Sorting Algorithms, Binary Search, Recursion

02-201 / 02-601

Recursion & The Stack

Computing Power(x,y)

- Write a function power(x,y) that returns x^y.

```
func power(x,y int) int {
    ans := 1
    for i := 1; i <= y; i++ {
        ans *= x
    }
    return ans
}
```

- How long will this take to run?

- Can write a function that will be faster?

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    ans := 1
    for i := 1; i <= y; i++ {
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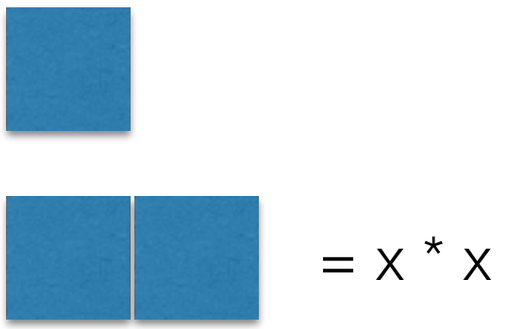
    return ans
}
```

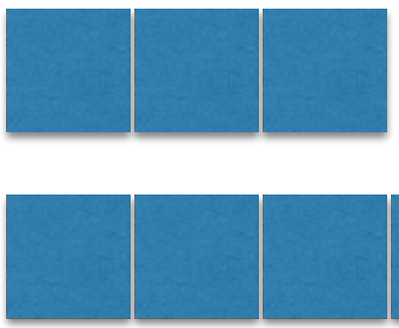
- How long will this take to run? **About y steps**

- Can write a function that will be faster?

Our previous version:



= x * x



= x * x * x

= x * x * x * x

= x * x * x * x * x

= x * x * x * x * x * x

= x * x * x * x * x * x * x

Multiple by x each time through the loop.

At the end, need to multiply x together y times — is there an way to do this with fewer than y multiplications?

Recursively Solve power(x, y/2)

= power(x, y/2) * power(x, y/2)

= power(x, y/2)

```
func power(x,y int) int {
    if y == 0 { return 1 }
    if y == 1 { return x }
    z := power(x, y/2)
    return z*z
}
```

What if y is odd?

Complete Function

```
func power(x,y int) int {
    if y == 0 { return 1 }
    if y == 1 { return x }
    z := power(x, y/2)
    z = z * z
    if y % 2 == 1 {
        z *= x
    }
    return z
}
```

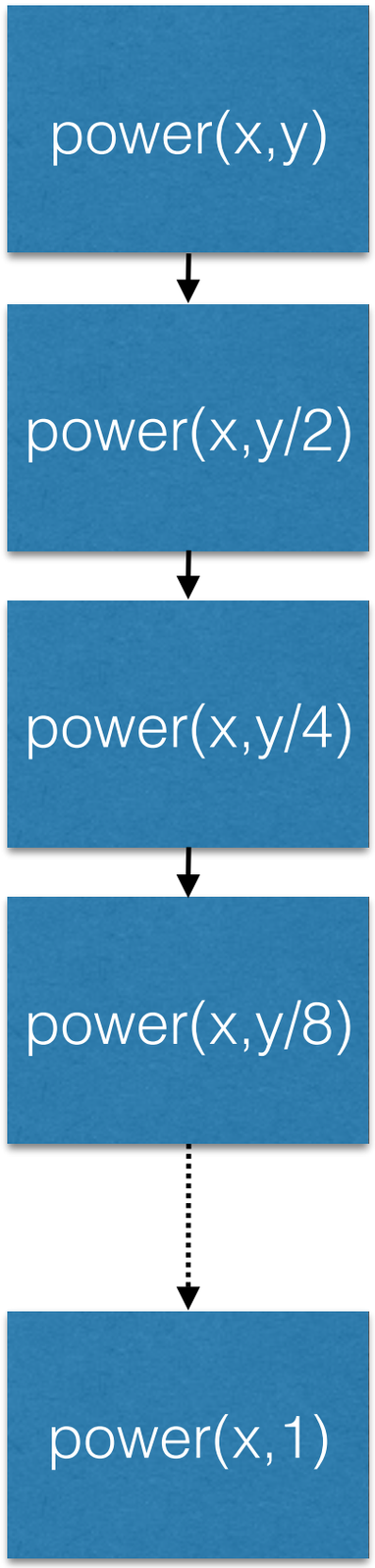
If y is odd, need to do one more multiplication

Running time of modified version

- Inside this function we do a constant amount of work (independent of x and y):
  - 3 if statements
  - 1 or 2 multiplications
  - 1 function call

```
func power(x,y int) int {
    if y == 0 { return 1 }
    if y == 1 { return x }
    z := power(x, y/2)
    z = z * z
    if y % 2 == 1 {
        z *= x
    }
    return z
}
```

How many times is power called?

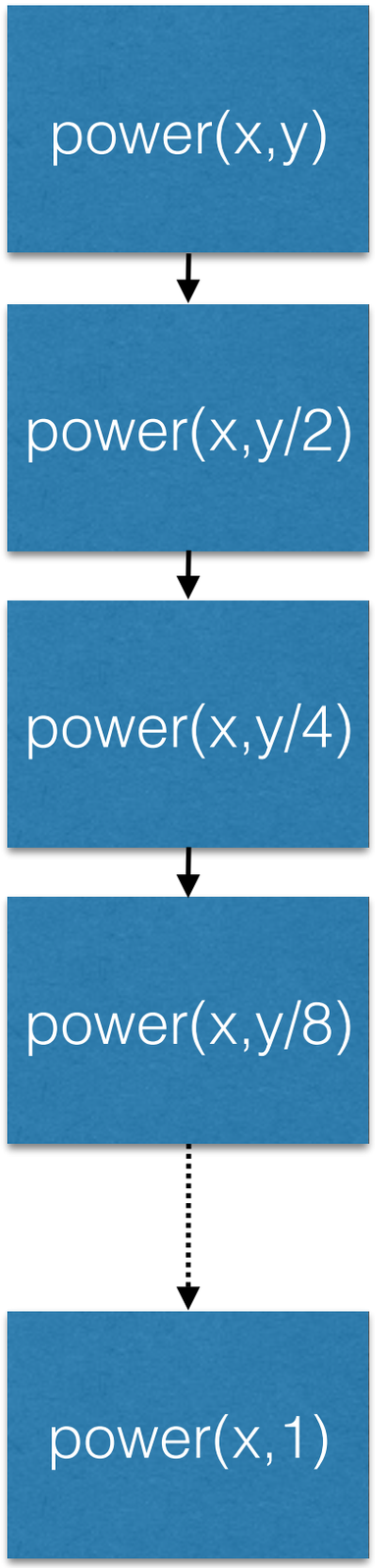


Every time we recurse, y is halved:

Base case: when y = 1, we stop the recursion:

- How many times can you halve a number y before you get to 1?

How many times is power called?



Every time we recurse, y is halved:

Base case: when y = 1, we stop the recursion:

- How many times can you halve a number y before you get to 1?

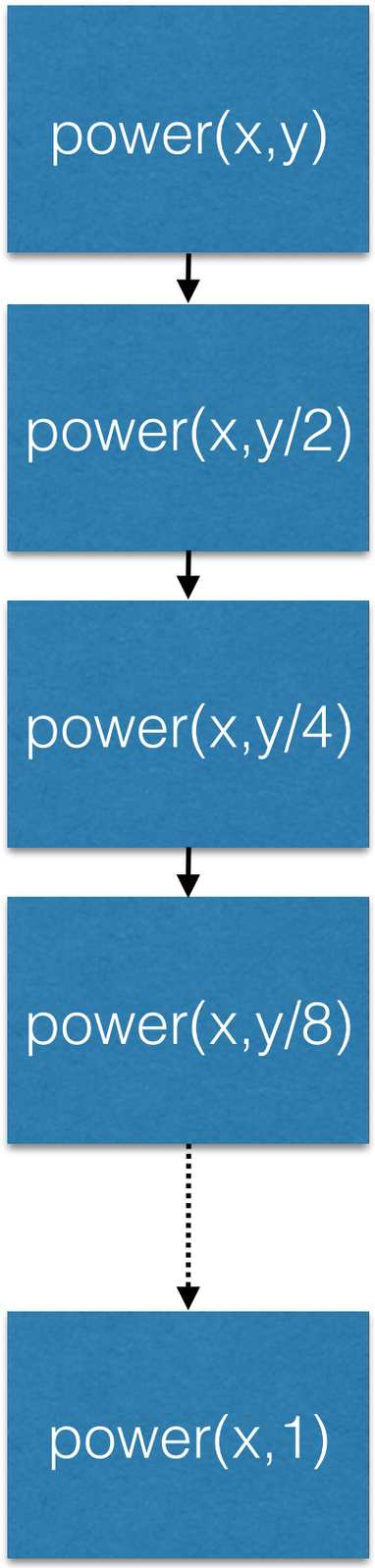
- Want *i* such that: $2^i = y$

- When this happens, the denominator will equal y and $y / 2^i$ will equal 1.

- Take log of both sides: $\log_2 2^i = \log_2 y$

- Therefore, $i = \log_2 y$

How many times is power called?



Every time we recurse, y is halved:

Base case: when y = 1, we stop the recursion:

- How many times can you halve a number y before you get to 1?

- Want *i* such that: $2^i = y$
- When this happens, the denominator will equal y and y / 2^i will equal 1.
- Take log of both sides: $\log_2 2^i = \log_2 y$
- Therefore, $i = \log_2 y$

Will recurse ≈$\log_2$ y times.

So total work is about $\log_2 y$.

Recursion

- How does it work for power to call itself?

- On one hand: nothing special is going on here. Power is a function and we can call the function like any other:

