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## **Decision Trees**

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July 30, 2001

#### Here is a dataset

age	employme	education	edur	marital		job	relation	race	gender	hour	country		wealth
39	State_gov	Bachelors	13	Never_mai		Adm_cleric	Not_in_far	White	Male	40	United_	Sta	poor
51	Self_emp_	Bachelors	13	Married		Exec_man	Husband	White	Male	13	United_	Sta	poor
39	Private	HS_grad	9	Divorced		Handlers_d	Not_in_far	White	Male	40	United_	Sta	poor
54	Private	11th	7	Married		Handlers_d	Husband	Black	Male	40	United_	Sta	poor
28	Private	Bachelors	13	Married		Prof_speci	Wife	Black	Female	40	Cuba		poor
38	Private	Masters	14	Married		Exec_man	Wife	White	Female	40	United_	Sta	poor
50	Private	9th	5	Married_sp		Other_serv	Not_in_far	Black	Female	16	Jamaica	1	poor
52	Self_emp_	HS_grad	9	Married		Exec_man	Husband	White	Male	45	United_	Sta	rich
31	Private	Masters	14	Never_mai		Prof_speci	Not_in_far	White	Female	50	United_	Sta	rich
42	Private	Bachelors	13	Married		Exec man	Husband	White	Male	40	United	Sta	rich
37	Private	Some_coll	10	Married		Exec_man	Husband	Black	Male	80	United_	Sta	rich
30	State gov	Bachelors	13	Married		Prof speci	Husband	Asian	Male	40	India		rich
24	Private	Bachelors	13	Never_mai		Adm_cleric	Own_child	White	Female	30	United_	Sta	poor
33	Private	Assoc ac	12	Never mai		Sales	Not in far	Black	Male	50	United	Sta	poor
41	Private	Assoc_voc	11	Married		Craft_repai	Husband	Asian	Male	40	*Missing	ĮV:	rich
34	Private	7th 8th	4	Married		Transport	Husband	Amer India	Male	45	Mexico		poor
26	Self_emp_	HS_grad	9	Never_mai		Farming_fi	Own_child	White	Male	35	United_	Sta	poor
33	Private	HS grad	9	Never_mai		Machine of	Unmarried	White	Male	40	United	Sta	poor
38	Private	11th		Married		Sales	Husband	White	Male		United_		
44	Self emp	Masters	14	Divorced		Exec man	Unmarried	White	Female	45	United	Sta	rich
	Private	Doctorate		Married		Prof speci	Husband	White	Male	60	United_	Sta	rich
	:	:		:	:		:	:	:				

48,000 records, 16 attributes [Kohavi 1995]

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#### Classification

- A Major Data Mining Operation
- Give one attribute (e.g wealth), try to predict the value of new people's wealths by means of some of the other available attributes.
- Applies to categorical outputs
  - Categorical attribute: an attribute which takes on two or more discrete values. Also known as a symbolic attribute.
  - · Real attribute: a column of real numbers

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Decision Trees: Slide 3

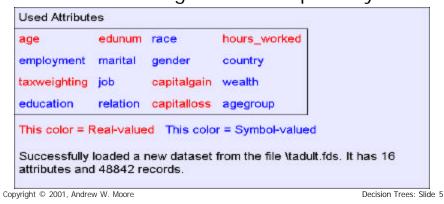
## Today's lecture

- Information Gain for measuring association between inputs and outputs
- Learning a decision tree classifier from data

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#### About this dataset

- It is a tiny subset of the 1990 US Census.
- It is publicly available online from the UCI Machine Learning Datasets repository



#### What can you do with a dataset? Well, you can look at histograms... Value Frequency Gender Female 16192 Male 32650 Value Frequency Divorced 6633 Married\_AF\_spouse 37 Marital Married 22379 Status Married\_spouse\_absent\_628 Never\_married 16117 Widowed 1518 Copyright © 2001, Andrew W. Moore Decision Trees: Slide 6

### **Contingency Tables**

A better name for a histogram:

A One-dimensional Contingency Table

- Recipe for making a k-dimensional contingency table:
  - 1. Pick *k* attributes from your dataset. Call them  $a_1, a_2, \dots a_k$ .
  - 2. For every possible combination of values,  $a_1, = x_1, a_2, = x_2, \dots a_k, = x_k$  record how frequently that combination occurs

Fun fact: A database person would call this a "k-dimensional datacube"

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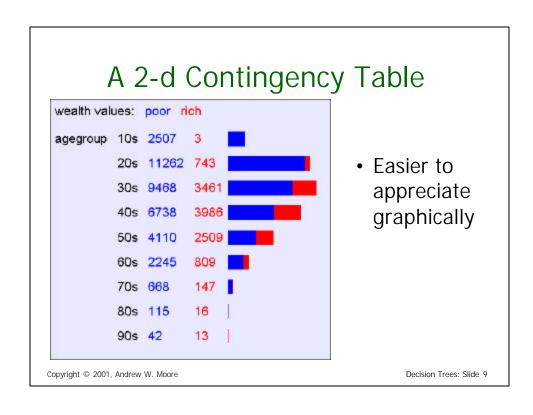
Decision Trees: Slide 7

