmatrix} \quad \mathbf{v}^{(n+1)} = \mathbf{v}^n - \left[\frac{[\partial^2 E_2]}{\partial^2 \mathbf{v}} \right]^{-1} \frac{\partial E_2}{\partial \mathbf{v}}$$

– **H** definite positive: $\mathbf{H} = \frac{\partial^2 E_2}{\partial^2 \mathbf{v}} + \lambda \mathbf{I}$

$repeat$

$$\mathbf{H} = \frac{\partial^2 E_2}{\partial^2 \mathbf{v}} \quad \mathbf{g} = \frac{\partial E_2}{\partial \mathbf{v}}$$

$repeat$

$\lambda = 10\lambda$

$\mathbf{y} = \mathbf{x} - (\mathbf{H} + \lambda I)^{-1}\mathbf{g}$

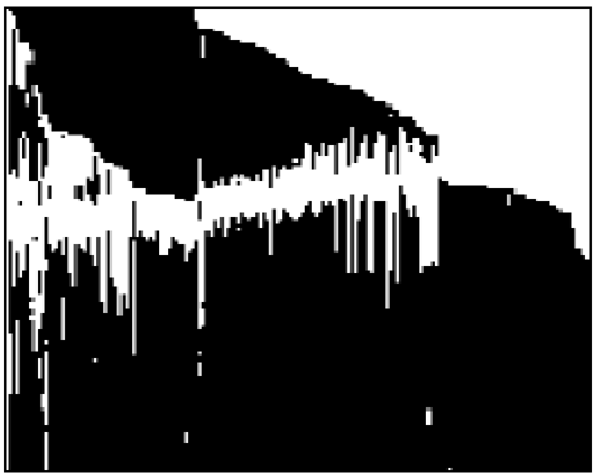
$until \;\; F(\mathbf{y}) < F(\mathbf{x})$

$\mathbf{x} = \mathbf{y}; \lambda = \frac{\lambda}{10}$

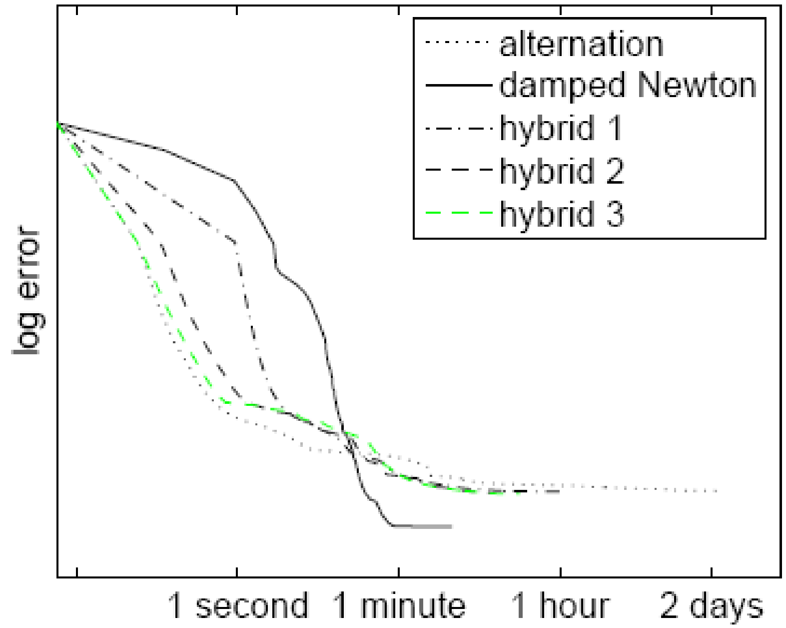
$until \;\; convergence$

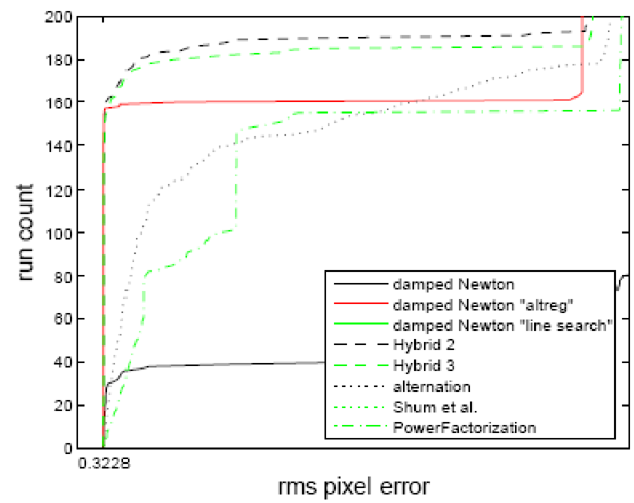
Experiments

240×167: 70% known



Total: 500 runs





Related work

- Iterative (Wiberg, 1976; Shum et al., 1995; Morris & Kanade, 1998; Aans et al.,2002; Guerreiro & Aguiar, 2002)
- Closed-form (Aguiar & Moura, 1999; Irani & Anandan, 2000)
- Power factorization (Hartley & Schaalitzky, 2003)
- Bayesian estimation (L.Torresani & Bregler, 2004)

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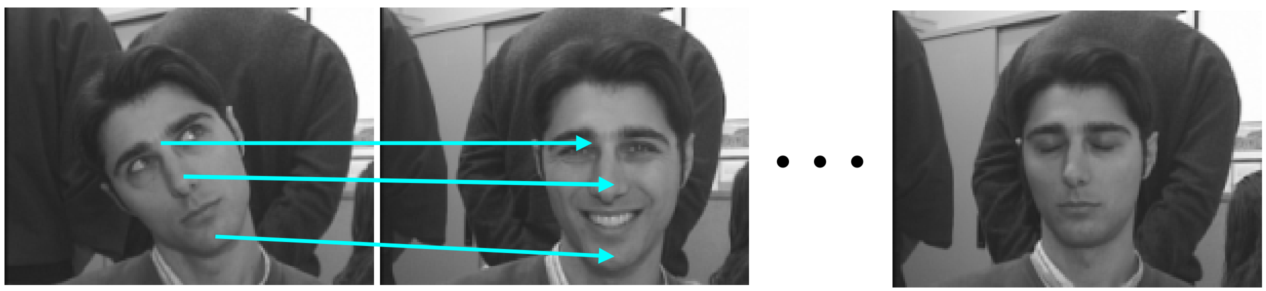
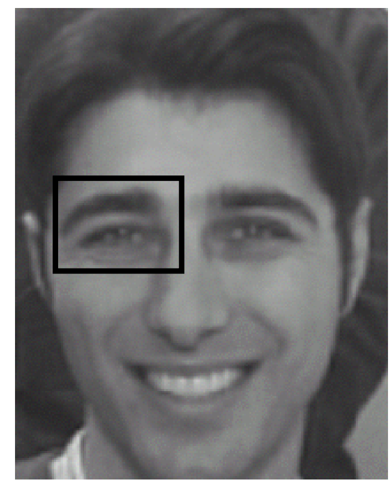
3- Parameterized Component Analysis (PaCA) (de la Torre & Black, 2003b)

- Learn a subspace invariant to geometric transformations?



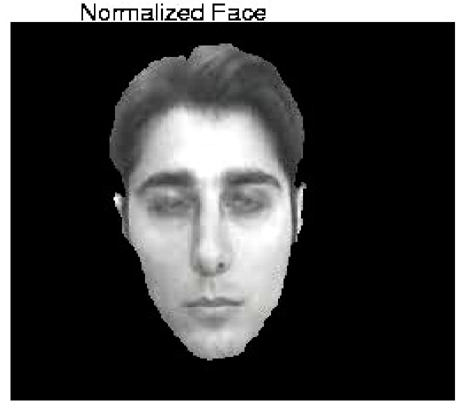
- Data has to be **geometrically** normalized
  - Tedious manual cropping.
  - Inaccuracies due to matching ambiguities.
  - Hard to achieve sub-pixel accuracy.



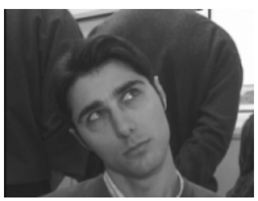
Error function for PaCA

Original Data

Normalized Face



$$E(\mathbf{B},\mathbf{C},\mathbf{a}) = \sum_{t=1}^{T} \left\| \mathbf{d}_t(\mathbf{f}(\mathbf{x},\mathbf{a}_t)) - \mathbf{B}\mathbf{c}_t \right\|_{\mathbf{W}_1}^2 + p_1(\mathbf{a}) + p_2(\mathbf{c})$$



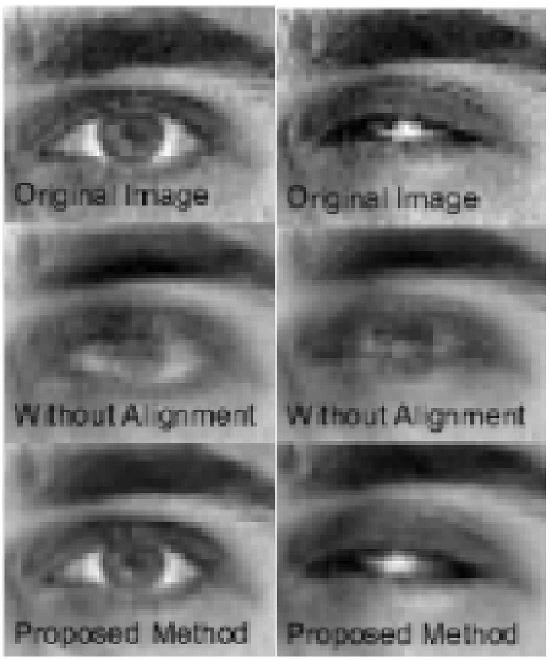
Motion (warping)

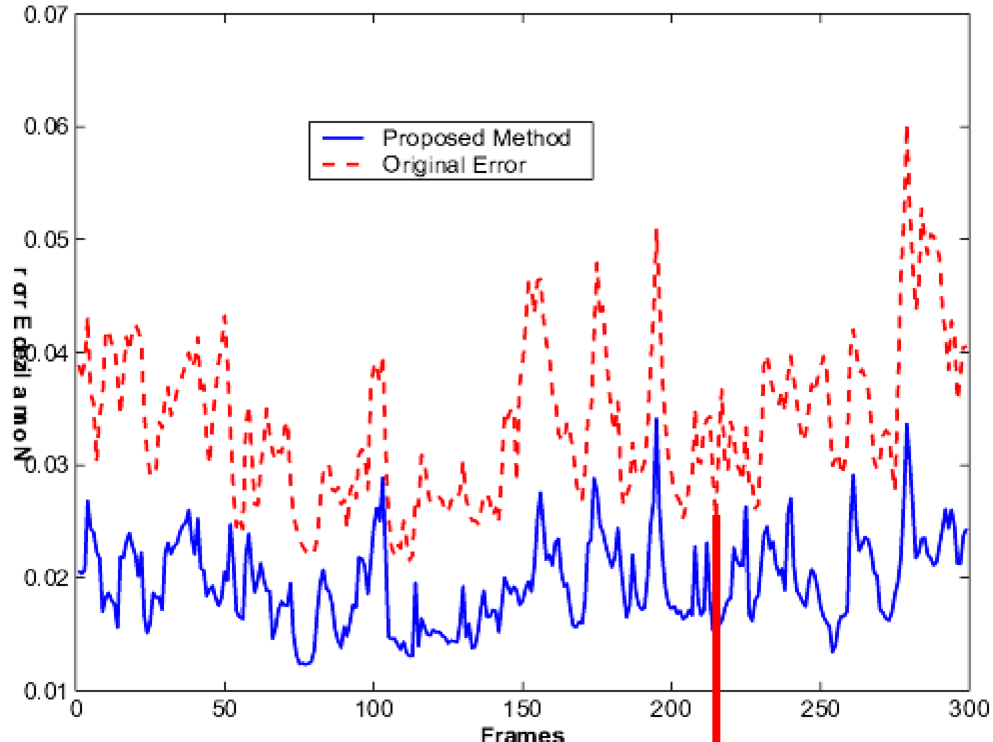
Basis (**B**) & coefficients (**c**)

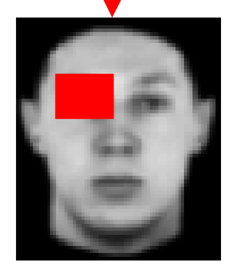
Regularization

$$\sum_{t=1}^{T}\sum_{l=1}^{L} \lambda_1 \left\| \mathbf{c}_t - \mathbf{\Gamma}_c \mathbf{c}_{t-1} \right\|_{\mathbf{W}2}^2 + \lambda_2 \left\| \mathbf{a}_t - \mathbf{\Gamma}_a \mathbf{a}_{t-1} \right\|_{\mathbf{W}3}^2$$

EigenEye Learning







More on Parameterized CA

- Probabilistic model
  - Search scales exponentially with the number of motion parameters (Frey & Jojic, 1999a; Frey & Jojic, 1999b; Williams & Titsias, 2004)
- Other continuous approaches.

