

Carnegie Mellon Univ.  
Dept. of Computer Science  
15-415/615 – DB Applications

C. Faloutsos & A. Pavlo  
Lecture#5: *Relational calculus*

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General Overview - rel. model

- history
- concepts
- Formal query languages
  - relational algebra
  - **rel. tuple calculus**
  - rel. domain calculus

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Overview - detailed

- rel. tuple calculus
  - why?
  - details
  - examples
  - equivalence with rel. algebra
  - more examples; ‘safety’ of expressions
- rel. domain calculus + QBE

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## Motivation

- Q: weakness of rel. algebra?
- A: procedural
  - describes the steps (ie., 'how')
  - (still useful, for query optimization)

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## Solution: rel. calculus

- describes **what** we want
- two equivalent flavors: 'tuple' and 'domain' calculus
- basis for SQL and QBE, resp.
- Useful for proofs (see query optimization, later)

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## Rel. tuple calculus (RTC)

- first order logic

$$\{t \mid P(t)\}$$

'Give me tuples 't', satisfying predicate P - eg:

$$\{t \mid t \in STUDENT\}$$

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## Details

- symbols allowed:
  - $\wedge, \vee, \neg, \Rightarrow$
  - $>, <, =, \neq, \leq, \geq,$
  - $(, ), \in$
- quantifiers  $\forall, \exists$

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## Specifically

- Atom
  - $t \in TABLE$
  - $t.attr \leq const$
  - $t.attr \leq s.attr'$

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## Specifically

- Formula:
  - atom
  - if P1, P2 are formulas, so are  $P1 \wedge P2; P1 \vee P2...$
  - if P(s) is a formula, so are  $\exists s(P(s))$   
 $\forall s(P(s))$

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## Specifically

- Reminders:
  - DeMorgan  $P1 \wedge P2 \equiv \neg(\neg P1 \vee \neg P2)$
  - implication:  $P1 \Rightarrow P2 \equiv \neg P1 \vee P2$
  - double negation:
    - $\forall s \in TABLE (P(s)) \equiv \neg \exists s \in TABLE (\neg P(s))$
    - 'every human is mortal : no human is immortal'

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## Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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## Examples

- find all student records

$\{t \mid t \in STUDENT\}$

output  
tuple

of type 'STUDENT'

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**(Goal: evidence that  $RTC = RA$ )**

- Full proof: complicated
- We'll just show examples of the 5 RA fundamental operators, and how RTC can handle them
- (Quiz: which are the 5 fundamental op's?)

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**FUNDAMENTAL Relational operators**

• selection	$\sigma_{condition}(R)$
• projection	$\pi_{att-list}(R)$
• cartesian product	MALE x FEMALE
• set union	$R \cup S$
• set difference	$R - S$

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**Examples**

- (selection) find student record with ssn=123

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**Examples**

- ✓  $\sigma$  selection
- $\pi$  projection
- X cartesian product
- U set union
- - set difference

- (selection) find student record with ssn=123

$$\{t \mid t \in STUDENT \wedge t.ssn = 123\}$$

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**Examples**

- (projection) find **name** of student with ssn=123

~~$$\{t \mid t \in STUDENT \wedge t.ssn = 123\}$$~~

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**Examples**

- ✓  $\sigma$  selection
- ✓  $\pi$  projection
- X cartesian product
- U set union
- - set difference

- (projection) find name of student with ssn=123

$$\{t \mid \exists s \in STUDENT (s.ssn = 123 \wedge t.name = s.name)\}$$

↑  
‘t’ has only one column

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### ‘Tracing’

$$\{t \mid \exists s \in STUDENT (s.ssn = 123 \wedge t.name = s.name)\}$$

Name
aaaa
....
jones
...
zzzz

s

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

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### Examples cont'd

- ✓  $\sigma$  selection
- ✓  $\pi$  projection
- $\times$  cartesian product
- ✓  $\cup$  set union
- - set difference