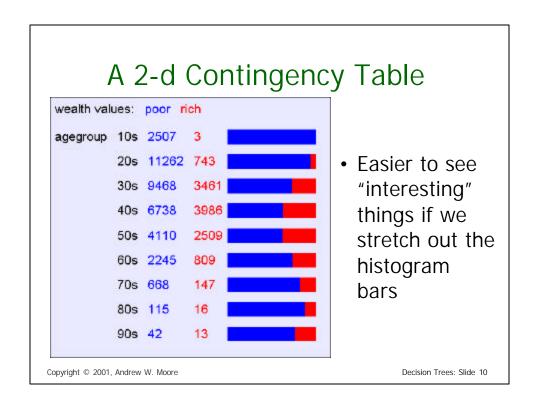
## A 2-d Contingency Table

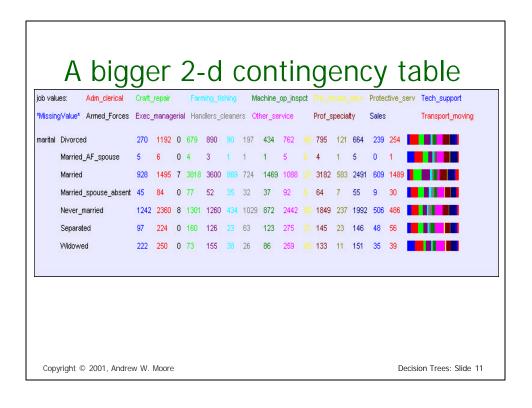
```
wealth values: poor rich
agegroup 10s 2507
         20s 11262 743
         30s 9468
                    3461
         40s 6738
                    3986
         50s 4110
                    2509
         60s 2245
                    809
         70s 668
                    147
         80s 115
                    16
         90s 42
                    13
```

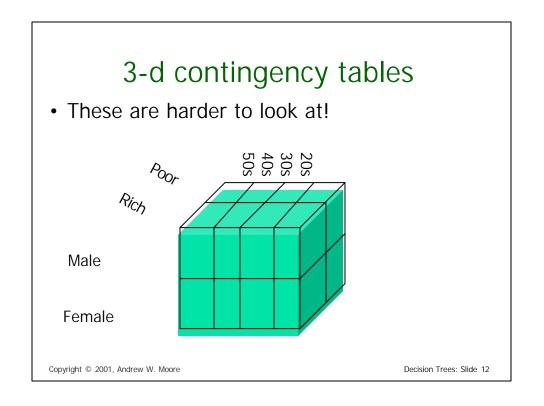
 For each pair of values for attributes (agegroup, wealth) we can see how many records match.

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# On-Line Analytical Processing (OLAP)

- Software packages and database add-ons to do this are known as OLAP tools
- They usually include point and click navigation to view slices and aggregates of contingency tables
- They usually include nice histogram visualization

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### Time to stop and think

 Why would people want to look at contingency tables?

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#### Let's continue to think

- With 16 attributes, how many 1-d contingency tables are there?
- How many 2-d contingency tables?
- How many 3-d tables?
- With 100 attributes how many 3-d tables are there?

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Decision Trees: Slide 15

#### Let's continue to think

- With 16 attributes, how many 1-d contingency tables are there? 16
- How many 2-d contingency tables? 16choose-2 = 16 \* 15 / 2 = 120
- How many 3-d tables? 560
- With 100 attributes how many 3-d tables are there? 161,700

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## Manually looking at contingency tables

- Looking at one contingency table: can be as much fun as reading an interesting book
- Looking at ten tables: as much fun as watching CNN
- Looking at 100 tables: as much fun as watching an infomercial
- Looking at 100,000 tables: as much fun as a three-week November vacation in Duluth with a dying weasel.

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Decision Trees: Slide 17

### Data Mining

 Data Mining is all about automating the process of searching for patterns in the data.

Which patterns are interesting? Which might be mere illusions? And how can they be exploited?

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#### **Data Mining**

 Data Mining is all about automating the process of searching for patterns in the data.

#### Which patterns are interesting?

That's what we'll look at right now.

And the answer will turn out to be the engine that drives decision tree learning.

Which might be mere illusions? And how can they be exploited?

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## Deciding whether a pattern is interesting

- We will use information theory
- A very large topic, originally used for compressing signals
- But more recently used for data mining...

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## Deciding whether a pattern is interesting

- We will use information theory
- A very large topic, originally used for compressing signals
- But more recently used for data mining...

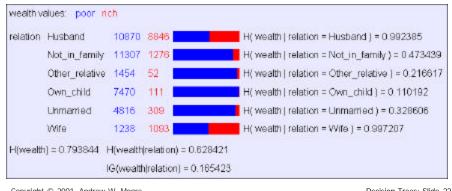
(The topic of Information Gain will now be discussed, but you will find it in a separate Andrew Handout)

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Decision Trees: Slide 21

### Searching for High Info Gains

• Given something (e.g. wealth) you are trying to predict, it is easy to ask the computer to find which attribute has highest information gain for it.



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### **Learning Decision Trees**

- A Decision Tree is a tree-structured plan of a set of attributes to test in order to predict the output.
- To decide which attribute should be tested first, simply find the one with the highest information gain.
- Then recurse...

