


CarnegieMellon

15-826: Multimedia Databases and Data Mining

Lecture #23: Multimedia indexing
C. Faloutsos

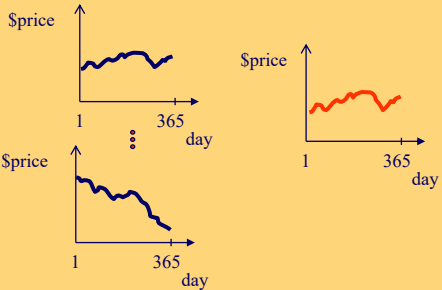
1

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Problem

- Q: Find stocks similar to <MSFT>



Price

1 365 day

Price

1 365 day


Price

1 365 day

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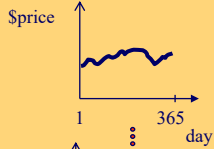
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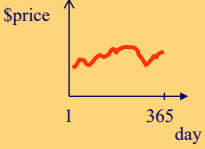


Solution

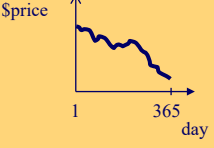
- Q: Find stocks similar to <MSFT>
- A: GEMINI: Extract features + SAM



Price vs day (1 to 365)



Price vs day (1 to 365)



Price vs day (1 to 365)

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Must-read Material

- [MM Textbook](#), chapters 7, 8, 9 and 10.
- Myron Flickner, et al: [Query by Image and Video Content: the QBIC System](#) IEEE Computer 28, 9, Sep. 1995, pp. 23-32.
- [Journal of Intelligent Inf. Systems, 3, 3/4, pp. 231-262, 1994](#) (An earlier, more technical version of the IEEE Computer '95 paper.)
- FastMap: [Textbook](#) chapter 11; Also in: C. Faloutsos and K.I. Lin *FastMap: A Fast Algorithm for Indexing, Data-Mining and Visualization of Traditional and Multimedia Datasets* ACM SIGMOD 95, pp. 163-174.

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Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- ➔ • Indexing - similarity search
- Data Mining

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Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
- fractals
- text
- Singular Value Decomposition (SVD)
- Multimedia
 - DSP
 - ➔ – indexing
- ...

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Multimedia - Detailed outline

- Multimedia indexing
 - ➔ – Motivation / problem definition
 - Main idea / time sequences
 - images
 - sub-pattern matching
 - automatic feature extraction / FastMap

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Problem

Given a large collection of (multimedia)
records (eg. stocks)
Allow fast, similarity queries

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Applications

- time series: financial, marketing (click-streams!), ECGs, sound;
- images: medicine, digital libraries, education, art
- higher-d signals: scientific db (eg., astrophysics), medicine (MRI scans), entertainment (video)

Sample queries

- find medical cases similar to Smith's
- Find pairs of stocks that move in sync
- Find pairs of documents that are similar (plagiarism?)
- find faces similar to 'Tiger Woods'

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Detailed problem defn.:

Problem:

- given a set of multimedia objects,
- find the ones similar to a desirable query object
- for example:

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Price

1 365 day

Price

1 365 day

Price

1 365 day

distance function: by **expert**
(eg, Euclidean distance)

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Types of queries

- whole match vs sub-pattern match
- range query vs nearest neighbors
- all-pairs query

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Design goals

- Fast (faster than seq. scan)
- 'correct' (ie., no false alarms; no false dismissals)

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Multimedia - Detailed outline

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 - ➔ – Main idea / time sequences
 - images
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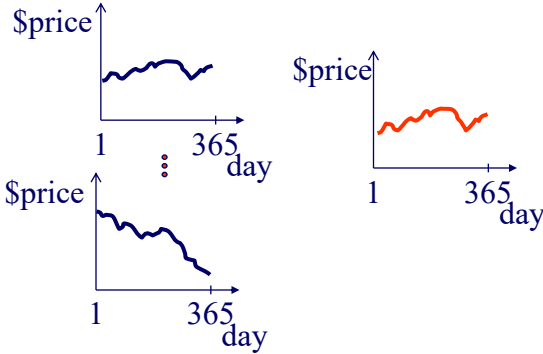
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Main idea

- Eg., time sequences, ‘whole matching’, range queries, Euclidean distance



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Main idea

- Seq. scanning works - how to do faster?

