## Advanced Component Analysis Methods for Signal Processing

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## Outline

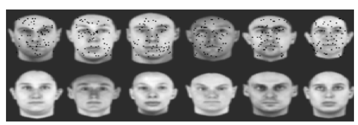
- Introduction
- Principal Component Analysis (PCA)
  - Robust Principal CA
  - PCA missing data
  - Parameterized Principal CA
- Linear Discriminant Analysis
  - Multimodal Oriented Discriminant Analysis
- Canonical Correlation Analysis
  - Canonical Time Warping
- K-means
  - Aligned Cluster Analysis (ACA)
  - Discriminative Cluster Analysis (DCA)



## Robust PCA

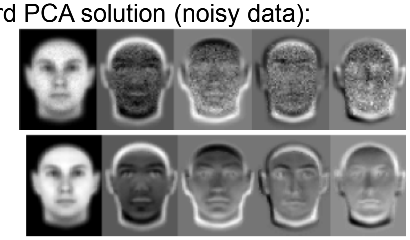
•Two types of outliers:

Sample outliers
(Xu & Yuille., 1995)



Intra-sample outliers
(de la Torre & Black, 2001b; Skocaj & Leonardis, 2003)

•Standard PCA solution (noisy data):



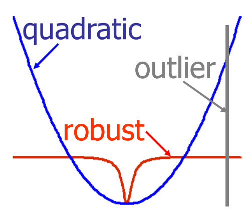


## Robust PCA

- Using robust statistics:
  (de la Torre & Black, 2001b; de la Torre & Black, 2003a)

Pixel residual

$$E_{rpca}(\mathbf{B},\mathbf{C},\boldsymbol{\mu}) = \sum_{i=1}^{n}\sum_{p=1}^{d}\rho\left(d_{pi}-\mu_p-\sum_{j=1}^{k} b_{pj}c_{ji},\sigma_p\right)$$




mean

Basis (**B**) &
Coefficients(**c**)

## Numerical Problems

- No closed form solution in terms of an eigen-equation.
- Deflation approaches do not hold.

$$\mathbf{A}' = \mathbf{A} - \lambda_1 \mathbf{u}_1\mathbf{u}_1^T$$ First eigenvector with highest eigenvalue.

$$\mathbf{A}'' = \mathbf{A}' - \lambda_2 \mathbf{u}_2\mathbf{u}_2^T$$ Second eigenvector with highest eigenvalue.

$$\ldots$$

- In the robust case all the basis have to be computed simultaneously (including the mean).



## How to Optimize it?

$$E_{rpca}(\mathbf{B},\mathbf{C},\boldsymbol{\mu}) = \sum_{i=1}^{n}\sum_{p=1}^{d}\rho(d_{pi}-\mu_p-\sum_{j=1}^{k} b_{pj}c_{ji},\sigma_p)$$

- Normalized Gradient descent

$$\mathbf{B}^{n+1} = \mathbf{B}^{n} - [\mathbf{H}_b]^{-1} \circ \frac{\partial E_{rpca}}{\partial \mathbf{B}} \qquad \mathbf{H}_{\mathbf{b}} = \max\ diag\ (\frac{\partial^2 E_{rpca}}{\partial \mathbf{b}_i \partial \mathbf{b}_i^T})$$

$$\mathbf{C}^{n+1} = \mathbf{C}^{n} - [\mathbf{H}_c]^{-1} \circ \frac{\partial E_{rpca}}{\partial \mathbf{C}} \qquad \mathbf{H}_{\mathbf{c}} = \max\ diag\ (\frac{\partial^2 E_{rpca}}{\partial \mathbf{c}_i \partial \mathbf{c}_i^T})$$

- Deterministic annealing methods to avoid local minima.

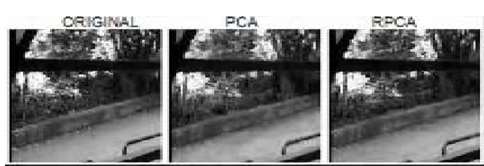
(Blake & Zisserman, 1987)



## Example

Statistical outlier

- Small region
- Short amount of time

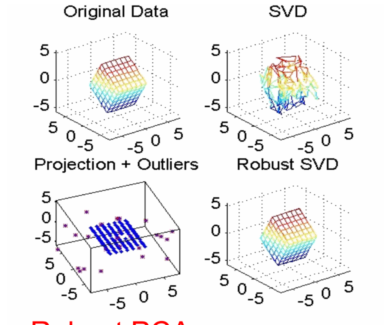





## Robust PCA

Original PCA RPCA Outliers

## Structure from Motion




## More work on Robust PCA

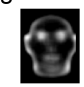
- Robust estimation of coefficients (Black & Jepson, 1998; Leonardis & Bischof, 2000; Ke & Kanade, 2004)
- Robust estimation of basis and coefficients (Gabriel & Odoro, 1984; Croux & Filzmoser., 1981; Skocaj et al., 2002;Skocaj & Leonardis, 2003; de la Torre & Black, 2001b; de la Torre & Black, 2003a)
- Other Robust PCA techniques (sample outliers) (Campbell, 1980; Ruymagaart, 1981; Xu & Yuille., 1995)



