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15-826: Multimedia Databases and Data Mining

Lecture #23: DSP tools –
Fourier and Wavelets

C. Faloutsos

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Must-read Material

- DFT/DCT: In PTVF ch. 12.1, 12.3, 12.4; in Textbook Appendix B.
- Wavelets: In PTVF ch. 13.10; in MM Textbook Appendix C

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Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- Indexing - similarity search
- Data Mining

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Indexing - Detailed outline

- primary key indexing
- ..
- multimedia
- Digital Signal Processing (DSP) tools
  - Discrete Fourier Transform (DFT)
  - Discrete Wavelet Transform (DWT)

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DSP - Detailed outline

- DFT
  - what
  - why
  - how
  - Arithmetic examples
  - properties / observations
  - DCT
  - 2-d DFT
  - Fast Fourier Transform (FFT)

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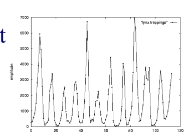
Introduction

Goal: given a signal (eg., sales over time and/or space)

Find: patterns and/or compress

count



lynx caught per year

year

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What does DFT do?

A: highlights the periodicities

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Why should we care?

A: several real sequences are periodic

Q: Such as?

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Why should we care?

A: several real sequences are periodic

Q: Such as?

A:

- sales patterns follow seasons;
- economy follows 50-year cycle
- temperature follows daily and yearly cycles

Many real signals follow (multiple) cycles

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Why should we care?

For example: human voice!

- Frequency analyzer http://www.relisoft.com/freeware/freq.html
- speaker identification
- impulses/noise -> flat spectrum
- high pitch -> high frequency

Freq.exe

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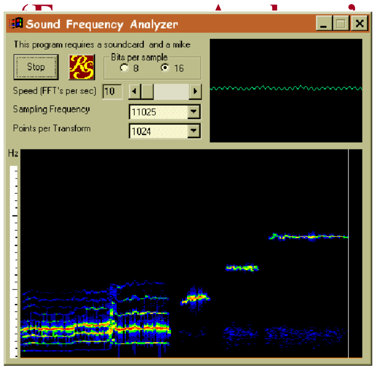

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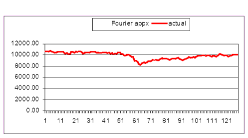
DFT and stocks



- Dow Jones Industrial index, 6/18/2001-12/21/2001

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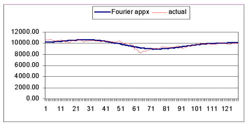
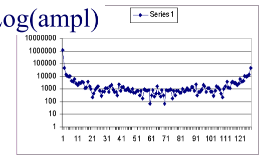
DFT and stocks



Log(ampl)



- Dow Jones Industrial index, 6/18/2001-12/21/2001
- just 3 DFT coefficients give very good approximation

freq

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DFT: definition

- Discrete Fourier Transform (n-point):

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j 2\pi f t / n)$$

$$(j = \sqrt{-1})$$

inverse DFT

$$x_t = 1/\sqrt{n} \sum_{f=0}^{n-1} X_f * \exp(+j 2\pi f t / n)$$

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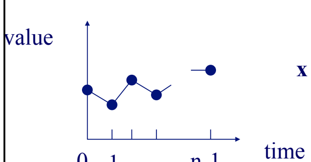
How does it work?

Decomposes signal to a sum of sine (and cosine) waves.

Q:How to assess 'similarity' of **x** with a wave?



x $=\{x_0, x_1, \ldots x_{n-1}\}$

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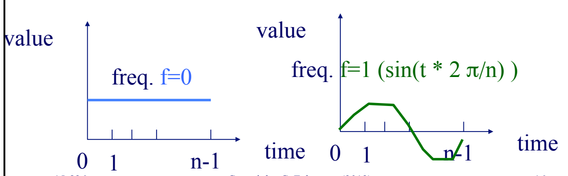
How does it work?

A: consider the waves with frequency 0, 1, ...; use the inner-product (~cosine similarity)



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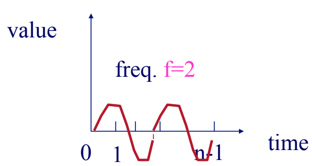

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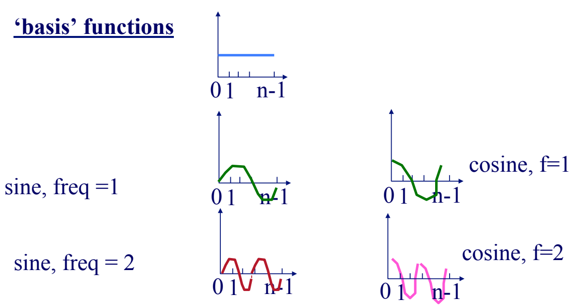
How does it work?

'basis' functions



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How does it work?

- Basis functions are actually n-dim vectors, **orthogonal** to each other
- 'similarity' of **x** with each of them: inner product
- DFT: ~ all the similarities of **x** with the basis functions

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How does it work?

Since $e^{jf} = cos(f) + j\ sin(f)$

(j=$sqrt$(-1)),

we finally have:

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DFT: definition

- Discrete Fourier Transform (n-point):

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j 2\pi\ f\, t / n)$$

$$(j = \sqrt{-1})$$

inverse DFT

$$x_t = 1/\sqrt{n} \sum_{f=0}^{n-1} X_f * \exp(+j 2\pi\ f\, t / n)$$

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DFT: definition

- **Good** news: Available in **all** symbolic math packages, eg., in 'mathematica'

```
x = [1,2,1,2];
X = Fourier[x];
Plot[ Abs[X] ];
```


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DFT: definition

(variations:

- 1/n instead of 1/sqrt(n)
- exp(-...) instead of exp(+...)

