Eigen representations; Detecting faces in images

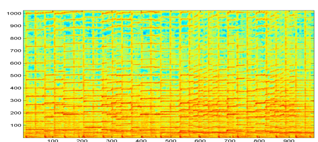
Class 5. 7 Sep 2010

Instructor: Bhiksha Raj

Administrivia

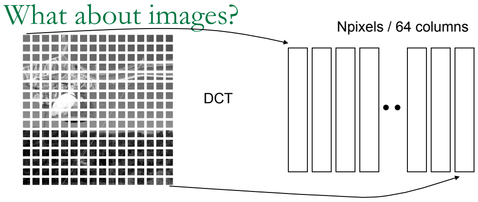
- Homeworks were up last week
  - Questions to be directed to Sourish/Sohail/Myself
  - Delays are worth negative points ☺
- Project ideas next class
  - Begin thinking about what project you will do
  - You are welcome to think up your own ideas/projects
  - Think workshop paper – novel problems, novel ideas can get published
- Projects will be done by teams
  - 2-4 people
  - Begin forming teams by yourselves
  - Students without teams will be assigned to teams
- Class of 28^{th}: Intel's open house

Last Class: Representing Audio



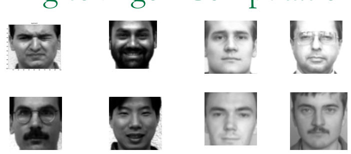
- Basic DFT
- Computing a Spectrogram
- Computing additional features from a spectrogram

What about images?



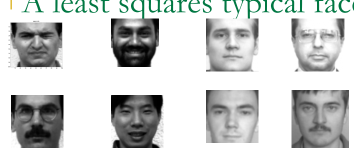
- DCT of small segments
  - 8x8
  - Each image becomes a matrix of DCT vectors
- DCT of the image
- Haar transform (checkerboard)
- ***Or data-driven representations..***

Returning to Eigen Computation

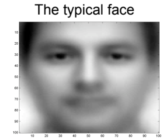


- A collection of faces
  - All normalized to 100x100 pixels
- What is common among all of them?
  - Do we have a common descriptor?

A least squares typical face

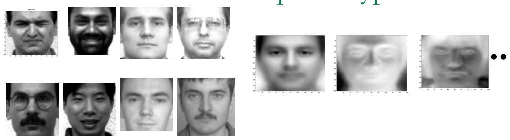


The typical face



- Can we do better than a blank screen to find the most common portion of faces?
  - The first checkerboard; the zeroth frequency component..
- Assumption: There is a "typical" face that captures most of what is common to all faces
  - Every face can be represented by a scaled version of a typical face
- Approximate **every** face f as $f = w_f V$
- Estimate V to minimize the squared error over all faces
  - How?
  - What is V?

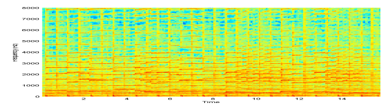
A collection of least squares typical faces



- Assumption: There are a set of K "typical" faces that captures most of all faces
- Approximate **every** face f as $f = w_{f,1} V_1 + w_{f,2} V_2 + w_{f,3} V_3 +.. + w_{f,k} V_k$
  - V_2 is used to "correct" errors resulting from using only V_1
    - So the total energy in $w_{f,2}$ must be lesser than the total energy in $w_{f,1}$
    - $\Sigma w_{f,2}^2 > \Sigma w_{f,1}^2$
  - V_3 corrects errors remaining after correction with V_2
    - The total energy in $w_{f,3}$ must be lesser than that even in $w_{f,2}$
  - And so on..
  - $V = [V_1 V_2 V_3]$
- Estimate V to minimize the squared error
  - How?
  - What is V?

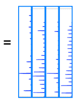
A recollection

M =



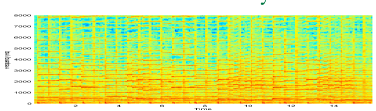
V=PINV(W)*M

W =

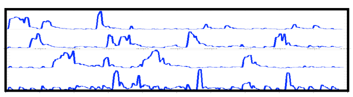


How about the other way?

M =



V =



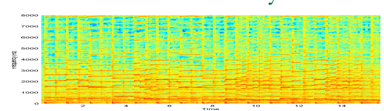
W = ?

U = ?

- W = M * Pinv(V)

How about the other way?

M =



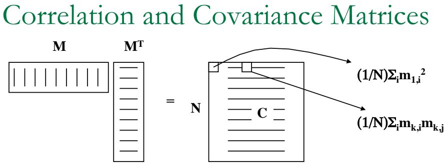
V = ?

W = ?

U = ?