  (Schewitzer, 1999; Rao, 1999; Shashua et al., 2002)
- Invariant clustering

  (Fitzgibbon & Zisserman, 2003)
- Non-rigid motion

  (Baker et al., 2004)
- Invariant recognition

  (Black & Jepson, 1998)
- Invariant support vector machines (Avidan, 2001)
- Parameterized Kernel Component Analysis (De la Torre, 2008)

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Problems of k-means

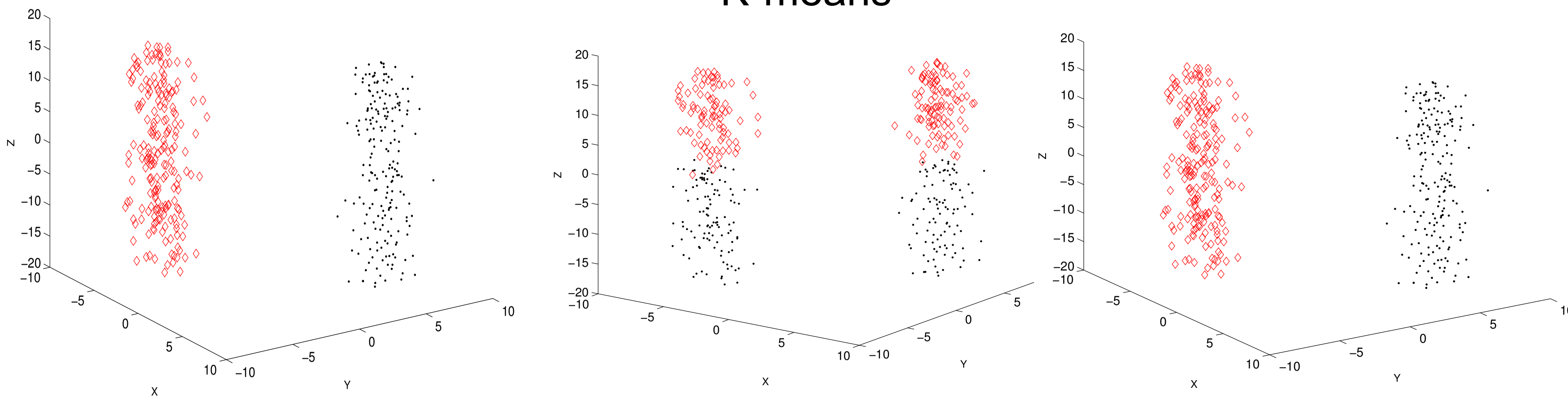
- K-means optimizes:

$$E(\mathbf{A},\mathbf{B}) = \| \mathbf{D} - \mathbf{B}\mathbf{A}^T \|_F = \sum_{i=1}^{c} \sum_{j \in C_i} \| \mathbf{d}_j - \mathbf{a}_i \|$$

  - Not efficient for high dimensional data.
  - Multiple local minima.
  - No mechanism to remove irrelevant features for clustering

K-means

DCA



Discriminative cluster analysis (DCA)

- LDA optimizes:

$$E(\mathbf{A},\mathbf{B}) = \| (\mathbf{G}^T\mathbf{G})^{-\frac{1}{2}}(\mathbf{G}^T - \mathbf{B}\mathbf{A}^{\mathbf{T}}\mathbf{D}) \|_F$$

Supervised

$$\mathbf{G}^T = \begin{bmatrix} 1 & \dots & 0 \\ 0 & \dots & 1 \\ 0 & \dots & 0 \end{bmatrix}$$

Unsupervised

- DCA optimizes:

$$E(\mathbf{A},\mathbf{B},\mathbf{G}) = \| (\mathbf{G}^T\mathbf{G})^{-\frac{1}{2}}(\mathbf{G}^T - \mathbf{B}\mathbf{A}^{\mathbf{T}}\mathbf{D}) \|_F$$

$$g_{ij} \in \{0,1\} \qquad \mathbf{G}\mathbf{1}_c = \mathbf{1}_n$$

- In LDA **G** is known because it is a supervised technique. In DCA **G** is estimated.
- Simultaneous dimensionality reduction and clustering.

Optimization

- Eliminate **A**

S_t S_b

$$E(\mathbf{B},\mathbf{G}) \propto tr((\mathbf{B}^T\mathbf{D}\mathbf{D}^T\mathbf{B})^{-1}(\mathbf{B}^T\mathbf{D}\mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{D}^T\mathbf{B}))$$

- Optimize for **B**

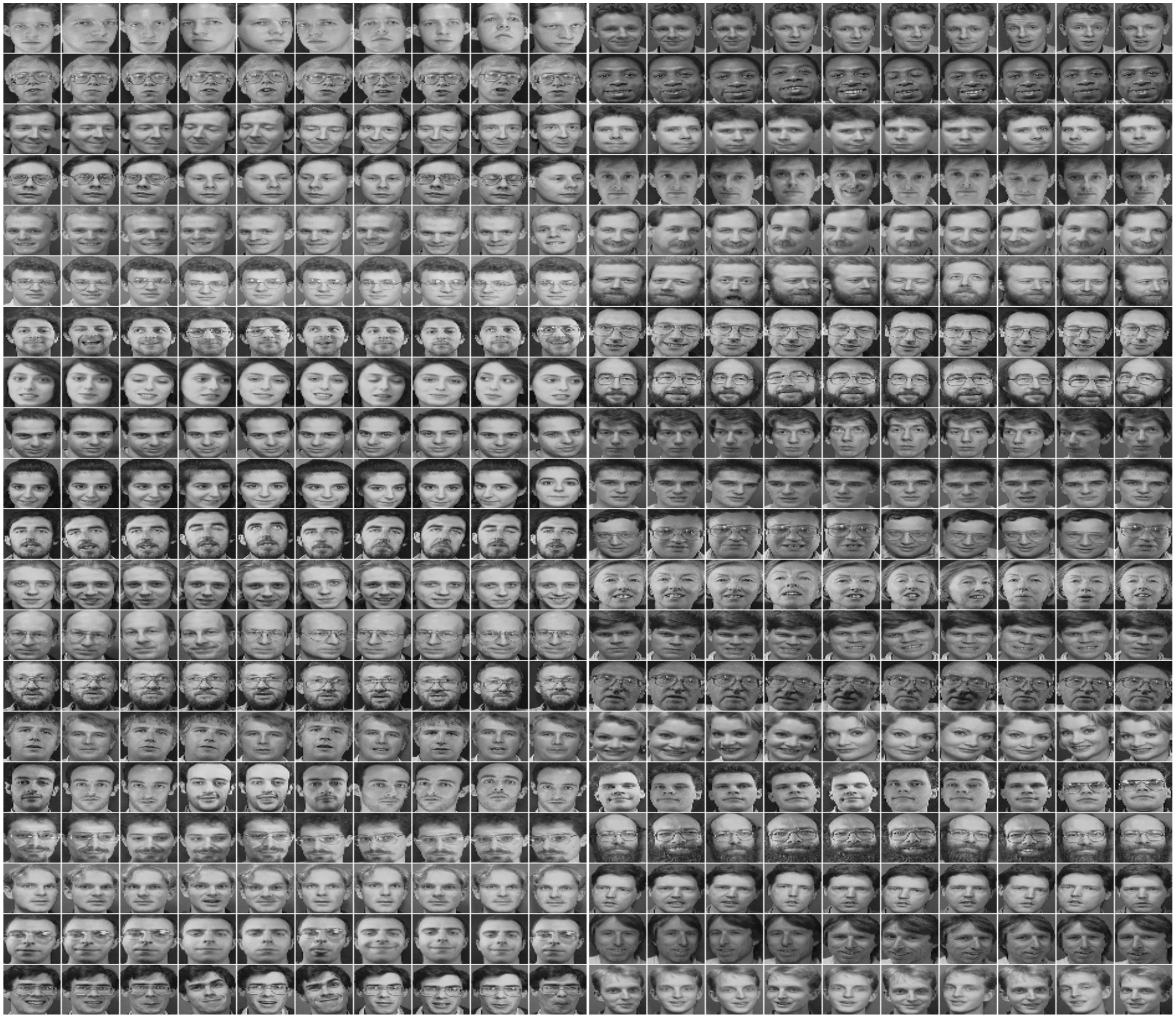
$$\mathbf{D}\mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{D}^T\mathbf{B} = \mathbf{D}\mathbf{D}^T\mathbf{B}\Lambda$$

- Optimize for **G** $\quad \mathbf{A} = \mathbf{C}^T(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C} \quad \mathbf{C} = \mathbf{B}^T\mathbf{D}$

$$\mathbf{G} = \mathbf{V} \circ \mathbf{V} \qquad \mathbf{V}^{(n+1)} = \mathbf{V}^{(n)} - \eta\frac{\partial E}{\partial \mathbf{V}}$$

$$\frac{\partial E}{\partial \mathbf{V}} = (\mathbf{I}_C - \mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T)\mathbf{A}\mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}$$

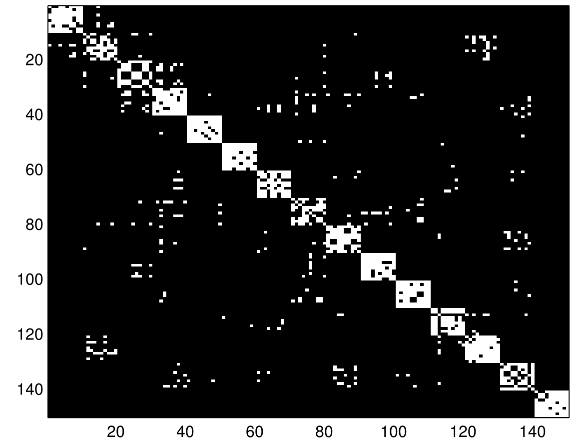
Clustering faces



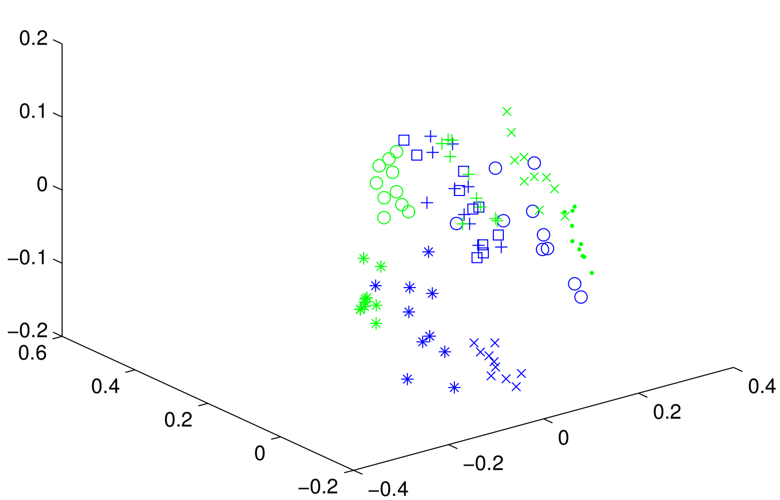
92x112 pixels.
40 people
10 samples/person

Experiments

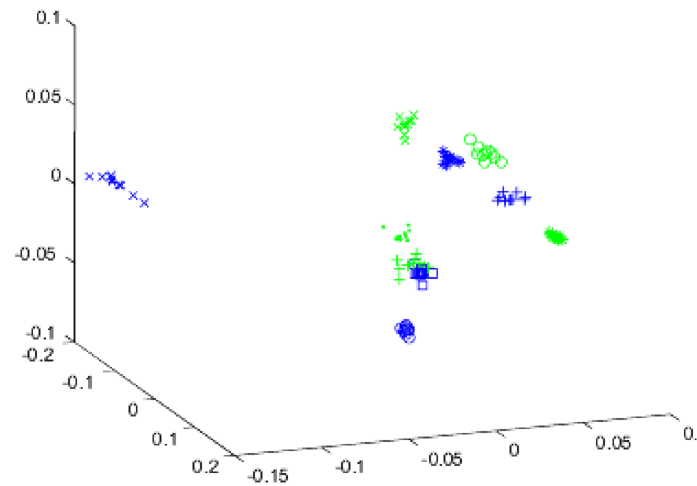
$$\mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T \approx$$



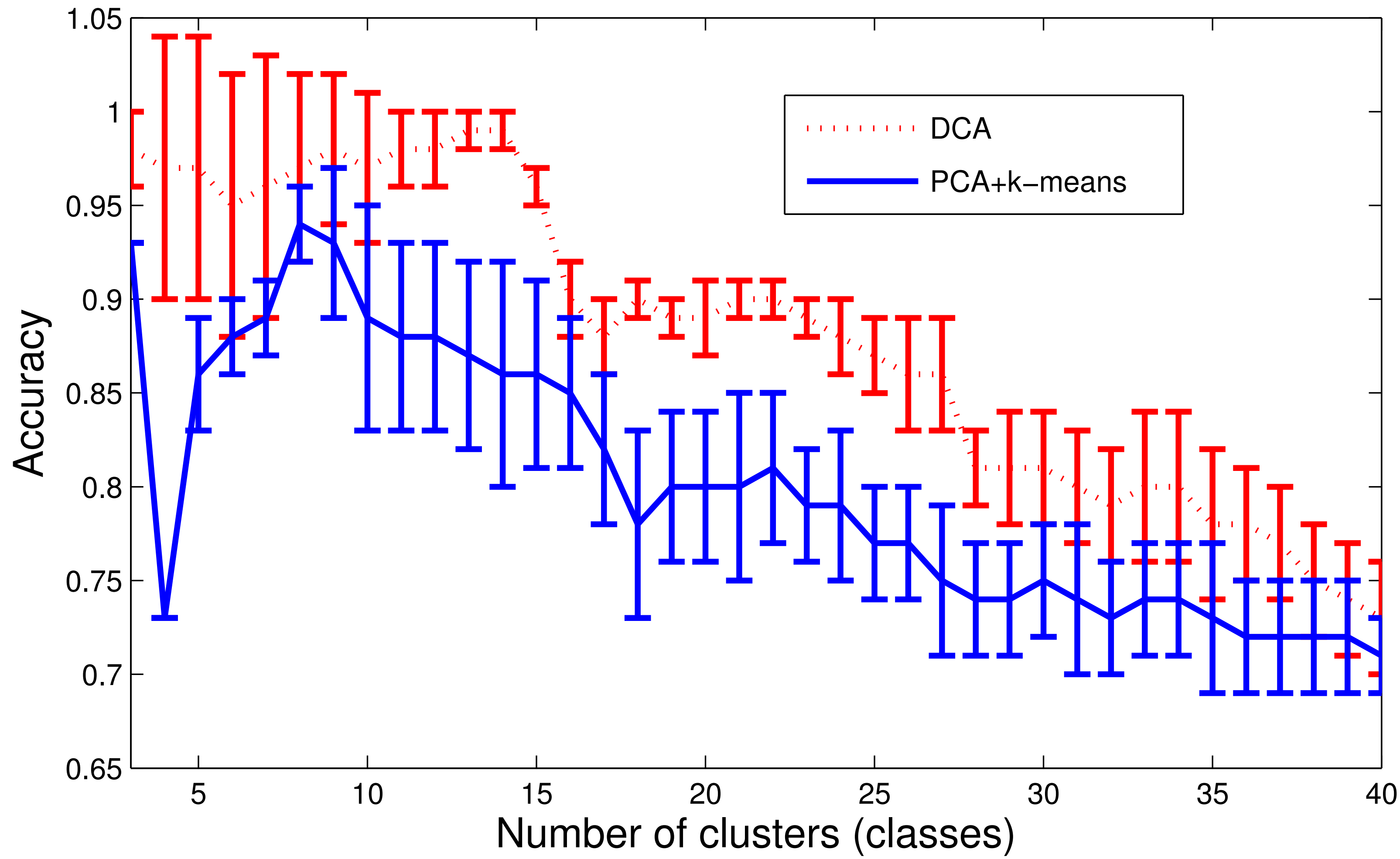
PCA

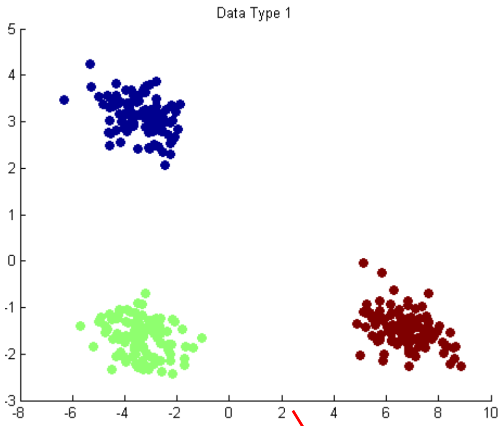


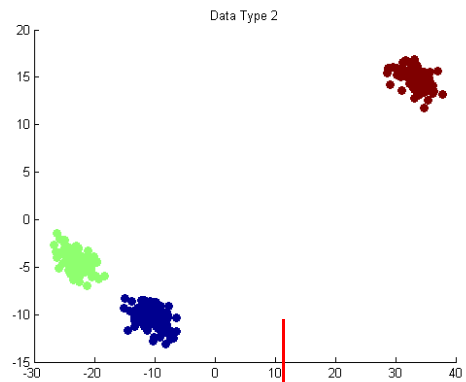
DCA

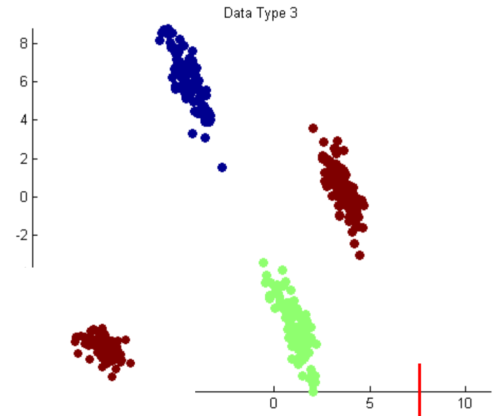


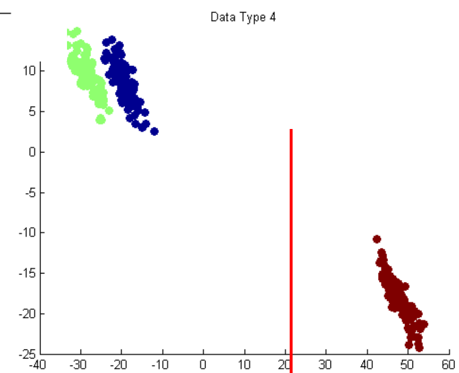
DCA vs. PCA+k-means











1000 realizations

	Datatype			
	1	2	3	4
k-Means	53.57%	38.22%	41.50%	11.88%
DCA	84.71%	82.11%	86.50%	85.55%