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DFT: definition

(variations:

…

or even

$$a_f = \sum_{i=0}^{N-1} x_i \cos(2\pi f i/N)$$

$$b_f = \sum_{i=0}^{N-1} x_i \sin(2\pi f i/N)$$

$$f = 0, 1, ...N/2$$

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DFT: definition

Observations:

- X_f : are complex numbers except
  - X_0 , who is real
- Im (X_f): ~ amplitude of sine wave of frequency *f*
- Re (X_f): ~ amplitude of cosine wave of frequency *f*
- **x:** is the sum of the above sine/cosine waves

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DFT: definition

Observation - SYMMETRY property:

$$X_f = (X_{n-f})^*$$

("*": complex conjugate*: (a + b j)* = a - b j)*

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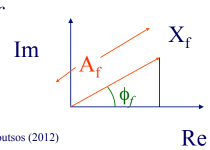
DFT: definition

Definitions

- $A_f = |X_f|$: amplitude of frequency *f*
- $|X_f|^2 = Re(X_f)^2 + Im(X_f)^2$ = energy of frequency *f*
- phase ϕ_f at frequency *f*



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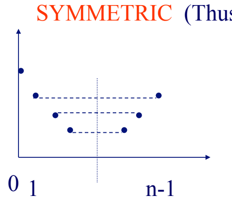
DFT: definition

Amplitude spectrum: $| X_f |$ vs f $(f=0, 1, \ldots n-1)$

SYMMETRIC (Thus, we plot the first half only)



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DFT: definition

Phase spectrum $| \phi_f |$ vs f $(f=0, 1, \ldots n-1)$:

Anti-symmetric

(Rarely used)

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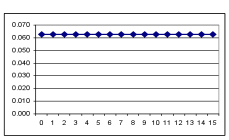
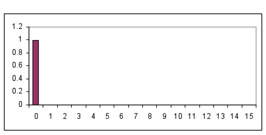
DFT: examples

flat

Amplitude





time

freq

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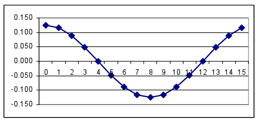
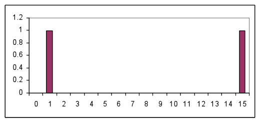
DFT: examples

Low frequency sinusoid





time freq

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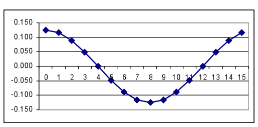
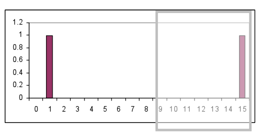
DFT: examples

- Sinusoid - symmetry property: $X_f = X^*_{n-f}$





time freq

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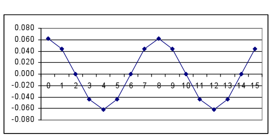
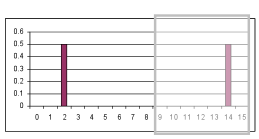
DFT: examples

- Higher freq. sinusoid





time freq

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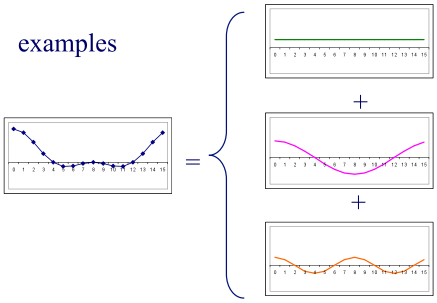
DFT: examples

examples



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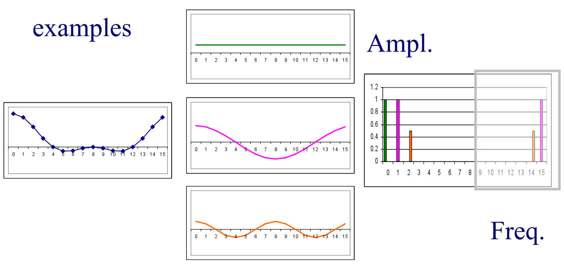
DFT: examples

examples

Ampl.



Freq.

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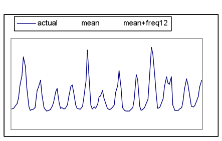
DFT: Amplitude spectrum

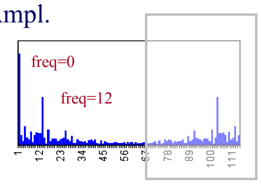
Amplitude: $A_f^2 = \mathrm{Re}^2(X_f) + \mathrm{Im}^2(X_f)$

count



year

Ampl.



Freq.

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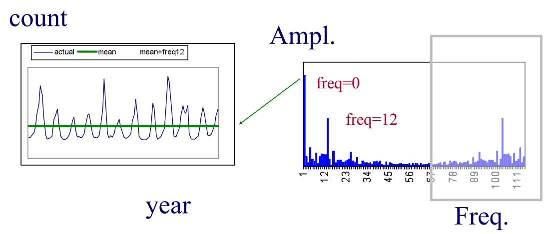
DFT: Amplitude spectrum



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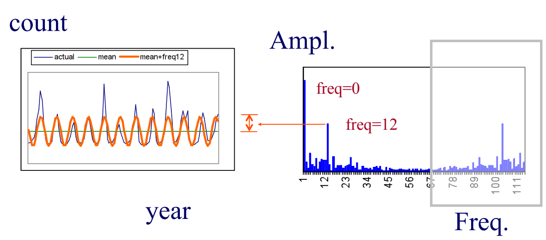
DFT: Amplitude spectrum



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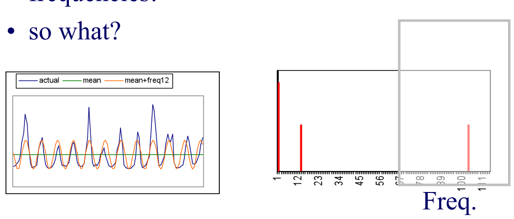
DFT: Amplitude spectrum

- excellent approximation, with only 2 frequencies!
- so what?