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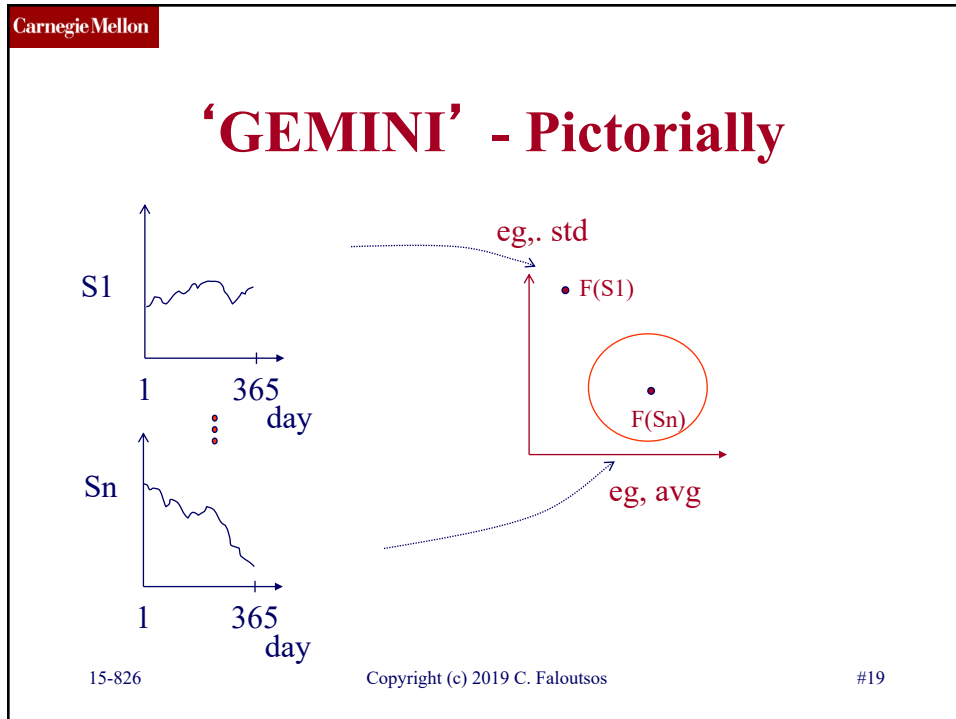
Idea: 'GEMINI'

(GENeric Multimedia INdexIng)

Extract a few numerical features, for a 'quick and dirty' test

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GEMINI

Solution: Quick-and-dirty' filter:

- extract n features (numbers, eg., avg., etc.)
- map into a point in n -d feature space
- organize points with off-the-shelf spatial access method (‘SAM’)
- discard false alarms

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GEMINI

Important: Q: how to guarantee no false dismissals?

A1: preserve distances (but: difficult/impossible)

A2: **Lower-bounding lemma**: if the mapping 'makes things look closer', then there are **no** false dismissals

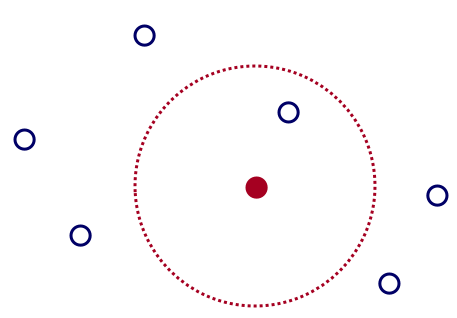
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GEMINI

- 'proof' of lower-bounding lemma



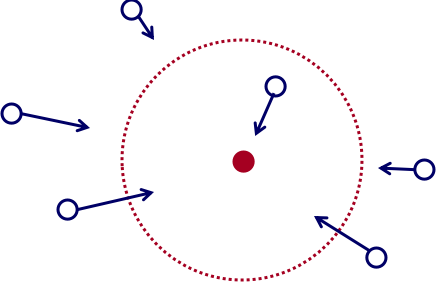
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GEMINI