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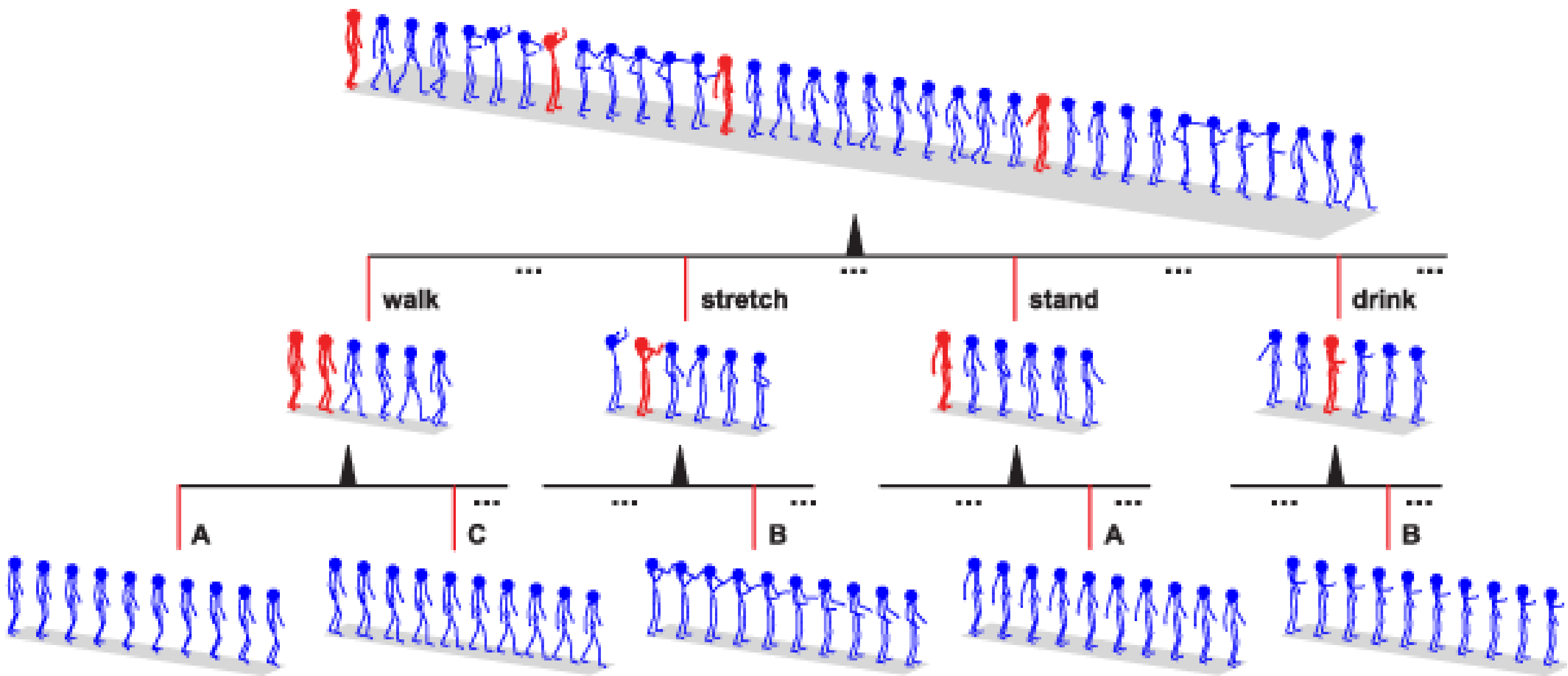
Problem I

(Learning motion primitives)

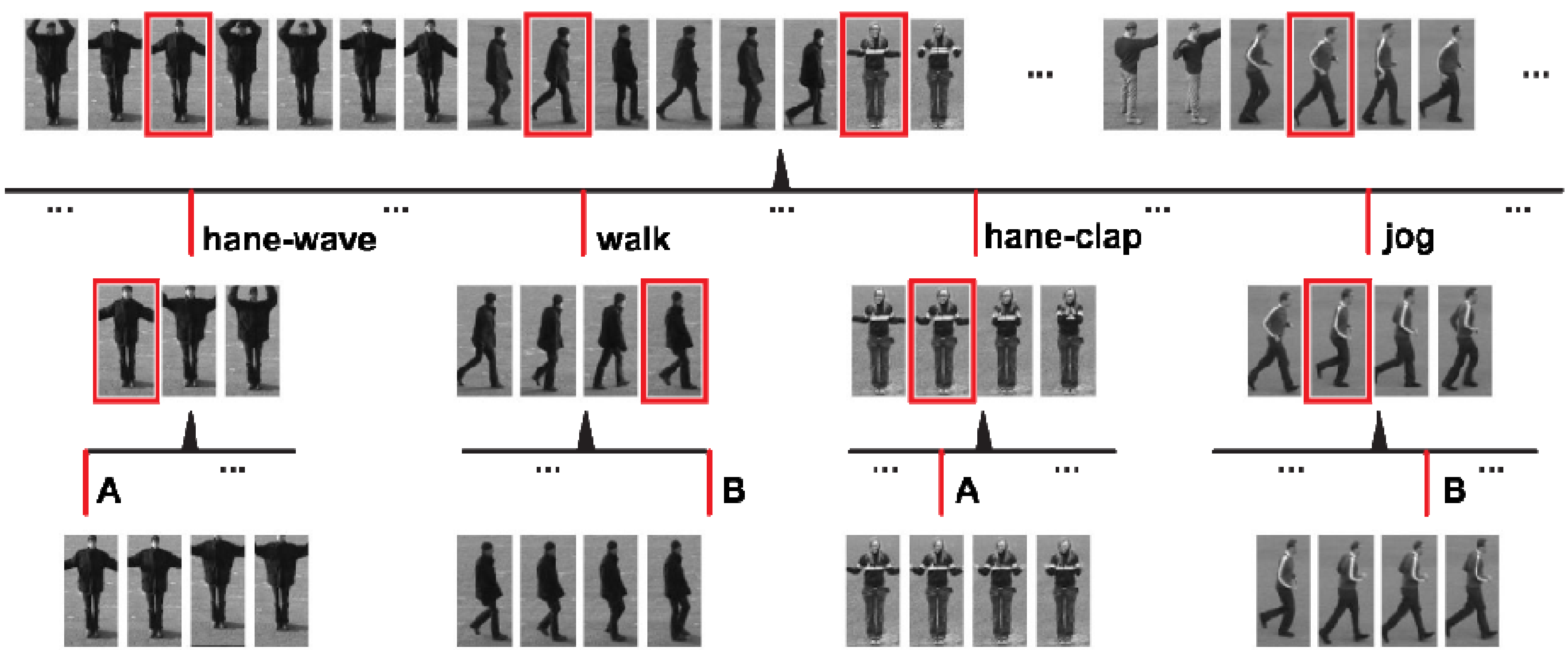
- How to factorize a stream of motion capture data into motion primitives?



Problem II

(Unsupervised segmentation of actions in video)

- How to factorize a stream of video of different people into actions?



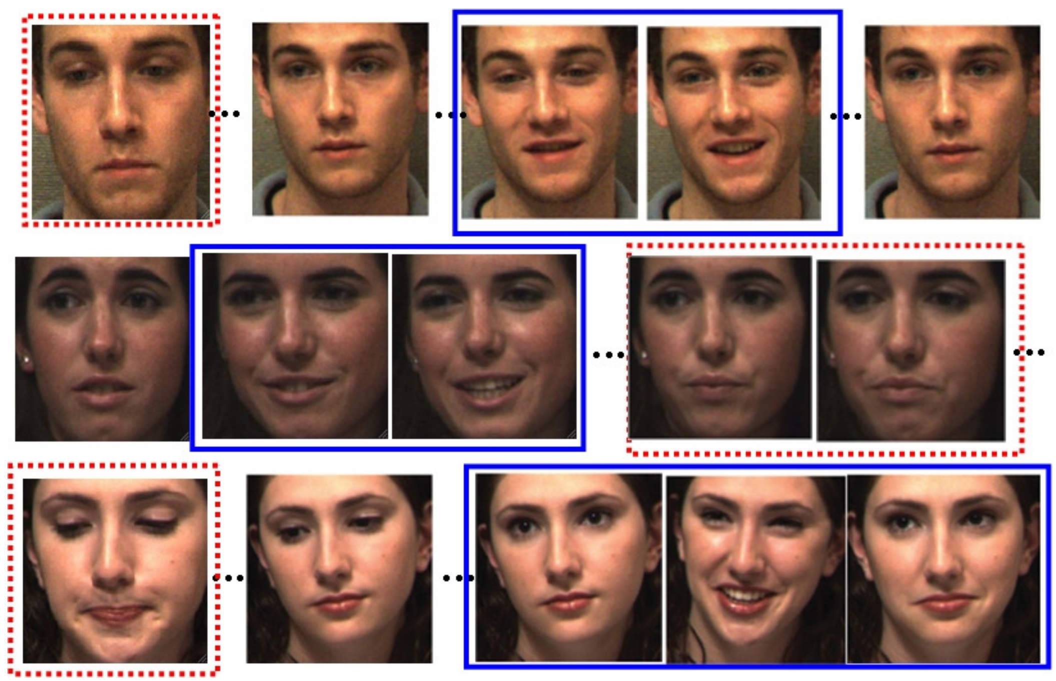
Problem III

(Learning Facial Action Coding System)

- How to learn a vocabulary for facial expressions?

Outer Brow Raiser **(AU2)** Brow lowerer **(AU4)** Upper lid raiser **(AU5)** Nose Wrinkler **(AU9)**

Lip corner puller **(AU12)** Chin raiser **(AU17)** Lip stretcher **(AU20)** Dimpler **(AU14)** Lip corner depressor **(AU15)**

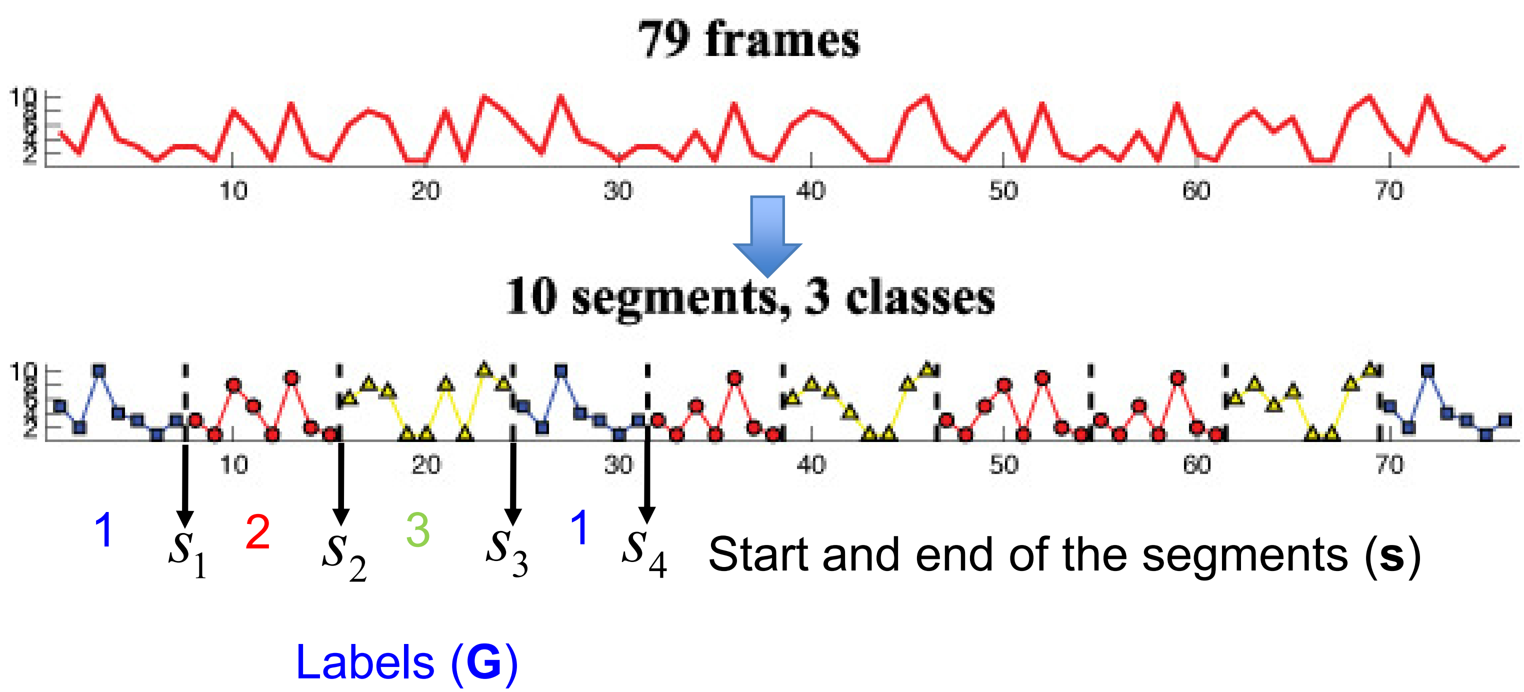


Previous Work

- **Change point detection** (Fearnhead 06, *Murphy et al. 07, Harchaoui et al. 09,* Boysen *et al. 09,)*
- **Bayesian networks:**
  - **HMMs** *(Kohlmorgen et al. 01)* , **switching LDS** *(Oh et al. 08, Fox et al. 08), **MM Markov Networks*** (Xu 06).
- **PCA, GMM, spectral Clustering** *(Barbic et al.04, Irani et al. 01, De la Torre et al. 07)*
- **Other methods: Zero-velocity detection** (Jenkins et al. 02)**, Frequency analysis** (Davis et al. 00)
- ***ACA***
  - ***Extension on kernel k-means to cluster time series.***
  - ***Efficient solution with dynamic programming.***