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DFT: Amplitude spectrum

- excellent approximation, with only 2 frequencies!
- so what?
- A1: compression
- A2: pattern discovery
- (A3: forecasting)

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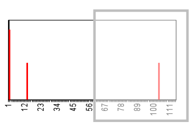
DFT: Amplitude spectrum

- excellent approximation, with only 2 frequencies!
- so what?
- A1: **(lossy) compression**
- A2: pattern discovery



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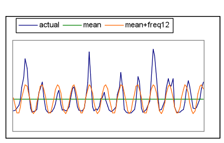
DFT: Amplitude spectrum

- excellent approximation, with only 2 frequencies!
- so what?
- A1: (lossy) compression
- A2: **pattern discovery**



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DFT: Amplitude spectrum

- Let's see it in action (defunct now…)
- (http://www.dsptutor.freeuk.com/jsanalyser/FFTSpectrumAnalyser.html)
- plain sine
- phase shift
- two sine waves
- the 'chirp' function
- http://ion.researchsystems.com/

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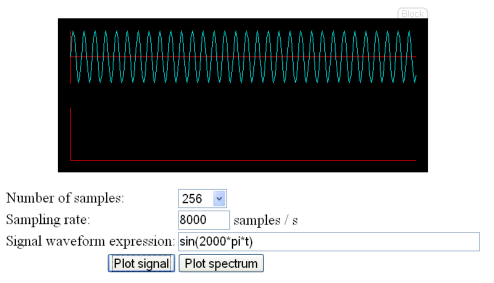
Plain sine



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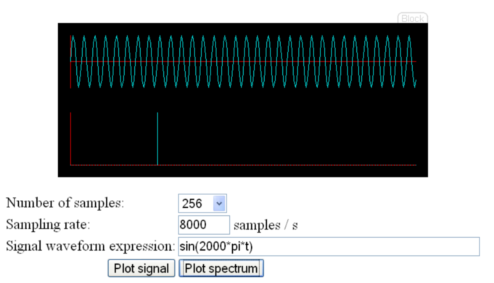
Plain sine



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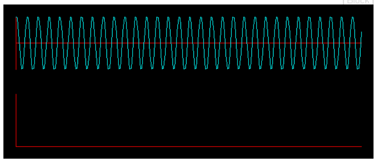
Plain sine – phase shift



Number of samples: 256
Sampling rate: 8000 samples / s
Signal waveform expression: sin(2000*pi*t+1.2)
Plot signal | Plot spectrum

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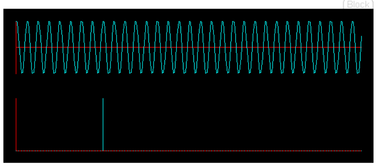
Plain sine – phase shift



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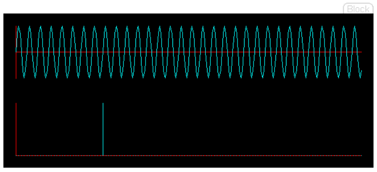

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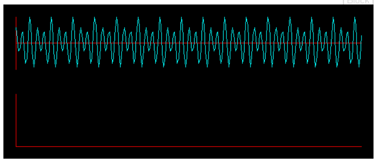
Sampling rate: 8000 samples / s
Signal waveform expression: sin(2000*pi*t)
Plot signal | Plot spectrum

Two sines



Number of samples: 256

Sampling rate: 8000 samples / s

Signal waveform expression: sin(2000*pi*t)+2*cos(3000*pi*t+0.5)

Plot signal | Plot spectrum

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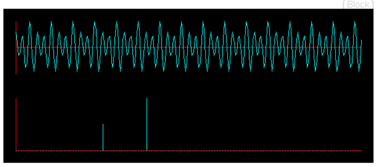
Two sines



Number of samples: 256

Sampling rate: 8000 samples / s

Signal waveform expression: sin(2000*pi*t)+2*cos(3000*pi*t+0.5)

Plot signal | Plot spectrum

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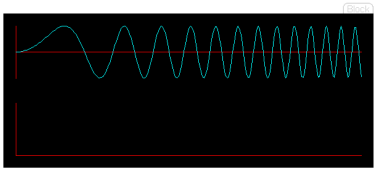
Chirp



Number of samples: 256

Sampling rate: 8000 samples / s

Signal waveform expression: sin(25000*pi*t*t)

Plot signal | Plot spectrum

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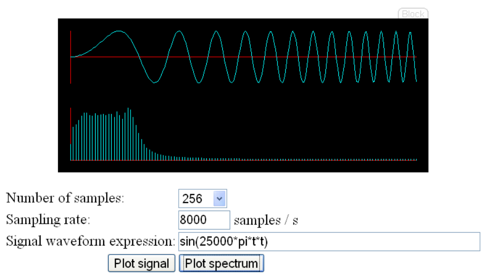
Chirp



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Another applet

http://www.falstad.com/fourier/

(seems virus-free – but scan, before you install)

FFT_applet/index.html

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Properties

- Time shift sounds the same
  - Changes only phase, not amplitudes
- Sawtooth has almost all frequencies
  - With decreasing amplitude
- Spike has all frequencies

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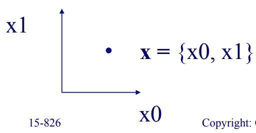
DFT: Parseval's theorem

sum($x_t^{\ 2}$) = sum (| X_f | ^{2})

Ie., DFT preserves the 'energy'

or, alternatively: it does an axis rotation:



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DFT: Parseval's theorem

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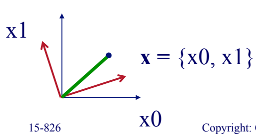

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DSP - Detailed outline

- DFT
  - what
  - why
  - how
  - Arithmetic examples
  - properties / observations
  - DCT
  - 2-d DFT
  - Fast Fourier Transform (FFT)

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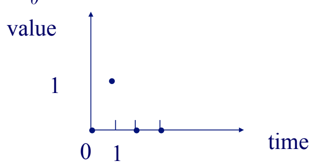
Skip

Arithmetic examples

- Impulse function: $\mathbf{x}$ = { 0, 1, 0, 0} (n = 4)
- X_0=?