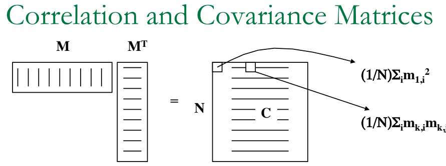
- $WV \approx M$

The Correlation and Covariance Matrices



- Consider a set of column vectors represented as a DxN matrix M
- The ***correlation*** matrix is
  - $C = (1/N)\ MM^T = (1/N)\Sigma_i\ M_i M_i^T$
  - Diagonal elements represent average of the squared value of each dimension
  - Off diagonal elements represent how two components are related
    - How much knowing one lets us guess the value of the other

The Correlation and Covariance Matrices



- If we "center" the data first, we get the **covariance** matrix
- Mean = $1/N\ \Sigma_i\ M_i$
  - M_i is the ith column of M
- The centered data are obtained by subtracting the mean from every column of M
  - $M_{centered}$ = M – Mean***1**
  - **1** is a 1xN row matrix
- The ***Covariance*** matrix is
  - $Cov = (1/N)\ M_{centered}\ M_{centered}^T$

Correlation / Covariance matrix

- Covariance and correlation matrices are symmetric
  - $C_{ij} = C_{ji}$
- Properties of symmetric matrices:
  - Eigenvalues and Eigenvectors are real
  - Can be expressed as
    - $C = VLV^T$
    - V is the matrix of Eigenvectors
    - L is a diagonal matrix of Eigenvalues
    - $V^T = V^{-1}$

Correlation / Covariance Matrix

$$C = VLV^T$$

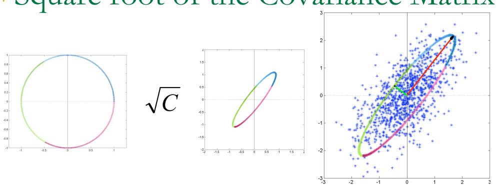
$$\boxed{Sqrt(C) = V.Sqrt(L).V^T}$$

$$\boxed{Sqrt(C).Sqrt(C)} = V.Sqrt(L).V^T V.Sqrt(L).V^T$$

$$= V.Sqrt(L).Sqrt(L)V^T = VLV^T = \boxed{C}$$

- The *square root* of a correlation or covariance matrix is easily derived from the eigen vectors and eigen values
  - The eigen values of the *square root* of the correlation matrix are the square roots of the eigen values of the correlation matrix
  - These are also the "singular values" of the data set

Square root of the Covariance Matrix

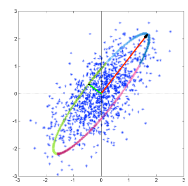


- The square root of the covariance matrix represents the elliptical scatter of the data
- The eigenvectors of the matrix represent the major and minor axes

PCA: The Covariance Matrix

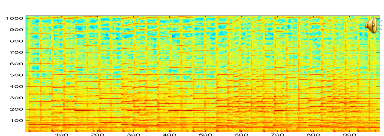
Any vector V = $a_{V,1}$ * eigenvec1 + $a_{V,2}$ *eigenvec2 + ..

$\Sigma_V a_{V,i}$ = eigenvalue(i)



- Projections along the N eigenvectors with the largest eigenvalues represent the N most ***informative*** components of the matrix
  - N directions along which variance is maximum
  - These represent the N ***principal components***

An audio example



- The spectrogram has 974 vectors of dimension 1025
- The covariance matrix is size 1025 x 1025
- There are 1025 eigenvectors

Eigen Reduction

$$M = spectrogram \quad \textbf{1025x1000}$$

$$C = M_{centered}.M_{centered}^T \quad \textbf{1025x1025}$$

$$\textbf{V = 1025x1025} \quad [V, L] = eig(C)$$

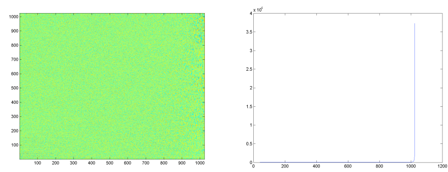
$$V_{reduced} = [V_1 \quad . \quad . \quad V_{25}] \quad \textbf{1025x25}$$

$$M_{lowdim} = Pinv(V_{reduced})M \quad \textbf{25x1000}$$

$$M_{reconstructed} = V_{reduced} M_{lowdim} \quad \textbf{1025x1000}$$

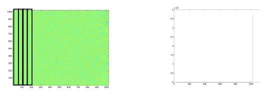
- Compute the ***Covariance***
- Compute Eigen vectors and values
- Create matrix from the 25 Eigen vectors corresponding to 25 highest Eigen values
- Compute the weights of the 25 eigenvectors
- To reconstruct the spectrogram – compute the projection on the 25 eigen vectors

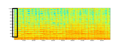
Eigenvalues and Eigenvectors



- Left panel: Matrix with 1025 eigen vectors
- Right panel: Corresponding eigen values
  - Most eigen values are close to zero
    - The corresponding eigenvectors are "unimportant"