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Decision Trees: Slide 23

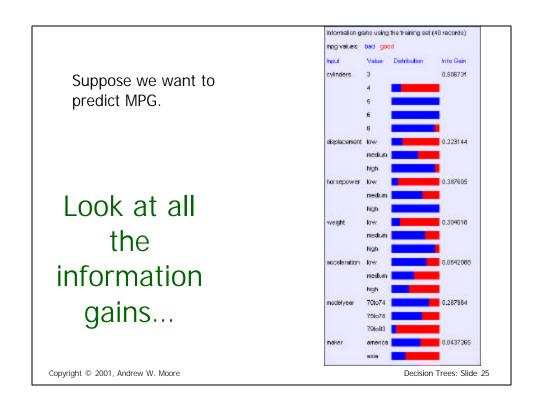
#### A small dataset: Miles Per Gallon

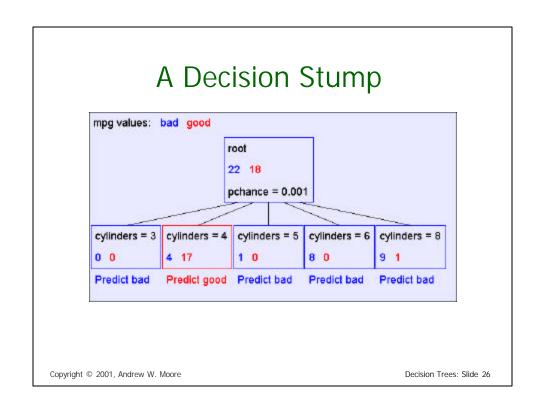
40 Records

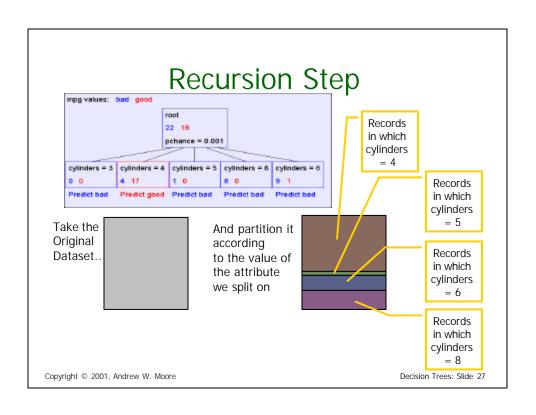
mpg	cylinders	displacement	horsepower	weight	acceleration	modelyear	maker
good	4	low	low	low	high	75to78	asia
bad	6	medium	medium	medium	medium	70to74	america
bad	4	medium	medium	medium	low	75to78	europe
bad	8	high	high	high	low	70to74	america
bad	6	medium	medium	medium	medium	70to74	america
bad	4	low	medium	low	medium	70to74	asia
bad	4	low	medium	low	low	70to74	asia
bad	8	high	high	high	low	75to78	america
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
bad	8	high	high	high	low	70to74	america
good	8	high	medium	high	high	79to83	america
bad	8	high	high	high	low	75to78	america
good	4	low	low	low	low	79to83	america
bad	6	medium	medium	medium	high	75to78	america
good	4	medium	low	low	low	79to83	america
good	4	low	low	medium	high	79to83	america
bad	8	high	high	high	low	70to74	america
good	4	low	medium	low	medium	75to78	europe
had	5	medium	medium	medium	medium	75to78	europe

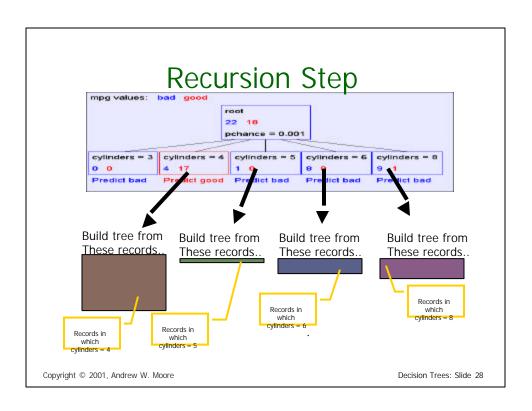
From the UCI repository (thanks to Ross Quinlan)

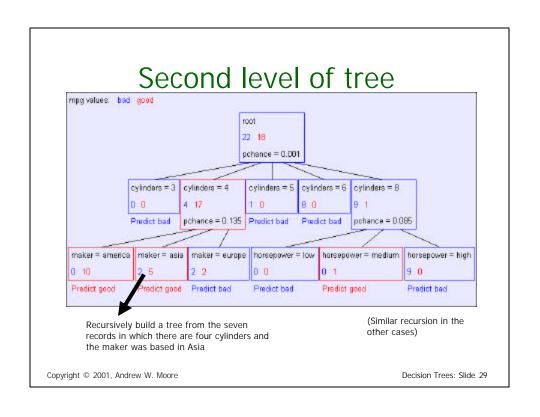
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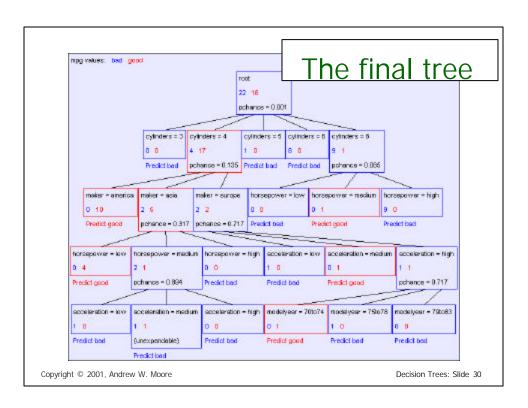