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Arithmetic examples

- Impulse function: $\mathbf{x}$ = { 0, 1, 0, 0} (n = 4)
- X_0=?
- A: X_0 = 1/sqrt(4) * 1* exp(-j 2 π 0 / n) = 1/2
- X_1=?
- X_2=?
- X_3=?

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Arithmetic examples

- Impulse function: $\mathbf{x}$ = { 0, 1, 0, 0} (n = 4)
- X_0=?
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- X_1= -1/2 j
- X_2= - 1/2
- X_3= + 1/2 j
- Q: does the ‘symmetry’ property hold?

Skip

Arithmetic examples

- Impulse function: $\mathbf{x}$ = { 0, 1, 0, 0} (n = 4)
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- X_1= -1/2 j
- X_2= - 1/2
- X_3= + 1/2 j
- Q: does the 'symmetry' property hold?
- A: Yes (of course)

Skip

Arithmetic examples

- Impulse function: $\mathbf{x}$ = { 0, 1, 0, 0} (n = 4)
- X_0=?
- A: X_0 = 1/sqrt(4) * 1* exp(-j 2 π 0 / n) = 1/2
- X_1= -1/2 j
- X_2= - 1/2
- X_3= + 1/2 j
- Q: check Parseval's theorem

Skip

Arithmetic examples

- Impulse function: $\mathbf{x}$ = { 0, 1, 0, 0} (n = 4)
- X_0=?
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- X_1= -1/2 j
- X_2= - 1/2
- X_3= + 1/2 j
- Q: (Amplitude) spectrum?

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Arithmetic examples

- Impulse function: **x** = { 0, 1, 0, 0} (n = 4)
- X_0=?
- A: X_0 = 1/sqrt(4) * 1* exp(-j 2 π 0 / n) = 1/2
- X_1= -1/2 j
- X_2= - 1/2
- X_3= + 1/2 j
- Q: (Amplitude) spectrum?
- A: FLAT!

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Arithmetic examples

- Q: What does this mean?

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Arithmetic examples

- Q: What does this mean?
- A: All frequencies are equally important ->
  – we need n numbers in the frequency domain to represent just one non-zero number in the time domain!
  – "*frequency leak*"

CMU SCS

DSP - Detailed outline

- DFT
  - what
  - why
  - how
  - Arithmetic examples
  - properties / observations
  - DCT
  - 2-d DFT
  - Fast Fourier Transform (FFT)

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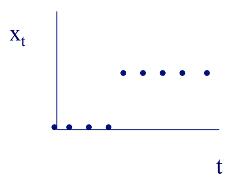
Observations

- DFT of 'step' function:
  x = { 0, 0, ..., 0, 1, 1, ... 1}



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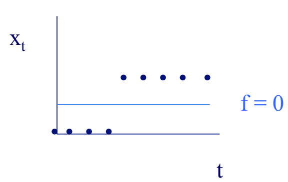
Observations

- DFT of 'step' function:
  x = { 0, 0, ..., 0, 1, 1, ... 1}



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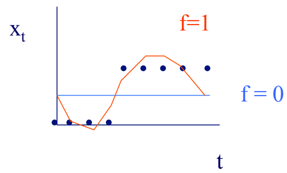
Observations

- DFT of ‘step’ function:
  x = { 0, 0, ..., 0, 1, 1, ... 1}



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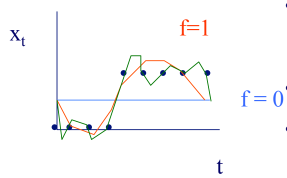
Observations

- DFT of ‘step’ function:
  x = { 0, 0, ..., 0, 1, 1, ... 1}



•the more frequencies,
the better the approx.
•‘ringing’ becomes worse
•reason: discontinuities; trends

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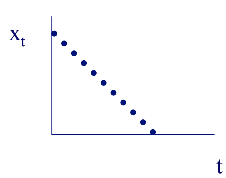
Observations

- Ringing for trends: because DFT ‘sub-consciously’ replicates the signal



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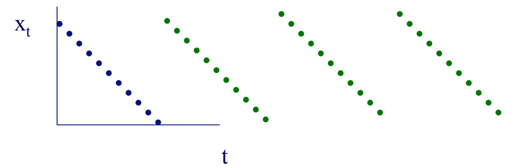
Observations

- Ringing for trends: because DFT 'sub-consciously' replicates the signal



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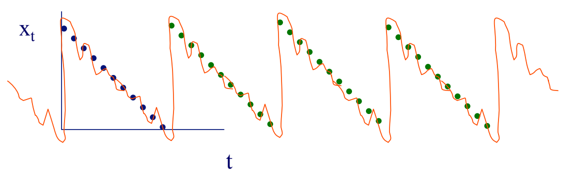
Observations

- Ringing for trends: because DFT 'sub-consciously' replicates the signal



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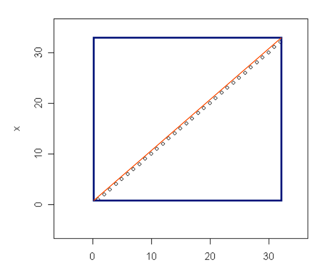
original



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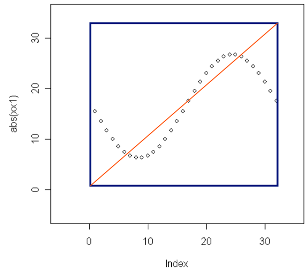
DC and 1st



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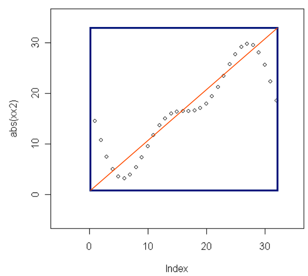
DC and 1st

And 2nd



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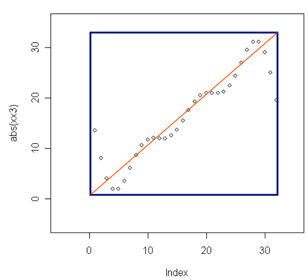
DC and 1st

And 2nd

And 3rd



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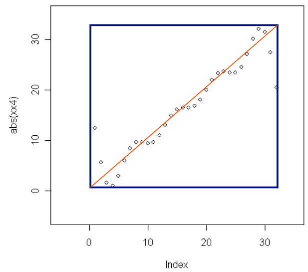
DC and 1st

And 2nd

And 3rd

And 4th



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Observations

- Q: DFT of a sinusoid, eg.

