## Decision Trees

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| age | employme | education | edur | marital | ... | job | relation | race | gender | hour | country | wealth |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | ... |  |  |  |  |  |  |  |
| 39 | State_gov | Bachelors | 13 | Never_mar. | ... | Adm_cleric | Not_in_far | White | Male | 40 | United_St: | poor |
| 51 | Self_emp_ | Bachelors | 13 | Married | ... | Exec_man | Husband | White | Male | 13 | United_Sti | poor |
| 39 | Private | HS_grad | 9 | Divorced | $\ldots$ | Handlers_( | Not_in_far | White | Male | 40 | United_St: | poor |
| 54 | Private | 11th | 7 | Married | $\ldots$ | Handlers_( | Husband | Black | Male | 40 | United_St: | poor |
| 28 | Private | Bachelors | 13 | Married | $\ldots$ | Prof_speci | Wife | Black | Female | 40 | Cuba | poor |
| 38 | Private | Masters | 14 | Married | ... | Exec_man | Wife | White | Female | 40 | United_Sti | poor |
| 50 | Private | 9th |  | Married_sp. | ... | Other_serv | Not_in_far | Black | Female | 16 | Jamaica | poor |
| 52 | Self_emp_ | HS_grad | 9 | Married | ... | Exec_man | Husband | White | Male | 45 | United_St |  |
| 31 | Private | Masters | 14 | Never_mar. | ... | Prof_speci | Not_in_far | White | Female | 50 | United_Sti | :rich |
| 42 | Private | Bachelors | 13 | Married | ... | Exec_man | Husband | White | Male | 40 | United_Sti | rich |
| 37 | Private | Some_coll | 10 | Married | ... | Exec_man | Husband | Black | Male | 80 | United_Sti | rich |
| 30 | State_gov | Bachelors | 13 | Married | ... | Prof_speci | Husband | Asian | Male | 40 | India | rich |
| 24 | Private | Bachelors | 13 | Never_mal. | ... | Adm_cleric | Own_child | White | Female | 30 | United_Sti | poor |
| 33 | Private | Assoc_ac | 12 | Never_mal. | ... | Sales | Not_in_far | Black | Male | 50 | United_St: | poor |
| 41 | Private | Assoc_voc | 11 | Married | ... | Craft_repai | Husband | Asian | Male | 40 | *MissingV |  |
| 34 | Private | 7th_8th | 4 | Married | ... | Transport_H | Husband | Amer_Indi | Male | 45 | Mexico | poor |
| 26 | Self_emp_ | HS_grad | 9 | Never_mar. | ... | Farming_fi | Own_child | White | Male | 35 | United_St: | poor |
| 33 | Private | HS_grad |  | Never_mar. | ... | Machine_c | Unmarried | White | Male | 40 | United_St: | poor |
| 38 | Private | 11th |  | Married | ... | Sales | Husband | White | Male | 50 | United_Sti | poor |
| 44 | Self_emp_ | Masters | 14 | Divorced | ... | Exec_man | Unmarried | White | Female | 45 | United_Sti | :rich |
| 41 | Private | Doctorate | 16 | Married | ... | Prof_speci | Husband | White | Male | 60 | United_St | :rich |
| : | \| | : |  | : | : | : $\quad$ : | : | : | : | : |  | : |

## Classification

- A Major Data Mining Operation
- Give one attribute (e.g wealth), try to predict the value of new people's wealths by means of some of the other available attributes.
- Applies to categorical outputs
- Categorical attribute: an attribute which takes on two or more discrete values. Also known as a symbolic attribute.
- Real attribute: a column of real numbers


## Today's lecture

- Information Gain for measuring association between inputs and outputs
- Learning a decision tree classifier from data


## About this dataset

- It is a tiny subset of the 1990 US Census.
- It is publicly available online from the UCI Machine Learning Datasets repository

| Used Attributes |  |  |  |
| :--- | :--- | :--- | :--- |
| age | edunum | race | hours_worked |
| employment | marital | gender | country |
| taxweighting | job | capitalgain | wealth |
| education | relation | capitalloss | agegroup |

This color $=$ Real-valued This color $=$ Symbol-valued
Successfully loaded a new dataset from the file Ytadult.fds. It has 16 attributes and 48842 records.