```
fibb(i)    = fibb(i-1) + fibb(i-2)
fibb(1)    = 1
fibb(2)    = 1
```

- This works out so long as eventually we get to a case where the function doesn’t call itself.

- On the other hand: each time you call the function, you need to create new variables (x, y, and z in power). How is this done?

THE Stack

- Behind the scenes, Go (and all other programming languages) maintain a stack that contains the variables associated with the functions you are calling

```
func factorial(x int) int {
    var f int = 0
    for i := 1; i <= x; i++ {
        f = f * i
    }
    return f
}

func nChooseK(n1, k1 int) int {
    var numerator, denominator int
    numerator = factorial(n1) // (2)
    denominator = factorial(k1) // (3)
    denominator = denominator * factorial(n1-k1) // (4)
    return numerator / denominator
}

func main() {
    var n, k, nCk int
    n, k = 10, 3
    nCk = nChooseK(n, k) // (1)
    fmt.Println(nCk)
}
```

main()

```
n = 10
k = 3
nCk = 0
```

- A function call issues a *push* of a record that contains the local variables of the called function.
- A return issues a *pop* of that record (since those local variables are no longer needed)

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}
```

(1) nChooseK(10,3)

```
n1 = 10
k1 = 3
numerator = 0
denominator = 0
```

main()

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func main() {
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    n, k = 10, 3
    nCk = nChooseK(n, k) // (1)
    fmt.Println(nCk)
}
```

(2) factorial(10)

```
x = 10
f = 0 … 3628800
i = 1 … 10
```

(1) nChooseK(10,3)

```
n1 = 10
k1 = 3
numerator = 0
denominator = 0
```

main()

```
n = 10
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n1 = 10
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    return numerator / denominator
}

func main() {
    var n, k, nCk int
    n, k = 10, 3
    nCk = nChooseK(n, k) // (1)
    fmt.Println(nCk)
}
```

(3) factorial(10)

```
x = 3
f = 0 … 6
i = 1 … 3
```

(1) nChooseK(10,3)

```
n1 = 10
k1 = 3
numerator = 0
denominator = 0
```

main()

```
n = 10
k = 3
nCk = 0
```

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}
```

(1) nChooseK(10,3)

```
n1 = 10
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}

func main() {
    var n, k, nCk int
    n, k = 10, 3
    nCk = nChooseK(n, k) // (1)
    fmt.Println(nCk)
}
```

(4) factorial(10)

```
x = 3
f = 0 … 5040
i = 1 … 3
```

(1) nChooseK(10,3)

```
n1 = 10
k1 = 3
numerator = 0
denominator = 0
```

main()

```
n = 10
k = 3
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(1) nChooseK(10,3)

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```

main()

```
n = 10
k = 3
nCk = 0
```

- A function call issues a *push* of a record that contains the local variables of the called function.
- A return issues a *pop* of that record (since those local variables are no longer needed)

Recursion works the same way

```
func power(x,y int) int {
    if y == 0 { return 1 }
    if y == 1 { return x }
    z := power(x, y/2)
    z = z * z
    if y % 2 == 1 {
        z *= x
    }
    return z
}

func main() {
    power(10, 8)
}
```

Recursion works the same way

```
func power(x,y int) int {
    if y == 0 { return 1 }
    if y == 1 { return x }
    z := power(x, y/2)
    z = z * z
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    z = z * z
    if y % 2 == 1 {
        z *= x
    }
    return z
}

func main() {
    power(10, 8)
}
```

power(10, 4)

```
x = 10
y = 4
z = 0
```

power(10, 8)

```
x = 10
y = 8
z = 0
```

main()

```
no local vars
```

Recursion works the same way

```
func power(x,y int) int {
    if y == 0 { return 1 }
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    if y % 2 == 1 {
        z *= x
    }
    return z
}

func main() {
    power(10, 8)
}
```

power(10, 2)

```
x = 10
y = 2
z = 0
```

power(10, 4)

```
x = 10
y = 4
z = 0
```

power(10, 8)

```
x = 10
y = 8
z = 0
```

main()

```
no local vars
```

Recursion works the same way

```
func power(x,y int) int {
    if y == 0 { return 1 }
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    z = z * z
    if y % 2 == 1 {
        z *= x
    }
    return z
}

func main() {
    power(10, 8)
}
```

power(10, 1)

```
x = 10
y = 2
z = 0
```

power(10, 2)

```
x = 10
y = 2
z = 0
```

power(10, 4)

```
x = 10
y = 4
z = 0
```

power(10, 8)

```
x = 10
y = 8
z = 0
```

main()

```
no local vars
```

Recursion works the same way

```
func power(x,y int) int {
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    z = z * z
    if y % 2 == 1 {
        z *= x
    }
    return z
}

func main() {
    power(10, 8)
}
```

power(10, 2)

```
x = 10
y = 2
z = 0
```

power(10, 4)

```
x = 10
y = 4
z = 0
```

power(10, 8)

```
x = 10
y = 8
z = 0
```

main()

```
no local vars
```

Recursion works the same way

```
func power(x,y int) int {
    if y == 0 { return 1 }
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    return z
}

func main() {
    power(10, 8)
}
```

power(10, 4)

```
x = 10
y = 4
z = 0
```

power(10, 8)

```
x = 10
y = 8
z = 0
```

main()

```
no local vars
```

Recursion works the same way

```
func power(x,y int) int {
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    }
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    power(10, 8)
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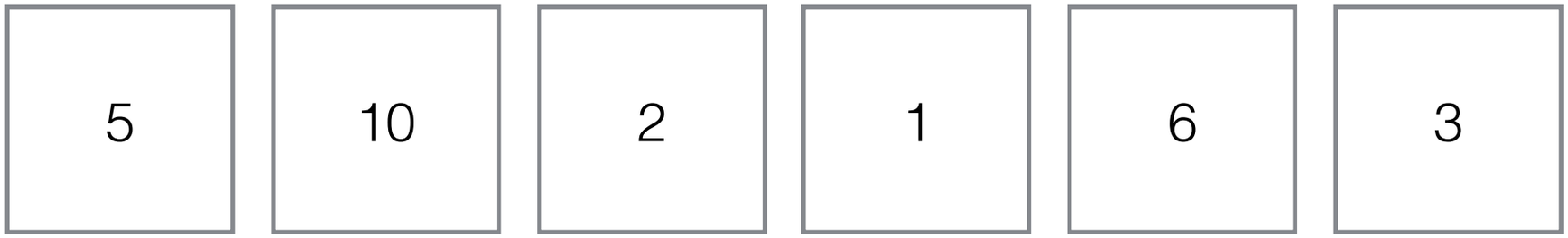
```

Sorting

The Sorting Problem

Given: a set of items k_1, k_2,… k_n, re-order them so that $k_{i1} < k_{i2} < \ldots < k_{in}$



Don’t have to be integers: can sort anything where < is defined.

Insertion Sort & Linked Lists

inList = list of items to sort
outList = empty list
for every item *k* in inList:
    walk down outList finding where *k* should go
    insert *k* into the middle of outList

```
func insertSort1(inList []int) []int {
  var outList []int = make([]int, 0)

  // for every item
  for j, k := range inList {
    if j == 0 {
      outList = append(outList, k)
    } else {
      // walk down outList
      for i := 0; i < len(outList); i++ {
        if outList[i] > k {
          // k belongs at position i
          outList = append(outList, 0)
          copy(outList[i+1:], outList[i:])
          outList[i] = k
          break
        }
      }
    }
  }
  return outList
}
```

copy(x,y) is a builtin function that copies the items from y into x

Here, we use it to make a "hole" at position *i* in order to store *k*

break stops the current **for** loop

How many steps does this insertSort() take?

Time for insertSort1

```
func insertSort1(inList []int) []int {
  var outList []int = make([]int, 0)