  $x_t = 3 \sin(2 \pi / 4 \, t)$

(t = 0, ... , 3)
- Q: X_0 = ?
- Q: X_1 = ?
- Q: X_2 = ?
- Q: X_3 = ?

CMU SCS

Observations

- Q: DFT of a sinusoid, eg.

  $x_t = 3 \sin(2 \pi / 4 \, t)$

(t = 0, ... , 3)
- Q: $X_0 = 0$
- Q: $X_1 = -3 j$
- Q: $X_2 = 0$
- Q: $X_3 = 3j$

•check ‘symmetry’
•check Parseval

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Observations

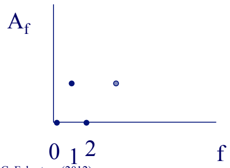
- Q: DFT of a sinusoid, eg.

$$x_t = 3 \sin(2 \pi / 4\ t)$$

(t = 0, ... , 3)

- Q: $X_0 = 0$
- Q: $X_1 = -3\,j$
- Q: $X_2 = 0$
- Q: $X_3 = 3j$

•Does this make sense?



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Property

- Shifting **x** in time does NOT change the amplitude spectrum
- eg., **x** = { 0 0 0 1} and **x'** = { 0 1 0 0 }: same (flat) amplitude spectrum
- (only the phase spectrum changes)
- Useful property when we search for patterns that may 'slide'

CMU SCS

DSP - Detailed outline

- DFT
  - what
  - why
  - how
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  - DCT
  - 2-d DFT
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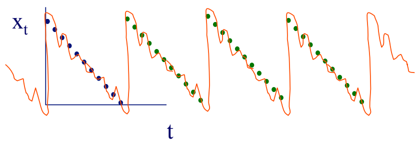
Skip

DCT

Discrete Cosine Transform

- motivation#1: DFT gives complex numbers
- motivation#2: how to avoid the 'frequency leak' of DFT on trends?



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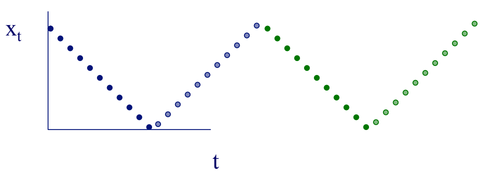
DCT

- brilliant solution to both problems: mirror the sequence, do DFT, and drop the redundant entries!



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DCT

- (see Numerical Recipes for exact formulas)

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DCT - properties

- it gives real numbers as the result
- it has no problems with trends
- it is very good when x_t and $x_{(t+1)}$ are correlated

(thus, is used in JPEG, for image compression)

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DSP - Detailed outline

- DFT
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2-d DFT

- Definition:

$$X_{f_1,f_2} = \frac{1}{\sqrt{n_1}} \frac{1}{\sqrt{n_2}} \sum_{i_1=0}^{n_1-1} \sum_{i_1=0}^{n_2-1} x_{i_1,i_2} \exp(-2\pi j i_1 f_1 / n_1) \exp(-2\pi j i_2 f_2 / n_2)$$

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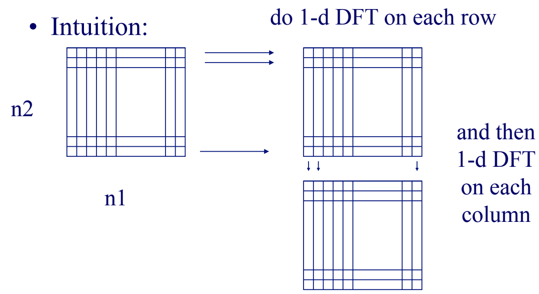
2-d DFT

- Intuition:



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2-d DFT

- Quiz: how do the basis functions look like?
- for f1= f2 =0
- for f1=1, f2=0
- for f1=1, f2=1

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2-d DFT

- Quiz: how do the basis functions look like?
- for f1= f2 =0 flat
- for f1=1, f2=0 wave on x; flat on y
- for f1=1, f2=1 ~ egg-carton

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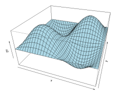
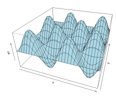
2-d DFT

- Quiz: how do the basis functions look like?
- for f1= f2 =0 flat
- for f1=1, f2=0 wave on x; flat on y
- for f1=1, f2=1 ~ egg-carton





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DSP - Detailed outline

- DFT
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FFT

- What is the complexity of DFT?

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j 2\pi\, tf / n)$$

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Skip

FFT

- What is the complexity of DFT?

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j2\pi\, tf/n)$$

- A: Naively, O(n^2)

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FFT

- However, if n is a power of 2 (or a number with many divisors), we can make it

O(n log n)

Main idea: if we know the DFT of the odd time-ticks, and of the even time-ticks, we can quickly compute the whole DFT

Details: in Num. Recipes

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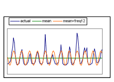
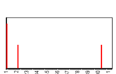
DFT - Conclusions

- It spots periodicities (with the '**amplitude spectrum'**)
- can be quickly computed (O(n log n)), thanks to the FFT algorithm.
- **standard** tool in signal processing (speech, image etc signals)





Freq.