## What can you do with a dataset?

- Well, you can look at histograms...


| Velle | Frequency |  |
| :---: | :---: | :---: |
| Disorced | 6633 |  |
| Married AF_spouse | 37 |  |
| Married | 22379 |  |
| Mamied_spouse_sbosent | 628 |  |
| Never_married | 16117 |  |
| Separated | 1530 |  |
| WViclowec | 1518 |  |

## Contingency Tables

- A better name for a histogram:


## A One-dimensional Contingency Table

- Recipe for making a k-dimensional contingency table:

1. Pick $k$ attributes from your dataset. Call them $a_{1}, a_{2}, \ldots a_{k}$.
2. For every possible combination of values, $a_{1},=x_{1}, a_{2},=x_{2}, \ldots a_{k},=x_{k}$, record how frequently that combination occurs
Fun fact: A database person would call this a "k-dimensional datacube"


## A 2-d Contingency Table



- Easier to appreciate graphically


## A 2-d Contingency Table



- Easier to see "interesting" things if we stretch out the histogram bars


## A bigger 2-d contingency table

| job values: <br> *MissingValue* |  | Adm_clerical | Craft_repair |  |  | Farming_fishing |  |  | Machine_op_inspct |  |  | Protective_sery |  |  |  |  | Tech_support |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Armed_Forces | Exec_managerial |  |  | Handers_cleaners |  |  | Other_service |  |  | Prof_specialty |  |  | Sales |  | Transport_moving |
| marital | Divorce |  | 270 | 1192 | 0 | 679 | 890 | 90 | 197 | 434 | 762 | 795 | 121 | 664 | 239 | 254 |  |
|  | Married | AF_spouse | 5 | 6 | 0 | 4 | 3 | 1 | 1 | 1 | 5 | 4 | 1 | 5 | 0 | 1 |  |
|  | Married |  | 928 | 1495 | 7 | 3818 | 3600 | 869 | 724 | 1469 | 1088 | 3182 | 583 | 2491 | 609 | 1489 |  |
|  | Married | _spouse_absent | 45 | 84 | 0 | 77 | 52 | 35 | 32 | 37 | 92 | 64 | 7 | 55 | 9 | 30 |  |
|  | Never | narried | 1242 | 2360 | 8 | 1301 | 1260 | 434 | 1029 | 872 | 2442 | 1849 | 237 | 1992 | 506 | 486 |  |
|  | Separa |  | 97 | 224 | 0 | 160 | 126 | 23 | 63 | 123 | 275 | 145 | 23 | 146 | 48 | 56 |  |
|  | Widow |  | 222 | 250 | 0 | 73 | 155 | 38 | 26 | 86 | 259 | 133 | 11 | 151 | 35 | 39 |  |

## 3-d contingency tables

## - These are harder to look at!



## On-Line Analytical Processing (OLAP)

- Software packages and database add-ons to do this are known as OLAP tools
- They usually include point and click navigation to view slices and aggregates of contingency tables
- They usually include nice histogram visualization


## Time to stop and think

- Why would people want to look at contingency tables?


## Let's continue to think

- With 16 attributes, how many 1-d contingency tables are there?
- How many 2-d contingency tables?
- How many 3-d tables?
- With 100 attributes how many 3 -d tables are there?


## Let's continue to think

- With 16 attributes, how many 1-d contingency tables are there? 16
- How many 2-d contingency tables? 16-choose-2 = 16 * $15 / 2=120$
- How many 3-d tables? 560
- With 100 attributes how many 3-d tables are there? 161,700


## Manually looking at contingency tables

- Looking at one contingency table: can be as much fun as reading an interesting book
- Looking at ten tables: as much fun as watching cnN
- Looking at 100 tables: as much fun as watching an infomercial
- Looking at 100,000 tables: as much fun as a threeweek November vacation in Duluth with a dying weasel.


## Data Mining

- Data Mining is all about automating the process of searching for patterns in the data.

Which patterns are interesting?
Which might be mere illusions?
And how can they be exploited?

## Data Mining

- Data Mining is all about automating the process of searching for patterns in the data.

Which patterns are interesting?
That's what we'll look at right now.
And the answer will turn out to be the engine that drives decision tree learning.
Which might be mere illusions?
And how can they be exploited?

## Deciding whether a pattern is interesting

- We will use information theory
- A very large topic, originally used for compressing signals
- But more recently used for data mining...