  // for every item
  for j, k := range inList {
    if j == 0 {
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          // k belongs at position i
          outList = append(outList, 0)
          copy(outList[i+1:], outList[i:])
          outList[i] = k
          break
        }
      }
    }
  }
  return outList
}
```

- Let n = len(inList)

n times through this loop

possibly n times through this loop

copy could have to copy n items

- About n^3 steps in total

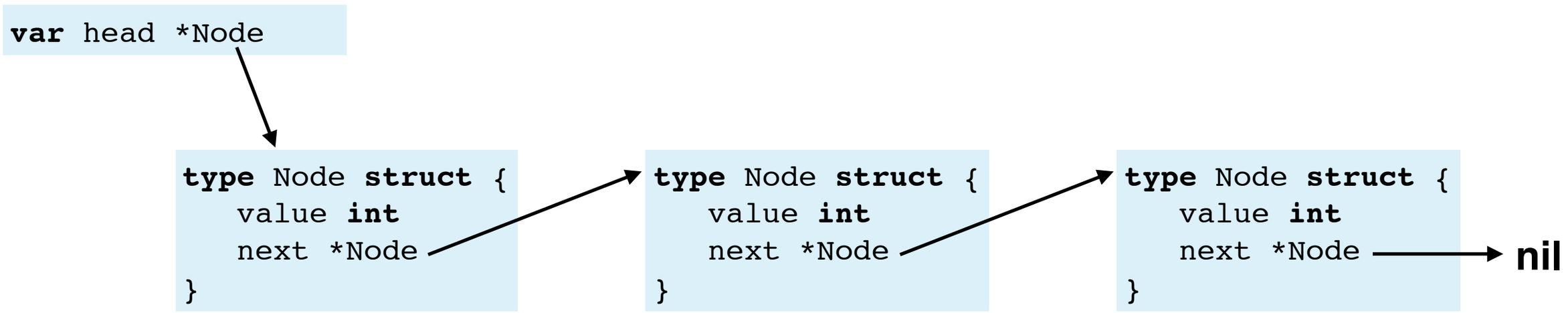
- This is pretty slow: to sort 100 items, might take 1 million steps!

Can we do better?

Linked Lists

- One big problem: we have to move all items after position *i* out of the way to insert something.

- Linked lists avoid this problem (and are another great example of the utility of pointers)



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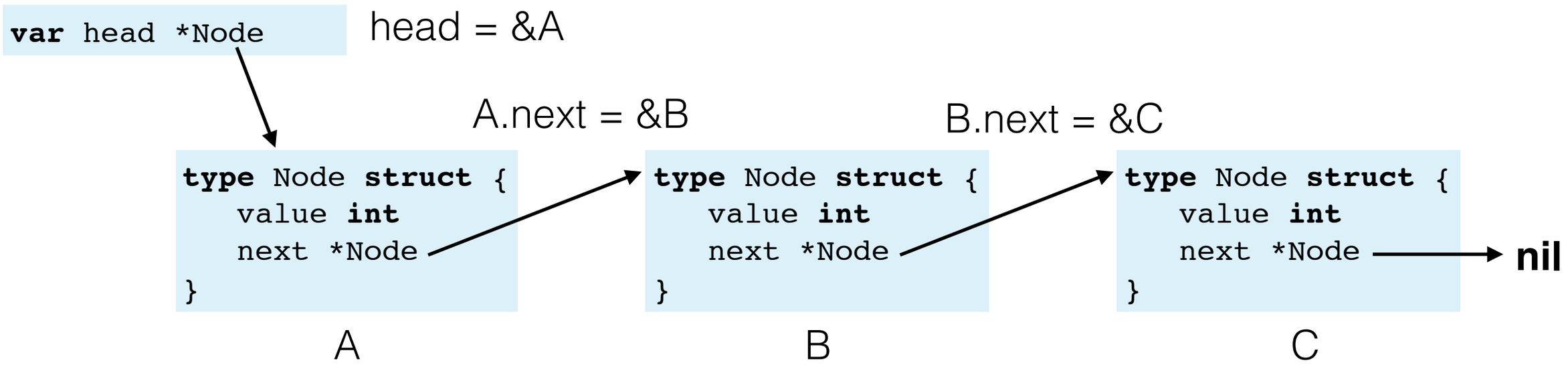

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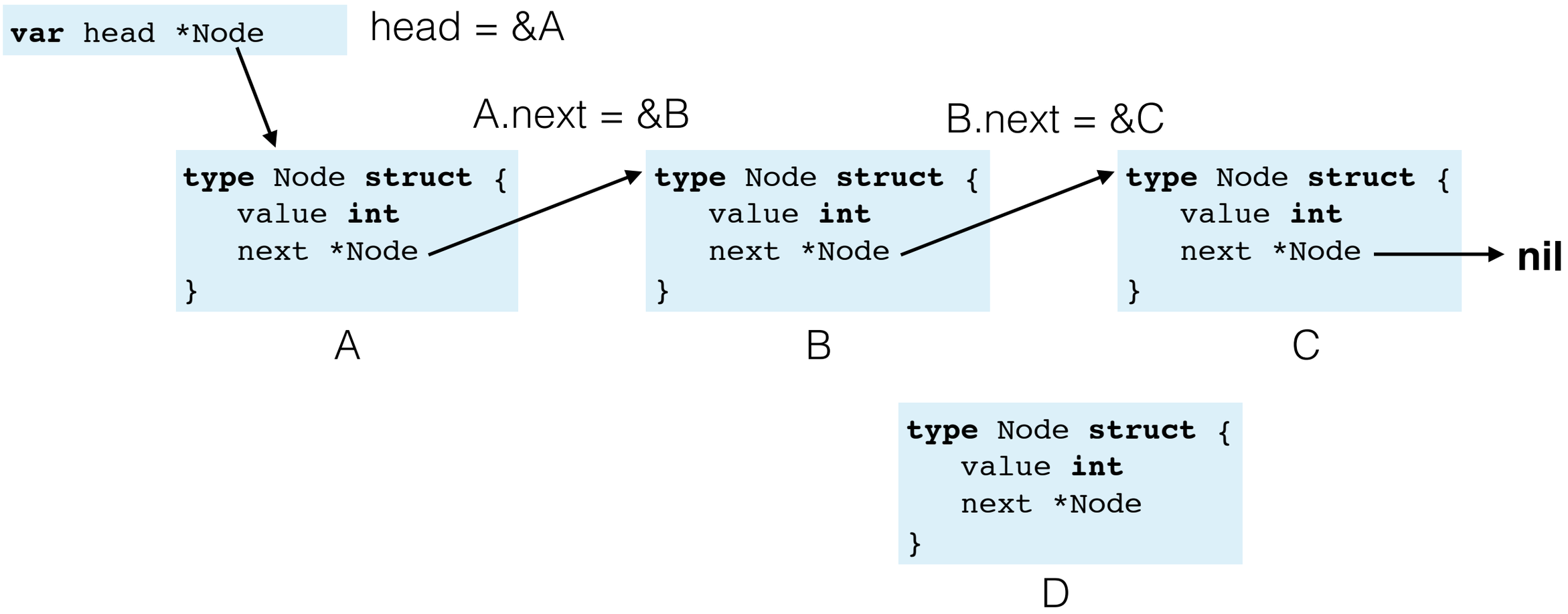
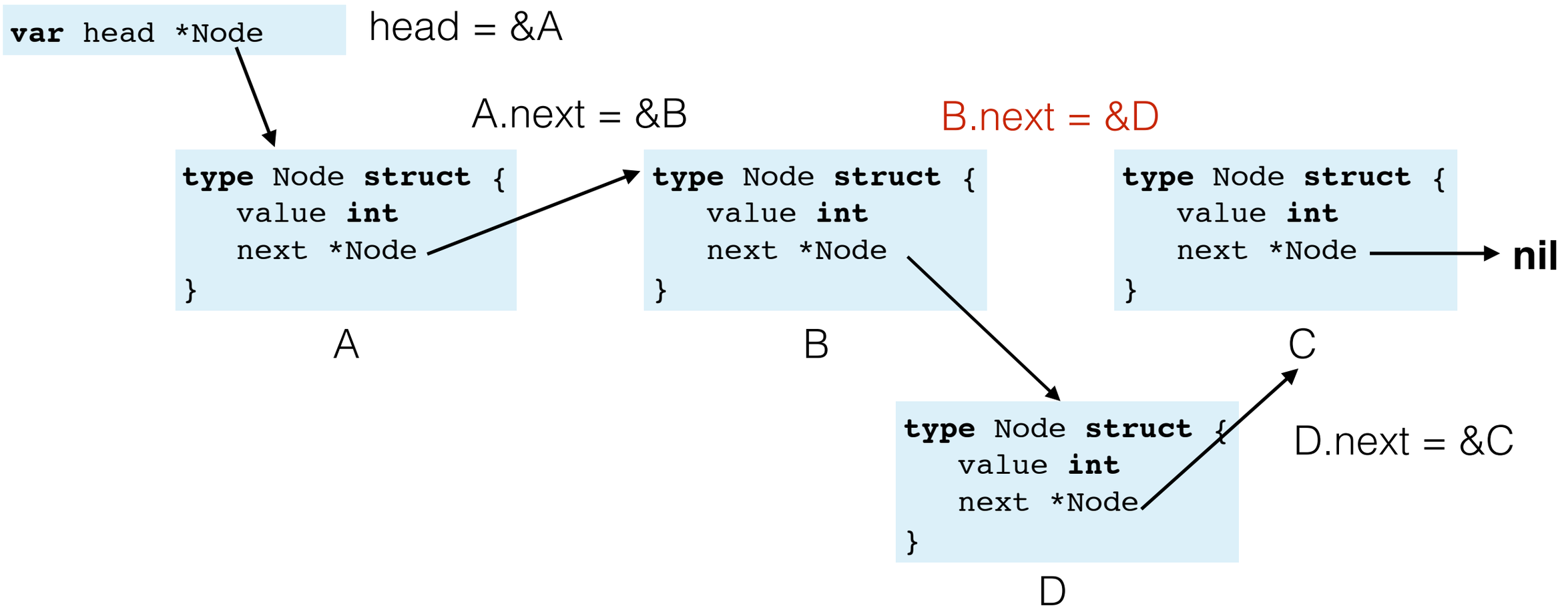


- How can we insert a node between B and C (say)?

Linked Lists

- One big problem: we have to move all items after position *i* out of the way to insert something.

- Linked lists avoid this problem (and are another great example of the utility of pointers)



- How can we insert a node between B and C (say)?

Linked List Insertion Sort

```
func insertionSort(inList []int) []int {
   // create a linked list with just one item in it
   var head *Node = createNode(inList[0])

   // for every remaining item
   for _, k := range inList[1:] {
      newNode := createNode(k)

      // walk down the linked list
      for prev, cur := (*Node)(nil), head; cur != nil; prev, cur = cur, cur.next {