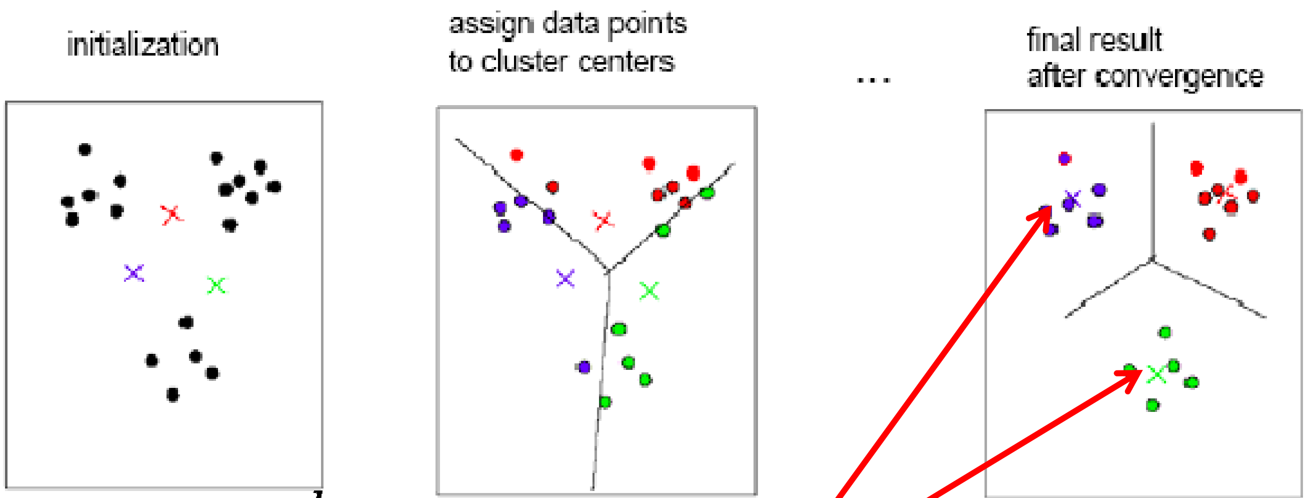
Problem Formulation for ACA

- Given a sequence $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_n] \in \Re^{d \times n}$, Aligned Cluster Analysis (ACA) decompose **X** into ***m*** disjoint segments belonging to one of ***k*** classes.



k-means Clustering

- Partition the data set in c-disjoint “clusters” of data points.



$$J_{km}(\mathbf{G},\mathbf{M}) = \sum_{c=1}^{k}\sum_{i=1}^{n} g_{ci}\|\mathbf{x}_i - \mathbf{m}_c\|_2^2 = \|\mathbf{X} - \mathbf{MG}\|_F^2$$

n=samples

d=features

$$\mathbf{X} = [x_1, \cdots, x_n] \in \mathbb{R}^{d\times n} \quad \mathbf{M} \in \mathbb{R}^{d\times k} \quad \mathbf{G}^T = \begin{array}{cc} \text{n} & \\ \begin{bmatrix} 1 & \dots & 0 \\ 0 & \dots & 1 \\ 0 & \dots & 0 \end{bmatrix} & \text{c} \end{array}$$

$$g_{ij} \in \{0,1\}$$

$$\mathbf{G1}_c = \mathbf{1}_n$$

k-means and kernel *k*-means

- K-means (Zha et al., 01;Ding & He,04; DelaTorre,06)

$$E_{km}(\mathbf{M},\mathbf{G}) = \sum_{c=1}^{k}\sum_{i=1}^{n} g_{ci}\left\|\mathbf{x}_i - \mathbf{m}_c\right\|_2^2 = \left\|\mathbf{X} - \mathbf{MG}\right\|_F^2$$

$$\mathbf{G} \in \{0,1\}^{k\times n} \quad \& \quad \mathbf{G}^T 1_k = 1_n \qquad \mathbf{G} = \begin{bmatrix} 1 & 1 \dots & 0 \\ 0 & 0 \dots & 1 \\ 0 & 0 \dots & 0 \end{bmatrix}$$

- Kernel K-means (Dhillon 04; DelaTorre,06)

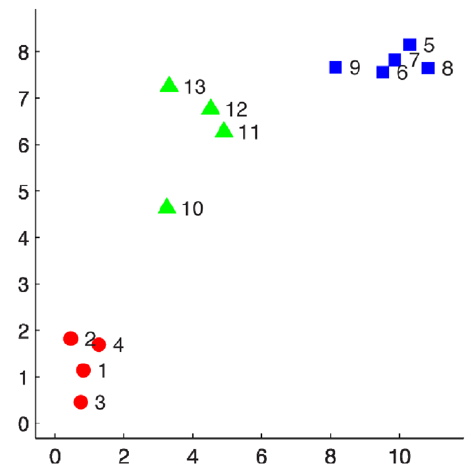
$$E_{kkm}(\mathbf{M},\mathbf{G}) = \sum_{c=1}^{k}\sum_{i=1}^{n} g_{ci}\left\|\varphi(\mathbf{x}_i) - \mathbf{m}_c\right\|_2^2 = \left\|\varphi(\mathbf{X}) - \mathbf{MG}\right\|_F^2$$

$$E_{kkm}(\mathbf{G}) = tr(\mathbf{K}(\mathbf{I}_n - \mathbf{G}^T(\mathbf{G}\mathbf{G}^T)^{-1}\mathbf{G}))$$

$\mathbf{K} = \varphi(\mathbf{X})^T \varphi(\mathbf{X}) \in \Re^{n\times n}$ is the kernel matrix

kernel *k*-means (Dhillon et al. 04, De la Torre 06)

$$J_{kkm}(\mathbf{G},\mathbf{M}) = \sum_{c=1}^{k}\sum_{i=1}^{n} g_{ci}\|\phi(\mathbf{x}_i) - \mathbf{m}_c\|_2^2 = \|\phi(\mathbf{X}) - \mathbf{MG}\|_F^2$$

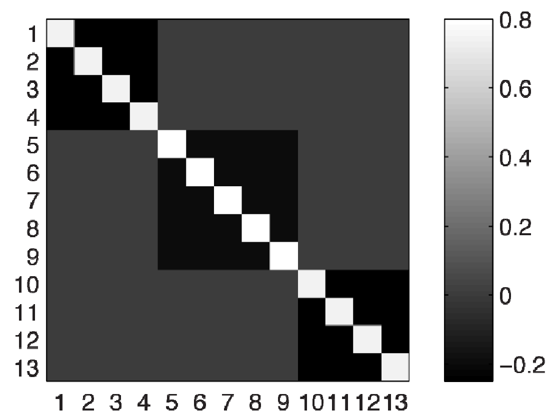


Matrix form

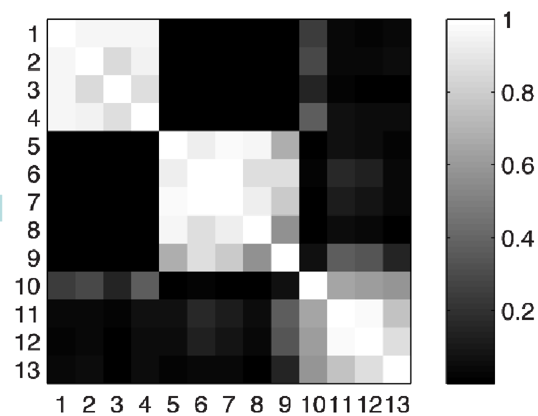
$$J_{kkm} = \mathrm{Tr}\left(\mathbf{LK}\right) \quad \text{with} \quad \mathbf{L} = \mathbf{I}_n - \mathbf{G}^T(\mathbf{G}\mathbf{G}^T)^{-1}\mathbf{G}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	1	1	1	1	1	0	0	0	0
3	0	0	0	0	0	0	0	0	0	1	1	1	1

$\mathbf{G} \in \{0,1\}^{k\times n}$



$\mathbf{L} \in \mathbb{R}^{n\times n}$



$\mathbf{K} = \phi(\mathbf{X})^T\phi(\mathbf{X}) \in \mathbb{R}^{n\times n}$

Dynamic Time Alignment Kernel (DTAK)

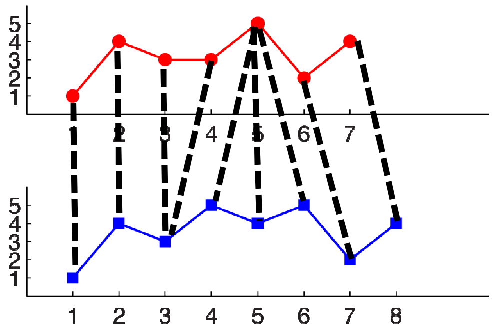
- How to measure a distance between two segment **X** and **Y**?
  - Use the Dynamic Time Alignment Kernel

$$\mathbf{X} \doteq [\mathbf{x}_1, \cdots, \mathbf{x}_{n_x}] \in \mathbb{R}^{d \times n_x} \qquad \mathbf{Y} \doteq [\mathbf{y}_1, \cdots, \mathbf{y}_{n_y}] \in \mathbb{R}^{d \times n_y}$$

$$\tau(\mathbf{X}, \mathbf{Y}) = \frac{p_{n_x, n_y}}{n_x + n_y} \quad \text{frame kernel: } \kappa_{ij} = \phi(\mathbf{x}_i)^T \phi(\mathbf{y}_j)$$

$$\text{with} \qquad p_{i,j} = \max \begin{cases} p_{i-1,j} + \kappa_{ij} \\ p_{i-1,j-1} + 2\kappa_{ij} \\ p_{i,j-1} + \kappa_{ij} \end{cases}$$

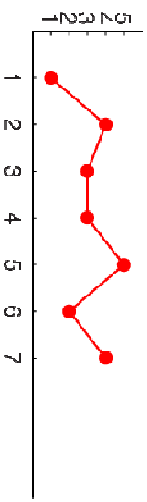
An example



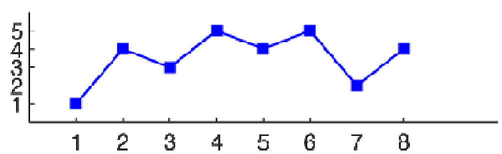
with

$$\tau(\mathbf{X},\mathbf{Y}) = \frac{p_{n_x,n_y}}{n_x+n_y}$$

$$p_{i,j} = \max \begin{cases} p_{i-1,j} + \kappa_{ij} \\ p_{i-1,j-1} + 2\kappa_{ij} \\ p_{i,j-1} + \kappa_{ij} \end{cases}$$

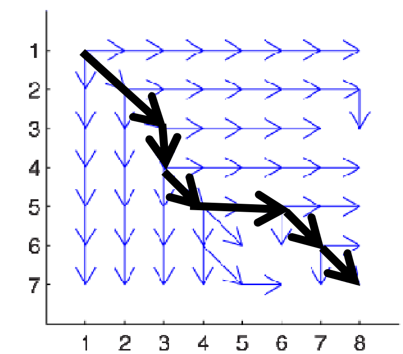


	1	2	3	4	5	6	7	8
1	1	0	0	0	0	0	0	0
2	0	1	0	0	1	0	0	1
3	0	0	1	0	0	0	0	0
4	0	0	1	0	0	0	0	0
5	0	0	0	1	0	1	0	0
6	0	0	0	0	0	0	1	0
7	0	1	0	0	1	0	0	1



Similarity matrix

		2		4		6		8
1	2	2	2	2	2	2	2	2
2	2	4	4	4	5	5	5	6
3	2	4	6	6	6	6	6	6
4	2	4	7	7	7	7	7	7
5	2	4	7	9	9	10	10	10
6	2	4	7	9	9	10	12	12
7	2	5	7	9	11	11	12	14



Error function for ACA

- Introduce a new variable **s** that defines the starting and ending of the segment $\mathbf{X}_{[s_i, s_{i+1})} = [\mathbf{x}_{s_i}, \mathbf{x}_{s_i+1}, \cdots, \mathbf{x}_{s_{i+1}-1}]$
- ACA minimizes:

$$J_{aca}(\mathbf{M}, \mathbf{G}, \mathbf{s}) = \sum_{c=1}^{k} \sum_{i=1}^{m} g_{ci} \|\psi(\mathbf{X}_{[s_i, s_{i+1})}) - \mathbf{m}_c\|^2 = \|\psi(\mathbf{X}) - \mathbf{MG}\|_F^2$$

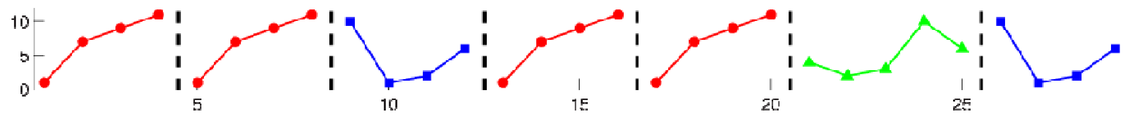
DTAK

	ACA	Kernel K-means
Features	segments with consecutive frames	single samples
Variables	s,G,M	G,M

Matrix form

$$J_{aca} = \mathrm{Tr}\Big((\mathbf{L} \circ \mathbf{W})\mathbf{K}\Big) \quad \text{with} \quad \mathbf{L} = \mathbf{I}_n - \mathbf{H}^T\mathbf{G}^T(\mathbf{G}\mathbf{G}^T)^{-1}\mathbf{G}\mathbf{H}$$

n = 28 (frames) m = 7 (segments) k = 3 (clusters)



clusters / segments

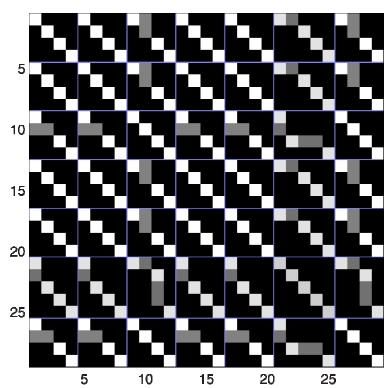
	1	2	3	4	5	6	7
1	1	1	0	1	1	0	0
2	0	0	1	0	0	0	1
3	0	0	0	0	0	1	0

$\mathbf{G} \in \{0,1\}^{k \times m}$

segments / samples

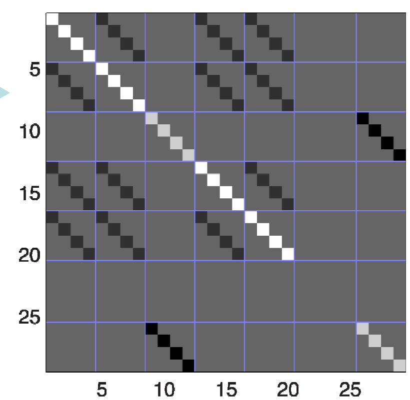
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1