Eigenvalues and Eigenvectors



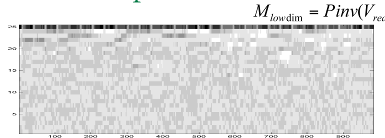


Vec = a1 *eigenvec1 + a2 * eigenvec2 + a3 * eigenvec3 …

- The vectors in the spectrogram are linear combinations of all 1025 eigen vectors
- The eigen vectors with low eigen values contribute very little
  - The average value of a_i is proportional to the square root of the eigenvalue
  - Ignoring these will not affect the composition of the spectrogram

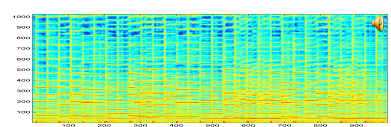
An audio example

$$V_{reduced} = [V_1 \quad . \quad . \quad V_{25}]$$
$$M_{lowdim} = Pinv(V_{reduced})M$$



- The same spectrogram projected down to the 25 principal eigenvectors with the highest eigenvalues
  - Only the 25-dimensional weights are shown
    - The weights with which the 25 eigen vectors must be added to compose a least squares approximation to the spectrogram

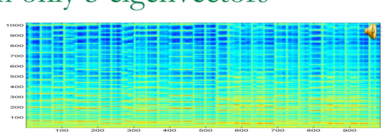
An audio example



$$M_{reconstructed} = V_{reduced} M_{lowdim}$$

- The same spectrogram constructed from only the 25 eigen vectors with the highest eigen values
  - Looks similar
    - With 100 eigenvectors, it would be indistinguishable from the original
  - Sounds pretty close
  - But now sufficient to store 25 numbers per vector (instead of 1024)

With only 5 eigenvectors

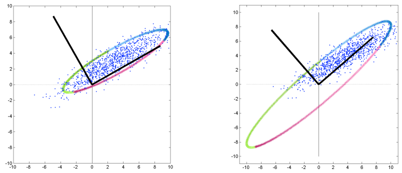


- The same spectrogram constructed from only the 5 eigen vectors with the highest eigen values
  - Highly recognizable

Covariance vs. Correlation

- If Eigenvectors are computed from the *correlation* matrix, they represent the most energy carrying bases
  - As opposed to the most informative bases obtained from the covariance
    - If the data are centered, the two are the same, but not otherwise
- Eigen decomposition of Correlations:
  - Direct computation using Singular Value Decomposition

Covariance vs. correlation



- Data are Gaussian, mean at [3,3]
- Left: Eigen vectors from covariance
  - Aligned to the direction of scatter
    - But not aligned to data
- Right: Eigen vectors from correlation (SVD)
  - Aligned to average direction of data
    - But not the scatter

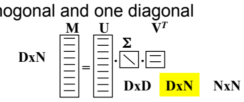
Singular Value Decomposition

- A matrix decomposition method

$$\mathbf{M} = \mathbf{U} \cdot \Sigma \cdot \mathbf{V}^T$$

$$\mathbf{U} \cdot \mathbf{U}^T = \mathbf{I},\ \mathbf{V} \cdot \mathbf{V}^T = \mathbf{I},\ \Sigma \text{ is diagonal}$$

- Breaks up the input into a product of three matrices, two orthogonal and one diagonal

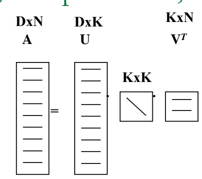


- The right matrix are Eigenvectors in row space
- The diagonal will represent how much spread is in each direction and contains the *singular values*
  - Also the square root of the eigen value matrix of the correlations
- The left matrix are the Eigen vectors of column space
  - Also Eigenvectors of correlation

SVD vs. Eigen decomposition

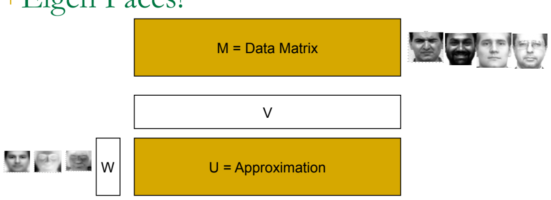
- Singluar value decomposition is analogous to the eigen decomposition of the correlation matrix of the data
- The "left" singular vectors are the eigenvectors of the correlation matrix
  - Show the directions of greatest importance
- The corresponding singular values are the square roots of the eigenvalues of the correlation matrix
  - Show the importance of the eigenvector

Thin SVD, compact SVD, reduced SVD



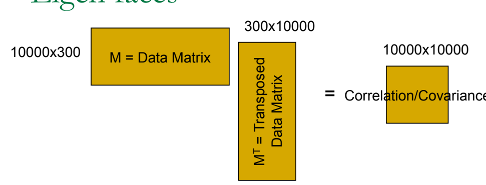
- Thin SVD: Only compute the first N columns of U
  - All that is required if N < M
- Compact SVD: Only the left and right eigen vectors corresponding to non-zero singular values are computed
- Reduced SVD: Only compute the columns of U corresponding to the K highest singular values

Eigen Faces!