- ‘proof’ of lower-bounding lemma



Lower-bounding:
Makes objects
look closer to each
other (& to query
object)

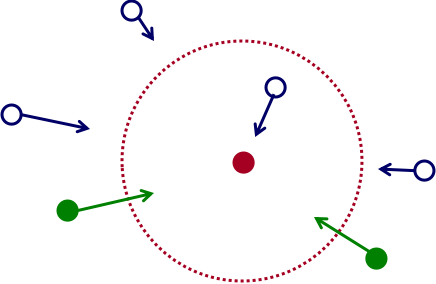
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GEMINI

- ‘proof’ of lower-bounding lemma



Lower-bounding:
Makes objects
look closer to each
other (& to query
object)
-> **ONLY false
alarms**

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GEMINI

Important:
Q: how to extract features?
A: *“if I have only one number to describe my object, what should this be?”*

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Time sequences

Q: what features?

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Time sequences


Q: what features?

A: Fourier coefficients (we'll see them in detail soon)

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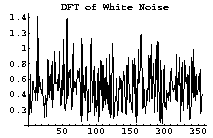
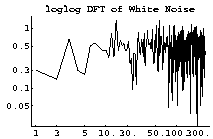
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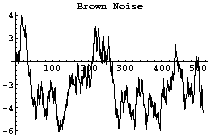
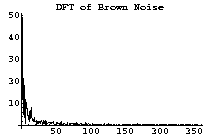
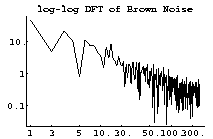
details

Time sequences

white noise

brown noise

Fourier spectrum

... in log-log

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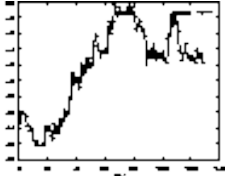
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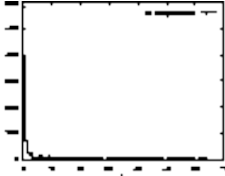
details

Time sequences

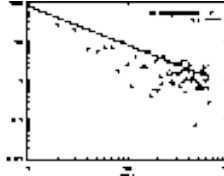
- Eg.:



(a) IBM stock



(b) spectrum
(linear scales)



(c) spectrum
(log scales)

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details

Time sequences

- conclusion: colored noises are well approximated by their first few Fourier coefficients
- colored noises appear in nature:

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Time sequences

- brown noise: stock prices ($1/f^2$ energy spectrum)
- pink noise: works of art ($1/f$ spectrum)
- black noises: water reservoirs ($1/f^b$, $b > 2$)
- (slope: related to 'Hurst exponent', for self-similar traffic, like, eg. Ethernet/web [Schroeder], [Leland+])

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Time sequences - results

- keep the first 2-3 Fourier coefficients
- faster than seq. scan
- NO false dismissals (see book)

time

coeff. kept

← total
← cleanup-time
← r-tree time

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Time sequences - improvements:


- improvements/variations:
[Kanellakis+Goldin], [Mendelzon+Rafiei]
- could use Wavelets, or DCT
- could use segment averages [Yi+2000]

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Multimedia - Detailed outline

- multimedia
 - Motivation / problem definition
 - Main idea / time sequences
 -  – images (color, shapes)
 - sub-pattern matching
 - automatic feature extraction / FastMap

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Images - color

what is an image?
A: 2-d array

COLOR IMAGE, eg. 256x256

i-th pixel:
(r_i, g_i, b_i)

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Images - color

Color histograms, and distance function

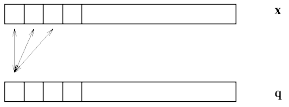
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Images - color

Mathematically, the distance function is:

$$distance_{histogram}(\vec{x}, \vec{q}) = (\vec{x} - \vec{q}) \begin{bmatrix} a_{RR} & a_{RP} & \dots \\ a_{PR} & a_{PP} & \dots \\ \dots & \dots & \dots \end{bmatrix} (\vec{x} - \vec{q})^t$$