## Deciding whether a pattern is interesting

- We will use information theory
- A very large topic, originally used for compressing signals
- But more recently used for data mining...
(The topic of Information Gain will now be discussed, but you will find it in a separate Andrew Handout)


## Searching for High Info Gains

- Given something (e.g. wealth) you are trying to predict, it is easy to ask the computer to find which attribute has highest information gain for it.

```
weathvalues: poor nch
```



```
H[\mathrm{ mealth }]=0.793844 H(wealth(relation) }=0.62842
    |G(wealih|;elation) =0.165423
```


## Learning Decision Trees

- A Decision Tree is a tree-structured plan of a set of attributes to test in order to predict the output.
- To decide which attribute should be tested first, simply find the one with the highest information gain.
- Then recurse...


# A small dataset: Miles Per Gallon 

40
Records


From the UCI repository (thanks to Ross Quinlan)


## A Decision Stump




## Second level of tree






## Base Cases

- Base Case One: If all records in current data subset have the same output then don't recurse
- Base Case Two: If all records have exactly the same set of input attributes then don't recurse


## Base Cases: An idea

- Base Case One: If all records in current data subset have the same output then don't recurse
- Base Case Two: If all records have exactly the same set of input attributes then don't recurse

-Is this a good idea?


## The problem with Base Case 3

| a | b |  |
| :--- | :--- | ---: |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

The information gains:


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The resulting decision tree:


Decision Trees: Slide 36

## If we omit Base Case 3:

| a | b | y |  |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | $y=a \operatorname{COR} b$ |
| 0 | 1 | 1 |  |
| 1 | 0 | 1 |  |
| 1 | 1 | 0 |  |

The resulting decision tree:


## Basic Decision Tree Building Summarized

## BuildTree( DataSet,Output)

- If all output values are the same in DataSet, return a leaf node that says "predict this unique output"
- If all input values are the same, return a leaf node that says "predict the majority output"
- Else find attribute X with highest Info Gain
- Suppose $X$ has $n_{X}$ distinct values (i.e. $X$ has arity $n_{X}$ ).
- Create and return a non-leaf node with $n_{x}$ children.
- The ith child should be built by calling

> BuildTree(DS_,Output)

Where $D S_{i}$ built consists of all those records in DataSet for which $X=$ ith distinct value of $X$.

## Training Set Error

- For each record, follow the decision tree to see what it would predict
For what number of records does the decision tree's prediction disagree with the true value in the database?
- This quantity is called the training set error. The smaller the better.




## Stop and reflect: Why are we doing this learning anyway?

- It is not usually in order to predict the training data's output on data we have already seen.


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Warning: A common data mining misperception is that the above two bullets are the only possible reasons for learning. There are at least a dozen others.

## Test Set Error

- Suppose we are forward thinking.
- We hide some data away when we learn the decision tree.
- But once learned, we see how well the tree predicts that data.
- This is a good simulation of what happens when we try to predict future data.
- And it is called Test Set Error.



## An artificial example

- We'll create a training dataset



## In our artificial example

- Suppose someone generates a test set according to the same method.
- The test set is identical, except that some of the $y$ 's will be different.
- Some y's that were corrupted in the training set will be uncorrupted in the testing set.
- Some y's that were uncorrupted in the training set will be corrupted in the test set.


## Building a tree with the artificial training set

- Suppose we build a full tree (we always split until base case 2 )



## Training set error for our artificial tree

All the leaf nodes contain exactly one record and so...

- We would have a training set error of zero


## Testing the tree with the test set

|  | $1 / 4$ of the tree nodes <br> are corrupted | $3 / 4$ are fine |
| :--- | :--- | :--- |
| $1 / 4$ of the test set <br> records are <br> corrupted | $1 / 16$ of the test set will <br> be correctly predicted <br> for the wrong reasons | $3 / 16$ of the test set will <br> be wrongly predicted <br> because the test record is <br> corrupted |
| $3 / 4$ are fine | $3 / 16$ of the test <br> predictions will be <br> wrong because the <br> tree node is corrupted | $9 / 16$ of the test <br> predictions will be fine |

In total, we expect to be wrong on $3 / 8$ of the test set predictions

## What's this example shown us?

- This explains the discrepancy between training and test set error
- But more importantly... ...it indicates there's something we should do about it if we want to predict well on future data.



## Without access to the irrelevant bits...



These nodes will be unexpandable


## Without access to the irrelevant bits...



In total, we expect to be wrong on only $1 / 4$ of the test set predictions

## Overfitting

- Definition: If your machine learning algorithm fits noise (i.e. pays attention to parts of the data that are irrelevant) it is overfitting.
- Fact (theoretical and empirical): If your machine learning algorithm is overfitting then it may perform less well on test set data.


