CMU SCS

15-826: Multimedia Databases and Data Mining

Lecture #11: Fractals: M-trees and dim. curse (case studies – Part II)

C. Faloutsos

CMU SCS

Must-read Material

- Alberto Belussi and Christos Faloutsos, Estimating the Selectivity of Spatial Queries Using the `Correlation' Fractal Dimension Proc. of VLDB, p. 299-310, 1995

CMU SCS

Optional Material

Optional, but **very** useful: Manfred Schroeder *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise* W.H. Freeman and Company, 1991

CMU SCS

Outline

Goal: ‘Find similar / interesting things’

- Intro to DB
- Indexing - similarity search
- Data Mining

CMU SCS

Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
  - z-ordering
  - R-trees
  - misc
- fractals
  - intro
  - applications
- text
- ...

CMU SCS

Indexing - Detailed outline

- fractals
  - intro
  - applications
    - disk accesses for R-trees (range queries)
    - dimensionality reduction
    - selectivity in M-trees
    - dim. curse revisited
    - “fat fractals”
    - quad-tree analysis [Gaede+]

CMU SCS

What else can they solve?

- ✓ separability [KDD’02]
- forecasting [CIKM’02]
- ✓ dimensionality reduction [SBBD’00]
- non-linear axis scaling [KDD’02]
- ✓ disk trace modeling [Wang+’02]
- ➡ selectivity of spatial/multimedia queries [PODS’94, VLDB’95, ICDE’00]
- ...

CMU SCS

Metric trees - analysis

- Problem: How many disk accesses, for an M-tree?
- Given:
  - N (# of objects)
  - C (fanout of disk pages)
  - r (radius of range query - BIASED model)

CMU SCS

Metric trees - analysis

- Problem: How many disk accesses, for an M-tree?
- Given:
  - N (# of objects)
  - C (fanout of disk pages)
  - r (radius of range query - BIASED model)
- NOT ENOUGH - what else do we need?

CMU SCS

Metric trees - analysis

- A: something about the distribution

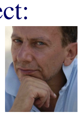
15-826 Copyright: C. Faloutsos (2009) 10

CMU SCS

Metric trees - analysis

- A: something about the distribution

[Ciaccia, Patella, Zezula, PODS98]: assumed that the distance distribution is the same, for every object:



Paolo Ciaccia Marco Patella

15-826 Copyright: C. Faloutsos (2009) 11

CMU SCS

Metric trees - analysis

- A: something about the distribution

[Ciaccia+, PODS98]: assumed that the distance distribution is the same, for every object:

F1(d) = Prob(an object is within d from object #1)

= F2(d) = ... = F(d)

15-826 Copyright: C. Faloutsos (2009) 12

CMU SCS

Metric trees - analysis

- A: something about the distribution
- Given our 'fractal' tools, we could try them - which one?

CMU SCS

Metric trees - analysis

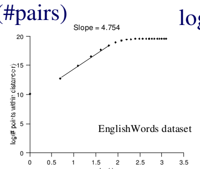
- A: something about the distribution
- Given our 'fractal' tools, we could try them - which one?
- A: Correlation integral [Traina+, ICDE2000]

CMU SCS

Metric trees - analysis

English dictionary

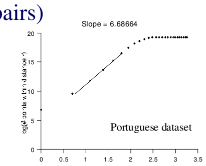
log(#pairs)



log(d)

Portuguese dictionary

log(#pairs)



log(d)

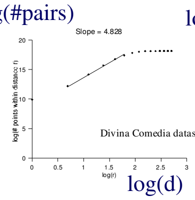
CMU SCS

Metric trees - analysis

Divina Comedia

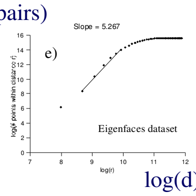
Eigenfaces

log(#pairs)



log(d)