- Here W, V and U are ALL unknown and must be determined
  - Such that the squared error between U and M is minimum
- Eigen analysis allows you to find W and V such that U = WV has the least squared error with respect to the original data M
- If the original data are a collection of faces, the columns of W are *eigen faces*
  - *Should the data be centered?*

Eigen faces

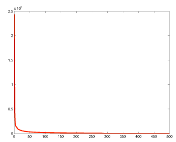


- Lay all faces side by side in vector form to form a matrix
  - In my example: 300 faces. So the matrix is 10000 x 300
- Multiply the matrix by its transpose
  - The correlation matrix is 10000x10000

Eigen faces

[U,S] = eig(correlation)

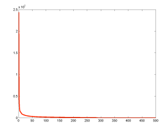
$$S = \begin{bmatrix} \lambda_1 & . & 0 & . & 0 \\ 0 & \lambda_2 & 0 & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & . & 0 & . & \lambda_{10000} \end{bmatrix} \qquad U = \begin{bmatrix} \text{eigenface1} & \text{eigenface2} & \cdots \end{bmatrix}$$



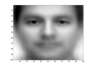
- Compute the eigen vectors
  - Only 300 of the 10000 eigen values are non-zero
    - Why?
- Retain eigen vectors with high eigen values (>0)
  - Could use a higher threshold

Eigen Faces

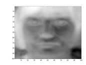
$$U = \begin{bmatrix} \text{eigenface1} & \text{eigenface2} & \cdots \end{bmatrix}$$



eigenface1



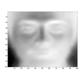
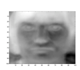
eigenface2



eigenface3

- The eigen vector with the highest eigen value is the first typical face
- The vector with the second highest eigen value is the second typical face.
- Etc.

Representing a face

= w_1  + w_2  + w_3  •••

Representation  = $[w_1\ w_2\ w_3\ \ldots.]^T$

- The weights with which the eigen faces must be combined to compose the face are used to represent the face!

SVD instead of Eigen

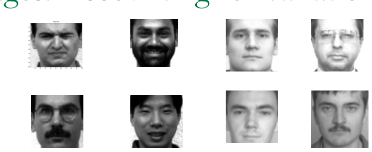
10000x300 M = Data Matrix	=	U=10000x300	S=300x300	V=300x300

$$U = \begin{bmatrix} \text{eigenface1} & \text{eigenface2} & \cdots \end{bmatrix}$$

- Do we need to compute a 10000 x 10000 correlation matrix and then perform Eigen analysis?
  - Will take a very long time on your laptop
- SVD
  - Only need to perform "Thin" SVD. Very fast
    - U = 10000 x 300
      - The columns of U are the eigen faces!
      - The Us corresponding to the "zero" eigen values are not computed
    - S = 300 x 300
    - V = 300 x 300

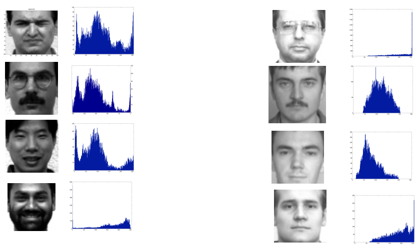
NORMALIZING OUT VARIATIONS

Images: Accounting for variations



- What are the obvious differences in the above images
- How can we capture these differences
  - Hint – image histograms..

Images -- Variations



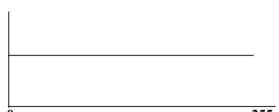
- Pixel histograms: what are the differences

Normalizing Image Characteristics

- Normalize the pictures
  - Eliminate lighting/contrast variations
  - All pictures must have “similar” lighting
    - How?
- Lighting and contrast are represented in the pixel value histograms:

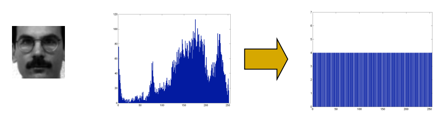
Histogram Equalization

- Normalize histograms of images
  - Maximize the contrast
    - Contrast is defined as the “flatness” of the histogram
    - For maximal contrast, every greyscale must happen as frequently as every other greyscale



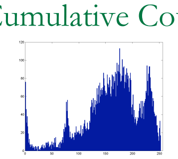
- Maximizing the contrast: Flattening the histogram
  - Doing it for every image ensures that every image has the same constrast
    - I.e. exactly the same histogram of pixel values
      - Which should be flat