## PCA with Uncertainty and Missing Data

- Adding uncertainty $E_2(\mathbf{B},\mathbf{C}) = \|\mathbf{W}\circ(\mathbf{D}-\mathbf{BC})\|_F = \sum_{i=1}^{d}\sum_{j=1}^{n} w_{ij}(d_{ij} - \sum_{s=1}^{k} b_{is}c_{sj})^2$

- If weights are separable $\mathbf{W} = \mathbf{w}_r\mathbf{w}_c^T$ closed-form solution.



$$\mathbf{w}^c = \begin{pmatrix} w_1^c & w_2^c & \dots & w_n^c \end{pmatrix}$$

$$\mathbf{w}^r = \begin{pmatrix} w_1^r \\ w_2^r \\ \dots \\ w_d^r \end{pmatrix} \quad \begin{pmatrix} d_{11} & \cdots & \cdots & d_{1n} \\ d_{21} & \ddots & & d_{2n} \\ \vdots & & \ddots & \vdots \\ d_{d1} & \cdots & \cdots & d_{dn} \end{pmatrix} = \mathbf{D}$$

$$\mathbf{W} \in \Re^{d\times n}$$

$$w_{ij} \geq 0$$

$$\circ \; Hadamard \quad product$$

- Generalized SVD

(Greenacre, 1984; Irani & Anandan, 2000;)



## General Case

- For arbitrary weights no closed-form solution.

$$E_2(\mathbf{B},\mathbf{C}) = \|\mathbf{W}\circ(\mathbf{D}-\mathbf{BC})\|_F = \sum_{i=1}^{n}(\mathbf{d}_i - \mathbf{Bc}_i)^T diag(\mathbf{w}_i)(\mathbf{d}_i - \mathbf{Bc}_i) =$$

$$\sum_{p=1}^{d}(\mathbf{d}^p - \mathbf{C}^T\mathbf{b}^p)^T diag(\mathbf{w}^p)(\mathbf{d}^p - \mathbf{C}^T\mathbf{b}^p)$$ (Torre & Black, 2003a)

- Alternated least squares algorithms
  - Slow convergence, easy implementation.
- Damped Newton Algorithm
  - Fast convergence. (Buchanan & Fitzgibbon., 2005)

$$E_2(\mathbf{B},\mathbf{C}) = \|\mathbf{W}\circ(\mathbf{D}-\mathbf{BC})\|_F + \lambda_1 \|\mathbf{B}\|_F + \lambda_2 \|\mathbf{C}\|_F$$

$$\mathbf{v} = \begin{bmatrix} vec(\mathbf{B}) \\ vec(\mathbf{C}) \end{bmatrix} \quad \mathbf{v}^{(n+1)} = \mathbf{v}^n - \begin{bmatrix} \dfrac{[\partial^2 E_2]}{\partial^2 \mathbf{v}} \end{bmatrix}^{-1} \frac{\partial E_2}{\partial \mathbf{v}}$$

- H definite positive: $\mathbf{H} = \dfrac{\partial^2 E_2}{\partial^2 \mathbf{v}} + \lambda \mathbf{I}$

*repeat*

$$\mathbf{H} = \frac{\partial^2 E_2}{\partial^2 \mathbf{v}} \quad \mathbf{g} = \frac{\partial E_2}{\partial \mathbf{v}}$$

*repeat*

$$\lambda = 10\lambda$$

$$\mathbf{y} = \mathbf{x} - (\mathbf{H} + \lambda I)^{-1}\mathbf{g}$$

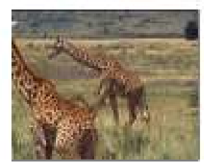
*until* $F(\mathbf{y}) < F(\mathbf{x})$

$$\mathbf{x} = \mathbf{y}; \lambda = \frac{\lambda}{10}$$

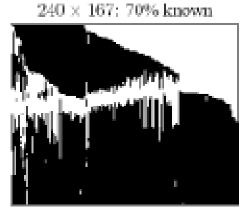
*until convergence*



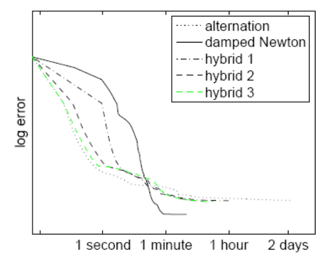
## Experiments

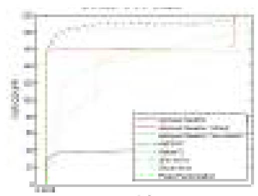


240 × 167: 70% known



Total: 500 runs



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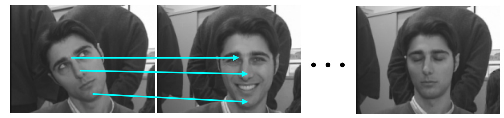
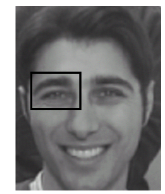
# Parameterized Component Analysis (PaCA)

(de la Torre & Black, 2003b)

- Learn a subspace invariant to geometric transformations?



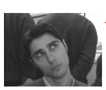
- Data has to be **geometrically** normalized
  - Tedious manual cropping.
  - Inaccuracies due to matching ambiguities.
  - Hard to achieve sub-pixel accuracy.