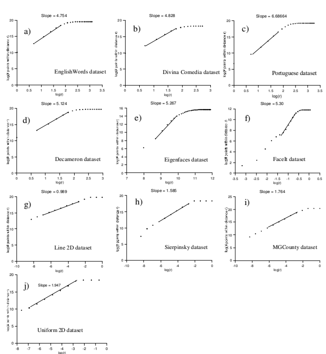
log(#pairs)



log(d)

15-826 Copyright: C. Faloutsos (2009) 16

CMU SCS



15-826 Copyright: C. Faloutsos (2009) 17

CMU SCS

Metric trees - analysis

	Data Set	N (# Objects)	Dimension	Distance Function	*Distance Exponent D*
Real Metric datasets	English	25,143	NA	L_{Edit}	4.753
	Divina Commedia	12,701	NA	L_{Edit}	4.827
	Decamerone	18,719	NA	L_{Edit}	5.124
	Portuguese	21,473	NA	L_{Edit}	6.686
	FaceIt	1,056	NA	Not divulged	6.821
Real vector datasets	MGCounty	15,559	2	L_2	1.752
	Eigenfaces	11,900	16	L_2	5.267
Synthetic datasets	Sierpinsky	9,841	2	L_2	1.584
	2D Line	20,000	2	L_2	0.989
	Uniform 2D	10,000	2	L_2	1.947

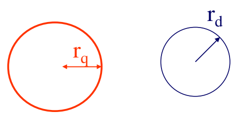
Table 2 - Datasets used in the experiments.

15-826 Copyright: C. Faloutsos (2009) 18

CMU SCS

Metric trees - analysis

- So, what is the # of disk accesses, for a node of radius r_d, on a query of radius r_q?

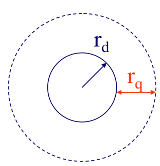


15-826 19

CMU SCS

Metric trees - analysis

- So, what is the # of disk accesses, for a node of radius r_d, on a query of radius r_q?
- A: ~ (r_d+r_q)....

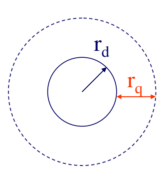


15-826 20

CMU SCS

Metric trees - analysis

- So, what is the # of disk accesses, for a node of radius r_d, on a query of radius r_q?
- A: ~ (r_d+r_q)^$\mathcal{D}$

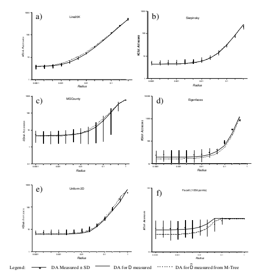


15-826 21

CMU SCS

Accuracy of selectivity formulas

log(#d.a.)



log(rq)

CMU SCS

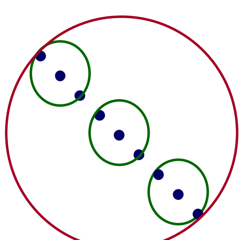
Fast estimation of $\mathcal{D}$

- Normally, $\mathcal{D}$ takes O(N^2) time
- Anything faster? suppose we have already built an M-tree

CMU SCS

Fast estimation of $\mathcal{D}$

- Hint:

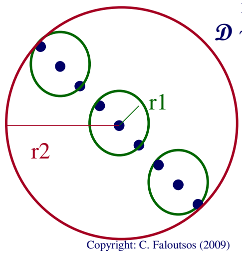


CMU SCS

Fast estimation of $\mathcal{D}$

- Hint:

ratio of radii:
$r1^{\mathcal{D}} * C = r2^{\mathcal{D}}$
$\mathcal{D} \sim \log(C) / \log(r2/r1)$



15-826 Copyright: C. Faloutsos (2009) 25

CMU SCS

Indexing - Detailed outline

- fractals
  - intro
  - applications
    - disk accesses for R-trees (range queries)
    - dimensionality reduction
    - selectivity in M-trees
    - dim. curse revisited
    - "fat fractals"
    - quad-tree analysis [Gaede+]

15-826 Copyright: C. Faloutsos (2009) 26

CMU SCS

Dim. curse revisited

- (Q: how serious is the dim. curse, e.g.:)
- Q: what is the search effort for k-nn?
  - given N points, in E dimensions, in an R-tree, with k-nn queries ('biased' model)