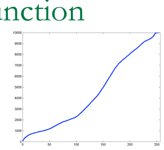
Histogram Equalization



- Modify pixel values such that histogram becomes “flat”.
- For each pixel
  - New pixel value = f(old pixel value)
  - What is f()?
- Easy way to compute this function: map cumulative counts

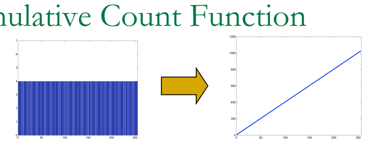
Cumulative Count Function



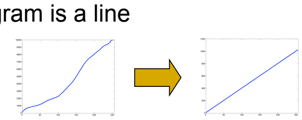


- The *histogram (count)* of a pixel value X is the number of pixels in the image that have value X
  - E.g. in the above image, the count of pixel value 180 is about 110
- The *cumulative count* at pixel value X is the total number of pixels that have values in the range 0 <= x <= X
  - CCF(X) = H(1) + H(2) + .. H(X)

Cumulative Count Function

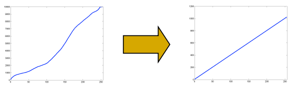


- The cumulative count function of a uniform histogram is a line



- We must modify the pixel values of the image so that its cumulative count is a line

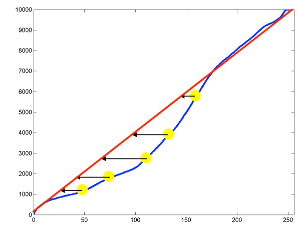
Mapping CCFs



Move x axis levels around until the plot to the left looks like the plot to the right

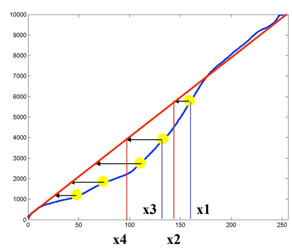
- CCF(f(x)) -> a*f(x)   [of a*(f(x)+1) if pixels can take value 0]
  - x = pixel value
  - f() is the function that converts the old pixel value to a new (normalized) pixel value
  - a = (total no. of pixels in image) / (total no. of pixel levels)
    - The no. of pixel levels is 256 in our examples
    - Total no. of pixels is 10000 in a 100x100 image

Mapping CCFs



- For each pixel value x:
  - Find the location on the red line that has the closet Y value to the observed CCF at x

Mapping CCFs



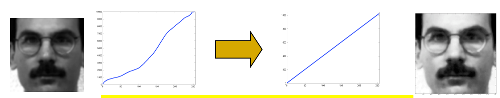
f(x1) = x2

f(x3) = x4

Etc.

- For each pixel value x:
  - Find the location on the red line that has the closet Y value to the observed CCF at x

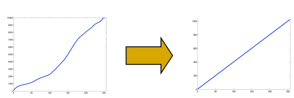
Mapping CCFs



Move x axis levels around until the plot to the left looks like the plot to the right

- For each pixel in the image to the left
  - The pixel has a value x
  - Find the CCF at that pixel value CCF(x)
  - Find x' such that CCF(x') in the plot to the right equals CCF(x)
    - x' such that CCF_flat(x') = CCF(x)
  - Modify the pixel value to x'

Doing it Formulaically



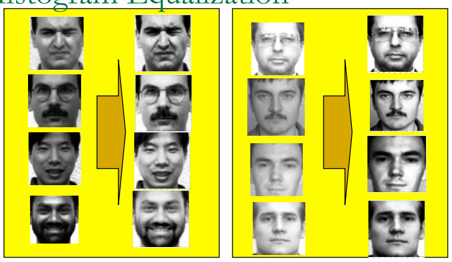
$$f(x) = round\left(\frac{CCF(x) - CCF_{\min}}{Npixels - CCF_{\min}} Max.pixel.value\right)$$

- CCF_{min} is the smallest non-zero value of CCF(x)
  - The value of the CCF at the smallest observed pixel value
- Npixels is the total no. of pixels in the image
  - 10000 for a 100x100 image
- Max.pixel.value is the highest pixel value
  - 255 for 8-bit pixel representations

Or even simpler

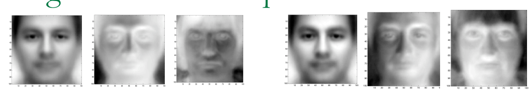
- Matlab:
  - `Newimage = histeq(oldimage)`

Histogram Equalization



- Left column: Original image
- Right column: Equalized image
- All images now have similar contrast levels

Eigenfaces after Equalization



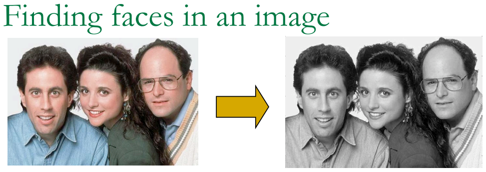
- Left panel : Without HEQ
- Right panel: With HEQ
  - Eigen faces are more face like..
    - Need not always be the case