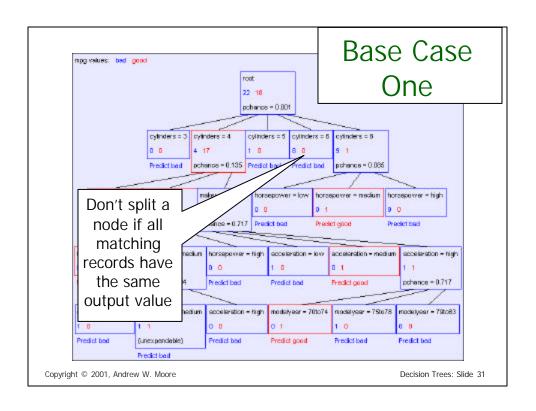


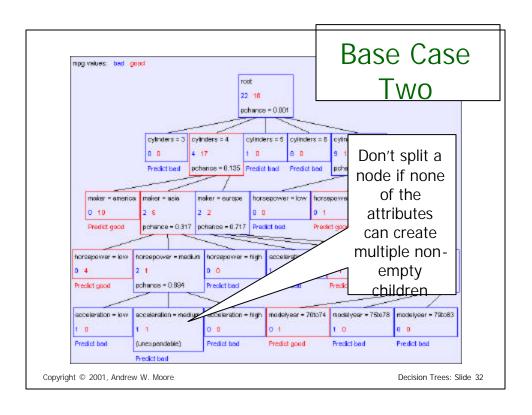


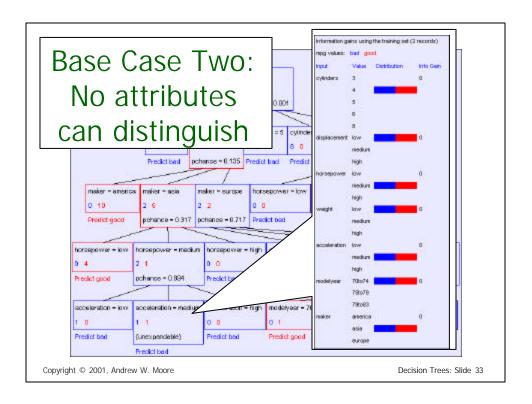












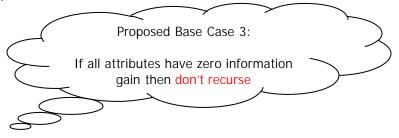
#### **Base Cases**

- Base Case One: If all records in current data subset have the same output then don't recurse
- Base Case Two: If all records have exactly the same set of input attributes then don't recurse

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#### Base Cases: An idea

- Base Case One: If all records in current data subset have the same output then don't recurse
- Base Case Two: If all records have exactly the same set of input attributes then don't recurse



· Is this a good idea?

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Decision Trees: Slide 35

### The problem with Base Case 3

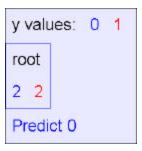
а	b	У
О	О	О
О	1	1
1	0	1
1	1	0

$$y = a XOR b$$

The information gains:



The resulting decision tree:

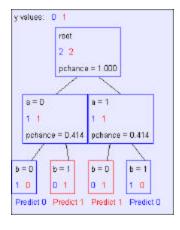


#### If we omit Base Case 3:

а	b	У		
О	О	0		
О	1	1		
1	0	1		
1	1	О		

y = a XOR b

The resulting decision tree:



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Decision Trees: Slide 37

## Basic Decision Tree Building Summarized

BuildTree(DataSet,Output)

- If all output values are the same in *DataSet*, return a leaf node that says "predict this unique output"
- If all input values are the same, return a leaf node that says "predict the majority output"
- Else find attribute X with highest Info Gain
- Suppose X has  $n_X$  distinct values (i.e. X has arity  $n_X$ ).
  - Create and return a non-leaf node with  $n_{\chi}$  children.
  - The ith child should be built by calling BuildTree(DS<sub>i</sub>,Output)

Where  $DS_i$  built consists of all those records in DataSet for which  $X = \hbar th$  distinct value of X.

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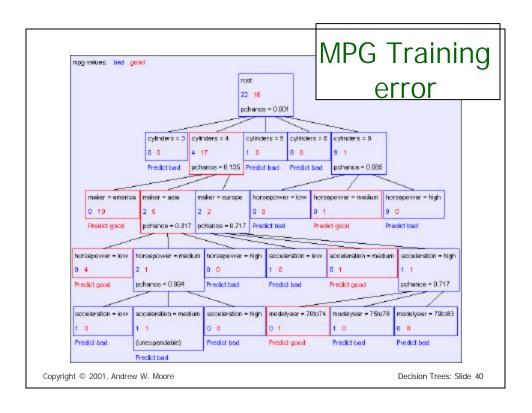
## Training Set Error

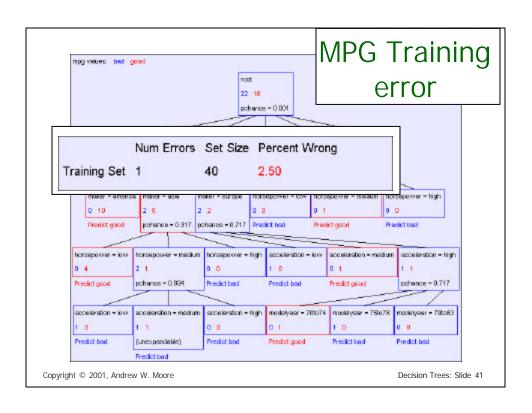
 For each record, follow the decision tree to see what it would predict

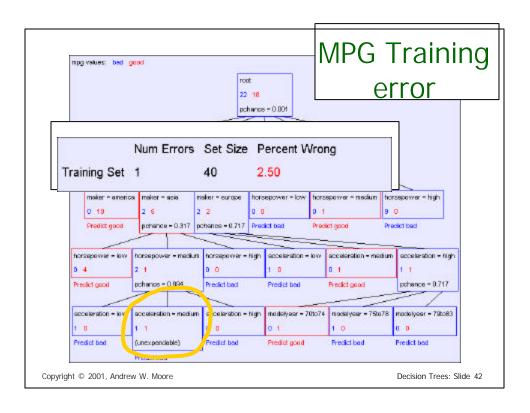
For what number of records does the decision tree's prediction disagree with the true value in the database?