$\dots = (\vec{x} - \vec{q}) \mathcal{A} (\vec{x} - \vec{q})^t$

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Images - color

Problem: ‘cross-talk’ :

- Features are not orthogonal ->
- SAMs will not work properly

- Q: what to do?
- A: feature-extraction question

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Images - color

possible answers:

- avg red, avg green, avg blue

it turns out that this lower-bounds the histogram distance ->

- no cross-talk
- SAMs are applicable

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Images - color

performance: time

The graph plots time in milliseconds on the y-axis (0 to 12000) against the fraction of database retrieved on the x-axis (0 to 0.05). Four data series are shown: Total time with filtering (solid line), CPU time with filtering (dashed line), Total time for naive sequential (dotted line), and CPU time for naive sequential (dash-dot line). The 'seq scan' method (Total time with filtering) shows a steep increase in total time, reaching approximately 10000 ms at a selectivity of 0.05. The 'w/ avg RGB' method (Total time for naive sequential) shows a much lower total time, reaching approximately 4000 ms at a selectivity of 0.05. The CPU time for both methods is relatively low and increases linearly with selectivity.

Fraction of database retrieved	Total time with filtering (seq scan)	Total time for naive sequential (w/ avg RGB)
0	0	0
0.005	~1000	~500
0.01	~2000	~1000
0.015	~3000	~1500
0.02	~4000	~2000
0.025	~5000	~2500
0.03	~6000	~3000
0.035	~7000	~3500
0.04	~8000	~4000
0.045	~9000	~4500
0.05	~10000	~5000

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Multimedia - Detailed outline

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Images - shapes

- distance function: Euclidean, on the area, perimeter, and 20 'moments'
- (Q: how to normalize them?)

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Images - shapes

- distance function: Euclidean, on the area, perimeter, and 20 'moments'
- (Q: how to normalize them?)
- A: divide by standard deviation)


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Images - shapes

- distance function: Euclidean, on the area, perimeter, and 20 'moments'
- (Q: other 'features' / distance functions?)




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Images - shapes

- distance function: Euclidean, on the area, perimeter, and 20 ‘moments’
- (Q: other ‘features’ / distance functions?
- A1: turning angle
- A2: dilations/erosions
- A3: ...)



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Images - shapes

- distance function: Euclidean, on the area, perimeter, and 20 ‘moments’
- Q: how to do dim. reduction?

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details

Images - shapes

- distance function: Euclidean, on the area, perimeter, and 20 ‘moments’
- Q: how to do dim. reduction?
- A: Karhunen-Loeve (= centered PCA/SVD)

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Images - shapes

- Performance: ~10x faster

log(# of I/Os)

Number of dimensions	Lange (I/Os)	Lange_2D (I/Os)	SVM (I/Os)	SVM_2D (I/Os)
1	5000	1000	1000	1000
2	1000	1000	100	100
4	1000	1000	100	100
6	1000	1000	100	100
10	1000	1000	100	100
15	1000	1000	100	100
20	1000	1000	100	100


of features kept

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Other shape features?




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Other shape features

- Morphology (dilations, erosions, openings, closings) [Korn+, VLDB96]

shape 

“structuring element”

R=1 

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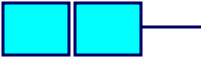
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details

Other shape features

- Morphology (dilations, erosions, openings, closings) [Korn+, VLDB96]

shape



“structuring element”

R=0.5	●
R=1	●
R=2	●

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details

Other shape features

- Morphology (dilations, erosions, openings, closings) [Korn+, VLDB96]

shape




“structuring element”

R=0.5	
R=1	
R=2	

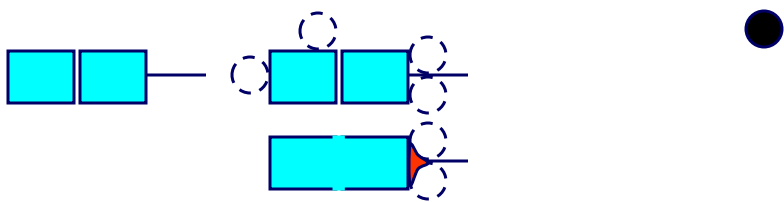
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Morphology: closing  details