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Detailed outline

- primary key indexing
- ..
- multimedia
- Digital Signal Processing (DSP) tools
  - Discrete Fourier Transform (DFT)
  - Discrete Wavelet Transform (DWT)

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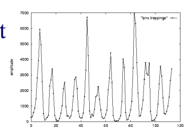
Reminder: Problem:

Goal: given a signal (eg., #packets over time)

Find: patterns, periodicities, and/or **compress**

count



year

lynx caught per year
(packets per day;
virus infections per month)

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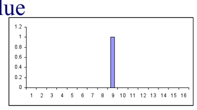
Wavelets - DWT

- DFT is great - but, how about compressing a spike?

value



time

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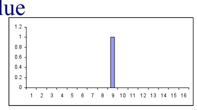
CMU SCS

Wavelets - DWT

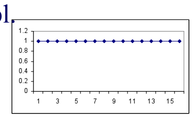
- DFT is great - but, how about compressing a spike?
- A: Terrible - all DFT coefficients needed!

value



time

Ampl.



Freq.

CMU SCS

Wavelets - DWT

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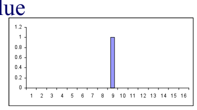
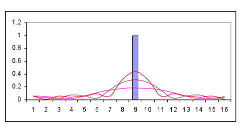


time



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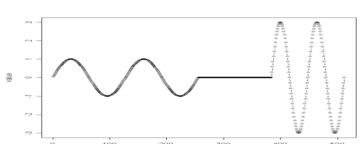
Wavelets - DWT

- Similarly, DFT suffers on short-duration waves (eg., baritone, silence, soprano)

value



time

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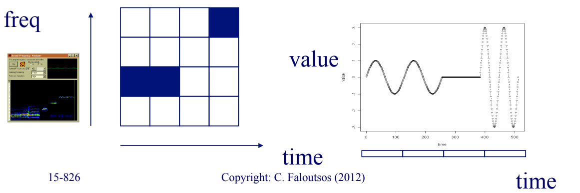
Wavelets - DWT

- Solution#1: Short window Fourier transform (SWFT)
- But: how short should be the window?



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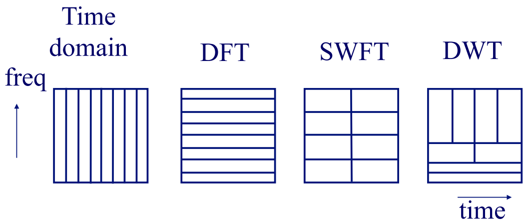
Wavelets - DWT

- Answer: **multiple** window sizes! -> DWT



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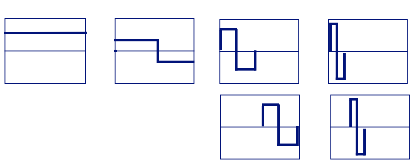
Haar Wavelets

- subtract sum of left half from right half
- repeat recursively for quarters, eight-ths, ...



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Wavelets - construction

x0 x1 x2 x3 x4 x5 x6 x7

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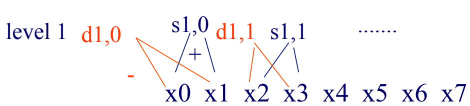
Wavelets - construction



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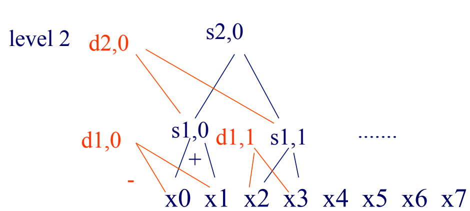
Wavelets - construction



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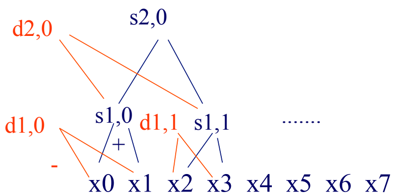
Wavelets - construction

etc ...



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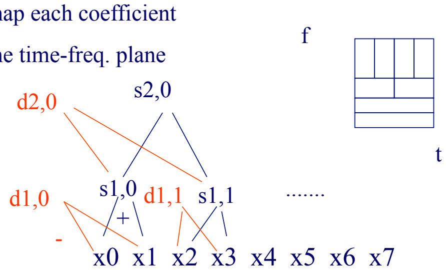
Wavelets - construction

Q: map each coefficient

on the time-freq. plane



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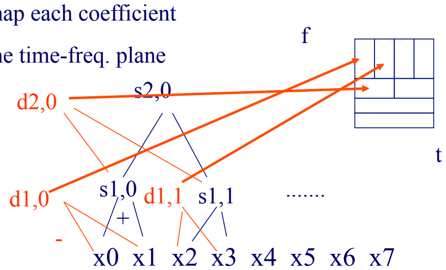
Wavelets - construction

Q: map each coefficient

on the time-freq. plane



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Haar wavelets - code

```
#!/usr/bin/perl5
# expects a file with numbers
# and prints the dwt transform
# The number of time-ticks should be a power of 2
# USAGE
#    haar.pl <fname>

my @vals=();
my @smooth; # the smooth component of the signal
my @diff;   # the high-freq. component

# collect the values into the array @val
while(<>){
        @vals = ( @vals ,  split );
}
```

```
my $len = scalar(@vals);
my $half = int($len/2);
while($half >= 1 ){
  for(my $i=0; $i< $half; $i++){
        $diff [$i] = ($vals[2*$i] - $vals[2*$i + 1] )/ sqrt(2);
        print "\t", $diff[$i];
        $smooth [$i] = ($vals[2*$i] + $vals[2*$i + 1] )/ sqrt(2);
  }
  print "\n";
  @vals = @smooth;
  $half = int($half/2);
}
print "\t", $vals[0], "\n" ;      # the final, smooth component
```

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Wavelets - construction

Observation1:

- '+' can be some weighted addition
- '-' is the corresponding weighted difference ('Quadrature mirror filters')

Observation2: unlike DFT/DCT,

- there are *many* wavelet bases: Haar, Daubechies-4, Daubechies-6, Coifman, Morlet, Gabor, ...