# Error function for PaCA

$$E(\mathbf{B},\mathbf{C},\mathbf{a}) = \sum_{t=1}^{T} \left\| \mathbf{d}_t (\mathbf{f}(\mathbf{x},\mathbf{a}_t)) - \mathbf{B}\mathbf{c}_t \right\|^2_{\mathbf{W}_1} + p_1(\mathbf{a}) + p_2(\mathbf{c})$$



Motion (warping)

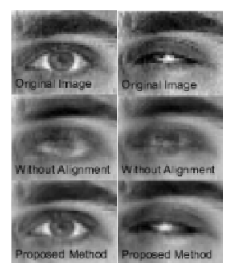
Basis (**B**) & coefficients (**c**)

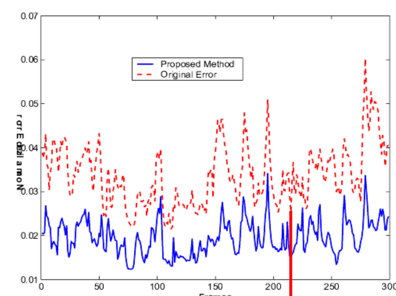
Regularization

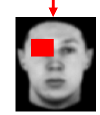
$$\sum_{t=1}^{T}\sum_{l=1}^{L} \lambda_1 \left\| \mathbf{c}_t - \Gamma_c \mathbf{c}_{t-1} \right\|^2_{\mathbf{W}2} + \lambda_2 \left\| \mathbf{a}_t - \Gamma_a \mathbf{a}_{t-1} \right\|^2_{\mathbf{W}3}$$



# EigenEye Learning



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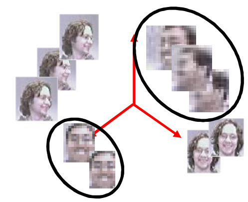
# Multimodal Oriented Component Analysis (MODA)

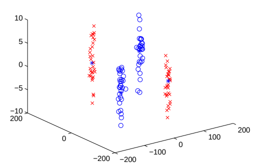
(de la Torre & Kanade, 2005a)

- How to extend LDA to deal with:
  - Model class covariances.
  - Multimodal classes.
  - Deal efficiently with huge covariance matrices (e.g. 100*100).

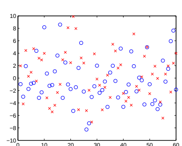




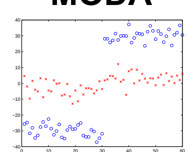
# Multimodality



**LDA**

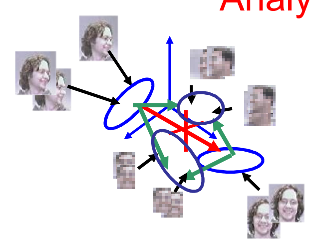


**MODA**





# Multimodal Oriented Discriminant Analysis (MODA)



- **B** that MAXIMIZES the Kullback-Leibler divergence between clusters among classes.

$$\sum_{i=1}^{classes}\sum_{j\neq i}^{classes}\sum_{r_1\in C_i}\sum_{r_2\in C_j} tr(\mathbf{B}^T(\boldsymbol{\Sigma}_j^{r_2}\boldsymbol{\Sigma}_i^{r_1\,-1}+\boldsymbol{\Sigma}_j^{r_2\,-1}\boldsymbol{\Sigma}_i^{r_1}+(\boldsymbol{\mu}_i^{r_1}-\boldsymbol{\mu}_j^{r_2})(\boldsymbol{\Sigma}_j^{r_1\,-1}+\boldsymbol{\Sigma}_i^{r_2\,-1})(\boldsymbol{\mu}_i^{r_1}-\boldsymbol{\mu}_j^{r_2})^T)\mathbf{B})$$

- 1 mode per class and equal covariances equivalent to LDA.

# Optimization

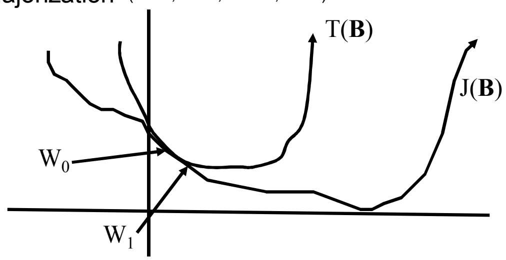
- Hard optimization problem

$$J(\mathbf{B}) = -\sum_{i=1} tr((\mathbf{B}^T \mathbf{\Sigma}_i \mathbf{B})^{-1} (\mathbf{B}^T \mathbf{A}_i \mathbf{B}))$$

- Iterative Majorization (Kiers, 1995; Leeuw, 1994)

$T(\mathbf{B}) \geq J(\mathbf{B}) \;\; \forall \mathbf{B}$

$T(\mathbf{B}_0) = J(\mathbf{B}_0)$






# Majorization

$$T(\mathbf{B}) = \sum_{i=1}^{classes} \| (\mathbf{B}^T \mathbf{\Sigma}_i \mathbf{B})^{-\frac{1}{2}} \mathbf{B}^T \mathbf{A}_i^{-\frac{1}{2}} - (\mathbf{B}^T \mathbf{\Sigma}_i \mathbf{B})^{\frac{1}{2}} (\mathbf{B}_0^T \mathbf{\Sigma}_i \mathbf{B}_0) \mathbf{B}^T \mathbf{A}_i^{\frac{1}{2}} \|$$

$$\geq -\sum_{i=1}^{classes} tr((\mathbf{B}^T \mathbf{\Sigma}_i \mathbf{B})^{-1} (\mathbf{B}^T \mathbf{A}_i \mathbf{B}))$$