## Avoiding overfitting

- Usually we do not know in advance which are the irrelevant variables
- ...and it may depend on the context

For example, if $y=a$ AND $b$ then $b$ is an irrelevant variable only in the portion of the tree in which $a=0$

But we can use simple statistics to warn us that we might be overfitting.


## A chi-squared test

| mpg values: bad good |  |
| :---: | :---: |
| maker america 010 | H ( mpg \| maker $=$ america $)=0$ |
| asia 25 | Hi mpg \| maker = asia ) = 0.883121 |
| europe 22 | Hy mpg \| maker $=$ europe $)=1$ |
| $\mathrm{H}(\mathrm{mpg})=0.702467 \mathrm{H}(\mathrm{mpg} \mid$ maker $)=0.478183$ |  |
| $1 \mathrm{G}(\mathrm{mpg} / \mathrm{maker})=0.224284$ |  |

- Suppose that mpg was completely uncorrelated with maker.
- What is the chance we'd have seen data of at least this apparent level of association anyway?


## A chi-squared test

| mpg values: bad good |  |
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- Suppose that mpg was completely uncorrelated with maker.
- What is the chance we'd have seen data of at least this apparent level of association anyway?
By using a particular kind of chi-squared test, the answer is $13.5 \%$.


## Using Chi-squared to avoid overfitting

- Build the full decision tree as before.
- But when you can grow it no more, start to prune:
- Beginning at the bottom of the tree, delete splits in which $\mathrm{p}_{\text {chance }}>$ MaxPchance.
- Continue working you way up until there are no more prunable nodes.

MaxPchance is a magic parameter you must specify to the decision tree, indicating your willingness to risk fitting noise.

## Pruning example

- With MaxPchance $=0.1$, you will see the following MPG decision tree:



## MaxPchance

- Good news: The decision tree can automatically adjust its pruning decisions according to the amount of apparent noise and data.
- Bad news: The user must come up with a good value of MaxPchance. (Note, Andrew usually uses 0.05 , which is his favorite value for any magic parameter).
- Good news: But with extra work, the best MaxPchance value can be estimated automatically by a technique called cross-validation.


## MaxPchance

- Technical note (dealt with in other lectures): MaxPchance is a regularization parameter.


High Bias
High Variance

## The simplest tree

- Note that this pruning is heuristically trying to find
The simplest tree structure for which all within-leafnode disagreements can be explained by chance
- This is not the same as saying "the simplest classification scheme for which..."
- Decision trees are biased to prefer classifiers that can be expressed as trees.


## Expressiveness of Decision Trees

- Assume all inputs are Boolean and all outputs are Boolean.
- What is the class of Boolean functions that are possible to represent by decision trees?
- Answer: All Boolean functions.

Simple proof:

1. Take any Boolean function
2. Convert it into a truth table
3. Construct a decision tree in which each row of the truth table corresponds to one path through the decision tree.

## Real-Valued inputs

- What should we do if some of the inputs are real-valued?

| mpg | cylinders | displacement | horsepower | weight | acceleration | modelyear | maker |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| good | 4 | 97 | 75 | 2265 | 18.2 | 77 | asia |
| bad | 6 | 199 | 90 | 2648 | 15 | 70 | america |
| bad | 4 | 121 | 110 | 2600 | 12.8 | 77 | europe |
| bad | 8 | 350 | 175 | 4100 | 13 | 73 | america |
| bad | 6 | 198 | 95 | 3102 | 16.5 | 74 | america |
| bad | 4 | 108 | 94 | 2379 | 16.5 | 73 | asia |
| bad | 4 | 113 | 95 | 2228 | 14 | 71 | asia |
| bad | 8 | 302 | 139 | 3570 | 12.8 | 78 | america |
| : | $:$ |  | : | : | : | : | : |
| : | : |  | : | : | $:$ | : | : |
| : | : | : | : | : | : | : | : |
| good | 4 | 120 | 79 | 2625 | 18.6 | 82 | america |
| bad | 8 | 455 | 225 | 4425 | 10 | 70 | america |
| good | 4 | 107 | 86 | 2464 | 15.5 | 76 | europe |
| bad | 5 | 131 | 103 | 2830 | 15.9 | 78 | europe |
|  |  |  |  |  |  |  |  |

Idea One: Branch on each possible real value

## "One branch for each numeric value" idea:



Hopeless: with such high branching factor will shatter the dataset and over fit

Note pchance is 0.222 in the above...if MaxPchance was 0.05 that would end up pruning away to a single root node.