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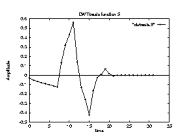
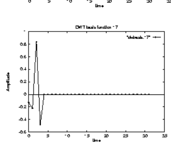
Wavelets - how do they look like?





- E.g., Daubechies-4

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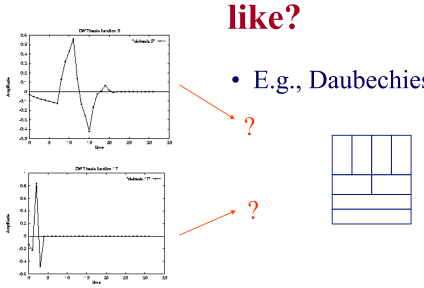
Wavelets - how do they look like?

- E.g., Daubechies-4



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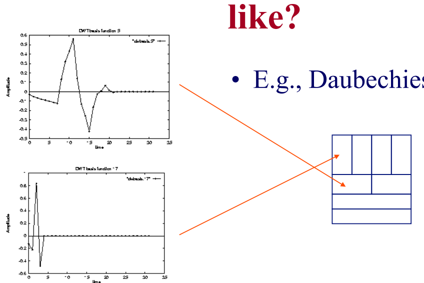
Wavelets - how do they look like?

- E.g., Daubechies-4



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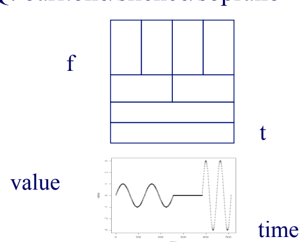
Wavelets - Drill#1:

- Q: baritone/silence/soprano - DWT?



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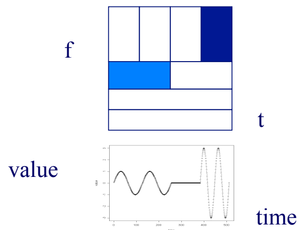
Wavelets - Drill#1:

- Q: baritone/silence/soprano - DWT?



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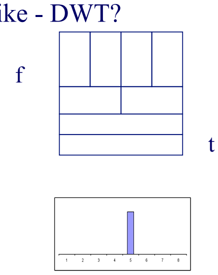
Wavelets - Drill#2:

- Q: spike - DWT?



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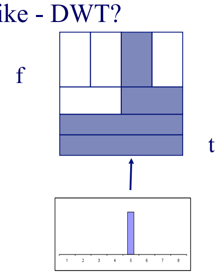
Wavelets - Drill#2:

- Q: spike - DWT?



0.00 0.00 **0.71** 0.00

0.00 **0.50**

-0.35

0.35

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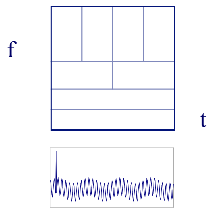
Wavelets - Drill#3:

- Q: weekly + daily periodicity, + spike - DWT?



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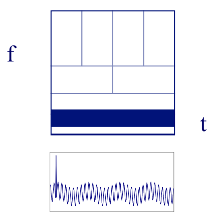
Wavelets - Drill#3:

- Q: **weekly** + daily periodicity, + spike - DWT?



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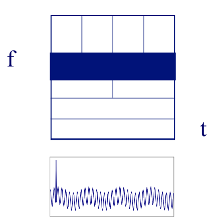
Wavelets - Drill#3:

- Q: weekly + **daily** periodicity, + spike - DWT?



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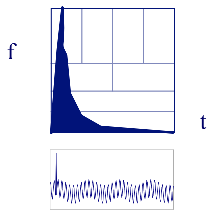
Wavelets - Drill#3:

- Q: weekly + daily periodicity, + **spike** - DWT?



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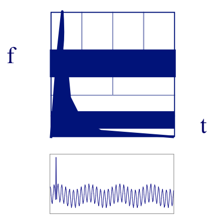
Wavelets - Drill#3:

- Q: weekly + daily periodicity, + spike - DWT?



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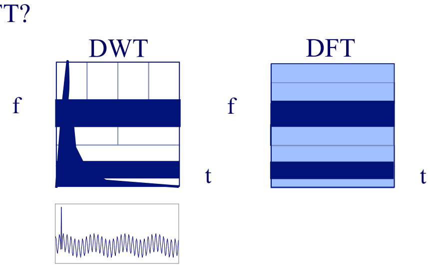
Wavelets - Drill#3:

- Q: DFT?



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Wavelets - Drill:

Let's see it live:

http://dsp.rice.edu/software/dsp-teaching-tools

delta; cosine; cosine2; chirp

- Haar vs Daubechies-4, -6, etc

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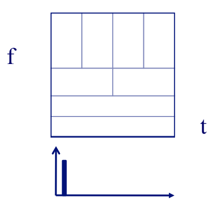
Delta?

x(0)=1; x(t)= 0 elsewhere



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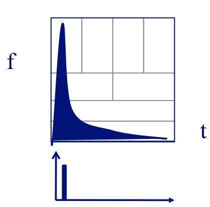
Delta?

x(0)=1; x(t)= 0 elsewhere



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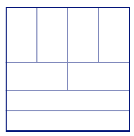
2 cosines?

x(t) = cos(2 * pi * 4 * t / 1024) +
5 * cos(2 * pi * 8 * t / 1024)

f

t

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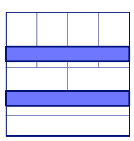
2 cosines?

x(t) = cos(2 * pi * 4 * t / 1024) +
5 * cos(2 * pi * 8 * t / 1024)

f

t

Which one
is for freq.=4?

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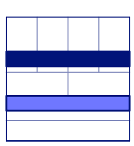
2 cosines?

x(t) = cos(2 * pi * 4 * t / 1024) +
5 * cos(2 * pi * 8 * t / 1024)

f~8 →
f~4 →
f

t

Which one
is for freq.=4?