- (union) get records of both PT and FT students

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### Examples cont'd

- (union) get records of both PT and FT students

$$\{t \mid t \in FT\_STUDENT \vee t \in PT\_STUDENT\}$$

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## Examples

- ✓  $\sigma$  selection
- ✓  $\pi$  projection
- $\times$  cartesian product
- ✓  $\cup$  set union
- ✓  $-$  set difference

- difference: find students that are not staff

(assuming that STUDENT and STAFF are union-compatible)

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## Examples

- difference: find students that are not staff

$$\{t \mid t \in STUDENT \wedge t \notin STAFF\}$$

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## Cartesian product

- eg., dog-breeding: MALE x FEMALE
- gives all possible couples

MALE
name
spike
spot

 $\times$ 

FEMALE
name
lassie
shiba

 $=$ 

M.name	F.name
spike	lassie
spike	shiba
spot	lassie
spot	shiba

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## Cartesian product

- ✓  $\sigma$  selection
- ✓  $\pi$  projection
- ✓ X cartesian product
- ✓  $\cup$  set union
- ✓ - set difference

- find all the pairs of (male, female)

$$\{t \mid \exists m \in MALE \wedge \exists f \in FEMALE$$

$$t.m - name = m.name \wedge$$

$$t.f - name = f.name\}$$

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## 'Proof' of equivalence

- rel. algebra  $\leftrightarrow$  rel. tuple calculus

- ✓  $\sigma$  selection
- ✓  $\pi$  projection
- ✓ X cartesian product
- ✓  $\cup$  set union
- ✓ - set difference

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## Overview - detailed

- rel. tuple calculus
  - why?
  - details
  - examples
  - equivalence with rel. algebra
  - **more examples**; 'safety' of expressions
- re. domain calculus + QBE

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## More examples

- join: find names of students taking 15-415

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## Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
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CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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## More examples

- join: find names of students taking 15-415

$$\{t \mid \exists s \in STUDENT$$

$$\wedge \exists e \in TAKES (s.ssn = e.ssn \wedge$$

$$t.name = s.name \wedge$$

$$e.c-id = 15-415)\}$$

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## More examples

- join: find names of students taking 15-415

$$\{t \mid \exists s \in STUDENT \wedge \exists e \in TAKES (s.ssn = e.ssn \wedge t.name = s.name \wedge e.c-id = 15-415)\}$$

join  
projection  
selection

(Remember: 'SPJ')

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## More examples

- 3-way join: find names of students taking a 2-unit course

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## Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
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CLASS		
c-id	c-name	units
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TAKES		
SSN	c-id	grade
123	15-413	A
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### More examples

- 3-way join: find names of students taking a 2-unit course

$$\{t \mid \exists s \in STUDENT \wedge \exists e \in TAKES$$

$\exists c \in CLASS( s.ssn = e.ssn \wedge$	join
$e.c - id = c.c - id \wedge$	
$t.name = s.name \wedge$	projection
$c.units = 2)\}$	selection

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### More examples

- 3-way join: find names of students taking a 2-unit course - in rel. algebra??

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### More examples

- 3-way join: find names of students taking a 2-unit course - in rel. algebra??

$$\pi_{name}(\sigma_{units=2}(STUDENT \bowtie TAKES \bowtie CLASS))$$

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### Even more examples:

- self -joins: find Tom's grandparent(s)

PC	
p-id	c-id
Mary	Tom
Peter	Mary
John	Tom

PC	
p-id	c-id
Mary	Tom
Peter	Mary
John	Tom

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### Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{t \mid \exists p \in PC \wedge \exists q \in PC$$

$$(p.c-id = q.p-id \wedge$$

$$p.p-id = t.p-id \wedge$$

$$q.c-id = "Tom")\}$$

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### Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

SHIPMENT	
s#	p#
s1	p1
s2	p1
s1	p2
s3	p1
s5	p3

$\div$

ABOMB
p#
p1
p2

$=$

BAD_S
s#
s1

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## Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

$$\{t \mid \forall p(p \in ABOMB \Rightarrow (\exists s \in SHIPMENT (t.s\# = s.s\# \wedge s.p\# = p.p\#)))\}$$