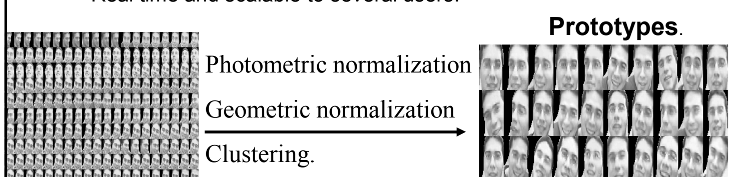
- Slow convergence…



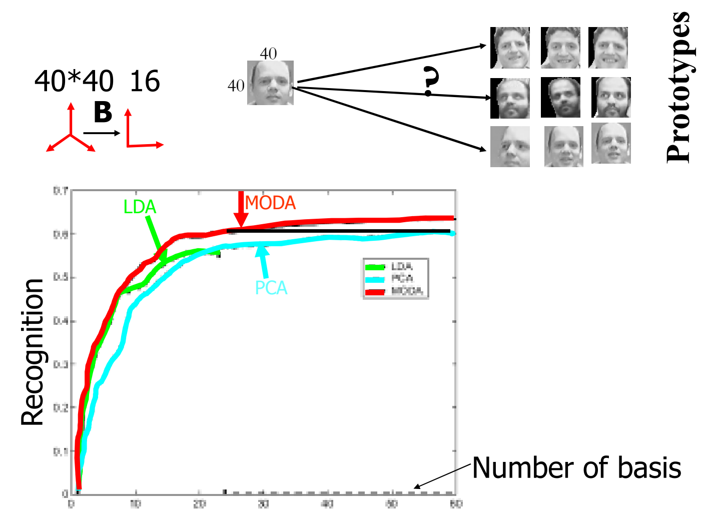
# Face recognition from video

(de la Torre et al., 2005b)


- Challenges
  - Low quality small images (40-50 pixels).
  - Changes in expression/pose/occlusion/illumination.
  - Real time and scalable to several users.





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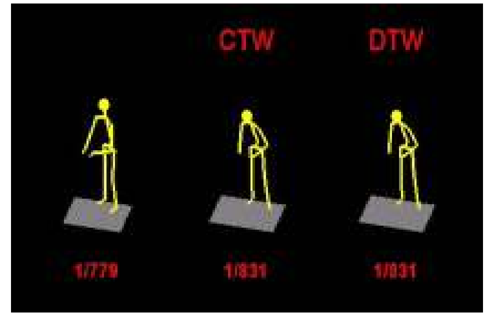
# Problem






# Canonical Correlation Analysis (CCA)

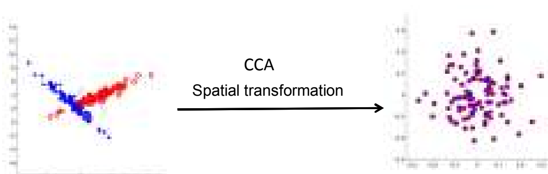
(Hotelling 1936)

- CCA minimizes:

different #rows, same #columns

$\mathbf{X} \in \Re^{d_x \times n}, \mathbf{Y} \in \Re^{d_y \times n}$

$$J_{cca}(\mathbf{V}_x, \mathbf{V}_y) = \left\| \mathbf{V}_x^T \mathbf{X} - \mathbf{V}_y^T \mathbf{Y} \right\|_F^2 \quad s.t. \left. \begin{matrix} \mathbf{V}_x^T \mathbf{X}\mathbf{X}^T \mathbf{V}_x \\ \mathbf{V}_y^T \mathbf{Y}\mathbf{Y}^T \mathbf{V}_y \end{matrix} \right\} = \mathbf{I}_b$$



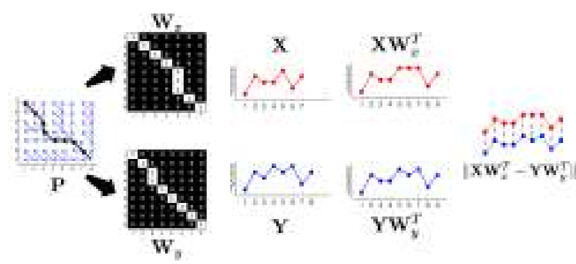



# A least-square formulation for DTW

same #rows, different #columns

$\mathbf{X} \in \Re^{d \times n_x}, \mathbf{Y} \in \Re^{d \times n_y}$

$$J_{dtw}(\mathbf{W}_x, \mathbf{W}_y) = \left\| \mathbf{X}\mathbf{W}_x^T - \mathbf{Y}\mathbf{W}_y^T \right\|_F^2$$