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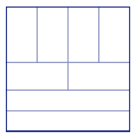
Chirp?

x(t) = cos(2 * pi * t * t / 1024)

f



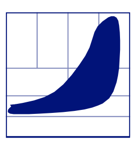
t

CMU SCS

Chirp?

x(t) = cos(2 * pi * t * t / 1024)

f



t

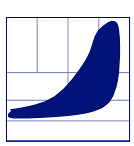
CMU SCS

Chirp?

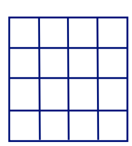
x(t) = cos(2 * pi * t * t / 1024)

SWFT?

f



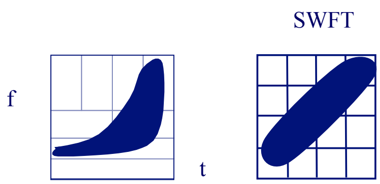
t



CMU SCS

Chirp?

x(t) = cos(2 * pi * t * t / 1024)



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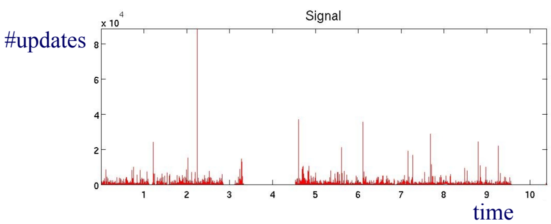
More examples (BGP updates)



BGP-lens: Patterns and Anomalies in Internet Routing Updates B. Aditya Prakash et al, SIGKDD 2009

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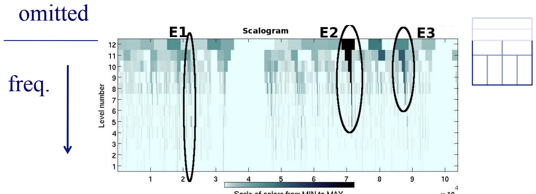
More examples (BGP updates)

Low freq.: omitted



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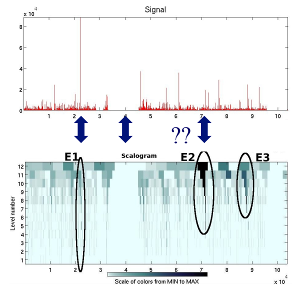
More examples (BGP updates)

freq.



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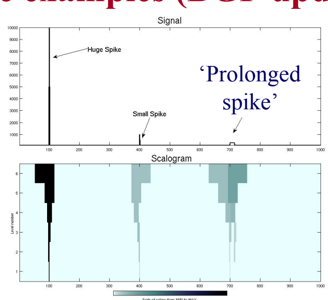
More examples (BGP updates)

‘Prolonged spike’

freq.



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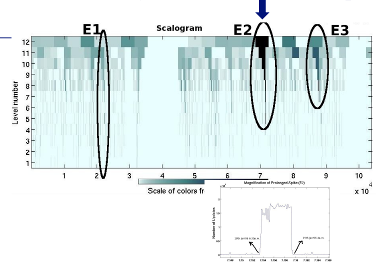
More examples (BGP updates)

15K msgs, for several hours: 6pm-4am

freq.



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Wavelets - Drill

- Or use 'R', 'octave' or 'matlab' – R:

```
install.packages("wavelets")
library("wavelets")
X1<-c(1,2,3,4,5,6,7,8)
dwt(X1, n.levels=3, filter="d4")
mra(X1, n.levels=3, filter="d4")
```


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Wavelets - k-dimensions?

- easily defined for any dimensionality (like DFT, DCT)

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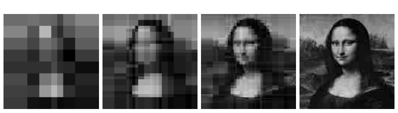
Wavelets - example

http://grail.cs.washington.edu/projects/query/

Wavelets achieve *great* compression:



20 100 400 16,000

coefficients

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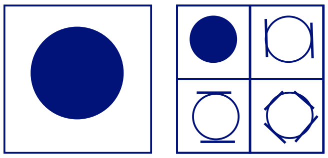
Wavelets - intuition

- Edges (horizontal; vertical; diagonal)



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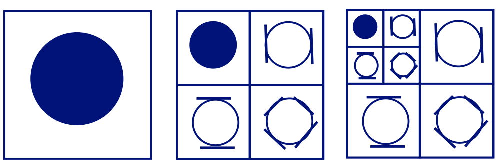
Wavelets - intuition

- Edges (horizontal; vertical; diagonal)
- recurse



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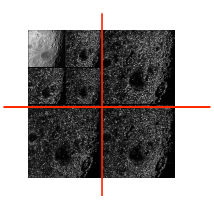
Wavelets - intuition

- Edges (horizontal; vertical; diagonal)
- http://www331.jpl.nasa.gov/public/wave.html



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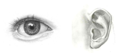
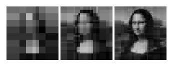
Advantages of Wavelets

- Better compression (better RMSE with same number of coefficients)
- closely related to the processing of the mammalian eye and ear



- Good for progressive transmission



- handle spikes well
- usually, fast to compute (O(*n*)!)

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Overall Conclusions

- DFT, DCT spot periodicities
- DWT : multi-resolution - matches processing of mammalian ear/eye better
- All three: powerful tools for compression, pattern detection in real signals
- All three: included in math packages (matlab, R, mathematica, ...)

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Resources

- Numerical Recipes in C: great description, intuition and code for all three tools
- *xwpl:* open source wavelet package from Yale, with excellent GUI.

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Resources (cont'd)

- (defunct?) http://www.dsptutor.freeuk.com/jsanalyser/FFTSpectrumAnalyser.html : Nice java applets
- http://www.relisoft.com/freeware/freq.html : voice frequency analyzer (needs microphone – MSwindows only)

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Resources (cont'd)

- www-dsp.rice.edu/software/EDU/mra.shtml (wavelets and other demos)
- R ('install.packages("wavelets"))

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