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## General pattern

- three equivalent versions:
  - 1) if it's bad, he shipped it  
 $\{t \mid \forall p(p \in ABOMB \Rightarrow (P(t)))\}$
  - 2) either it was good, or he shipped it  
 $\{t \mid \forall p(p \notin ABOMB \vee (P(t)))\}$
  - 3) there is no bad shipment that he missed  
 $\{t \mid \neg \exists p(p \in ABOMB \wedge (\neg P(t)))\}$

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## General pattern

- three equivalent versions:
  - 1) if it's bad, he shipped it  
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 $\{t \mid \forall p(p \notin ABOMB \vee (P(t)))\}$
  - 3) there is no bad shipment that he missed  
 $\{t \mid \neg \exists p(p \in ABOMB \wedge (\neg P(t)))\}$

$a \Rightarrow b$   
 $\neg a \vee b$

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### General pattern

- three equivalent versions:
  - 1) if it's bad, he shipped it  
 $\{t \mid \forall p(p \in ABOMB \Rightarrow (P(t)))\}$
  - 2) either it was good, or he shipped it  
 $\{t \mid \forall p(p \notin ABOMB \vee (P(t)))\}$
  - 3) there is no bad shipment that he missed  
 $\{t \mid \neg \exists p(p \in ABOMB \wedge (\neg P(t)))\}$

$\forall x(P(x)) = \neg \exists x(\neg P(x))$

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### $a \Rightarrow b$ is the same as $\neg a \vee b$

		b	
		T	F
a	T	T	F
	F	T	T

- If a is true, b must be true for the implication to be true. If a is true and b is false, the implication evaluates to false.
- If a is not true, we don't care about b, the expression is always true.

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### More on division

- find (SSNs of) students that take all the courses that ssn=123 does (and maybe even more)  
 find students 's' so that  
 if 123 takes a course => so does 's'

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### More on division

- find students that take all the courses that ssn=123 does (and maybe even more)

$$\{o \mid \forall t((t \in TAKES \wedge t.ssn = 123) \Rightarrow \exists t \in TAKES (t.c - id = t.c - id \wedge t.ssn = o.ssn))\}$$

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### Safety of expressions

- FORBIDDEN:  ~~$\{t \mid t \notin STUDENT\}$~~

It has infinite output!!

- Instead, always use  $\{t \mid \dots t \in SOME-TABLE\}$

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### Overview - conclusions

- rel. tuple calculus: DECLARATIVE
  - dfn
  - details
  - equivalence to rel. algebra
- rel. domain calculus + QBE

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## General Overview

- relational model
- Formal query languages
  - relational algebra
  - rel. tuple calculus
  - **rel. domain calculus**

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## Rel. domain calculus (RDC)

- Q: why?
- A: slightly easier than RTC, although equivalent - basis for QBE.
- idea: domain variables (w/ F.O.L.) - eg:
- ‘find STUDENT record with ssn=123’

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## Rel. Dom. Calculus

- find STUDENT record with ssn=123’

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \wedge s = 123 \}$$

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## Details

- Like R.T.C - symbols allowed:
  - $\wedge, \vee, \neg, \Rightarrow$
  - $>, <, =, \neq, \leq, \geq,$
  - $(, ), \in$
- quantifiers  $\forall, \exists$

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## Details

- but: domain (= column) variables, as opposed to tuple variables, eg:

$\langle s, n, a \rangle \in STUDENT$

SSN

↑

name

↑

address

↑

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## Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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## Examples

- find all student records

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \}$$

**RTC:**  $\{ t \mid t \in STUDENT \}$

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## Examples

- (selection) find student record with ssn=123

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## ('Proof' of RDC = RA)