## Canonical Time Warping (CTW)

**Reminder**

$$J_{cca}(\mathbf{V}_x, \mathbf{V}_y) = \left\|\mathbf{V}_x^T \mathbf{X} - \mathbf{V}_y^T \mathbf{Y}\right\|_F^2$$

$$J_{dtw}(\mathbf{W}_x, \mathbf{W}_y) = \left\|\mathbf{X}\mathbf{W}_x^T - \mathbf{Y}\mathbf{W}_y^T\right\|_F^2$$

different #rows, different #columns

$$\mathbf{X} \in \Re^{d_x \times n_x}, \mathbf{Y} \in \Re^{d_y \times n_y}$$

spatial transformation

$$J_{ctw}(\mathbf{W}_x, \mathbf{W}_y, \mathbf{V}_x, \mathbf{V}_y) = \left\|\mathbf{V}_x^T \mathbf{X} \mathbf{W}_x^T - \mathbf{V}_y^T \mathbf{Y} \mathbf{W}_y^T\right\|_F^2$$

temporal alignment

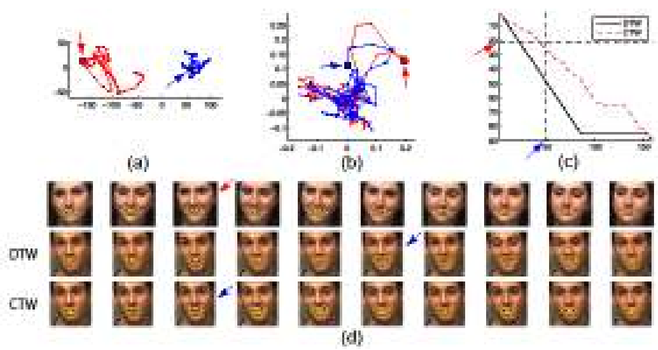
$$s.t. \quad \left.\begin{matrix} \mathbf{V}_x^T \mathbf{X} \mathbf{W}_x^T \mathbf{W}_x \mathbf{X}^T \mathbf{V}_x \\ \mathbf{V}_y^T \mathbf{Y} \mathbf{W}_y^T \mathbf{W}_y \mathbf{Y}^T \mathbf{V}_y \end{matrix}\right\} = \mathbf{I}_b$$



## Facial expression alignment

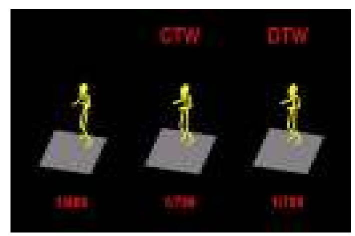



## Facial expression alignment

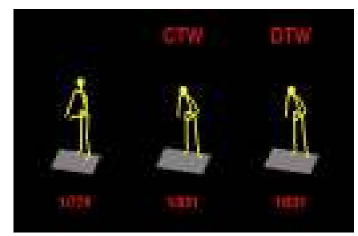




## Aligning human motion

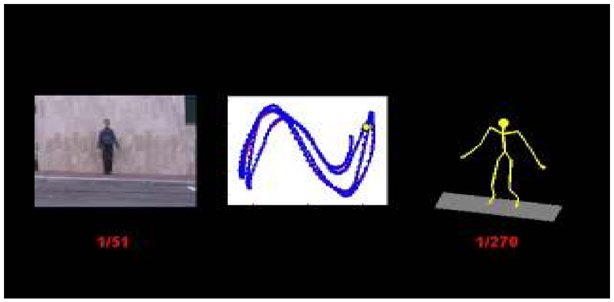



Boxing




Opening a cabinet

# Aligning motion capture and video



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# Problem

- Mining facial expression




# Problem

- Mining facial expression for one subject


- **Summarization**

# Problem

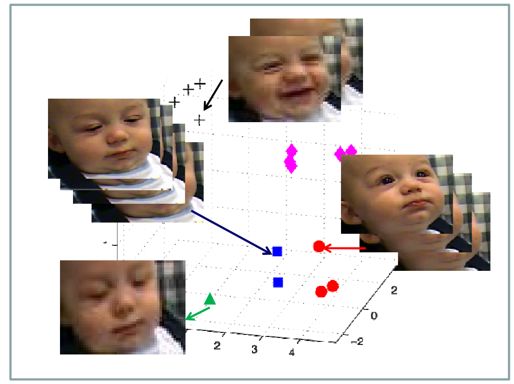
- Mining facial expression for one subject


- **Summarization**



# Problem

- Mining facial expression of one subject



☑ **Summarization**

- **Embedding**



# Problem

- Mining facial expression for one subject

☑ **Summarization**

☑ **Embedding**

- **Indexing**

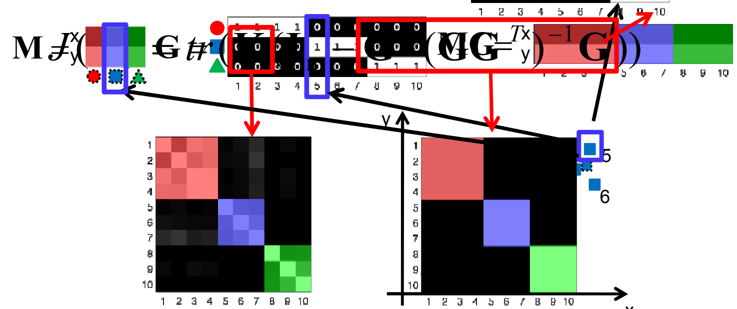


# *k*-means and kernel k-means

(MacQueen 67, Ding et al. 02, Dhillon et al. 04, Zass and Shashua 05, De la Torre 06)

