Carnegie Mellon DSP-I Electrical & Computer ENGINEERING

Digital Signal Processing I (18-791)
Fall Semester, 2002

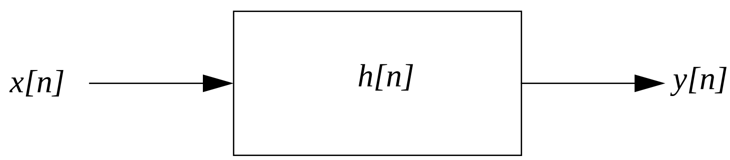
SOME ADDITIONAL DTFT PROPERTIES

Distributed: September 6, 2002

Introduction:

These notes briefly summarize the properties discussed in the recitation of September 6. Two additional properties are noted as well.

6. Convolution



If an LSI system has $x[n]$ as its input, $h[n]$ as the unit sample response, and $y[n]$ as the system output, these three quantities are related by the convolution sum,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \equiv x[n]*h[n]$$

We showed in class that if two time functions are convolved, the corresponding DTFTs are *multiplied* in frequency:

$$x[n]*h[n] \Leftrightarrow X(e^{j\omega})H(e^{j\omega})$$

This equation, of course, is the basis for our representation of linear filtering in the frequency domain.

7. Multiplication

As in the case of continuous-time linear system theory, multiplication is the dual of convolution, and just as convolution in time corresponds to multiplication in frequency, multiplication in time corresponds to convolution in frequency. Specifically,

$$x_1[n]x_2[n] \Leftrightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\omega$$

The expression on the right side of the equation is a form of *circular convolution*, which will be discussed in greater detail later in the course. Circular convolution, rather than the conventional linear convolution, must be performed when the functions to be convolved are periodic. The operation is similar to convolultion of the two (continuous) frequency functions, except that the integration is performed over only a single period (and can be performed over any contiguous strip of the dummy frequency variable θ of extent 2π.

We will encounter multiplication of two time functions in this course primarily in the operations of *modulation* and *windowing.*

8. Parseval's theorem

Parseval's theorem states

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega$$

Note that this is an equation and not a transform pair, as both sides of the equation are constants. Since the quantity on the left side of the equation is the *energy* of the time function $x[n]$, the expression on the right side states that the total energy of the function can also be obtained by integrating the squared magnitude of its DTFT over all frequencies (once) and dividing by 2π. This suggests that the energy of the function within a *range* of frequencies can be similarly obtained by integrating the squared magnitude of the DTFT over that frequency range, taking care to include negative as well as positive frequencies if appropriate.

9. Initial value theorems

The traditional DTFT equations are

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \text{ and}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n} d\omega$$

Setting the variable ω in the first equation and *n* in the second equation equal to zero, we easily obtain

$$X(0) = \sum_{n=-\infty}^{\infty} x[n] \text{ and}$$

$$x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$$

Do not confuse these "initial value" equations with Parseval's theorem!

This theorem is useful in working Problem 2.3 on this week's homework assignment.