- Again, we show examples of the 5 fundamental operators, in RDC

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## Examples

- (selection) find student record with ssn=123

**RTC:**  $\{t \mid t \in STUDENT \wedge t.ssn = 123\}$

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## Examples

- (selection) find student record with ssn=123

$\{ \langle 123, n, a \rangle \mid \langle 123, n, a \rangle \in STUDENT \}$

or

$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \wedge s = 123 \}$

**RTC:**  $\{t \mid t \in STUDENT \wedge t.ssn = 123\}$

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## Examples

- (projection) find name of student with ssn=123

$\{ \langle n \rangle \mid \langle 123, n, a \rangle \in STUDENT \}$

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### Examples

- (projection) find name of student with ssn=123

$$\{ \langle n \rangle \mid \exists a (\langle 123, n, a \rangle \in STUDENT) \}$$

↑  
need to 'restrict' "a"

**RTC:**  $\{ t \mid \exists s \in STUDENT (s.ssn = 123 \wedge t.name = s.name) \}$

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### Examples cont'd

- (union) get records of both PT and FT students

**RTC:**  $\{ t \mid t \in FT\_STUDENT \vee t \in PT\_STUDENT \}$

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### Examples cont'd

- (union) get records of both PT and FT students

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in FT\_STUDENT \vee \langle s, n, a \rangle \in PT\_STUDENT \}$$

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## Examples

- difference: find students that are not staff

**RTC:**  $\{t \mid t \in STUDENT \wedge t \notin STAFF\}$

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## Examples

- difference: find students that are not staff

$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \wedge \langle s, n, a \rangle \notin STAFF \}$

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## Cartesian product

- eg., dog-breeding: MALE x FEMALE
- gives all possible couples

MALE	x	FEMALE	=	<table border="1" style="border-collapse: collapse; text-align: left;"> <thead> <tr> <th style="padding: 2px;">M.name</th> <th style="padding: 2px;">F.name</th> </tr> </thead> <tbody> <tr> <td style="padding: 2px;">spike</td> <td style="padding: 2px;">lassie</td> </tr> <tr> <td style="padding: 2px;">spike</td> <td style="padding: 2px;">shiba</td> </tr> <tr> <td style="padding: 2px;">spot</td> <td style="padding: 2px;">lassie</td> </tr> <tr> <td style="padding: 2px;">spot</td> <td style="padding: 2px;">shiba</td> </tr> </tbody> </table>	M.name	F.name	spike	lassie	spike	shiba	spot	lassie	spot	shiba
M.name	F.name													
spike	lassie													
spike	shiba													
spot	lassie													
spot	shiba													

name
spike
spot

name
lassie
shiba

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## Cartesian product

- find all the pairs of (male, female) - RTC:

$$\{t \mid \exists m \in MALE \wedge \exists f \in FEMALE \\ t.m - name = m.name \wedge t.f - name = f.name\}$$

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## Cartesian product

- find all the pairs of (male, female) - RDC:

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## Cartesian product

- find all the pairs of (male, female) - RDC:

$$\{ \langle m, f \rangle \mid \langle m \rangle \in MALE \wedge \langle f \rangle \in FEMALE \}$$

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## 'Proof' of equivalence

- rel. algebra  $\leftrightarrow$  rel. domain calculus
- $\leftrightarrow$  rel. tuple calculus

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## Overview - detailed

- rel. domain calculus
  - why?
  - details
  - examples
  - equivalence with rel. algebra
  - **more examples**; 'safety' of expressions

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## More examples

- join: find names of students taking 15-415

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## Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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## More examples

- join: find names of students taking 15-415 - in RTC

$$\{t \mid \exists s \in STUDENT$$

$$\wedge \exists e \in TAKES (s.ssn = e.ssn \wedge$$

$$t.name = s.name \wedge$$

$$e.c-id = 15-415)\}$$

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## More examples

- join: find names of students taking 15-415 - in RDC

$$\{ \langle n \rangle \mid \exists s \exists a \exists g (\langle s, n, a \rangle \in STUDENT$$

$$\wedge \langle s, 15-415, g \rangle \in TAKES) \}$$

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### Sneak preview of QBE:

$$\{ \langle n \rangle \mid \exists s \exists a \exists g (\langle s, n, a \rangle \in STUDENT \wedge \langle s, 15-415, g \rangle \in TAKES) \}$$

STUDENT		
Ssn	Name	Address
_x	P.	