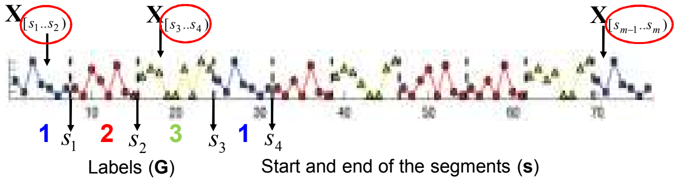
$$J(\mathbf{M},\mathbf{G}) = \| \boxed{\phi}(\mathbf{X}) - \cancel{\mathbf{M}}\mathbf{G} \|_F^2$$

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$



$$\mathbf{K} = \phi(\mathbf{X})^T \phi(\mathbf{X})$$

## Problem formulation for ACA (I)




$$J_{aca}(\mathbf{M},\mathbf{G},\mathbf{S}) = \| \varphi(\mathbf{X}_{[s_1..s_2)}, \mathbf{X}_{[s_2..s_3)}, \ldots, \mathbf{X}_{[s_{m-1}..s_m)}) - \mathbf{MG} \|_F^2$$

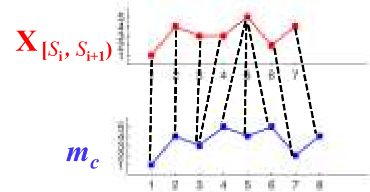


## Problem formulation for ACA (II)

$$J_{aca}(\mathbf{M},\mathbf{G},\mathbf{S}) = \| \varphi(\mathbf{X}_{[s_1..s_2)}, \mathbf{X}_{[s_2..s_3)}, \ldots, \mathbf{X}_{[s_{m-1}..s_m)}) - \mathbf{MG} \|_F^2$$

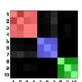
$$= \sum_{c=1}^{k}\sum_{i=1}^{m} g_{ci} \| \underbrace{\varphi(\mathbf{X}_{[S_i,\, S_{i+1})}) - \boldsymbol{m}_c}_{} \|_2^2$$

Dynamic Time Alignment Kernel (Shimodaira et al. 01)

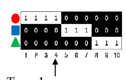





## Matrix formulation for ACA

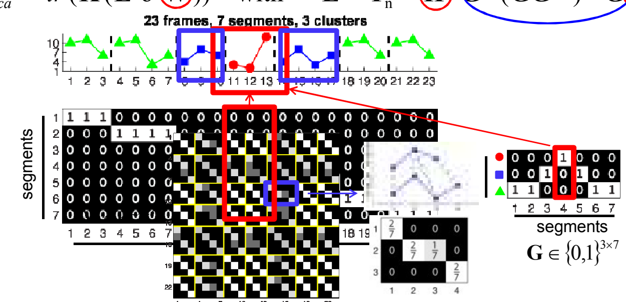


$$\mathbf{K} = \phi(\mathbf{X})^T \phi(\mathbf{X})$$



$$J_{kkm} = tr(\mathbf{KL}) \quad \text{with} \quad \mathbf{L} = \mathbf{I}_n - \mathbf{G}^T(\mathbf{GG}^T)^{-1}\mathbf{G}$$

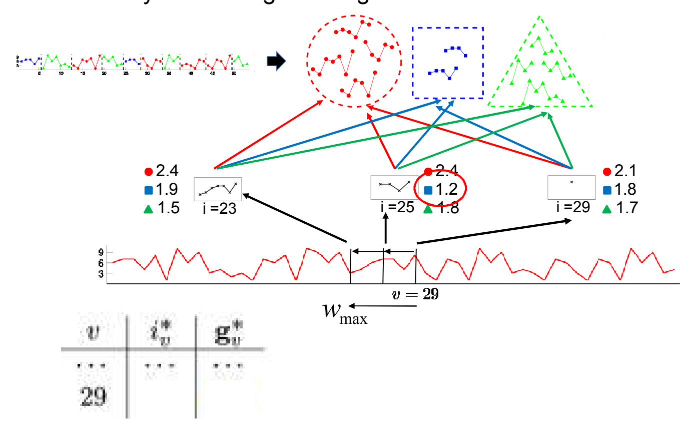
$$J_{aca} = tr(\mathbf{K}(\mathbf{L} \circ \mathbf{W})) \quad \text{with} \quad \mathbf{L} = \mathbf{I}_n - \mathbf{H}^T\mathbf{G}^T(\mathbf{GG}^T)^{-1}\mathbf{GH}$$