Detecting Faces in Images

Detecting Faces in Images

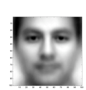
- Finding face like patterns
  - How do we find if a picture has faces in it
  - Where are the faces?
- A simple solution:
  - Define a “typical face”
  - Find the “typical face” in the image

Finding faces in an image



- Picture is larger than the “typical face”
  - E.g. typical face is 100x100, picture is 600x800
- First convert to greyscale
  - R + G + B
  - Not very useful to work in color

Finding faces in an image



- Goal .. To find out if and where images that look like the “typical” face occur in the picture

Finding faces in an image

- Try to “match” the typical face to each location in the picture

Finding faces in an image

- Try to “match” the typical face to each location in the picture

Finding faces in an image

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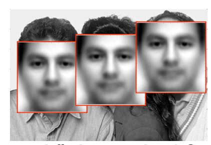
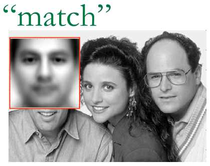
Finding faces in an image



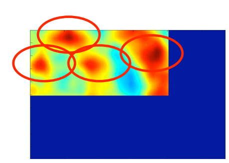
- Try to "match" the typical face to each location in the picture
- The "typical face" will explain some spots on the image much better than others
  - These are the spots at which we probably have a face!

How to "match"



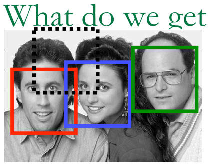
- What exactly is the "match"
  - What is the match "score"
- The DOT Product
  - Express the typical face as a vector
  - Express the region of the image being evaluated as a vector
    - But first histogram equalize the region
      - Just the section being evaluated, without considering the rest of the image
  - Compute the dot product of the typical face vector and the "region" vector

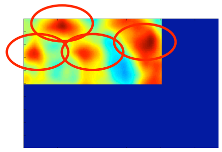
What do we get



- The right panel shows the dot product at various locations
  - Redder is higher
    - The locations of peaks indicate locations of faces!
- **This is a Matched Filter**

What do we get





- The right panel shows the dot product a various loctions
  - Redder is higher
    - The locations of peaks indicate locations of faces!
- Correctly detects all three faces
  - Likes George's face most
    - He looks most like the typical face
- Also finds a face where there is none!
  - A false alarm

Scaling and Rotation Problems

- Scaling
  - Not all faces are the same size
  - Some people have bigger faces
  - The size of the face on the image changes with perspective
  - Our "typical face" only represents one of these sizes
- Rotation
  - The head need not always be upright!
    - Our typical face image was upright

Solution

- Create many "typical faces"
  - One for each scaling factor
  - One for each rotation
    - How will we do this?
- Match them all
- Does this work
  - Kind of .. Not well enough at all
  - We need more sophisticated models

Face Detection: A Quick Historical Perspective

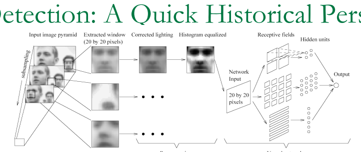


Figure 1: The basic algorithm used for face detection.

- Many more complex methods
  - Use edge detectors and search for face like patterns
  - Find "feature" detectors (noses, ears..) and employ them in complex neural networks..
- The Viola Jones method
  - Boosted cascaded classifiers
- But first, what is boosting

And even before that – what is classification?

- Given "features" describing an entity, determine the category it belongs to
  - Walks on two legs, has no hair. Is this
    - A Chimpanizee
    - A Human
  - Has long hair, is 5'4" tall, is this
    - A man
    - A woman
  - Matches "eye" pattern with score 0.5, "mouth pattern" with score 0.25, "nose" pattern with score 0.1. Are we looking at
    - A face
    - Not a face?

Classification

- Multi-class classification
  - Many possible categories
    - E.g. Sounds "AH, IY, UW, EY.."
    - E.g. Images "Tree, dog, house, person.."
- Binary classification
  - Only two categories
    - Man vs. Woman
    - Face vs. not a face..
- Face detection: Recast as binary face classification
  - For each little square of the image, determine if the square represents a face or not

Face Detection as Classification

For each square, run a classifier to find out if it is a face or not

- Faces can be many sizes
- They can happen anywhere in the image
- For each face size
  - For each location
    - Classify a rectangular region of the face size, at that location, as a face or not a face
- This is a series of ***binary*** classification problems

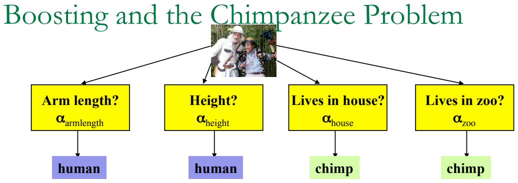
Introduction to Boosting

- An *ensemble* method that sequentially combines many simple ***BINARY*** classifiers to construct a final complex classifier
  - Simple classifiers are often called "weak" learners
  - The complex classifiers are called "strong" learners
- Each weak learner focuses on instances where the previous classifier failed
  - Give greater weight to instances that have been incorrectly classified by previous learners
- Restrictions for weak learners
  - Better than 50% correct
- Final classifier is *weighted* sum of weak classifiers