         // if this is where we should insert
         if cur.value > k {

            // if not at the start of the list
            if prev != nil {
               prev.next = newNode
            } else {
               // otherwise, we're at the start of the list
               head = newNode
            }
            newNode.next = cur
            break
         }
      }
   }

   return convertLinkedListToSlice(head)
}
```

try to insert each item

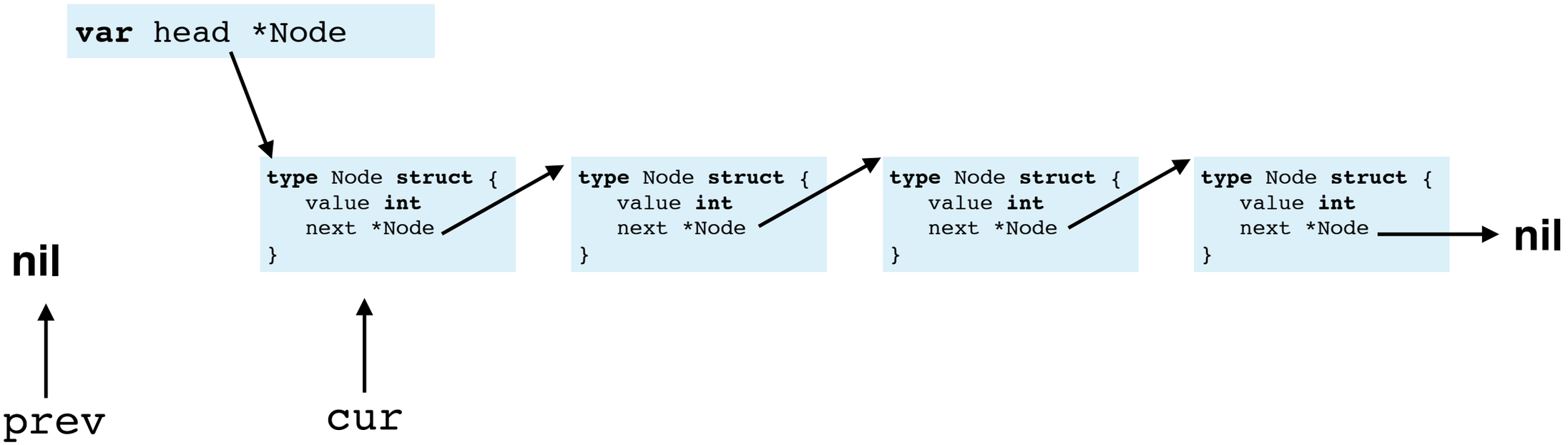
walk down the list, maintaining *two* pointers into it: one for the current node and one for the previous node

If we're not trying to put this at the start

If we need to change head because k belongs at the start

link newNode to cur so that it points to the rest of the list

Linked List Insertion In Pictures



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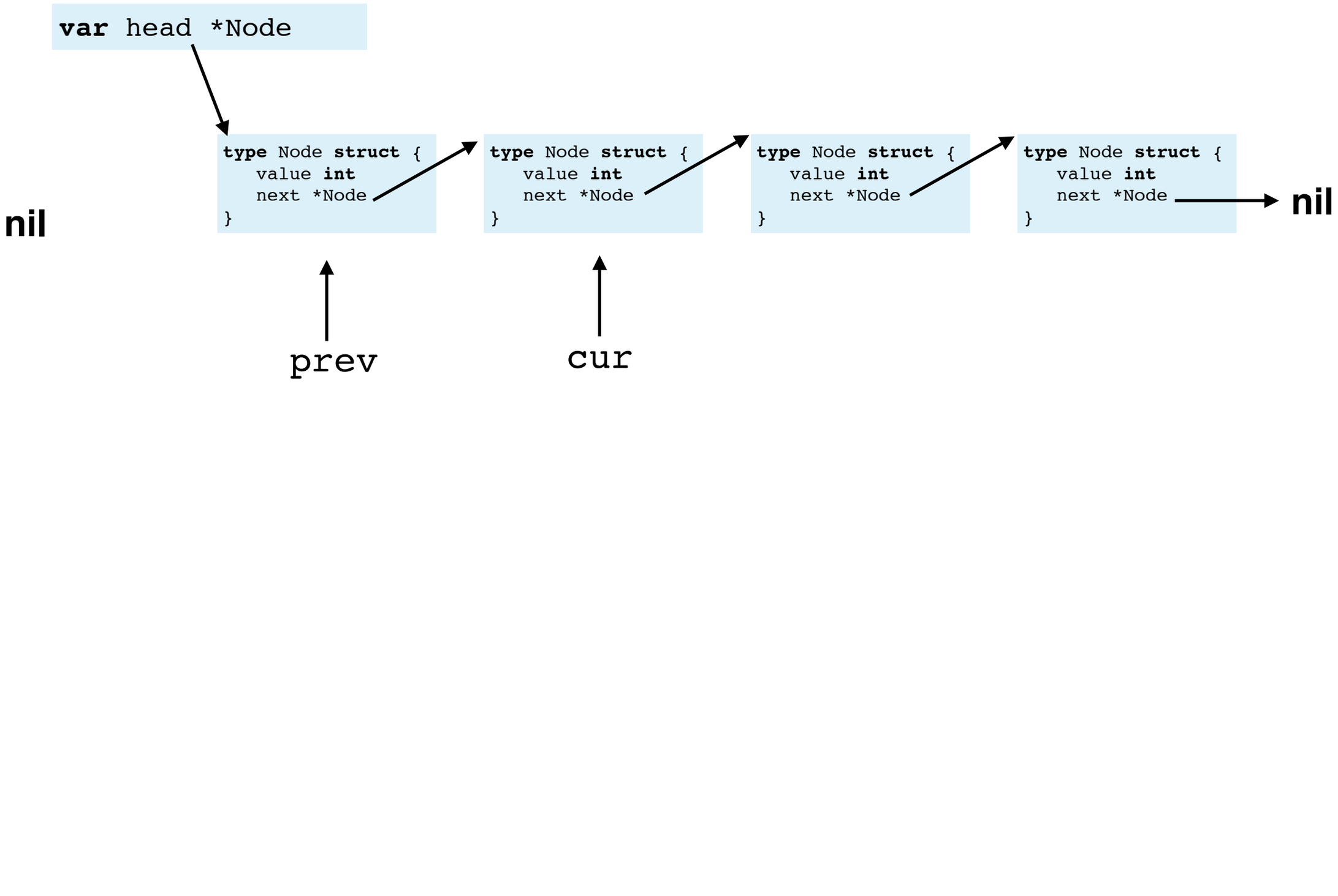

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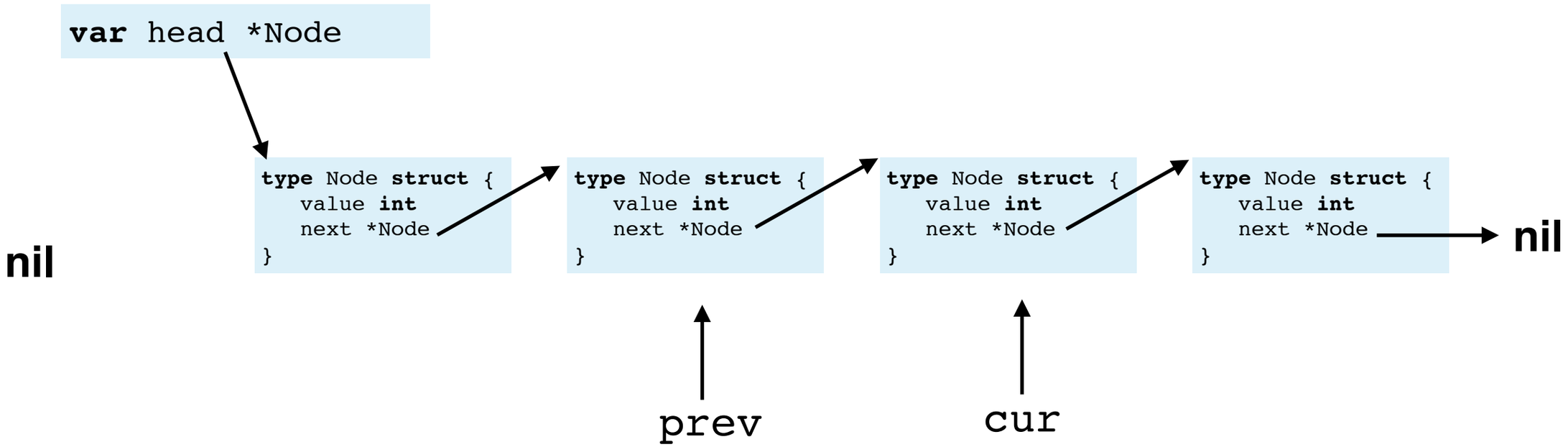

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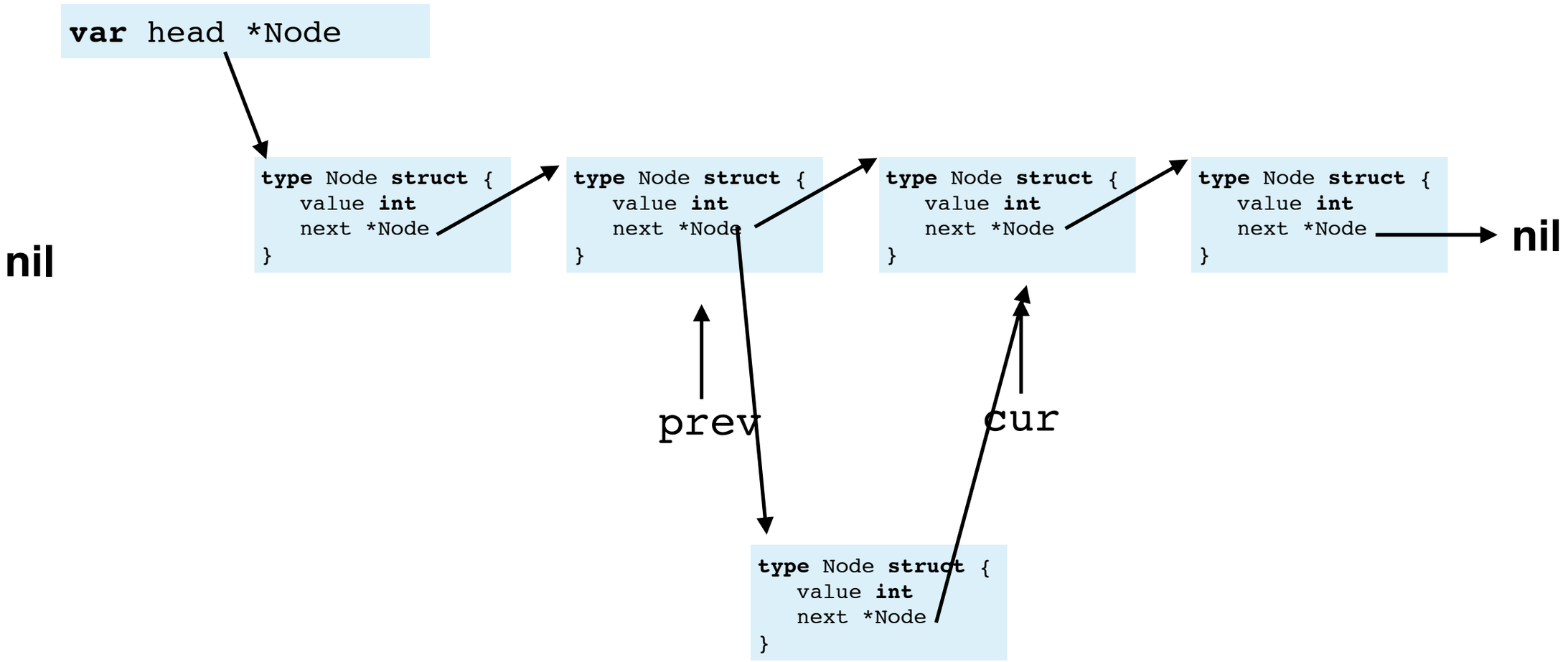

Worst-case runtime for Linked List Insertion Sort

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   for _, k := range inList[1:] {
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     // walk down the linked list
     for prev, cur := (*Node)(nil), head; cur != nil; prev, cur = cur, cur.next {

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        if cur.value > k {

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           if prev != nil {
              prev.next = newNode
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              // otherwise, we're at the start of the list
              head = newNode
           }
           newNode.next = cur
           break
        }
     }
  }

  return convertLinkedListToSlice(head)
}
```