• This quantity is called the *training set error*. The smaller the better.

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# Stop and reflect: Why are we doing this learning anyway?

 It is not usually in order to predict the training data's output on data we have already seen.

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Decision Trees: Slide 43

# Stop and reflect: Why are we doing this learning anyway?

- It is not usually in order to predict the training data's output on data we have already seen.
- It is more commonly in order to predict the output value for future data we have not yet seen.

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## Stop and reflect: Why are we doing this learning anyway?

- It is not usually in order to predict the training data's output on data we have already seen.
- It is more commonly in order to predict the output value for future data we have not yet seen.

Warning: A common data mining misperception is that the above two bullets are the only possible reasons for learning. There are at least a dozen others.

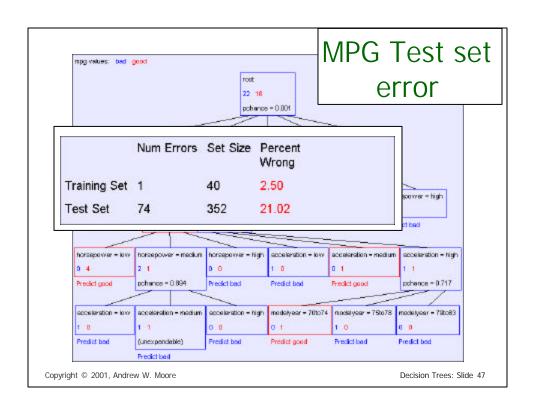
Copyright © 2001, Andrew W. Moore

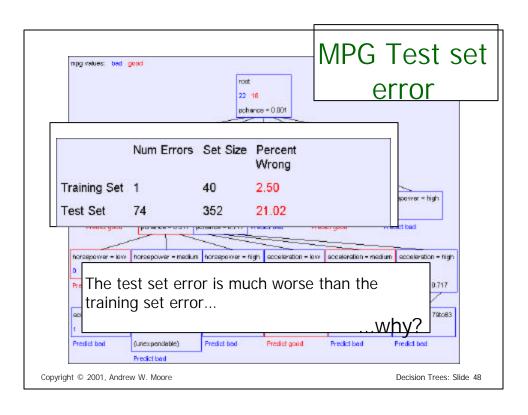
Decision Trees: Slide 45

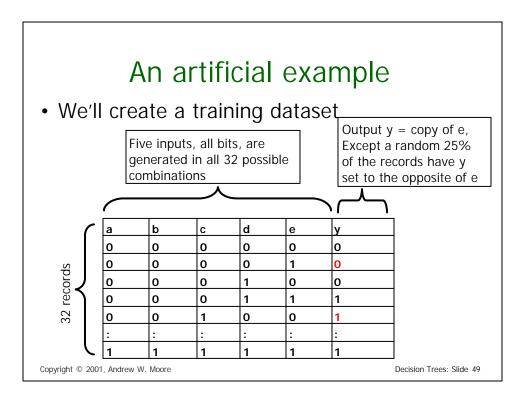
#### Test Set Error

- Suppose we are forward thinking.
- We hide some data away when we learn the decision tree.
- But once learned, we see how well the tree predicts that data.
- This is a good simulation of what happens when we try to predict future data.
- And it is called Test Set Error.

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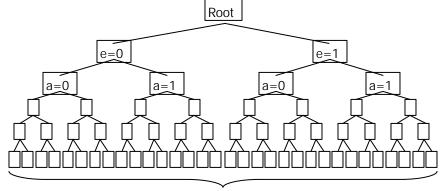
### In our artificial example

- Suppose someone generates a test set according to the same method.
- The test set is identical, except that some of the y's will be different.
- Some y's that were corrupted in the training set will be uncorrupted in the testing set.
- Some y's that were uncorrupted in the training set will be corrupted in the test set.

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# Building a tree with the artificial training set

Suppose we build a full tree (we always split until base case 2)



25% of these leaf node labels will be corrupted

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## Training set error for our artificial tree

All the leaf nodes contain exactly one record and so...

 We would have a training set error of zero

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## Testing the tree with the test set

	1/4 of the tree nodes are corrupted	3/4 are fine
1/4 of the test set records are corrupted	1/16 of the test set will be correctly predicted for the wrong reasons	3/16 of the test set will be wrongly predicted because the test record is corrupted
3/4 are fine	3/16 of the test predictions will be wrong because the tree node is corrupted	9/16 of the test predictions will be fine