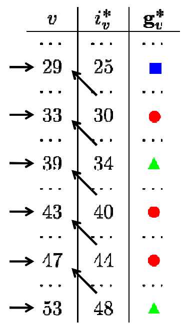



## Optimizing ACA (forward step)

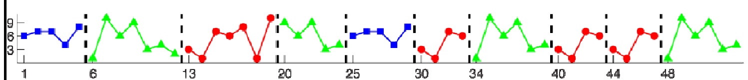
- Efficient Dynamic Programming



# Optimizing ACA (backward step)

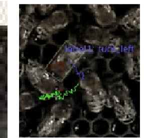


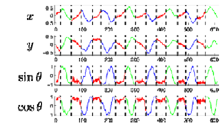

$O(n^2 w_{\max})$






# Honey bee dance data (Oh et al. 08)






Three behaviors:
1-waggle, 2-left turn, 3-right turn

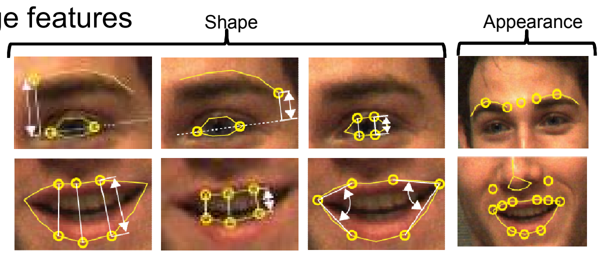
| | Seq 1 | Seq 2 | Seq 3 | Seq 4 | Seq 5 | Seq 6 |
|---|---|---|---|---|---|---|
| ACA | **0.845** | **0.925** | 0.600 | 0.922 | 0.878 | **0.928** |
| PS- SLDS (Oh et al 08) | 0.759 | 0.924 | **0.831** | **0.934** | 0.904 | 0.910 |
| HDP- VAR(1)-HMM (Fox et al 08) | 0.465 | 0.441 | 0.456 | 0.832 | **0.932** | 0.887 |
| Spectral Clustering | 0.698 | 0.631 | 0.509 | 0.671 | 0.577 | 0.649 |



# Facial image features

- Active Appearance Models (Baker and Matthews '04)

- Image features



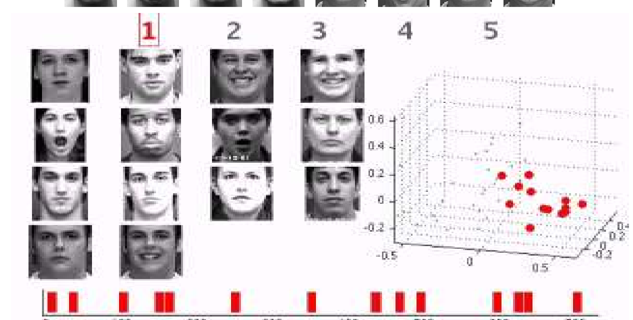

Upper face

Lower face



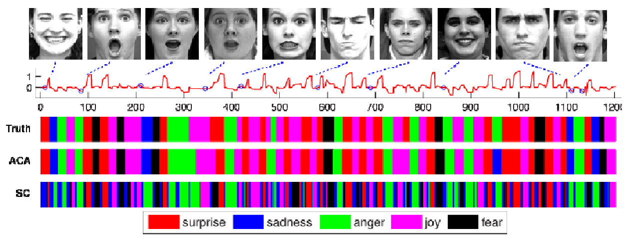
# Facial event discovery across subjects

- Cohn-Kanade: 30 people and five different expressions (surprise, joy, sadness, fear, anger)

# Facial event discovery across subjects

- Cohn-Kanade: 30 people and five different expressions (surprise, joy, sadness, fear, anger)