Boosting: A very simple idea

- One can come up with many rules to classify
  - E.g. Chimpanzee vs. Human classifier:
  - If arms == long, entity is chimpanzee
  - If height > 5'6" entity is human
  - If lives in house == entity is human
  - If lives in zoo == entity is chimpanzee
- Each of them is a reasonable rule, but makes many mistakes
  - Each rule has an intrinsic error rate
- *Combine* the predictions of these rules
  - But not equally
  - Rules that are less accurate should be given lesser weight

Boosting and the Chimpanzee Problem



- The total confidence in all classifiers that classify the entity as a chimpanzee is

$$Score_{chimp} = \sum_{classifier\ favors\ chimpanzee} \alpha_{classifier}$$

- The total confidence in all classifiers that classify it as a human is

$$Score_{human} = \sum_{classifier\ favors\ human} \alpha_{classifier}$$

- If $Score_{chimpanzee} > Score_{human}$ then the our belief that we have a chimpanzee is greater than the belief that we have a human

Boosting as defined by Freund

- A gambler wants to write a program to predict winning horses. His program must encode the expertise of his brilliant winner friend
- The friend has no single, encodable algorithm. Instead he has many rules of thumb
  - He uses a different rule of thumb for each set of races
    - E.g. "in this set, go with races that have black horses with stars on their foreheads"
  - But cannot really enumerate what rules of thumbs go with what sets of races: he simply "knows" when he encounters a set
    - A common problem that faces us in many situations
- Problem:
  - How best to combine all of the friend's rules of thumb
  - What is the best set of races to present to the friend, to extract the various rules of thumb

Boosting

- The basic idea: Can a "weak" learning algorithm that performs just slightly better than random guessing be *boosted* into an arbitrarily accurate "strong" learner
  - Each of the gambler's rules may be just better than random guessing
- This is a "meta" algorithm, that poses no constraints on the form of the weak learners themselves
  - The gambler's rules of thumb can be anything

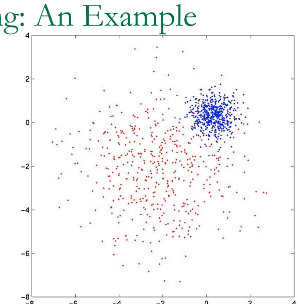
Boosting: A Voting Perspective

- Boosting can be considered a form of voting
  - Let a number of different classifiers classify the data
  - Go with the majority
  - Intuition says that as the number of classifiers increases, the dependability of the majority vote increases
- The corresponding algorithms were called Boosting by majority
  - A (weighted) majority vote taken over all the classifiers
  - How do we compute weights for the classifiers?
  - How do we actually train the classifiers

ADA Boost: Adaptive algorithm for learning the weights

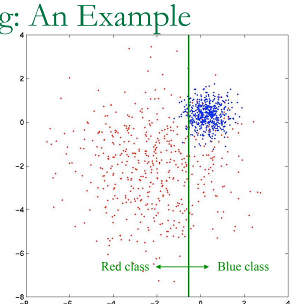
- ADA Boost: Not named of ADA Lovelace
- An *adaptive* algorithm that learns the weights of each classifier sequentially
  - Learning adapts to the current accuracy
- Iteratively:
  - Train a simple classifier from training data
    - It will make errors even on training data
    - Train a new classifier that focuses on the training data points that have been misclassified

Boosting: An Example



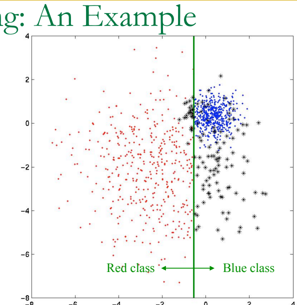
- Red dots represent training data from Red class
- Blue dots represent training data from Blue class

Boosting: An Example



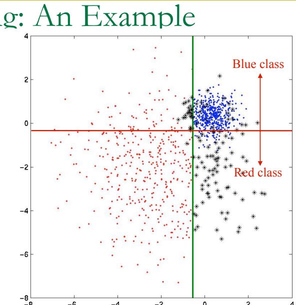
- Very simple weak learner
  - A line that is parallel to one of the two axes

Boosting: An Example



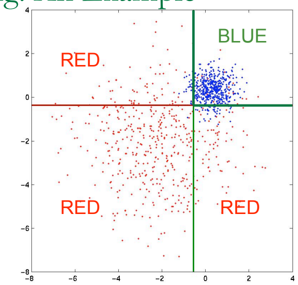
- First weak learner makes many mistakes
  - Errors coloured black