In total, we expect to be wrong on 3/8 of the test set predictions

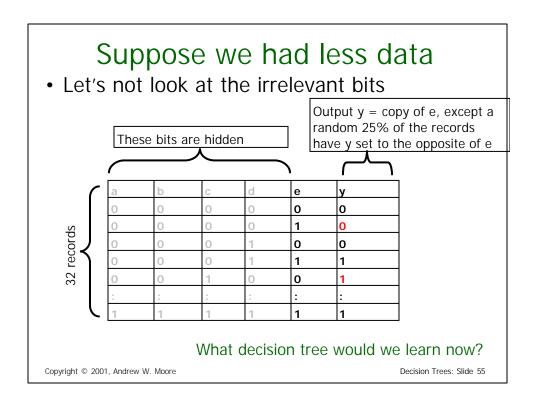
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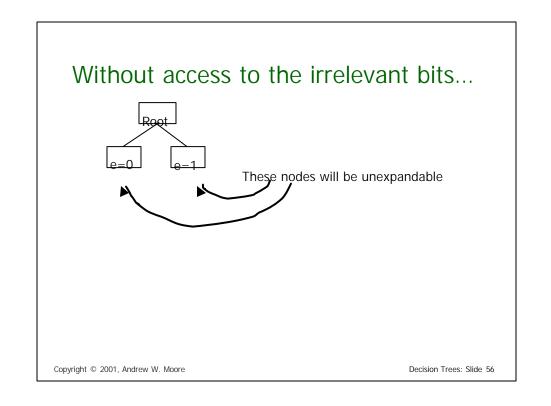
Decision Trees: Slide 53

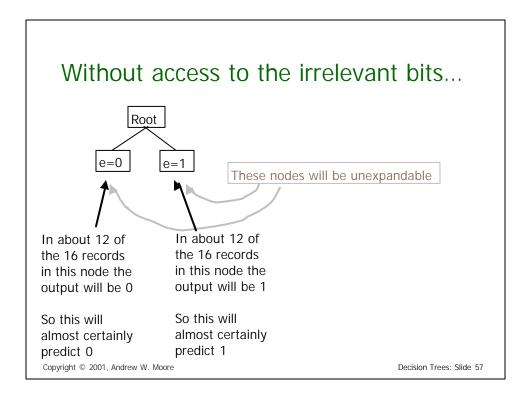
### What's this example shown us?

- This explains the discrepancy between training and test set error
- But more importantly... ...it indicates there's something we should do about it if we want to predict well on future data.

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#### Without access to the irrelevant bits... almost certainly almost certainly all none of the tree are fine nodes are corrupted 1/4 of the test ln/a 1/4 of the test set set records will be wrongly are corrupted predicted because the test record is 3/4 are fine 3/4 of the test n/a predictions will be In total, we expect to be wrong on only 1/4 of the test set predictions Copyright © 2001, Andrew W. Moore Decision Trees: Slide 58

#### Overfitting

- Definition: If your machine learning algorithm fits noise (i.e. pays attention to parts of the data that are irrelevant) it is overfitting.
- Fact (theoretical and empirical): If your machine learning algorithm is overfitting then it may perform less well on test set data.

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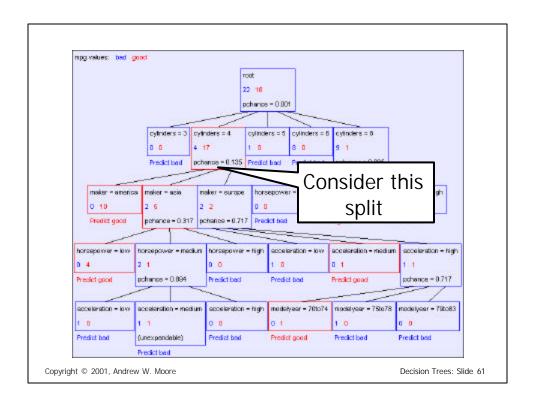
Decision Trees: Slide 59

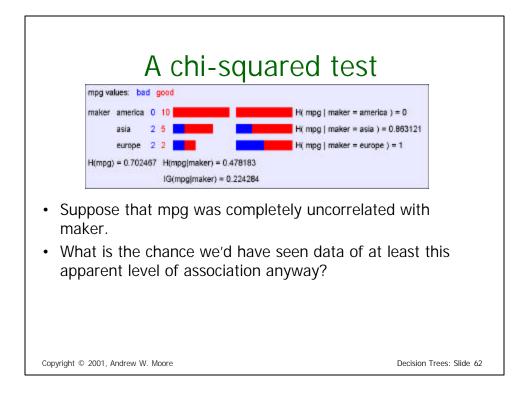
## Avoiding overfitting

- Usually we do not know in advance which are the irrelevant variables
- ...and it may depend on the context
   For example, if y = a AND b then b is an irrelevant variable only in the portion of the tree in which a=0

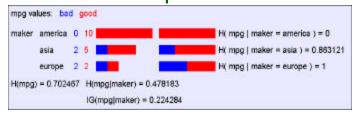
But we can use simple statistics to warn us that we might be overfitting.

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#### A chi-squared test



- Suppose that mpg was completely uncorrelated with maker.
- What is the chance we'd have seen data of at least this apparent level of association anyway?

By using a particular kind of chi-squared test, the answer is 13.5%.

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Decision Trees: Slide 63

# Using Chi-squared to avoid overfitting

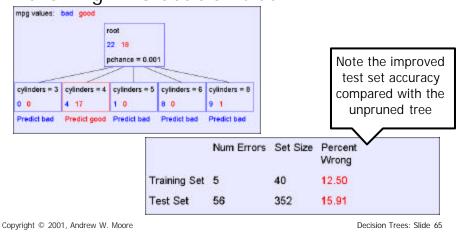
- Build the full decision tree as before.
- But when you can grow it no more, start to prune:
  - Beginning at the bottom of the tree, delete splits in which  $p_{chance} > MaxPchance$ .
  - Continue working you way up until there are no more prunable nodes.

*MaxPchance* is a magic parameter you must specify to the decision tree, indicating your willingness to risk fitting noise.