[Pagel, Korn + ICDE 2000]


15-826 Copyright: C. Faloutsos (2009) 27

CMU SCS

(Overview of proofs)

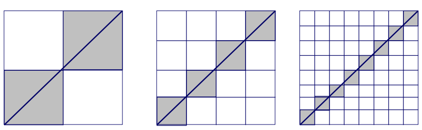
- assume that your points are uniformly distributed in a d-dimensional manifold (= hyper-plane)
- derive the formulas
- substitute d for the fractal dimension

CMU SCS

proof

Reminder: Hausdorff Dimension (D_0)

- r = side length (each dimension)
- $B(r)$ = # boxes containing points $\propto r^{D0}$



$r = 1/2$ $B = 2$ $r = 1/4$ $B = 4$ $r = 1/8$ $B = 8$

$\log r = -1$, $\log B = 1$ $\log r = -2$, $\log B = 2$ $\log r = -3$, $\log B = 3$

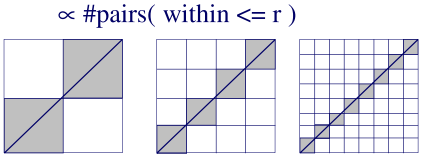
CMU SCS

proof

Reminder: Correlation Dimension (D_2)

- $S(r) = \sum p_i^2$ (squared % pts in box) $\propto r^{D2}$

  $\propto$ #pairs(within <= r)



$r = 1/2$ $S = 1/2$ $r = 1/4$ $S = 1/4$ $r = 1/8$ $S = 1/8$

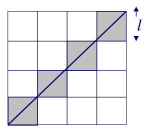
$\log r = -1$, $\log S = -1$ $\log r = -2$, $\log S = -2$ $\log r = -3$, $\log S = -3$

CMU SCS

proof

Observation #1

- How to determine avg MBR side l?
  - N = #pts, C = MBR capacity



Hausdorff dimension: $B(r) \propto r^{D0}$

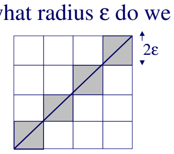
$B(l) = N/C = l^{-D0} \Rightarrow \boxed{l = (N/C)^{-1/D0}}$

CMU SCS

proof

Observation #2

- k-NN query → ε-range query
  - For k pts, what radius ε do we expect?



Correlation dimension: $S(r) \propto r^{D2}$

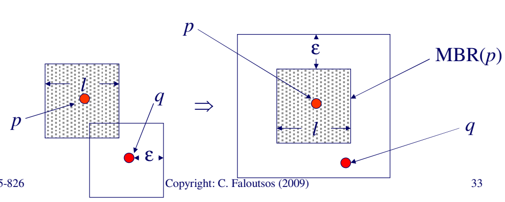
$$S(\varepsilon) = \frac{k}{N-1} = (2\varepsilon)^{D2}$$

CMU SCS

proof

Observation #3

- Estimate avg # query-sensitive anchors:
  - How many **expected** q will touch **avg** page?
  - Page touch: q stabs ε-dilated MBR(p)



CMU SCS

Asymptotic Formula

- k-NN page accesses as N $\rightarrow \infty$
  - C = capacity
  - D = fractal dimension (=$D0$ ~ $D2$)

$$P_{all}^{L\infty}(k) \approx \sum_{j=0}^{h} \left\{ \frac{1}{C^{h-j}} + \left[1 + \left(\frac{k}{C^{h-j}} \right)^{1/D} \right]^{D} \right\}$$

15-826 Copyright: C. Faloutsos (2009) 34

CMU SCS

Asymptotic Formula

$$P_{all}^{L\infty}(k) \approx \sum_{j=0}^{h} \left\{ \frac{1}{C^{h-j}} + \left[1 + \left(\frac{k}{C^{h-j}} \right)^{1/D} \right]^{D} \right\}$$