$\mathbf{H} \in \{0,1\}^{m \times n}$

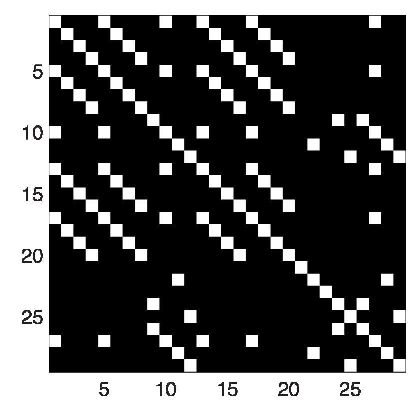


$\mathbf{W} \in \mathbb{R}^{n \times n}$

$\mathbf{L} \circ \mathbf{W} \in \mathbb{R}^{n \times n}$



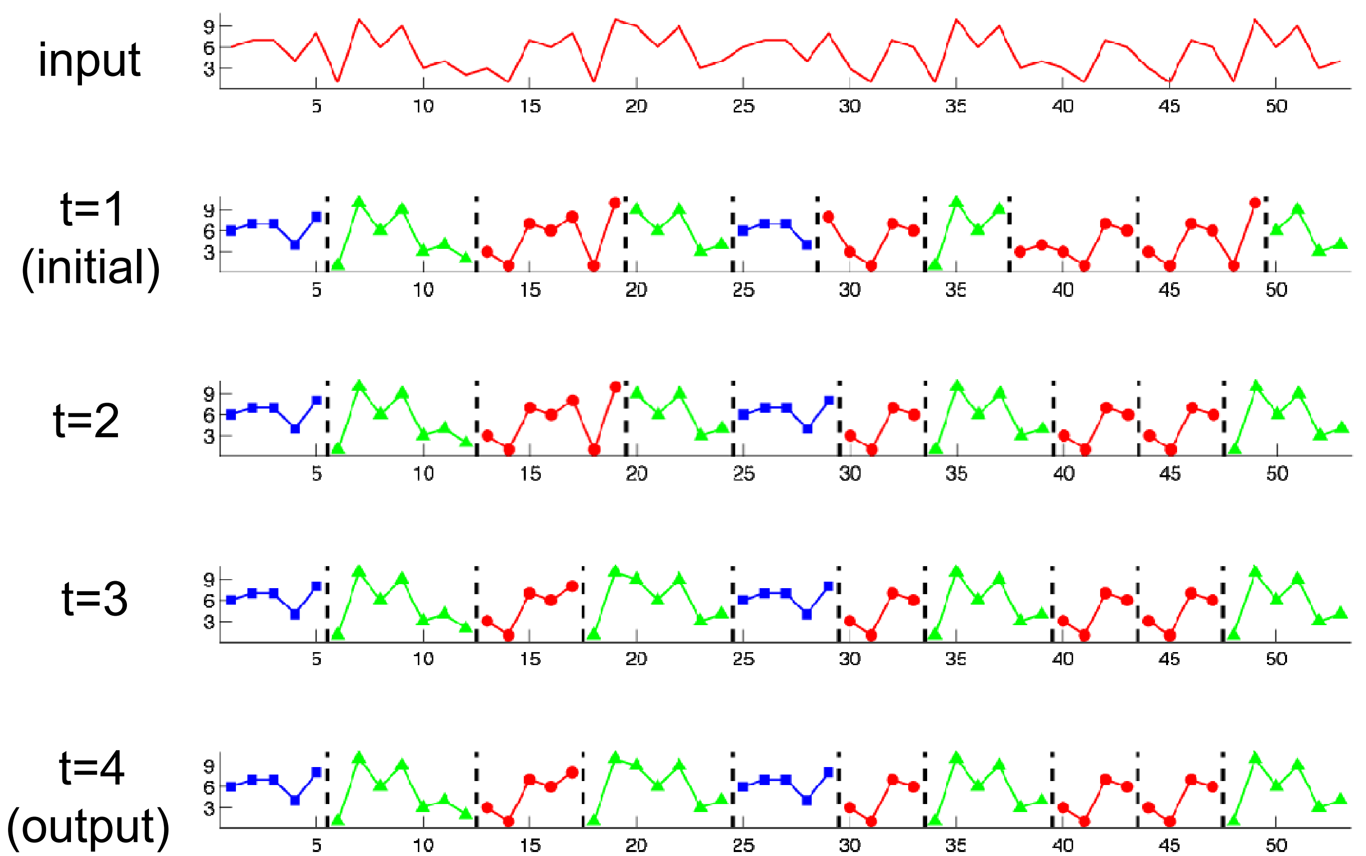
$\mathbf{K} \in \mathbb{R}^{n \times n}$

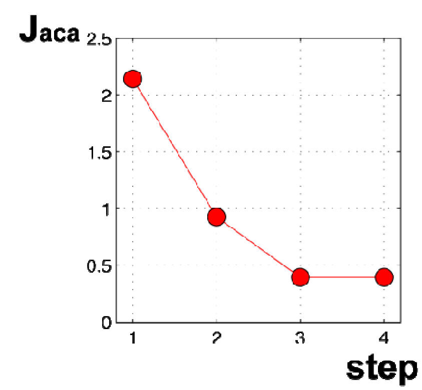


Optimizing ACA

- ACA is optimized by coordinate-descent

$$\mathbf{G}^t, \mathbf{s}^t = \arg\min_{\mathbf{G},\mathbf{s}} \sum_{c=1}^{k} \sum_{i=1}^{m} g_{ci} \|\psi(\mathbf{Y}_i) - \mathbf{m}_c^{t-1}\|^2$$

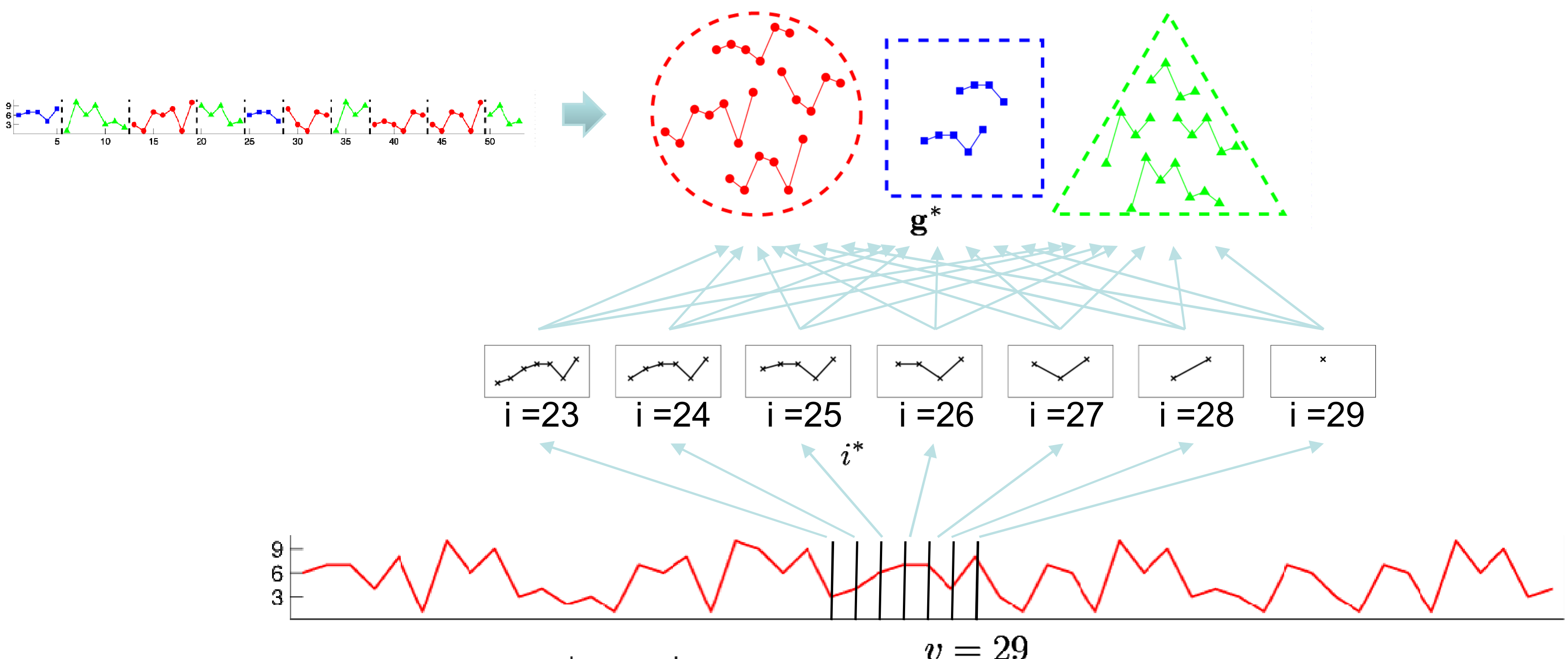




Optimizing ACA (Forward step)

- Each step is solved with Dynamic Programming.

$$\mathbf{L}(1,v) = \min_{v-w_{\max}<i\leq v}\left(\mathbf{L}(1,i-1) + \min_{\mathbf{g}}\sum_{c=1}^{k} g_c\|\psi(\mathbf{X}_{[i,v]}) - \mathbf{m}_c^0\|^2\right)$$

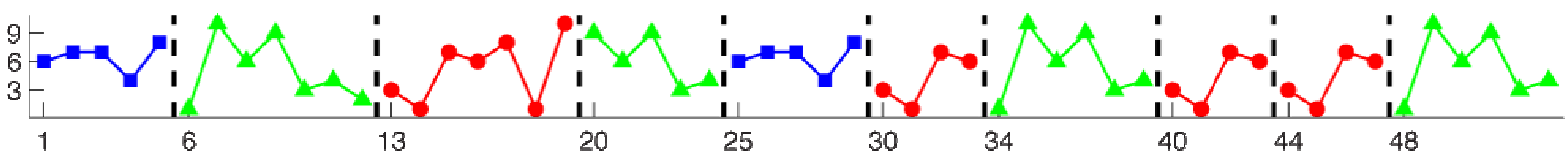


v	i_v^*	$\mathbf{g}_v^*$
$\cdots$	$\cdots$	$\cdots$
29	25	■

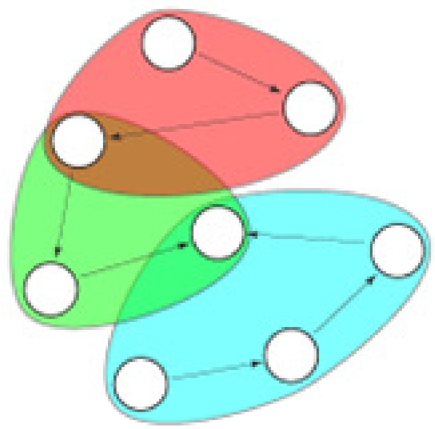
Move forward for v > 29

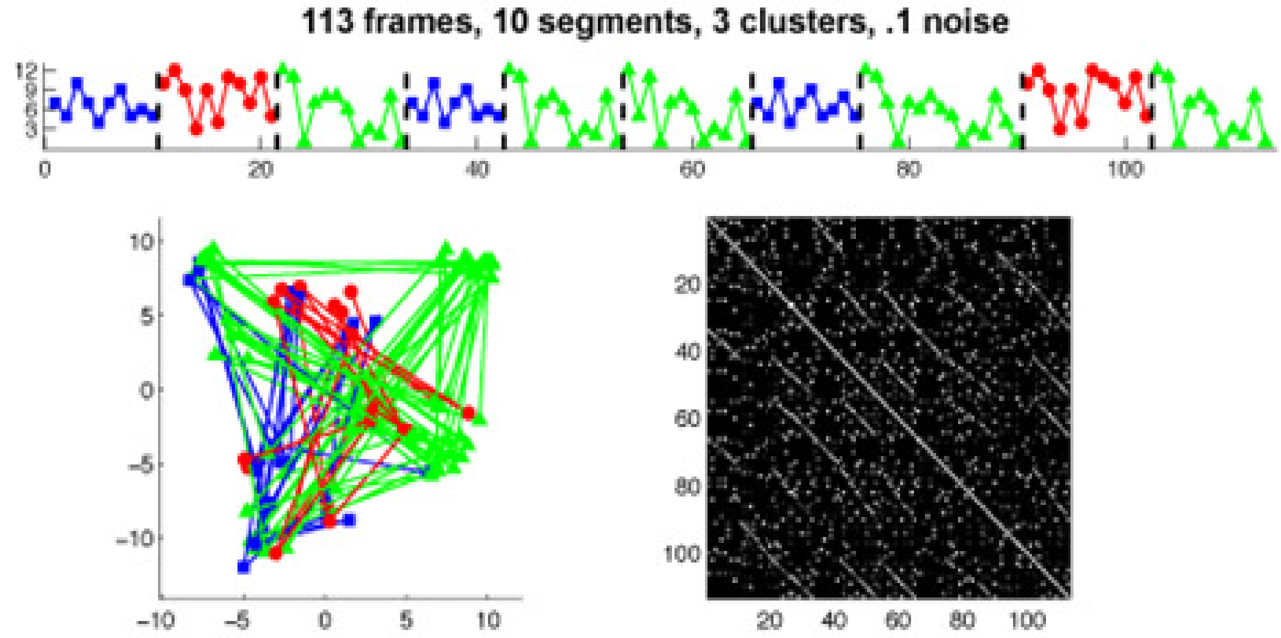
Optimizing ACA (Backward step)

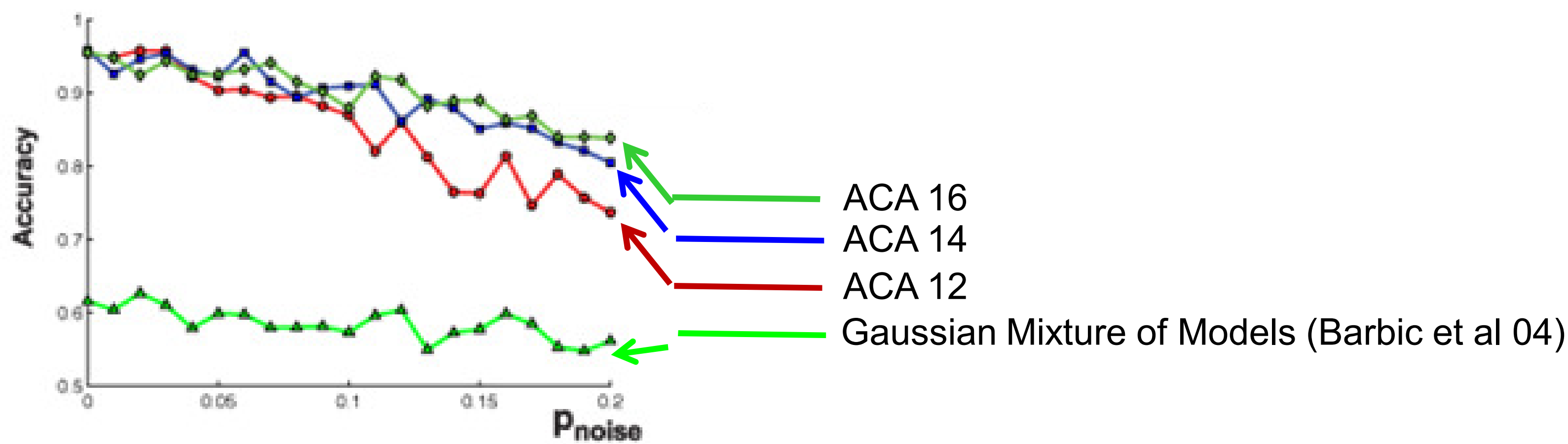
v	i_v^*	$\mathbf{g}_v^*$
...	...	...
29	25	■
...	...	...
33	30	●
...	...	...
39	34	▲
...	...	...
43	40	●
...	...	...
47	44	●
...	...	...
53	48	▲