TAKES		
SSN	c-id	grade
_x	15-415	

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### Sneak preview of QBE:

- very user friendly
- heavily based on RDC
- very similar to MS Access interface and pgAdminIII

STUDENT		
Ssn	Name	Address
_x	P.	

TAKES		
SSN	c-id	grade
_x	15-415	

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### More examples

- 3-way join: find names of students taking a 2-unit course - in RTC:

$$\{ t \mid \exists s \in STUDENT \wedge \exists e \in TAKES \wedge \exists c \in CLASS ( s.ssn = e.ssn \wedge e.c-id = c.c-id \wedge t.name = s.name \wedge c.units = 2) \}$$

| join  
| projection  
| selection

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### Reminder: our Mini-U db

**STUDENT**

Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

**CLASS**

c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

**TAKES**

SSN	c-id	grade
123	15-413	A
234	15-413	B

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### More examples

- 3-way join: find names of students taking a 2-unit course

$$\{ \langle n \rangle \mid \dots \dots \dots \}$$

$$\langle s, n, a \rangle \in STUDENT \wedge$$

$$\langle s, c, g \rangle \in TAKES \wedge$$

$$\langle c, cn, 2 \rangle \in CLASS \}$$

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### More examples

- 3-way join: find names of students taking a 2-unit course

$$\{ \langle n \rangle \mid \exists s, a, c, g, cn ($$

$$\langle s, n, a \rangle \in STUDENT \wedge$$

$$\langle s, c, g \rangle \in TAKES \wedge$$

$$\langle c, cn, 2 \rangle \in CLASS$$

$$) \}$$

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### Even more examples:

- self -joins: find Tom's grandparent(s)

PC	
p-id	c-id
Mary	Tom
Peter	Mary
John	Tom

PC	
p-id	c-id
Mary	Tom
Peter	Mary
John	Tom

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### Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{t \mid \exists p \in PC \wedge \exists q \in PC$$

$$(p.c-id = q.p-id \wedge$$

$$p.p-id = t.p-id \wedge$$

$$q.c-id = "Tom")\}$$

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### Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{t \mid \exists p \in PC \wedge \exists q \in PC$$

$$(p.c-id = q.p-id \wedge$$

$$p.p-id = t.p-id \wedge$$

$$q.c-id = "Tom")\}$$

$$\{ \langle g \rangle \mid \exists p (\langle g, p \rangle \in PC \wedge$$

$$\langle p, "Tom" \rangle \in PC) \}$$

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### Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{ \langle g \rangle \mid \exists p (\langle g, p \rangle \in PC \wedge \langle p, \text{"Tom"} \rangle \in PC) \}$$

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### Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

SHIPMENT	
s#	p#
s1	p1
s2	p1
s1	p2
s3	p1
s5	p3

$\div$

ABOMB	
p#	
p1	
p2	

$=$

BAD_S	
s#	
s1	

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### Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

$$\{ t \mid \forall p (p \in ABOMB \Rightarrow (\exists s \in SHIPMENT (t.s\# = s.s\# \wedge s.p\# = p.p\#))) \}$$

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### Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

$$\{t \mid \forall p(p \in ABOMB \Rightarrow (\exists s \in SHIPMENT (t.s\# = s.s\# \wedge s.p\# = p.p\#)))\}$$

$$\{< s \mid \forall p(< p \> \in ABOMB \Rightarrow < s, p \> \in SHIPMENT)\}$$

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### More on division

- find students that take all the courses that ssn=123 does (and maybe even more)

$$\{o \mid \forall t((t \in TAKES \wedge t.ssn = 123) \Rightarrow \exists t1 \in TAKES (t1.c-id = t.c-id \wedge t1.ssn = o.ssn))\}$$