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## Pruning example

• With MaxPchance = 0.1, you will see the following MPG decision tree:



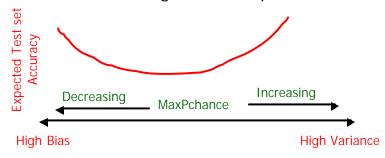
#### MaxPchance

- Good news: The decision tree can automatically adjust its pruning decisions according to the amount of apparent noise and data.
- Bad news: The user must come up with a good value of MaxPchance. (Note, Andrew usually uses 0.05, which is his favorite value for any magic parameter).
- Good news: But with extra work, the best MaxPchance value can be estimated automatically by a technique called cross-validation.

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#### MaxPchance

• Technical note (dealt with in other lectures): MaxPchance is a regularization parameter.



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Decision Trees: Slide 67

### The simplest tree

 Note that this pruning is heuristically trying to find

The simplest tree structure for which all within-leafnode disagreements can be explained by chance

- This is not the same as saying "the simplest classification scheme for which..."
- Decision trees are biased to prefer classifiers that can be expressed as trees.

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## Expressiveness of Decision Trees

- Assume all inputs are Boolean and all outputs are Boolean.
- What is the class of Boolean functions that are possible to represent by decision trees?
- Answer: All Boolean functions.

#### Simple proof:

- 1. Take any Boolean function
- 2. Convert it into a truth table
- 3. Construct a decision tree in which each row of the truth table corresponds to one path through the decision tree.

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Decision Trees: Slide 69

### Real-Valued inputs

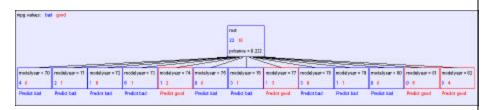
 What should we do if some of the inputs are real-valued?

mpg	cylinders	displacement	horsepower	weight	acceleration	modelyear	maker
good	4	97	75	2265	18.2	77	asia
bad	6	199	90	2648	15	70	america
bad	4	121	110	2600	12.8	77	europe
bad	8	350	175	4100	13	73	america
bad	6	198	95	3102	16.5	74	america
bad	4	108	94	2379	16.5	73	asia
bad	4	113	95	2228	14	71	asia
bad	8	302	139	3570	12.8	78	america
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
good	4	120	79	2625	18.6	82	america
bad	8	455	225	4425	10	70	america
good	4	107	86	2464	15.5	76	europe
bad	5	131	103	2830	15.9	78	europe

Idea One: Branch on each possible real value

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## "One branch for each numeric value" idea:



Hopeless: with such high branching factor will shatter the dataset and over fit

Note pchance is 0.222 in the above...if MaxPchance was 0.05 that would end up pruning away to a single root node.

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Decision Trees: Slide 71

### A better idea: thresholded splits

- Suppose X is real valued.
- Define IG(Y|X:t) as H(Y) H(Y|X:t)
- Define H(Y|X:t) = H(Y|X < t) P(X < t) + H(Y|X >= t) P(X >= t)
  - IG(Y|X:t) is the information gain for predicting Y if all you know is whether X is greater than or less than t
- Then define  $IG^*(Y|X) = max_t IG(Y|X:t)$
- For each real-valued attribute, use  $IG^*(Y|X)$  for assessing its suitability as a split

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## Computational Issues

• You can compute  $IG^*(Y|X)$  in time  $R \log R + 2 R n_y$ 

#### Where

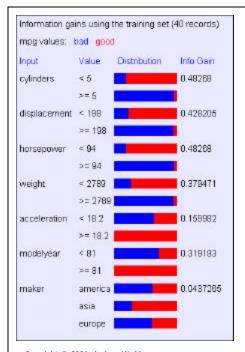
R is the number of records in the node under consideration  $n_v$  is the arity (number of distinct values of) Y

#### How?

Sort records according to increasing values of X. Then create a  $2xn_y$  contingency table corresponding to computation of  $IG(Y|X:x_{min})$ . Then iterate through the records, testing for each threshold between adjacent values of X, incrementally updating the contingency table as you go. For a minor additional speedup, only test between values of Y that differ.

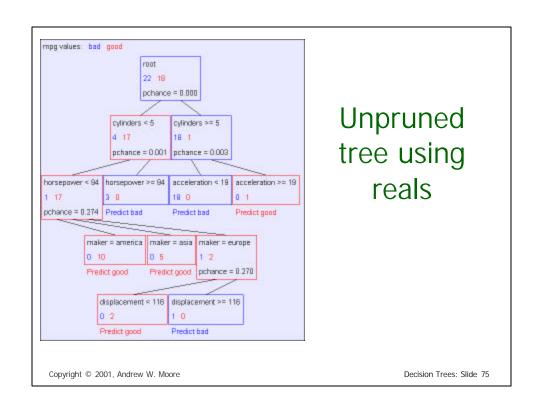
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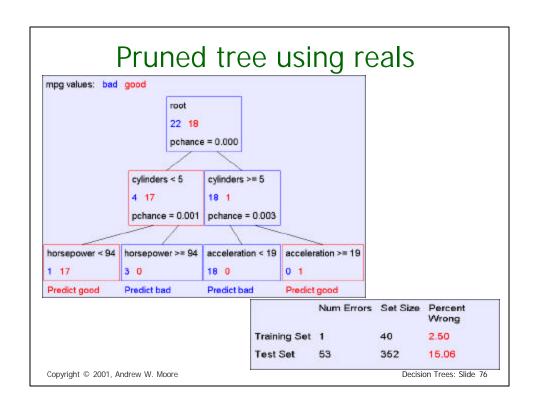
Decision Trees: Slide 73