## A better idea: thresholded splits

- Suppose $X$ is real valued.
- Define $\mathrm{IG}(\mathrm{Y} \mid \mathrm{X}: \mathrm{t})$ as $\mathrm{H}(\mathrm{Y})$ - $\mathrm{H}(\mathrm{Y} \mid \mathrm{X}: \mathrm{t})$
- Define $\mathrm{H}(\mathrm{Y} \mid \mathrm{X}: \mathrm{t})=$
$H(Y \mid X<t) P(X<t)+H(Y \mid X>=t) P(X>=t)$
- $I G(Y \mid X: t)$ is the information gain for predicting $Y$ if all you know is whether $X$ is greater than or less than $t$
- Then define $I G^{*}(Y \mid X)=\max _{t} I G(Y \mid X: t)$
- For each real-valued attribute, use IG* $(Y \mid X)$ for assessing its suitability as a split


## Computational Issues

## - You can compute $I G^{*}(Y \mid X)$ in time $R \log R+2 R n_{y}$

## - Where

$R$ is the number of records in the node under consideration $\mathrm{n}_{\mathrm{y}}$ is the arity (number of distinct values of) Y

How?
Sort records according to increasing values of $X$. Then create a $2 \mathrm{xn}_{y}$ contingency table corresponding to computation of $\mathrm{IG}\left(\mathrm{Y} \mid \mathrm{X}: \mathrm{X}_{\text {min }}\right)$. Then iterate through the records, testing for each threshold between adjacent values of $X$, incrementally updating the contingency table as you go. For a minor additional speedup, only test between values of $Y$ that differ.


## Example with MPG



## Unpruned tree using <br> reals

## Pruned tree using reals



|  |  | Num Errors | Set Size | Percent Wrong |
| :---: | :---: | :---: | :---: | :---: |
|  | Training Set | 1 | 40 | 2.50 |
|  | Test Set | 53 | 352 | 15.06 |

## Binary categorical splits

- One of Andrew's favorite tricks
- Allow splits of the following form




## Predicting age from census



## Predicting gender from census



## Conclusions

- Decision trees are the single most popular data mining tool
- Easy to understand
- Easy to implement
- Easy to use
- Computationally cheap
- It's possible to get in trouble with overfitting
- They do classification: predict a categorical output from categorical and/or real inputs


## What you should know

- What's a contingency table?
- What's information gain, and why we use it
- The recursive algorithm for building an unpruned decision tree
- What are training and test set errors
- Why test set errors can be bigger than training set
- Why pruning can reduce test set error
- How to exploit real-valued inputs


## What we haven't discussed

- It's easy to have real-valued outputs too---these are called Regression Trees*
- Bayesian Decision Trees can take a different approach to preventing overfitting
- Computational complexity (straightforward and cheap) *
- Alternatives to Information Gain for splitting nodes
- How to choose MaxPchance automatically *
- The details of Chi-Squared testing *
- Boosting---a simple way to improve accuracy *
* $=$ discussed in other Andrew lectures


## For more information

## - Two nice books

- L. Breiman, J. H. Friedman, R. A. Olshen, and C. J. Stone. Classification and Regression Trees. Wadsworth, Belmont, CA, 1984.
- C4.5 : Programs for Machine Learning (Morgan Kaufmann Series in Machine Learning) by J. Ross Quinlan


## - Dozens of nice papers, including

- Learning Classification Trees, Wray Buntine, Statistics and Computation (1992), Vol 2, pages 63-73
- Kearns and Mansour, On the Boosting Ability of Top-Down Decision Tree Learning Algorithms, STOC: ACM Symposium on Theory of Computing, 1996"
- Dozens of software implementations available on the web for free and commercially for prices ranging between $\$ 50-\$ 300,000$


## Discussion

- Instead of using information gain, why not choose the splitting attribute to be the one with the highest prediction accuracy?
- Instead of greedily, heuristically, building the tree, why not do a combinatorial search for the optimal tree?
- If you build a decision tree to predict wealth, and marital status, age and gender are chosen as attributes near the top of the tree, is it reasonable to conclude that those three inputs are the major causes of wealth?
- ..would it be reasonable to assume that attributes not mentioned in the tree are not causes of wealth?
- ..would it be reasonable to assume that attributes not mentioned in the tree are not correlated with wealth?