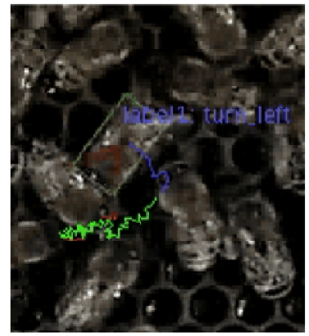
Synthetic data

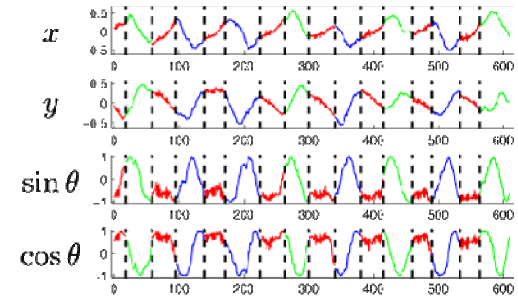




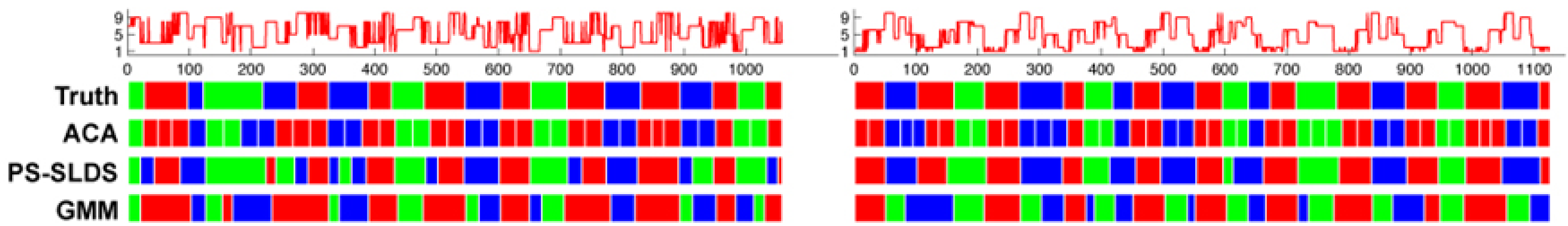


Honey bee dance data





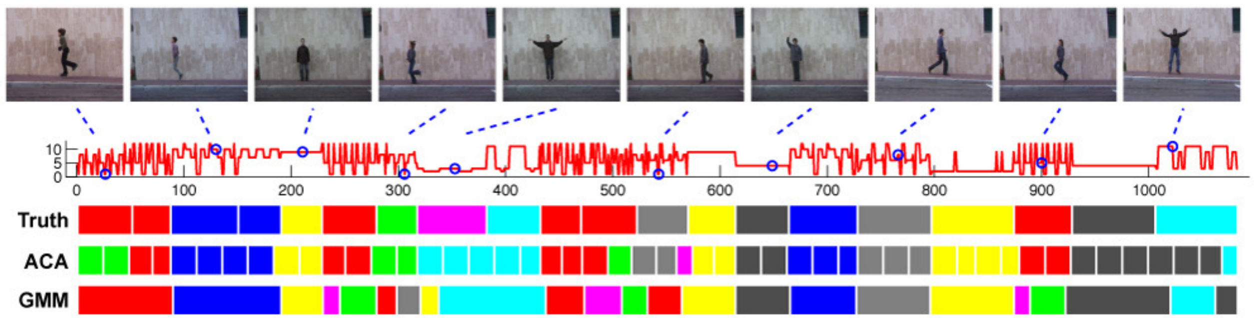
3 clusters:
waggle, left turn and right turn.



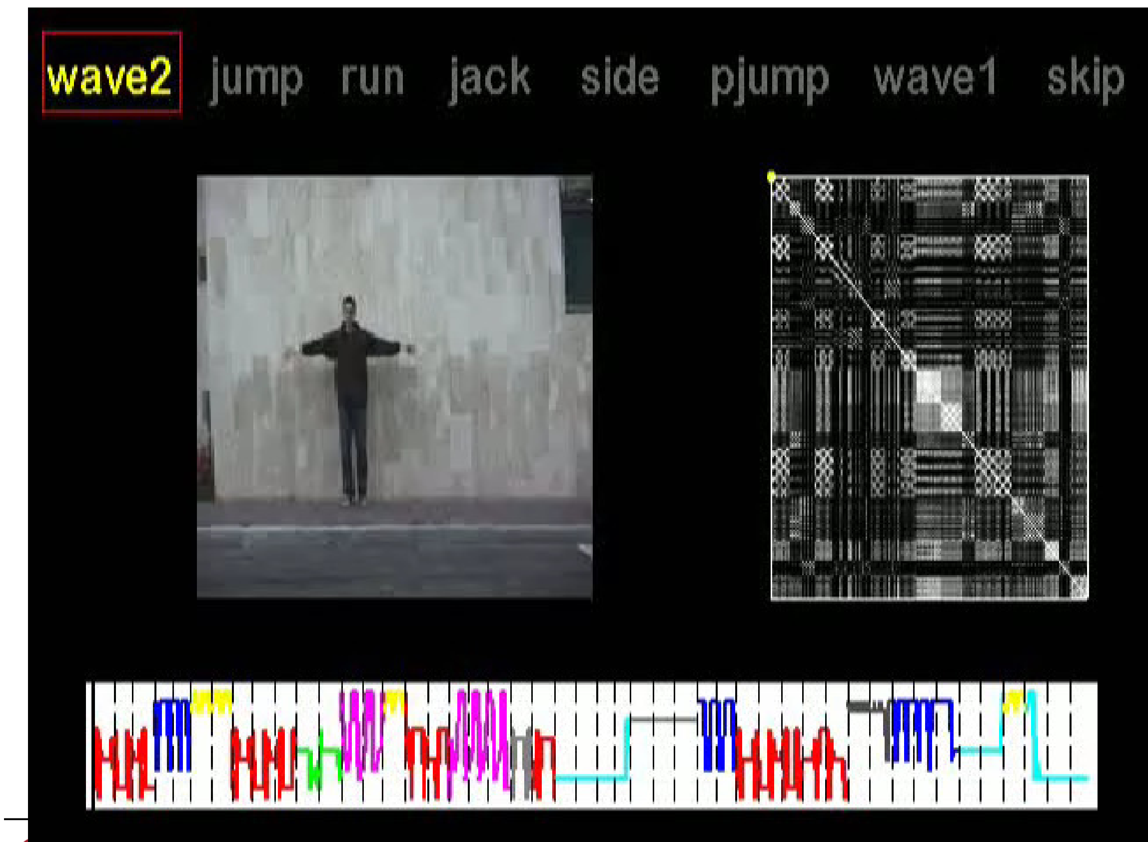
Parametric Segmental Switching Linear Dynamic System (PS-SLDS) (Oh et al. 08)

	Seq 1	Seq 2	Seq 3	Seq 4	Seq 5	Seq 6
ACA	**0.86**	**0.94**	0.67	0.92	**0.90**	**0.91**
PS-SLDS	0.76	0.92	**0.83**	**0.93**	**0.90**	0.90
GMM	0.74	0.67	0.44	0.71	0.58	0.71

Video data



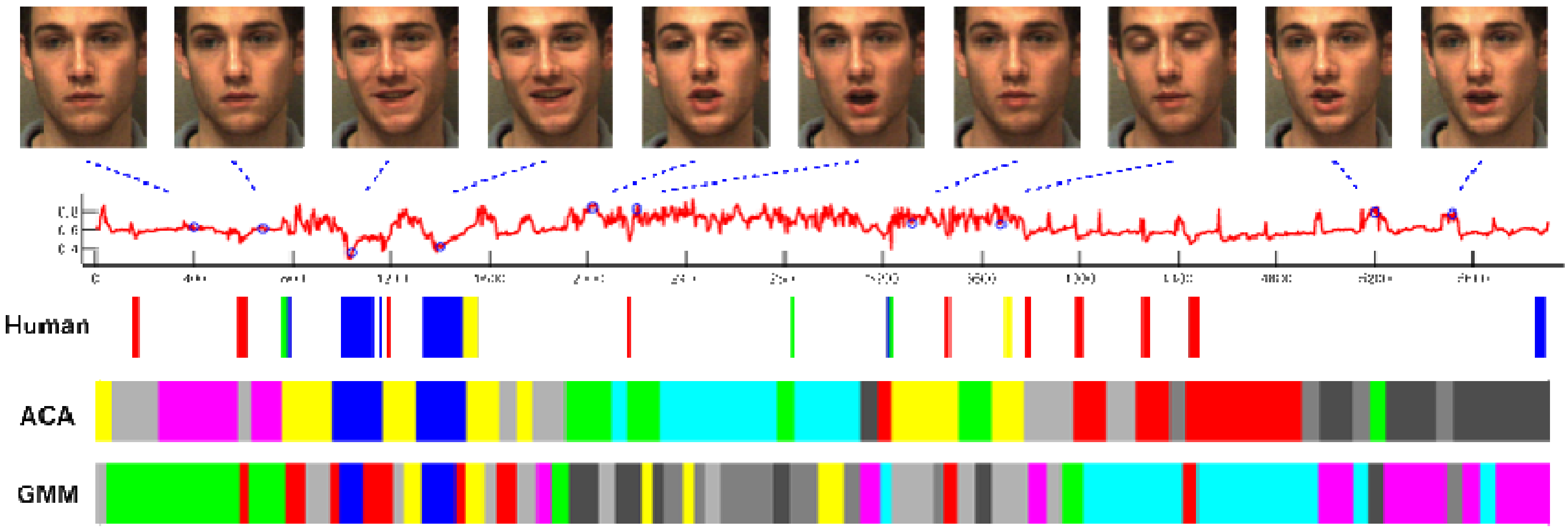
Weizmann dataset
(90 videos, 9 actions,
10 subjects):