about n times through this loop

possibly n times through this loop

A constant amount of work inside here

This insertion sort implementation takes about n^2 steps:
about 10,000 steps to stort 100 numbers.

createNode and convertLinkedListToSlice

```
func createNode(v int) *Node {
    return &Node{value: v, next: nil}
}

func convertLinkedListToSlice(head *Node) []int {
    out := make([]int, 0)
    for p := head; p != nil; p = p.next {
        out = append(out, p.value)
    }
    return out
}
```

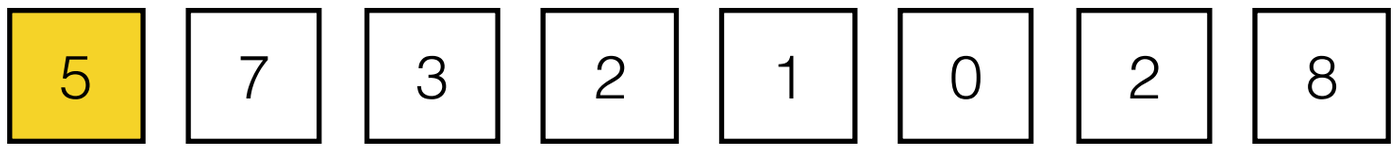
walk down the list with a single pointer

Can we sort faster?

Quicksort

- Quicksort is often the fastest sort in practice.
- Based on the idea of “divide and conquer”: break the problem of sorting *n* numbers into two subproblems of sorting fewer numbers
- Based on the *partition* operation:

partition: Let p be the first item in the list.
Rearrange the list so that items < p are to the left of p and items > p are to the right



- After this, 5 is in the right place in the sorted order
- And everything that should be before 5 is before it, and everything that is after it should be after it.

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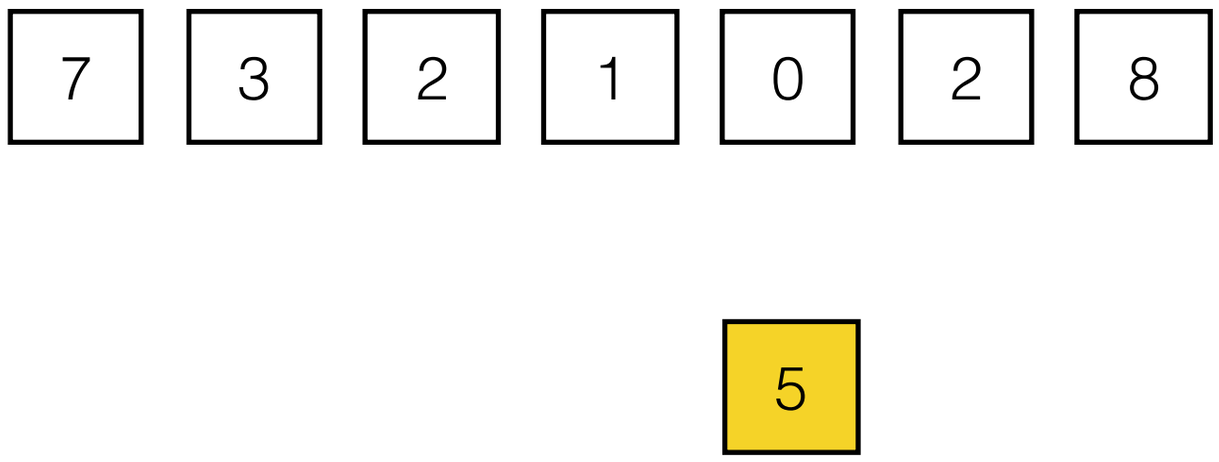

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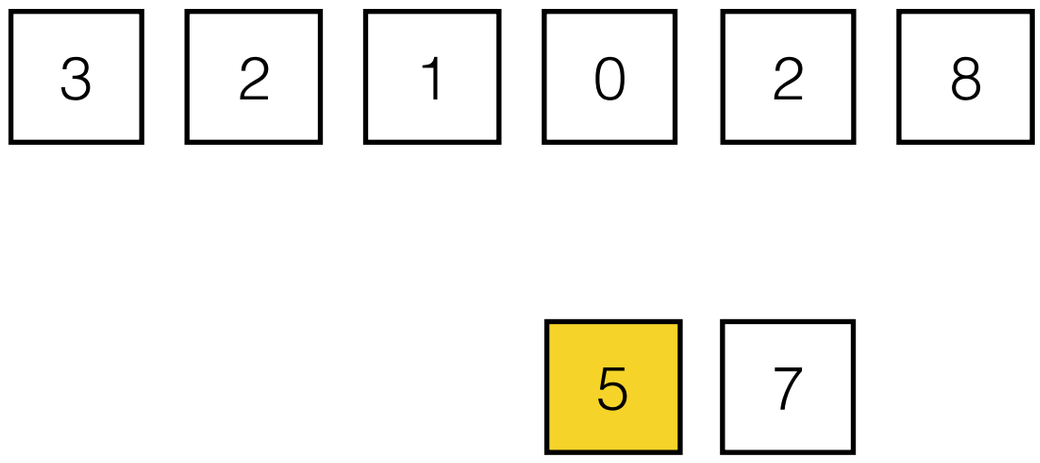

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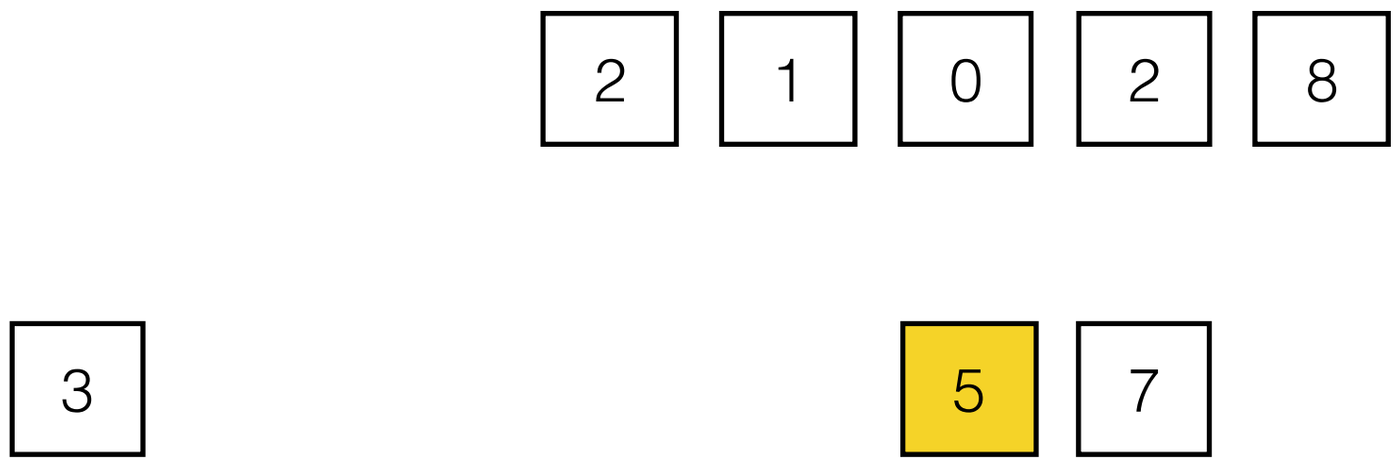