Boosting: An Example



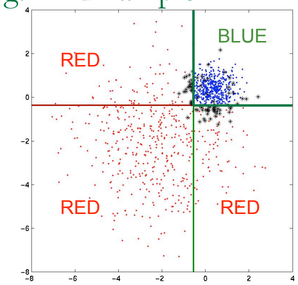
- Second weak learner focuses on errors made by first learner

Boosting: An Example



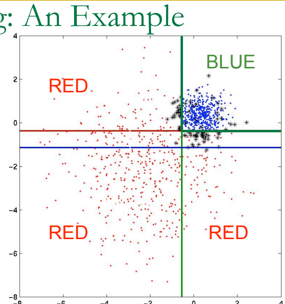
- Second strong learner: weighted combination of first and second weak learners
  - Decision boundary shown by black lines

Boosting: An Example



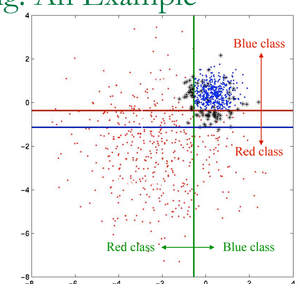
- The second strong learner also makes mistakes
  - Errors colored black

Boosting: An Example



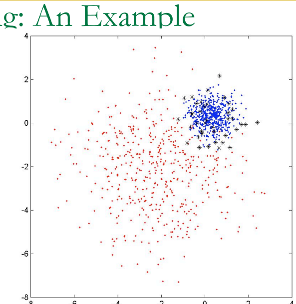
- Third weak learner concentrates on errors made by second strong learner

Boosting: An Example



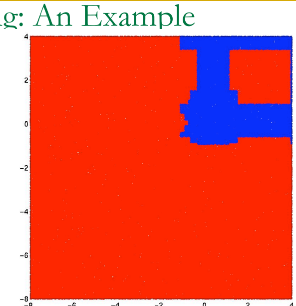
- Third weak learner concentrates on errors made by combination of previous weak learners
- Continue adding weak learners until….

Boosting: An Example



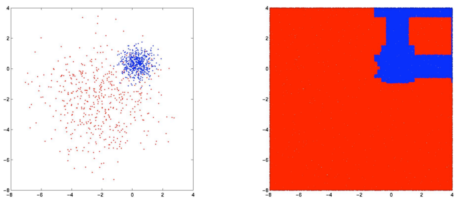
- Voila! Final strong learner: very few errors on the training data

Boosting: An Example



- The final strong learner has learnt a complicated decision boundary

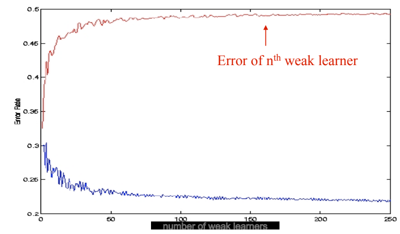
Boosting: An Example



- The final strong learner has learnt a complicated decision boundary
- Decision boundaries in areas with low density of training points assumed inconsequential

Overall Learning Pattern

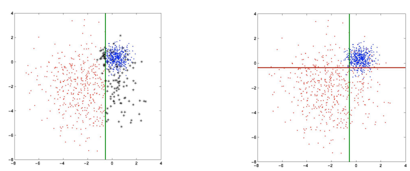
- Strong learner increasingly accurate with increasing number of weak learners
- Residual errors increasingly difficult to correct
  - Additional weak learners less and less effective



ADABoost

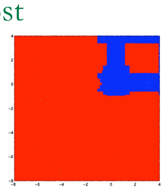
- Cannot just add new classifiers that work well only the the previously misclassified data
- Problem: The new classifier will make errors on the points that the **earlier** classifiers got right
  - Not good
  - On test data we have no way of knowing which points were correctly classified by the first classifier
- Solution: Weight the data when training the second classifier
  - Use all the data but assign them weights
    - Data that are already correctly classified have less weight
    - Data that are currently incorrectly classified have more weight

ADA Boost



- The red and blue points (correctly classified) will have a weight $\alpha < 1$
- Black points (incorrectly classified) will have a weight β $(= 1/\alpha) > 1$
- To compute the optimal second classifier, we minimize the total weighted error
  - Each data point contributes α or β to the total count of correctly and incorrectly classified points
    - E.g. if one of the red points is misclassified by the new classifier, the total error of the new classifier goes up by α

ADA Boost



- Each new classifier modifies the weights of the data points based on the accuracy of the *current* classifier
- ***The final classifier too is a weighted combination of all component classifiers***

Will continue next week

- Next class: Project ideas.
- Today's lecture and the next lecture are the basis for HW 2.