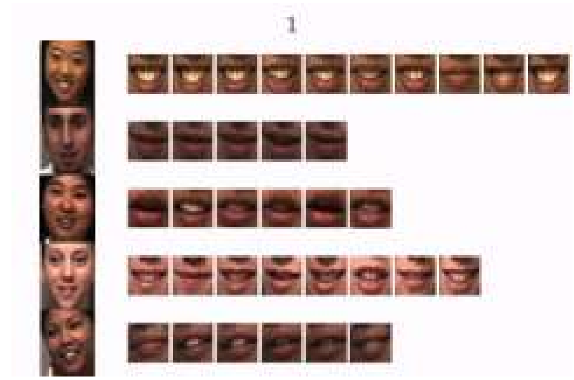



- 10 sets of 30 people

| ACA | Spectral Clustering (SC) |
|---|---|
| **0.87(.05)** | 0.56(.04) |

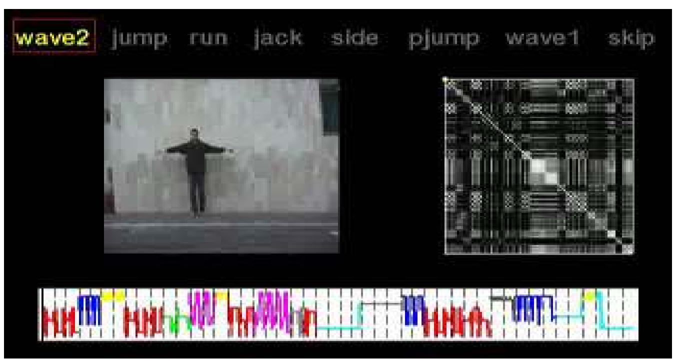


# Unsupervised facial event discovery



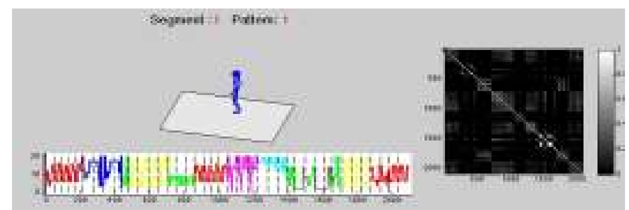


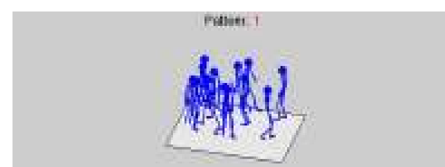
# Clustering human motion






# clustering of human motion II



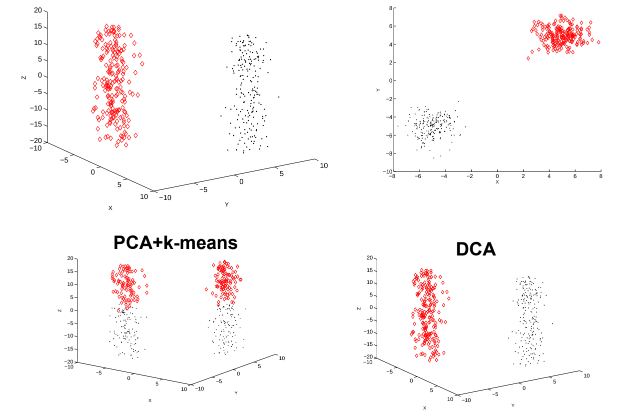


# Outline

- Introduction
- Principal Component Analysis (PCA)
  - Robust Principal CA
  - PCA missing data
  - Parameterized Principal CA
- Linear Discriminant Analysis
  - Multimodal Oriented Discriminant Analysis
- Canonical Correlation Analysis
  - Canonical Time Warping
- K-means
  - Aligned Cluster Analysis (ACA)
  - Discriminative Cluster Analysis (DCA)



# Problem






# Discriminative Cluster Analysis (DCA)

(de la Torre & Kanade, 2006)

- Generative clustering (e.g. k-means):

$$E(\mathbf{G},\mathbf{B}) = \|\mathbf{D}-\mathbf{B}\mathbf{G}^T\|_F = \sum_{i=1}^{c}\sum_{j\in C_i}\|\mathbf{d}_j-\mathbf{b}_i\|$$

$$g_{ij}\in\{0,1\} \qquad \mathbf{G}\mathbf{1}_k = \mathbf{1}_n$$

  - Not efficient for high dimensional data.
  - Multiple local minima.
- Discriminative clustering (de la Torre & Kanade, 2006):

$$E(\mathbf{V},\mathbf{B},\mathbf{G}) = \|(\mathbf{G}^T\mathbf{G})^{-\frac{1}{2}}(\mathbf{G}^T-\mathbf{V}\mathbf{B}^{\mathbf{T}}\mathbf{D})\|_F$$

  - Simultaneous dimensionality reduction and clustering.



# Optimization

- Eliminate **V**

$$E(\mathbf{B},\mathbf{G}) \propto tr((\mathbf{B}^T\overbrace{\mathbf{D}\mathbf{D}^T}^{\mathbf{S}_t}\mathbf{B})^{-1}(\mathbf{B}^T\overbrace{\mathbf{D}\mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{D}^T}^{\mathbf{S}_b}\mathbf{B}))$$

- Optimize for **B**

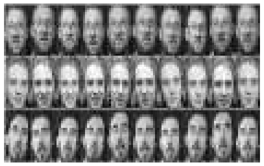
$$\mathbf{D}\mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{D}^T\mathbf{B} = \mathbf{D}\mathbf{D}^T\mathbf{B}\mathbf{\Lambda}$$

- Optimize for **G** $\quad \mathbf{A} = \mathbf{C}^T(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C} \qquad \mathbf{C} = \mathbf{B}^T\mathbf{D}$

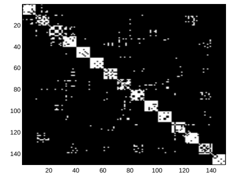
$$\mathbf{G} = \mathbf{V}\circ\mathbf{V} \qquad \mathbf{V}^{(n+1)} = \mathbf{V}^{(n)} - \eta\frac{\partial E}{\partial \mathbf{V}}$$

$$\frac{\partial E}{\partial \mathbf{V}} = (\mathbf{I}_C - \mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T)\mathbf{A}\mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}$$

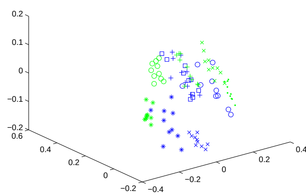
# Experiments



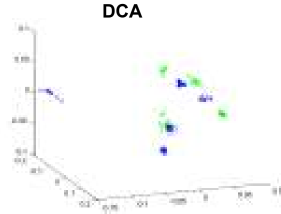
$$\mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T \approx$$



**PCA**

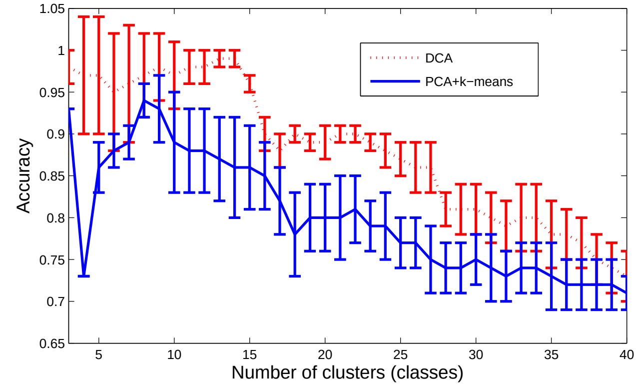


**DCA**





# DCA vs. PCA+k-means






# What's next?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