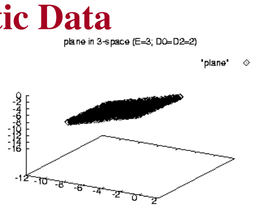
- NO mention of the embedding dimensionality!!
- Still have dim. curse, but on f.d. D

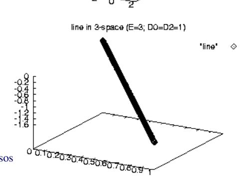
15-826 Copyright: C. Faloutsos (2009) 35

CMU SCS

Synthetic Data

- `plane`
  - $D_0 = D_2 = 2$
  - embedded in E-space
  - N = 100K
- `manifold`
  - $E = 8$
  - D_0=D_2 varies from 1-6
  - line, plane, *etc*. (in 8-d)

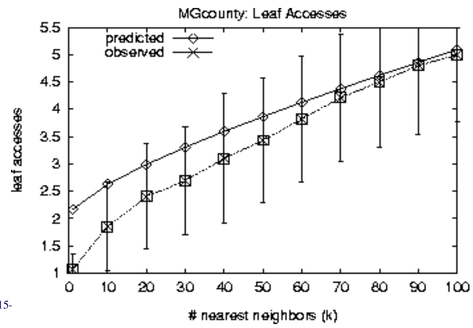




15-826 Copyright: C. Faloutsos

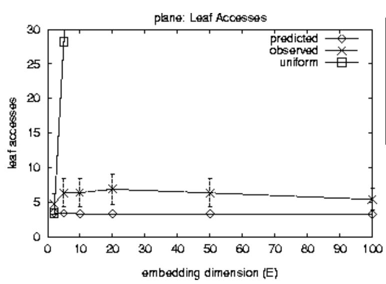
CMU SCS

Accuracy of L_∞ Formula



CMU SCS

Embedding Dimension

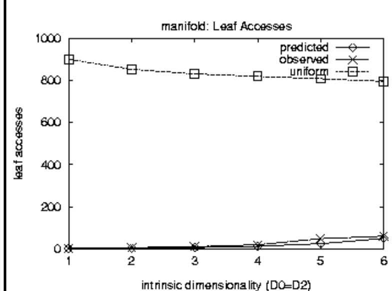


E	*unif/ind*	*fractal*	*leaf*
2	3.49	3.49	4.75
5	28.26	3.45	6.40
10	847.26	3.34	6.42
20	All	3.36	6.9
50	All	3.32	6.37
100	All	3.32	5.43

plane
$k = 50$
L_∞ dist

CMU SCS

Intrinsic Dimensionality



D	*unif/ind*	*fractal*	*leaf*
1	899.31	1.79	3.55
2	851.1	3.36	6.56
3	829.89	6.53	11.04
4	818.32	12.85	20.21
5	806.89	25.30	48.9
6	794.35	49.77	60.94

manifold
$k = 50$
L_∞ dist

CMU SCS

Non-Euclidean Data Set

E	*unif/ind*	*fractal*	*leaf*
2	3.49	2.53	4.72±1.81
10	847.26	2.53	6.42±2.11
20	all	2.53	7.76±4.12
50	all	2.53	6.15±2.82
100	all	2.53	5.64±2.32

`sierpinski`, $k = 50$, L_∞ dist

CMU SCS

Conclusions

- Worst-case theory is **over-pessimistic**
- High dimensional data can exhibit good performance if **correlated, non-uniform**
- Many real data sets are **self-similar**
- Determinant is **intrinsic** dimensionality
  - multiple fractal dimensions (D_0 and D_2)
  - indication of how far one can go

CMU SCS

References

• Ciaccia, P., M. Patella, et al. (1998). *A Cost Model for Similarity Queries in Metric Spaces*. PODS.
• Pagel, B.-U., F. Korn, et al. (2000). *Deflating the Dimensionality Curse Using Multiple Fractal Dimensions*. ICDE, San Diego, CA.
•Traina, C., A. J. M. Traina, et al. (2000). *Distance Exponent: A New Concept for Selectivity Estimation in Metric Trees*. ICDE, San Diego, CA.