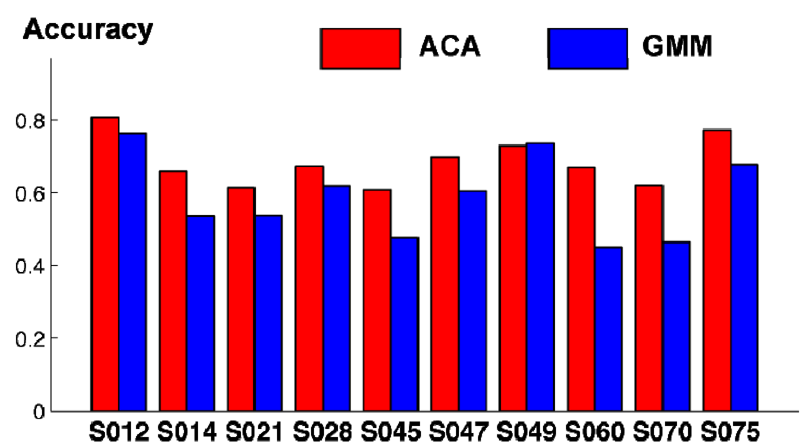


	Weizmann
ACA	0.76
GMM	0.63

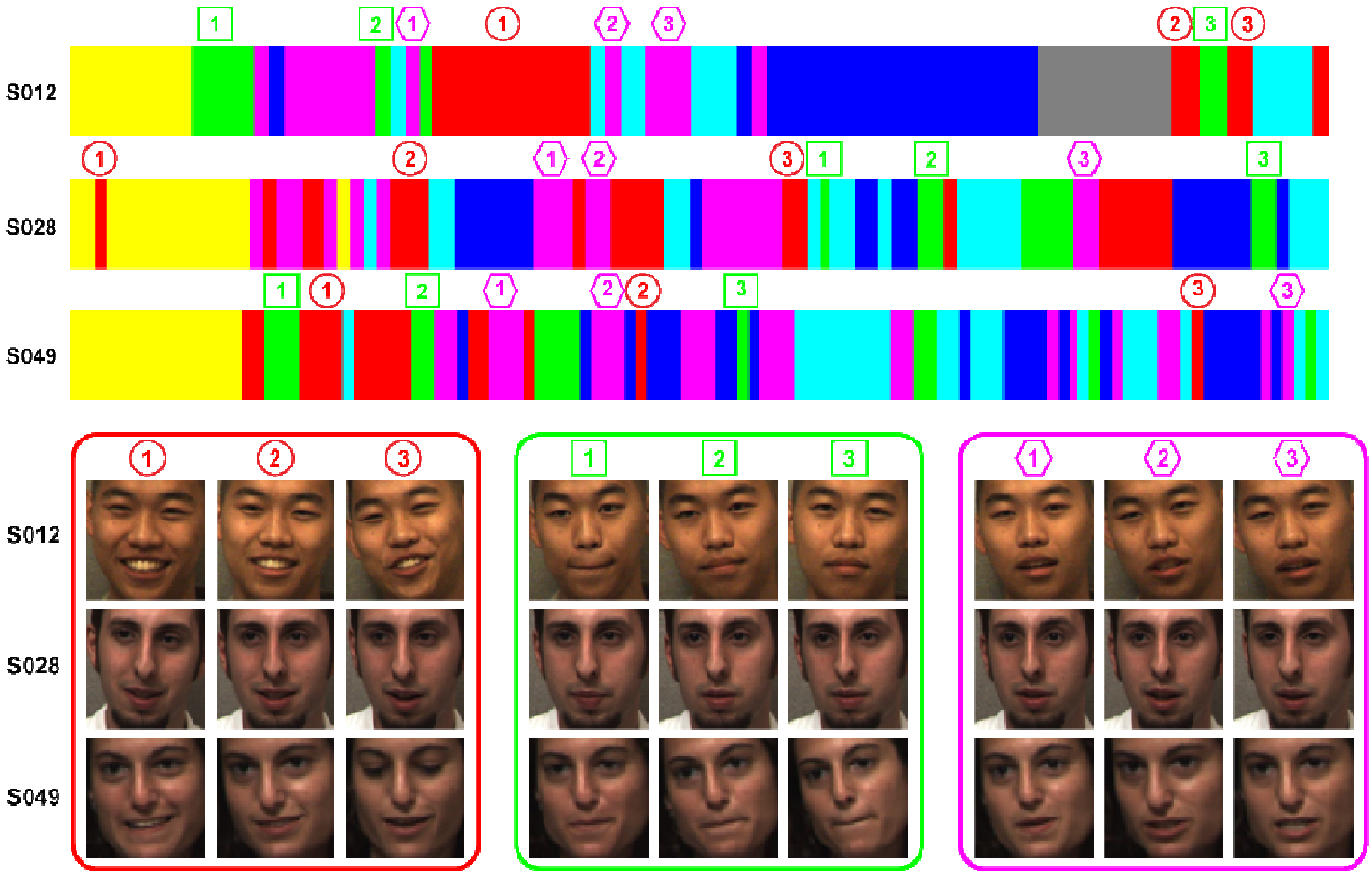
150 videos, 6
actions, 25
subjects

Unsupervised facial action discovery





Learning a vocabulary for facial expression

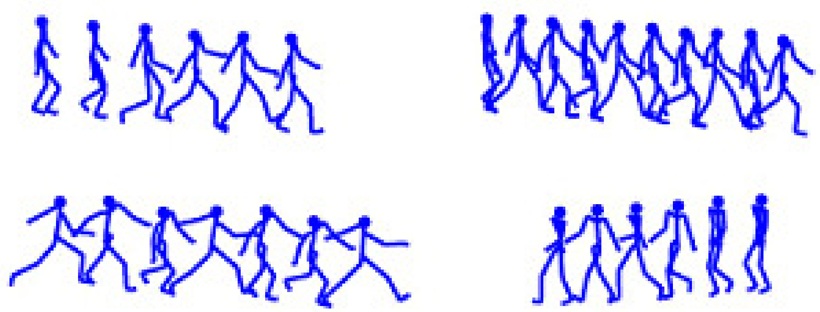


Outline

- Introduction
- Principal component analysis (PCA)
  – Robust Principal Component Analysis (RPCA)
  – PCA with uncertainty and missing data.
  – Parameterized Principal Component Analysis (PaPCA)
- Linear discriminant analysis (LDA)
  – Discriminative cluster analysis (DCA).
- K-means
  – Hierarchical aligned cluster analysis (HACA).
- Canonical correlation analysis (CCA)
  – Canonical time warping (CTW)
  – Dynamic coupled component analysis (DCCA)

Alignment of human behavior

- Temporal alignment of human behavior is useful in many human sensing tasks:

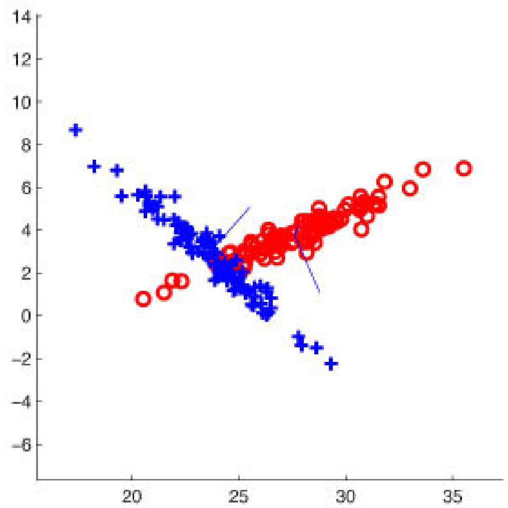


Previous work

- Data mining: *Derivative Dynamic Time Warping* *(Keogh et al. 01)*
  - DTW + Derivatives of features
- Computer Graphics: *Style Translation* *(Hsu et al. 05)*
  - DTW + Least-square
- Computer Vision: *View-invariant Action Recognition* *(Rao et al. 03 09)*
  - DTW + Homography
- Other Area: *Multiple Alignment of Continuous Time Series* *(Listgarten et al. 04)*
  - HMM + Account for changes in the amplitude of the signals
- Our work: *Canonical Time Warping* *(appear in NIPS 2009)*
  - DTW + Canonical Correlation Analysis (CCA)

Canonical Correlation Analysis (CCA)

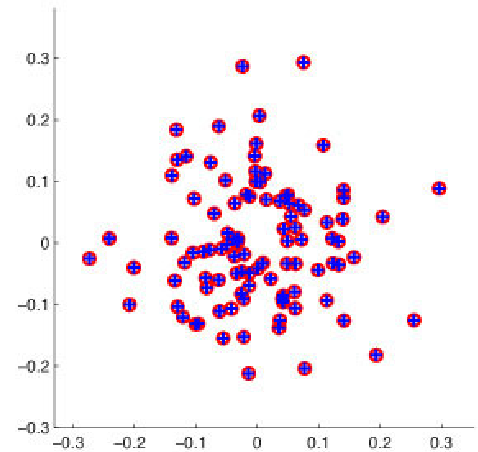
- Canonical Correlation Analysis *(Hotelling 1936)*



$$\mathbf{X} \in \mathbb{R}^{d_x \times n}, \mathbf{Y} \in \mathbb{R}^{d_y \times n}$$

Same #columns (samples)
Different #rows (dimensionality)

CCA projection



$$J_{cca}(\mathbf{V}_x, \mathbf{V}_y) = \|\mathbf{V}_x^T \mathbf{X} - \mathbf{V}_y^T \mathbf{Y}\|_F^2$$

$$\mathbf{V}_x \in \mathbb{R}^{d_x \times b}, \mathbf{V}_y \in \mathbb{R}^{d_y \times b}$$

$$\text{s.t. } \mathbf{V}_x^T \mathbf{X}\mathbf{X}^T \mathbf{V}_x = \mathbf{V}_y^T \mathbf{Y}\mathbf{Y}^T \mathbf{V}_y = \mathbf{I}_b$$

A least-square formulation for DTW

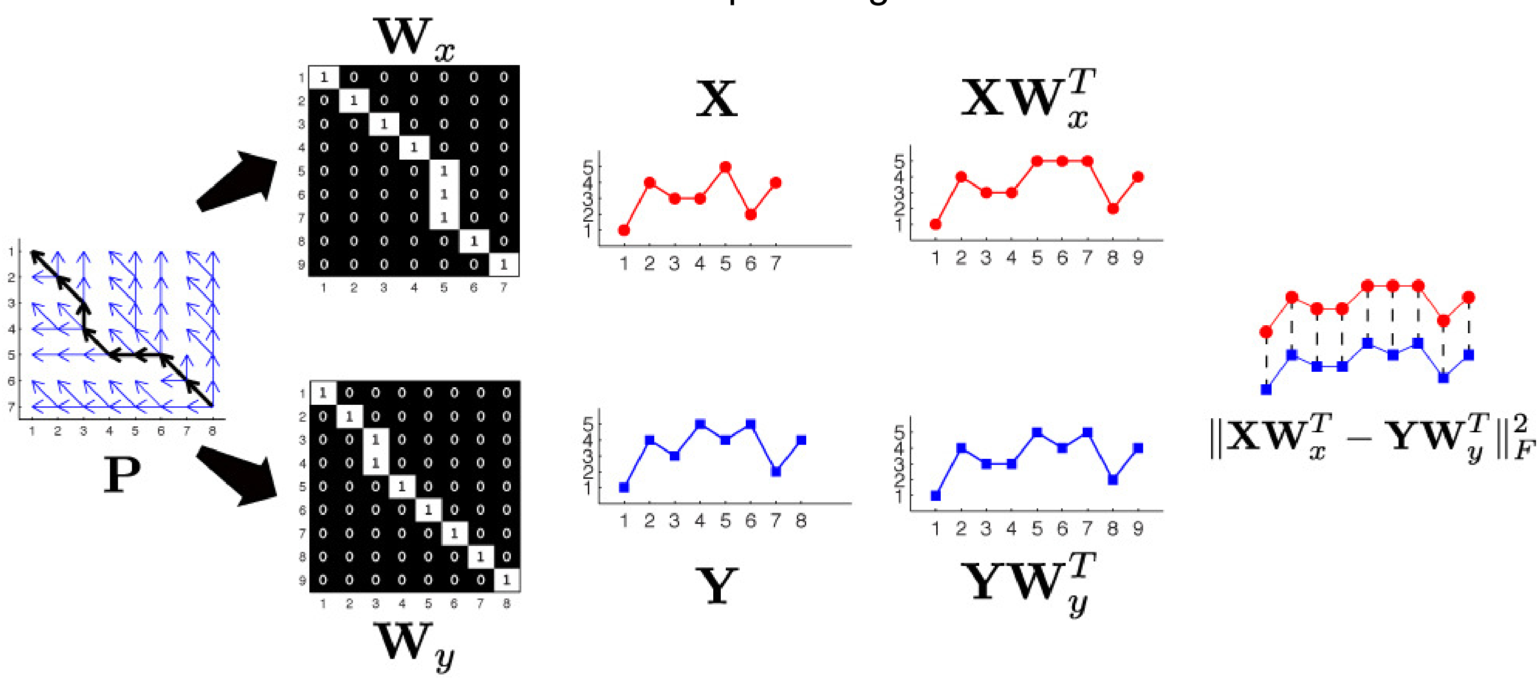
same #rows, different #columns

$$\mathbf{X} \in \mathbb{R}^{d \times n_x}, \mathbf{Y} \in \mathbb{R}^{d \times n_y}$$

$$J_{dtw}(\mathbf{W}_x, \mathbf{W}_y) = \|\mathbf{X}\mathbf{W}_x^T - \mathbf{Y}\mathbf{W}_y^T\|_F^2$$

$$\mathbf{W}_x \in \{0,1\}^{m \times n_x}, \mathbf{W}_y \in \{0,1\}^{m \times n_y}$$

temporal alignment



Canonical Time Warping (CTW)

Reminder

$$J_{dtw}(\mathbf{W}_x, \mathbf{W}_y) = \|\mathbf{X}\mathbf{W}_x^T - \mathbf{Y}\mathbf{W}_y^T\|_F^2$$

$$J_{cca}(\mathbf{V}_x, \mathbf{V}_y) = \|\mathbf{V}_x^T\mathbf{X} - \mathbf{V}_y^T\mathbf{Y}\|_F^2$$

different #rows, different #columns

$$\mathbf{X} \in \mathbb{R}^{d_x \times n_x}, \mathbf{Y} \in \mathbb{R}^{d_y \times n_y}$$

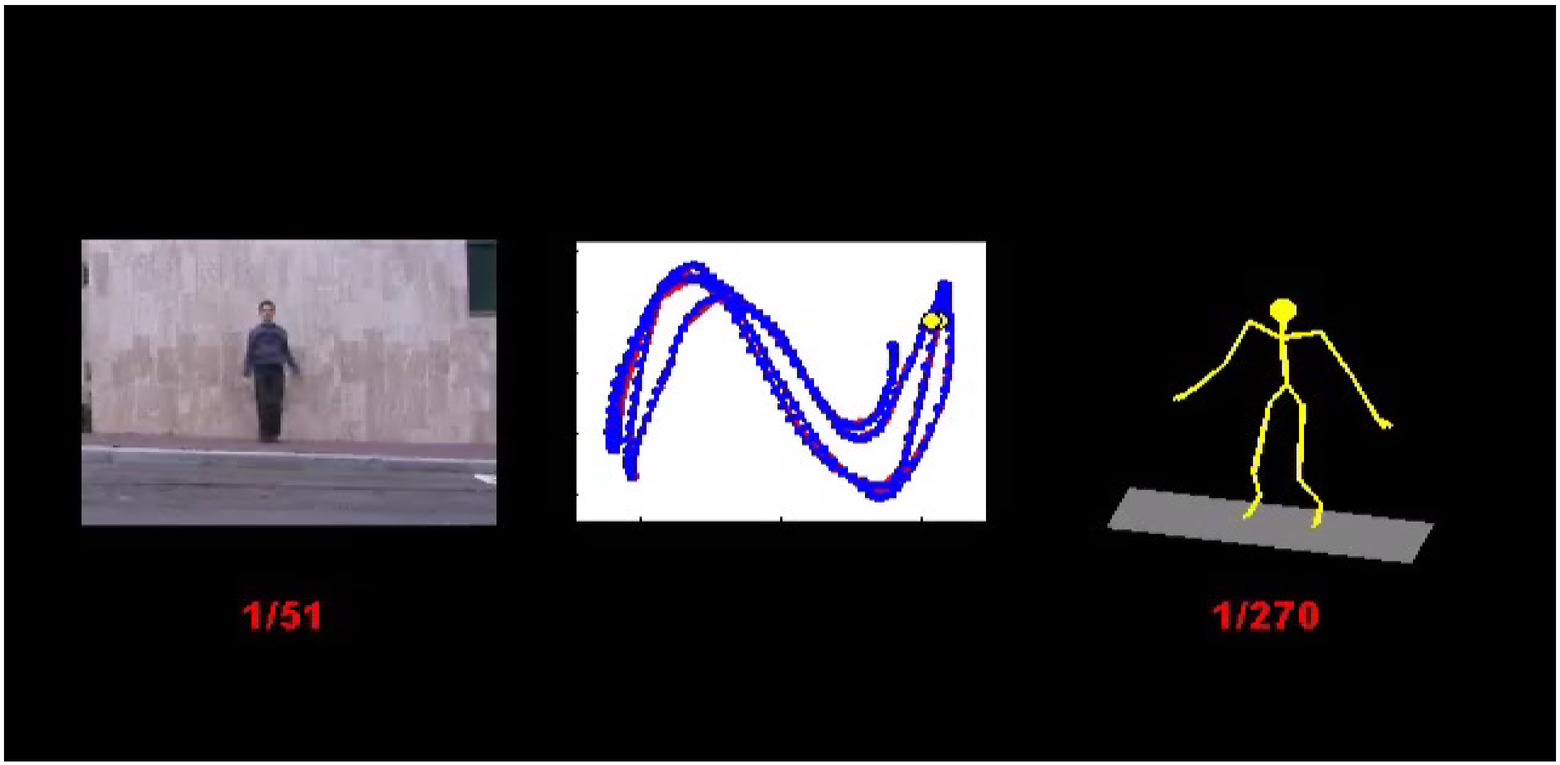
spatial transformation

$$J_{ctw}(\mathbf{W}_x, \mathbf{W}_y, \mathbf{V}_x, \mathbf{V}_y) = \|\mathbf{V}_x^T\mathbf{X}\mathbf{W}_x^T - \mathbf{V}_y^T\mathbf{Y}\mathbf{W}_y^T\|_F^2$$