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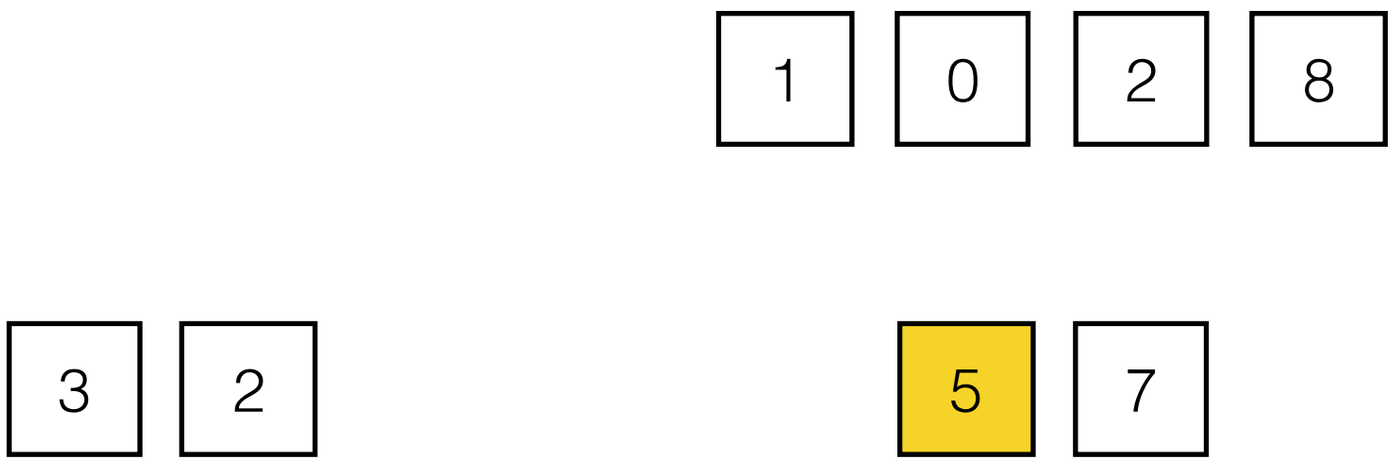

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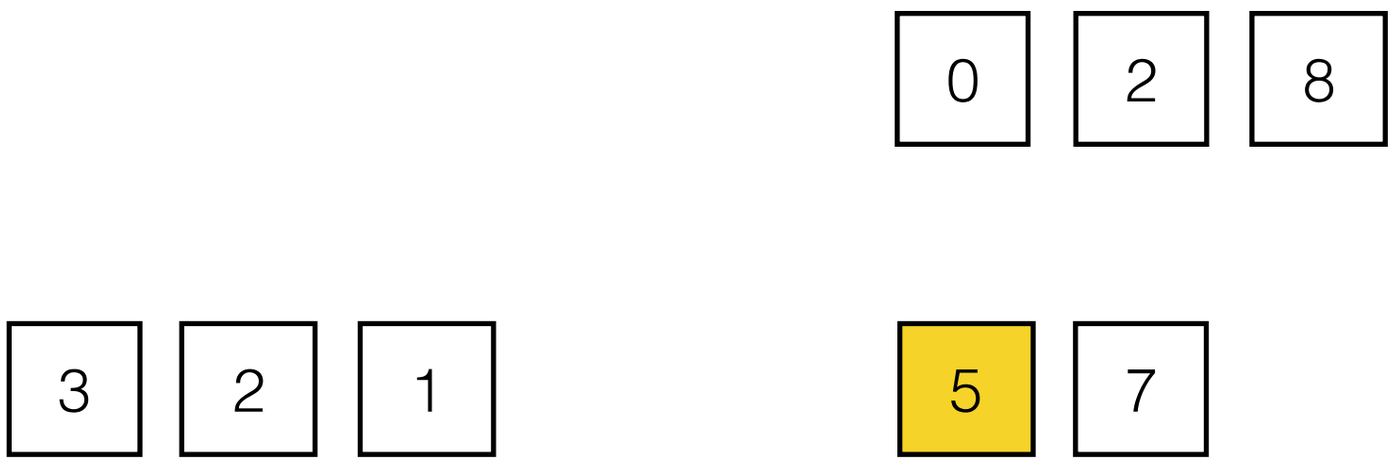

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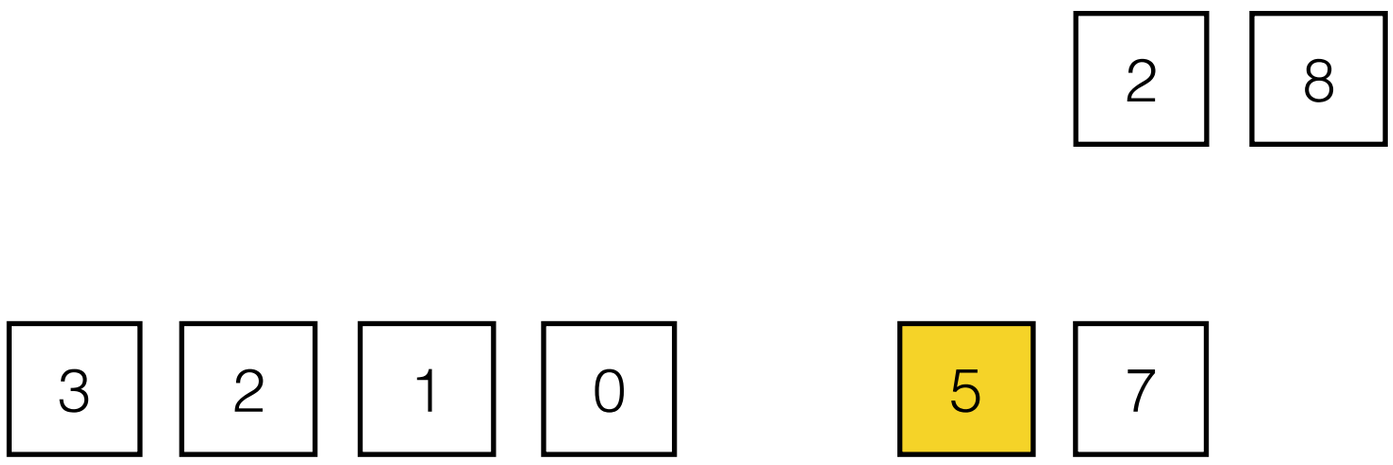

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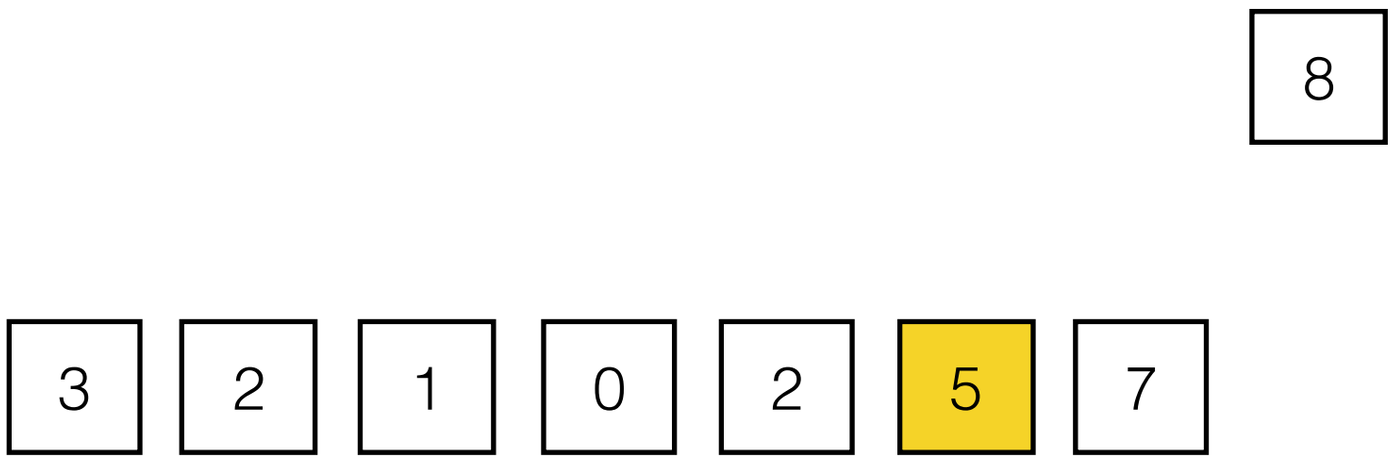