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### More on division

- find students that take all the courses that ssn=123 does (and maybe even more)

$$\{< s \mid \forall c(\exists g(< 123, c, g \> \in TAKES) \Rightarrow \exists g'(< s, c, g' \> \in TAKES))\}$$

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## Safety of expressions

- similar to RTC
- FORBIDDEN:

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \notin STUDENT \}$$

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## Overview - detailed

- **rel. domain calculus + QBE**
  - dfn
  - details
  - equivalence to rel. algebra

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## Fun Drill: Your turn ...

- Schema:
  - Movie(title, year, studioName)
  - ActsIn(movieTitle, starName)
  - Star(name, gender, birthdate, salary)

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## Your turn ...

- Queries to write in TRC:
  - Find all movies by Paramount studio
  - ... movies starring Kevin Bacon
  - Find stars who have been in a film w/Kevin Bacon
  - Stars within six degrees of Kevin Bacon\*
  - Stars connected to K. Bacon via any number of films\*\*

\* Try *two* degrees for starters    \*\* Good luck with this one!

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## Answers ...

- Find all movies by Paramount studio

$$\{M \mid M \in \text{Movie} \wedge M.\text{studioName} = \text{'Paramount'}\}$$

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## Answers ...

- Movies starring Kevin Bacon

$$\{M \mid M \in \text{Movie} \wedge \exists A \in \text{ActsIn}(A.\text{movieTitle} = M.\text{title} \wedge A.\text{starName} = \text{'Bacon'})\}$$

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### Answers ...

- Stars who have been in a film w/ Kevin Bacon

$$\{S \mid S \in \text{Star} \wedge \exists A \in \text{ActsIn}(A.\text{starName} = S.\text{name} \wedge \exists A2 \in \text{ActsIn}(A2.\text{movieTitle} = A.\text{movieTitle} \wedge A2.\text{starName} = \text{'Bacon'}))\}$$

S: 


name	...
------	-----

A: 

movie	star
-------	------

A2: 

movie	star
-------	------

 'Bacon' #97

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### Answers ...

- Stars within <sup>two</sup> ~~six~~ degrees of Kevin Bacon

$$\{S \mid S \in \text{Star} \wedge \exists A \in \text{ActsIn}(A.\text{starName} = S.\text{name} \wedge \exists A2 \in \text{ActsIn}(A2.\text{movieTitle} = A.\text{movieTitle} \wedge \exists A3 \in \text{ActsIn}(A3.\text{starName} = A2.\text{starName} \wedge \exists A4 \in \text{ActsIn}(A4.\text{movieTitle} = A3.\text{movieTitle} \wedge A4.\text{starName} = \text{'Bacon'}))\}$$

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### Two degrees:

S: 


name	...
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A3: 

movie	star
-------	------

A4: 

movie	star
-------	------

 'Bacon' #99

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## Two degrees:

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## Answers ...

- Stars connected to K. Bacon via any number of films
- **Sorry ... that was a trick question**
  - Not expressible in relational calculus!!
- **What about in relational algebra?**
  - No – RA, RTC, RDC are equivalent

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## Expressive Power

- Expressive Power (Theorem due to Codd):
  - Every query that can be expressed in relational algebra can be expressed as a safe query in RDC / RTC; the converse is also true.
- Relational Completeness:
  - Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus. (actually, SQL is more powerful, as we will see...)

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### Summary

- The relational model has rigorously defined query languages — simple and powerful.
- Relational algebra is more operational/procedural
  - useful as internal representation for query evaluation plans
- Relational calculus is **declarative**
  - users define queries in terms of what they want, not in terms of how to compute it.

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### Summary - cnt'd

- Several ways of expressing a given query
  - a *query optimizer* should choose the most efficient version.
- Algebra and safe calculus have same *expressive power*
  - leads to the notion of *relational completeness*.

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