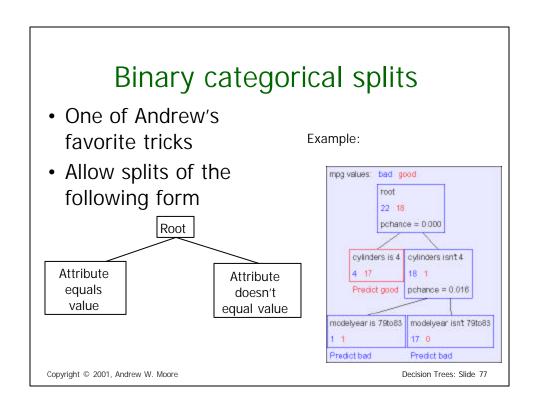


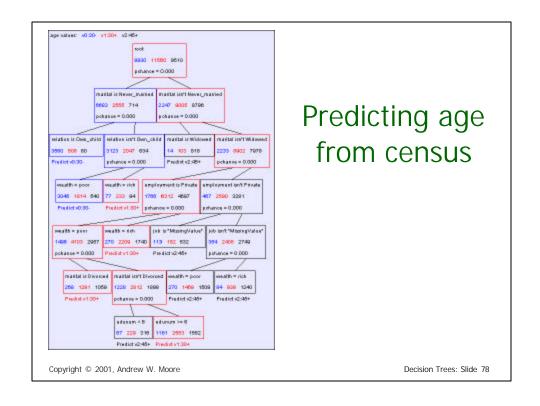
## Example with MPG

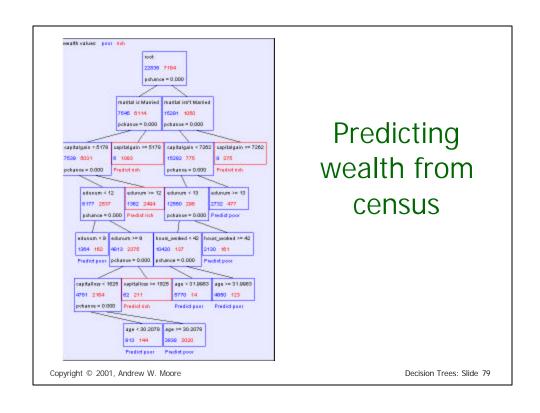
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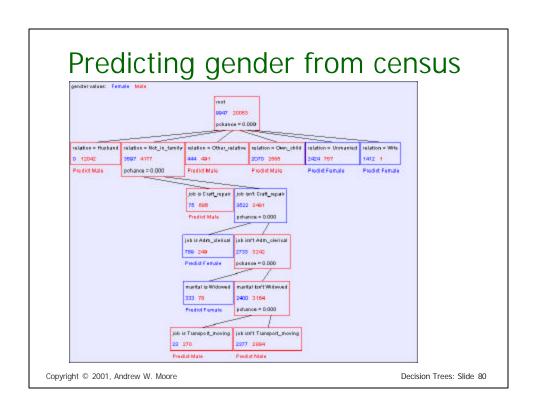












#### Conclusions

- Decision trees are the single most popular data mining tool
  - · Easy to understand
  - Easy to implement
  - · Easy to use
  - Computationally cheap
- It's possible to get in trouble with overfitting
- They do classification: predict a categorical output from categorical and/or real inputs

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Decision Trees: Slide 81

### What you should know

- What's a contingency table?
- · What's information gain, and why we use it
- The recursive algorithm for building an unpruned decision tree
- What are training and test set errors
- Why test set errors can be bigger than training set
- Why pruning can reduce test set error
- How to exploit real-valued inputs

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#### What we haven't discussed

- It's easy to have real-valued outputs too---these are called Regression Trees\*
- Bayesian Decision Trees can take a different approach to preventing overfitting
- Computational complexity (straightforward and cheap) \*
- Alternatives to Information Gain for splitting nodes
- How to choose MaxPchance automatically \*
- The details of Chi-Squared testing \*
- Boosting---a simple way to improve accuracy \*

\* = discussed in other Andrew lectures

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Decision Trees: Slide 83

#### For more information

- Two nice books
  - L. Breiman, J. H. Friedman, R. A. Olshen, and C. J. Stone. Classification and Regression Trees. Wadsworth, Belmont, CA, 1984.
  - C4.5 : Programs for Machine Learning (Morgan Kaufmann Series in Machine Learning) by J. Ross Quinlan
- Dozens of nice papers, including
  - Learning Classification Trees, Wray Buntine, Statistics and Computation (1992), Vol 2, pages 63-73
  - Kearns and Mansour, On the Boosting Ability of Top-Down Decision Tree Learning Algorithms, STOC: ACM Symposium on Theory of Computing, 1996"
- Dozens of software implementations available on the web for free and commercially for prices ranging between \$50 - \$300,000

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#### Discussion

- Instead of using information gain, why not choose the splitting attribute to be the one with the highest prediction accuracy?
- Instead of greedily, heuristically, building the tree, why not do a combinatorial search for the optimal tree?
- If you build a decision tree to predict wealth, and marital status, age and gender are chosen as attributes near the top of the tree, is it reasonable to conclude that those three inputs are the major causes of wealth?
- ..would it be reasonable to assume that attributes not mentioned in the tree are not causes of wealth?
- ..would it be reasonable to assume that attributes not mentioned in the tree are not correlated with wealth?

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