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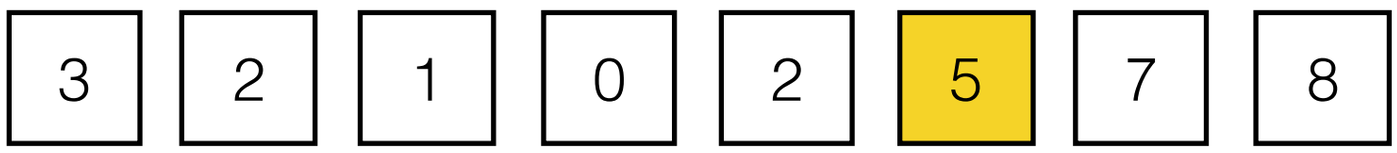

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Quicksort Using partition

```
func quickSort(inList []int) {
    if len(inList) > 1 {
        p := partition(inList)
        quickSort(inList[:p])
        quickSort(inList[p+1:])
    }
}
```

If the list contains 0 or 1 items, it's already sorted

Otherwise, partition, putting inList[0] in the right place

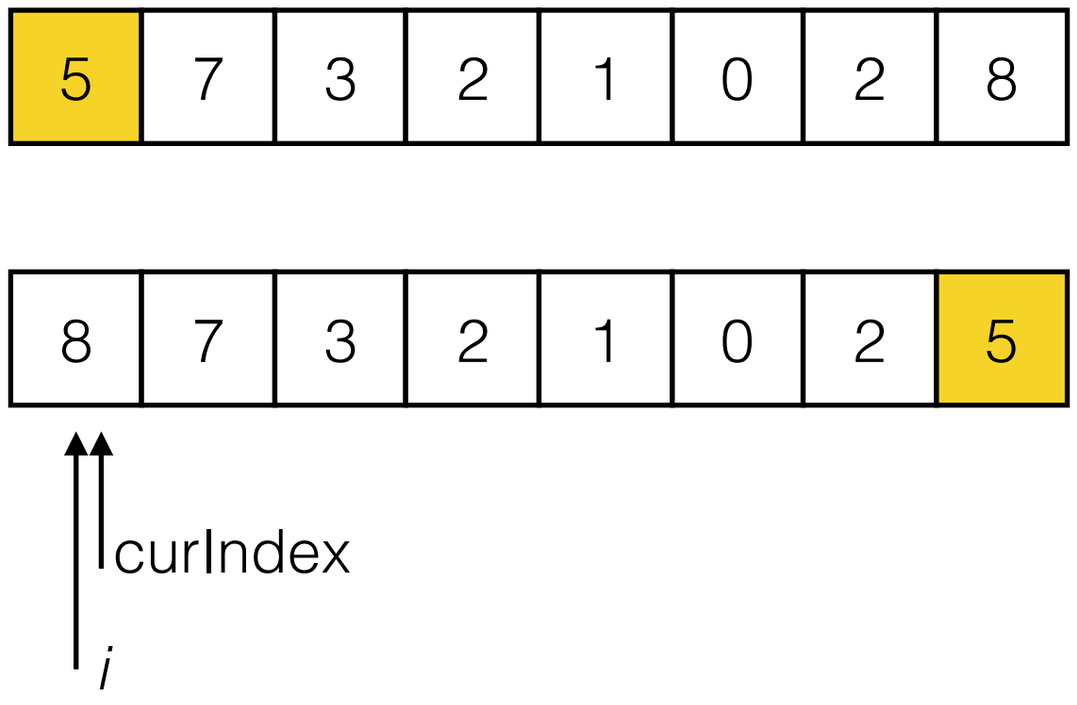
Recursively partition the left half and the right half

Partition

```
func partition(inList []int) int {
    pivot := inList[0]
    lastPos := len(inList)-1

    // swap the first and list items
    inList[0], inList[lastPos] = inList[lastPos], inList[0]

    curIndex := 0
    for i := 0; i < lastPos; i++ {
        if inList[i] < pivot {
            inList[i], inList[curIndex] = inList[curIndex], inList[i]
            curIndex++
        }
    }
    inList[curIndex], inList[lastPos] = inList[lastPos], inList[curIndex]
    return curIndex
}
```