- fill in small gaps
- **very similar** to ‘alpha contours’



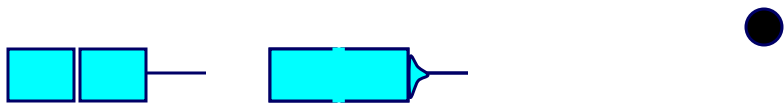
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Morphology: closing  details

- fill in small gaps



‘closing’,
with $R=1$

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details

Morphology: opening

- ‘closing’, for the complement =
- trim small extremities

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details

Morphology: opening

- ‘closing’, for the complement =
- trim small extremities

‘opening’
with $R=1$


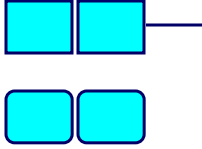

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details

Morphology

- Closing: fills in gaps 
- Opening: trims extremities 
- All wrt a structuring element: 

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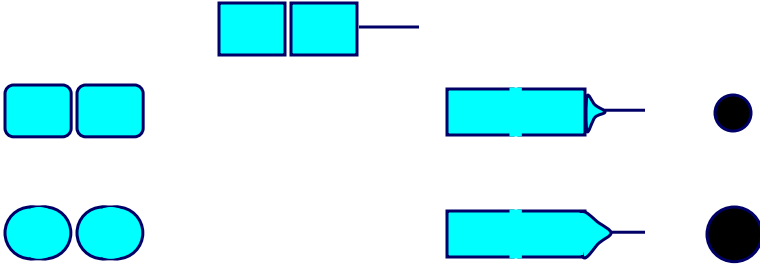
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details

Morphology

- Features: areas of openings ($R=1, 2, \dots$) and closings



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Morphology

- Powerful method:
- ‘pattern spectrum’ [Maragos+]
- ‘skeletonization’ of images
- ‘Alpha-shapes’ [Edelsbrunner]
- Book: *An introduction to morphological image processing*, by Edward R. Dougherty

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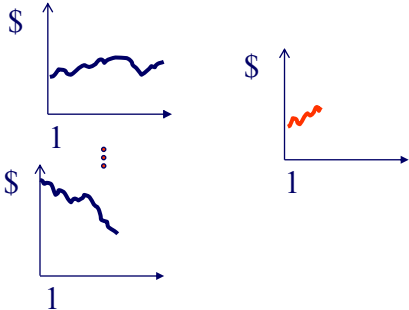
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Sub-pattern matching

- Problem: find **sub**-sequences that match the given query pattern



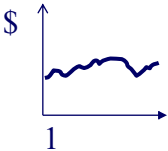
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Sub-pattern matching

- Q: how to proceed?
- Hint: try to turn it into a 'whole-matching' problem (how?)



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Sub-pattern matching

- Assume that queries have minimum duration w ; (eg., $w=7$ days)
- divide data sequences into windows of width w (overlapping, or not?)

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Sub-pattern matching

- Assume that queries have minimum duration w ; (eg., $w=7$ days)
- divide data sequences into windows of width w (overlapping, or not?)
- A: sliding, overlapping windows. Thus: trails

Pictorially:

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Sub-pattern matching

Offset $c=0$ ——— time
 $c=1$ ———
 ———

feature2 $c=1$
 $c=2$
 feature1

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Sub-pattern matching

F_2 C_1 C_2 F_1 MBR_1 MBR_2 F_1 F_2

sequences -> trails -> MBRs in feature space

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Sub-pattern matching

Q: do we store all points? why not?

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Sub-pattern matching

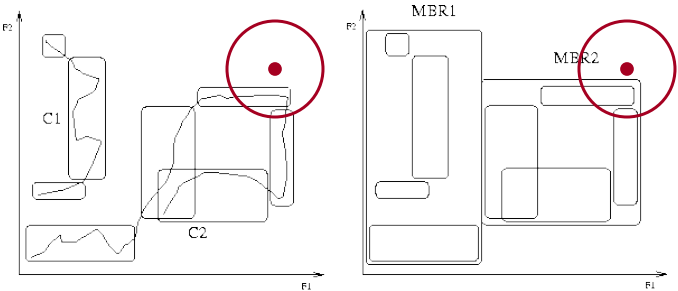
Q: how to do range queries of duration w ?