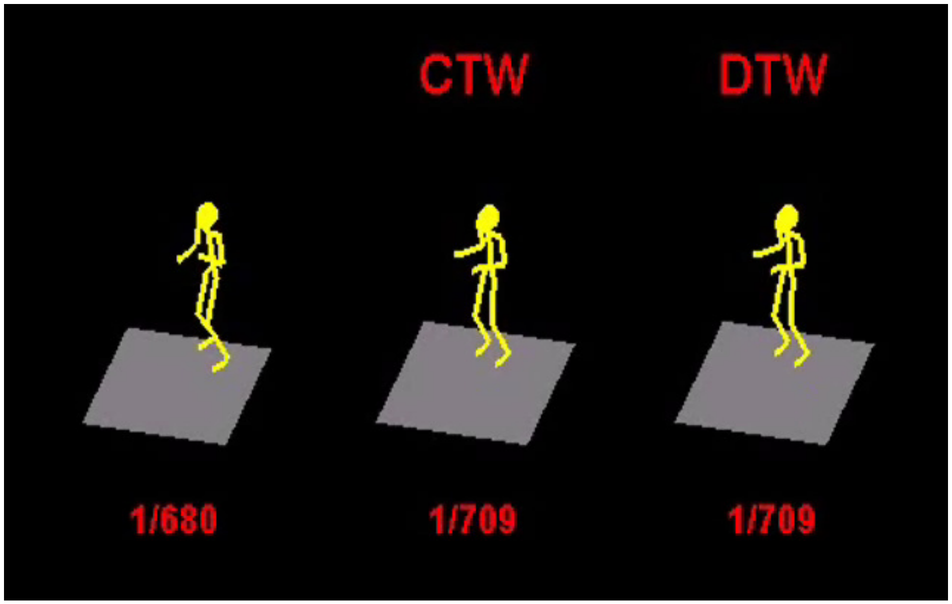
temporal alignment

$$\mathbf{V}_x^T\mathbf{X}\underbrace{\mathbf{W}_x^T\mathbf{W}_x}_{\mathbf{D}_x}\mathbf{X}^T\mathbf{V}_x - \mathbf{V}_y^T\mathbf{Y}\underbrace{\mathbf{W}_y^T\mathbf{W}_y}_{\mathbf{D}_y}\mathbf{Y}^T\mathbf{V}_y - \mathbf{I}_b$$

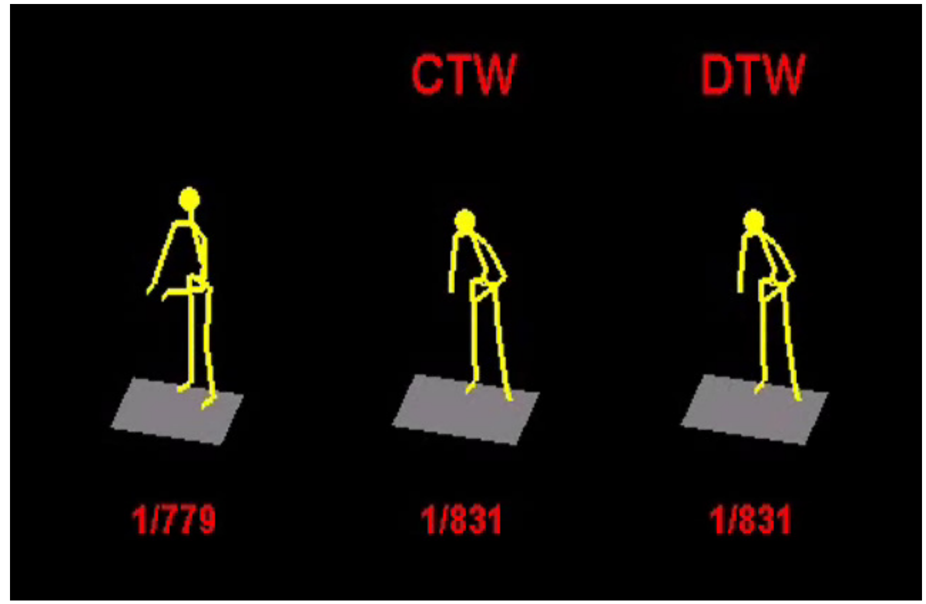
Aligning motion capture and video



Aligning motion capture data of same action (different people)

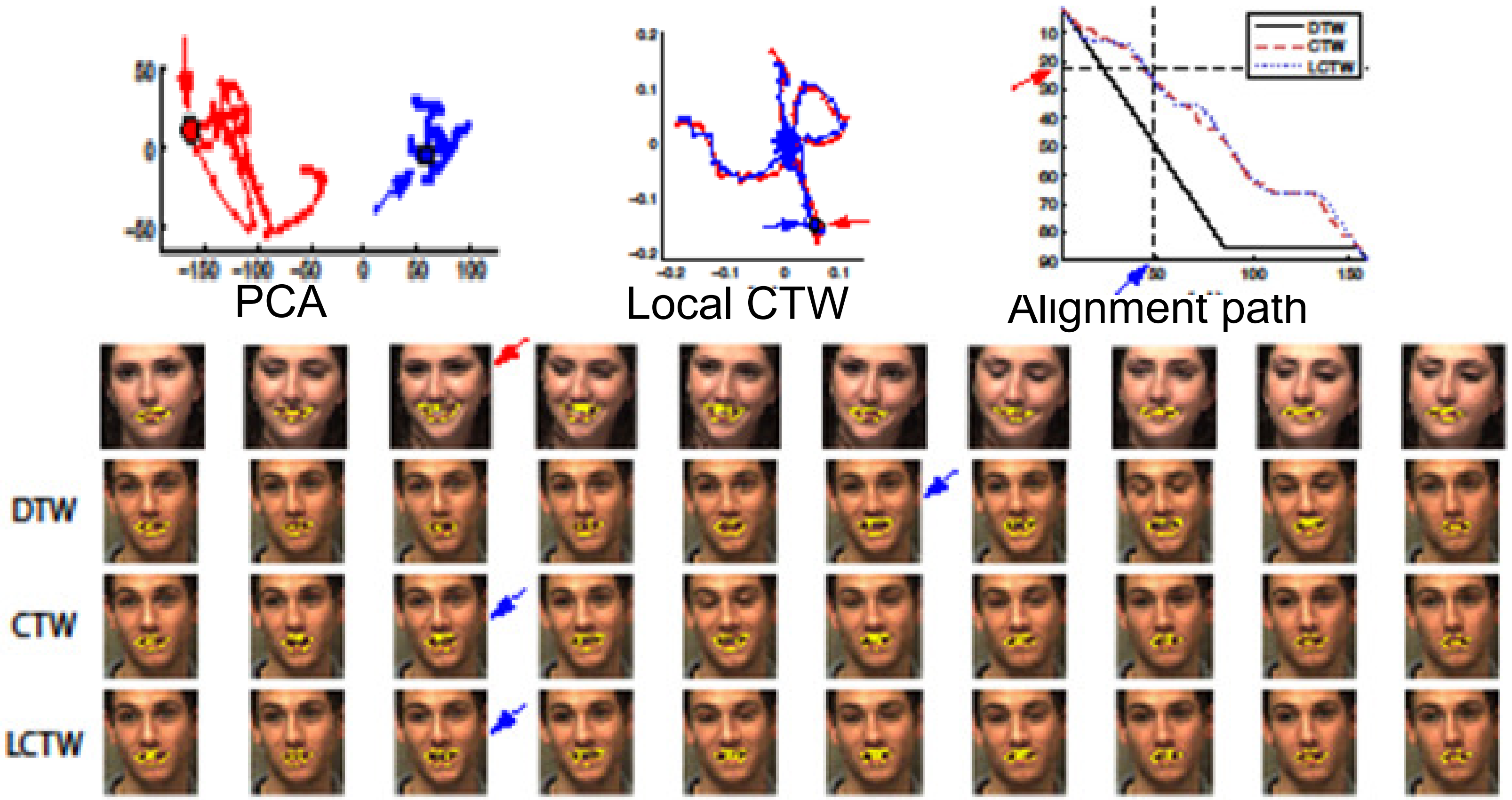


Boxing



Opening a cabinet

Facial expression alignment



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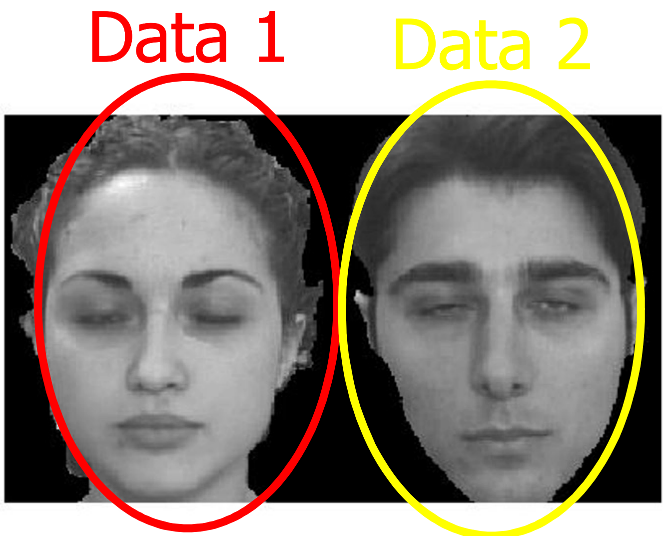
- Canonical correlation analysis (CCA)
  - Canonical time warping (CTW)
  - Dynamic coupled component analysis (DCCA)

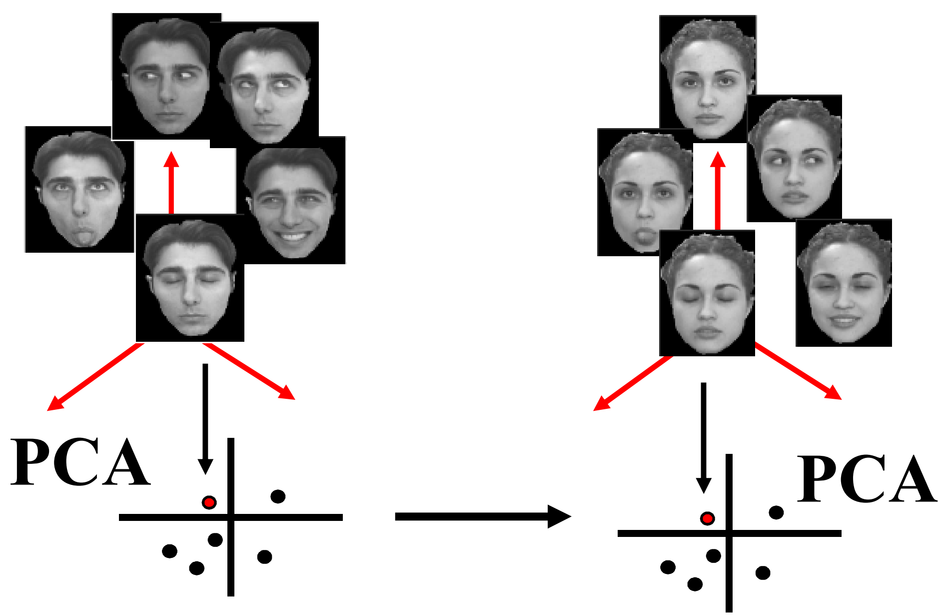
Dynamic Coupled Component Analysis (DCCA) (de la Torre & Black, 2001a)



- Learning the coupling.
- High dimensional data.
- Limited training data.

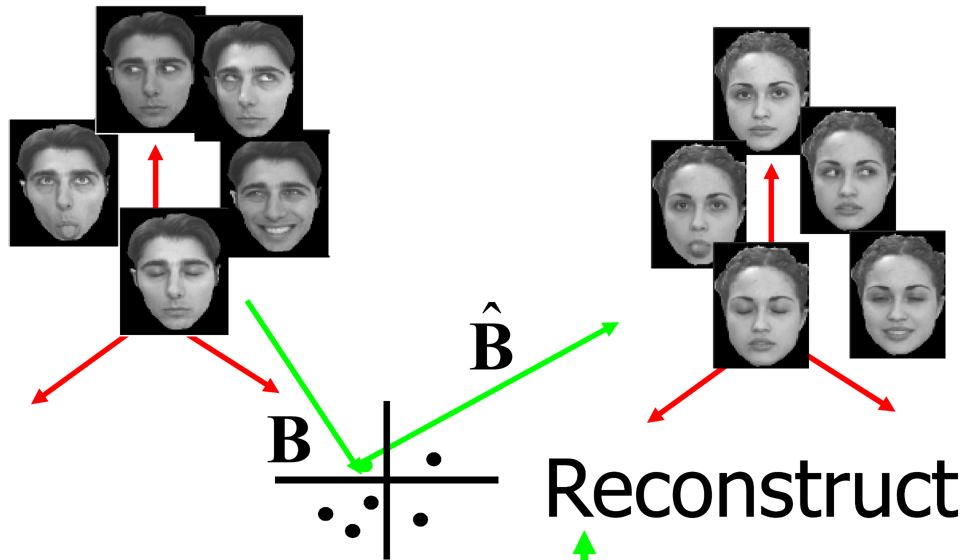
Solutions?

- PCA independently and general mapping



- Signals dependent signals with small energy can be lost.

DCCA



Reconstruction

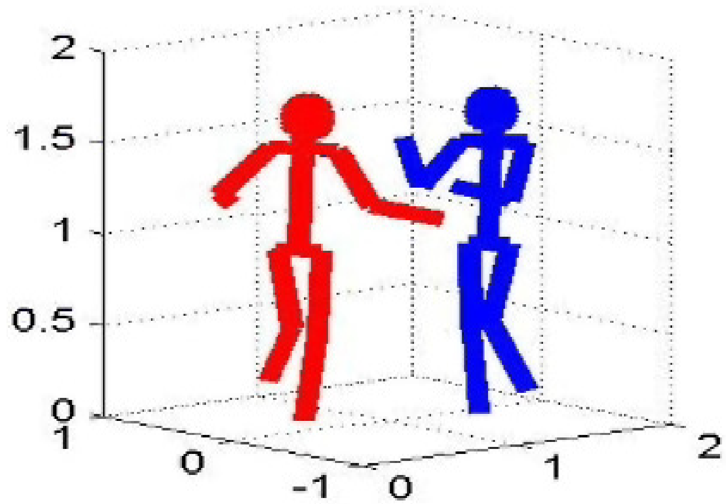
$$E_{cca}(\mathbf{B},\hat{\mathbf{B}},\mathbf{A},\mathbf{C},\boldsymbol{\mu},\hat{\boldsymbol{\mu}}) = \sum_{i=1}^{n} \left\| \hat{\mathbf{d}}_i - \hat{\boldsymbol{\mu}} - \hat{\mathbf{B}}\mathbf{c}_i \right\|_{\mathbf{W}1_i}^2$$

Projection

$$+ \lambda \sum_{i=1}^{n} \left\| \mathbf{c}_i - \mathbf{B}^T(\mathbf{d}_i - \boldsymbol{\mu}) \right\|_{\mathbf{W}2_i}^2 + \lambda_2 \sum_{i=1}^{n} \left\| \mathbf{c}_i - \mathbf{A}\mathbf{c}_{i-1} \right\|_{\mathbf{W}3_i}^2$$

Dynamics

Dynamic Coupled Component Analysis

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