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Sub-pattern matching



Q: how to do range queries of duration w ?

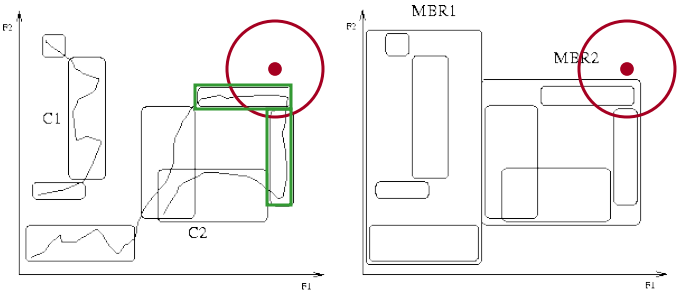
A: R-tree; find qualifying stocks and intervals

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Sub-pattern matching



Q: how to do range queries of duration w ?

A: R-tree; find qualifying stocks and intervals

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Sub-pattern matching

Q: how to do range queries of duration $>w$ (say, $2*w$)?

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Sub-pattern matching

Q: how to do range queries of duration $>w$ (say, $2*w$)?

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Sub-pattern matching

Q: how to do range queries of duration $>w$ (say, $2*w$)?
 A: Two range queries of radius epsilon and intersect
 (or two queries of smaller radius and union – see paper)

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Sub-pattern matching

(improvement [Moon+2001])

- use non-overlapping windows, for data

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Conclusions

- GEMINI works for any setting (time sequences, images, etc)
- uses a ‘quick and dirty’ filter
- faster than seq. scan
- (but: how to extract features automatically?)

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Multimedia - Detailed outline

- multimedia
 - Motivation / problem definition
 - Main idea / time sequences
 - images (color; shape)
 - sub-pattern matching
 - ➔ – automatic feature extraction / FastMap

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FastMap

Automatic feature extraction:

- Given a dissimilarity function of objects
- Quickly map the objects to a (k-d) 'feature' space.
- (goals: indexing and/or visualization)

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FastMap

	O1	O2	O3	O4	O5
O1	0	1	1	100	100
O2	1	0	1	100	100
O3	1	1	0	100	100
O4	100	100	100	0	1
O5	100	100	100	1	0

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FastMap

- Multi-dimensional scaling (MDS) can do that, but in $O(N^2)$ time

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MDS

Multi Dimensional Scaling

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Main idea: projections

We want a **linear** algorithm: FastMap
[SIGMOD95]

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FastMap - next iteration

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Results

Documents / cosine similarity ->
Euclidean distance (how?)

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Results

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Applications: time sequences

- given n co-evolving time sequences
- visualize them + find rules [ICDE00]

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Applications - financial

- currency exchange rates [ICDE00]

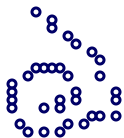
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Variations

- Isomap [Tenenbaum, de Silva, Langford, 2000]
- LLE (Local Linear Embedding) [Roweis, Saul, 2000]
- MVE (Minimum Volume Embedding) [Shaw & Jebara, 2007]



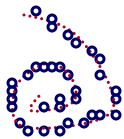
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Variations

- Isomap [Tenenbaum, de Silva, Langford, 2000]
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Conclusions

- GEMINI works for multiple settings
- FastMap can extract ‘features’ automatically (-> indexing, visual d.m.)

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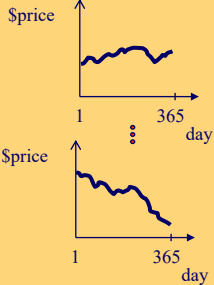
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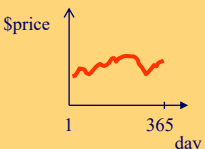
Solution

- Q: Find stocks similar to <MSFT>
- A: GEMINI: Extract features + SAM
 - A’: and FastMap etc, for feature extraction



Price

1 365 day



Price

1 365 day

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