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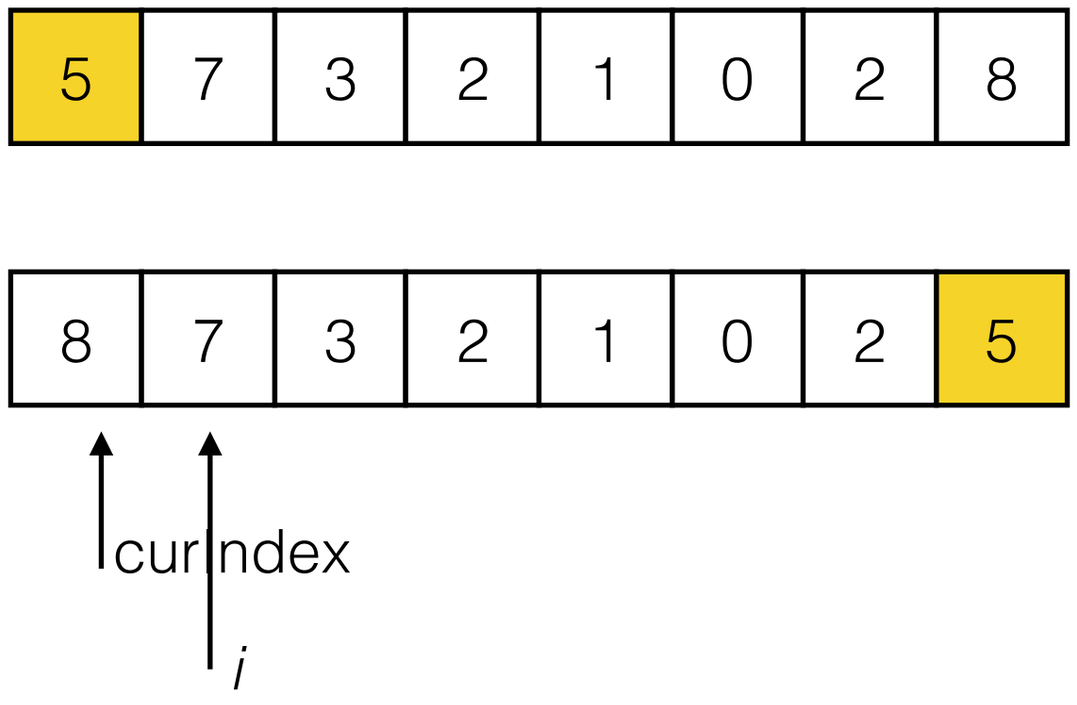
```



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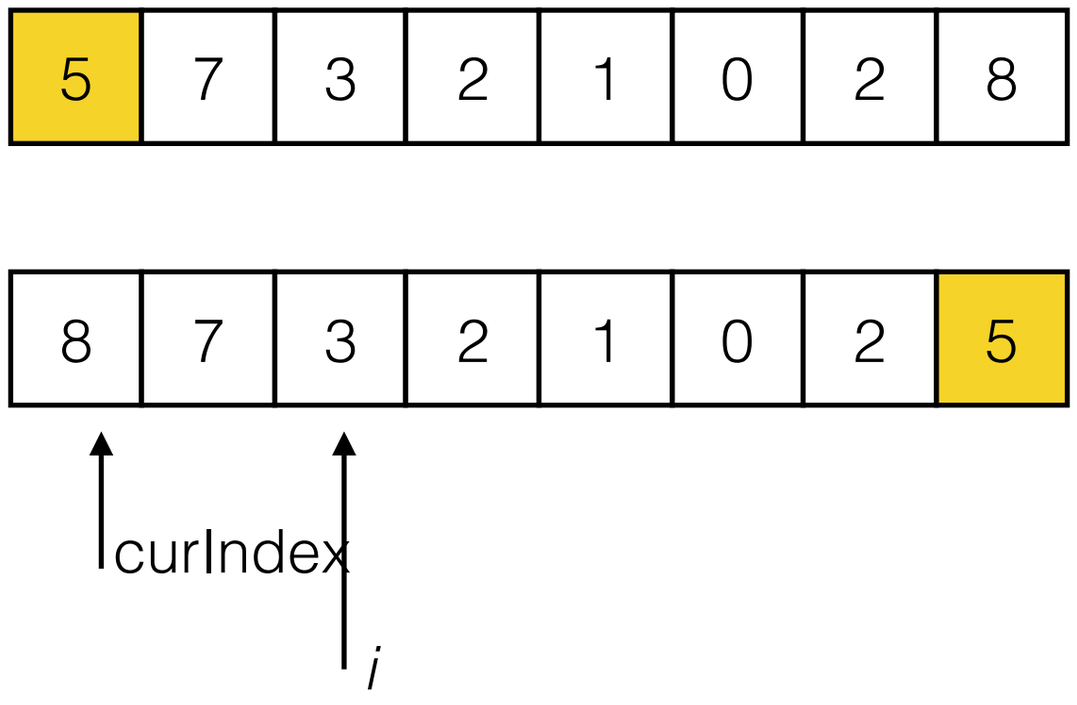

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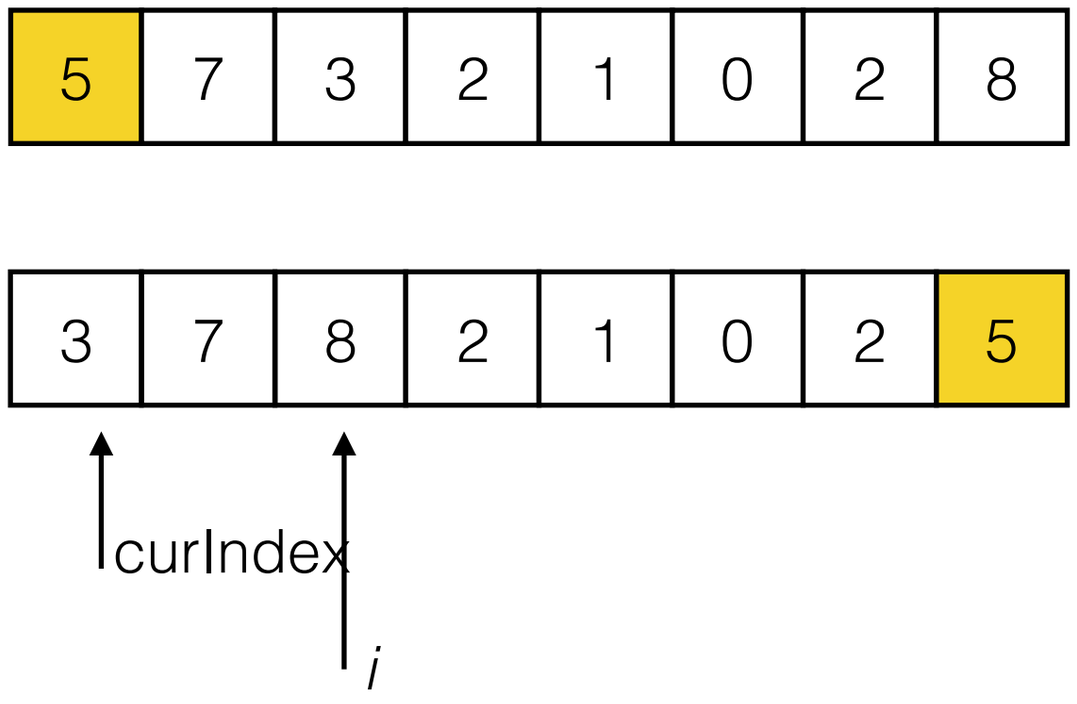

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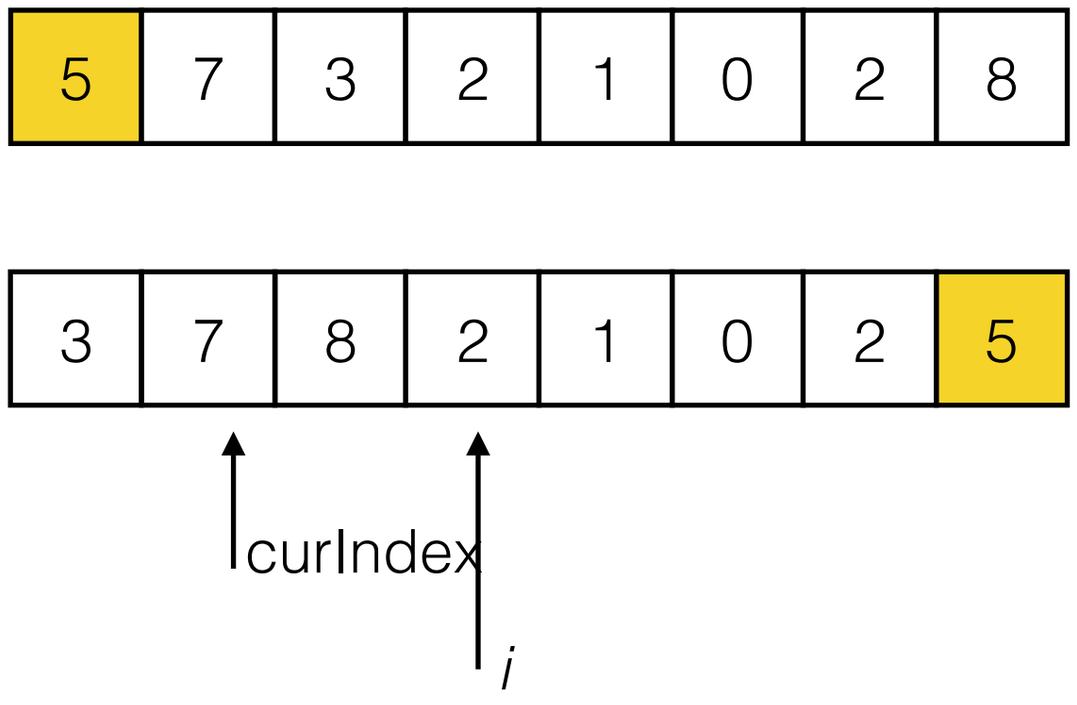

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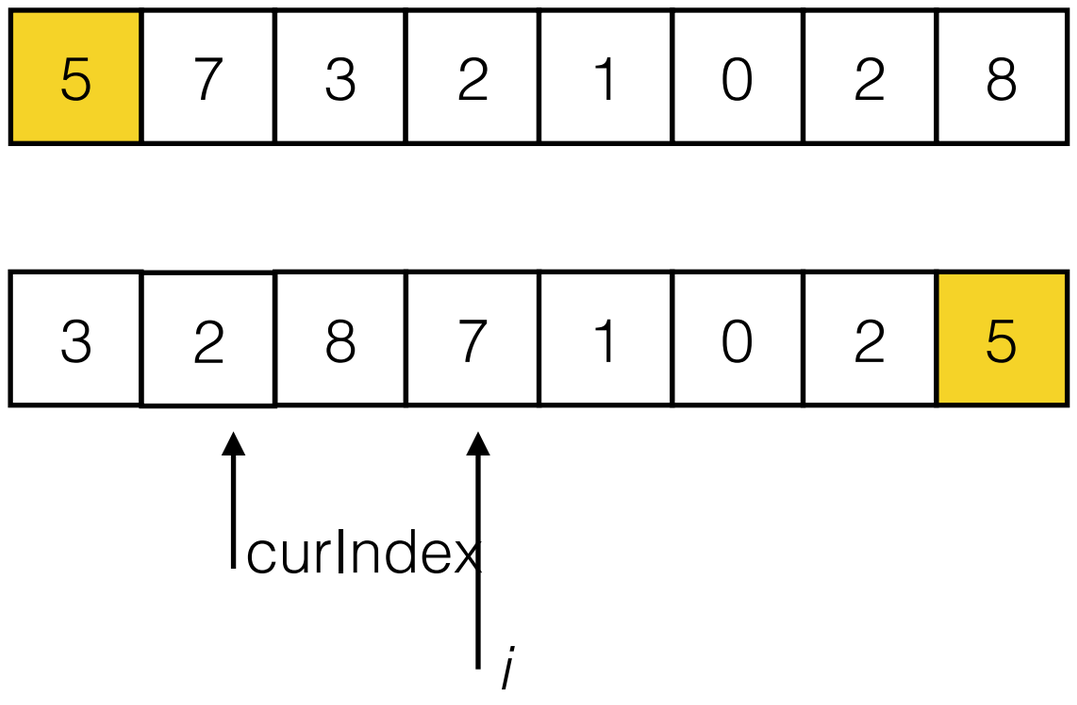

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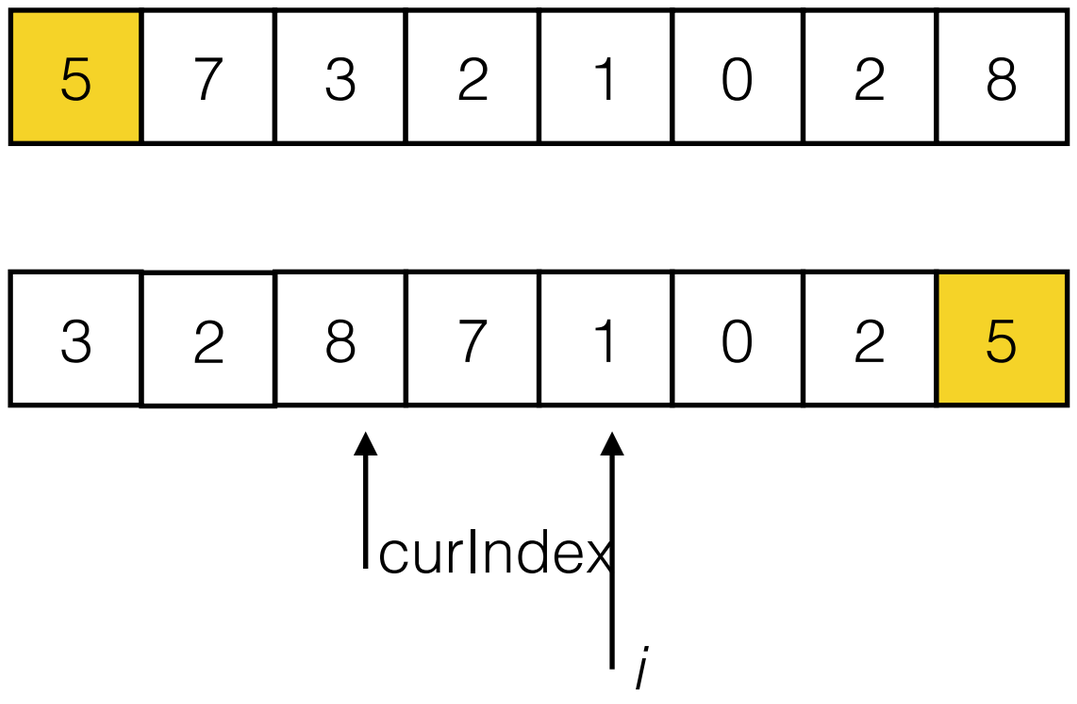

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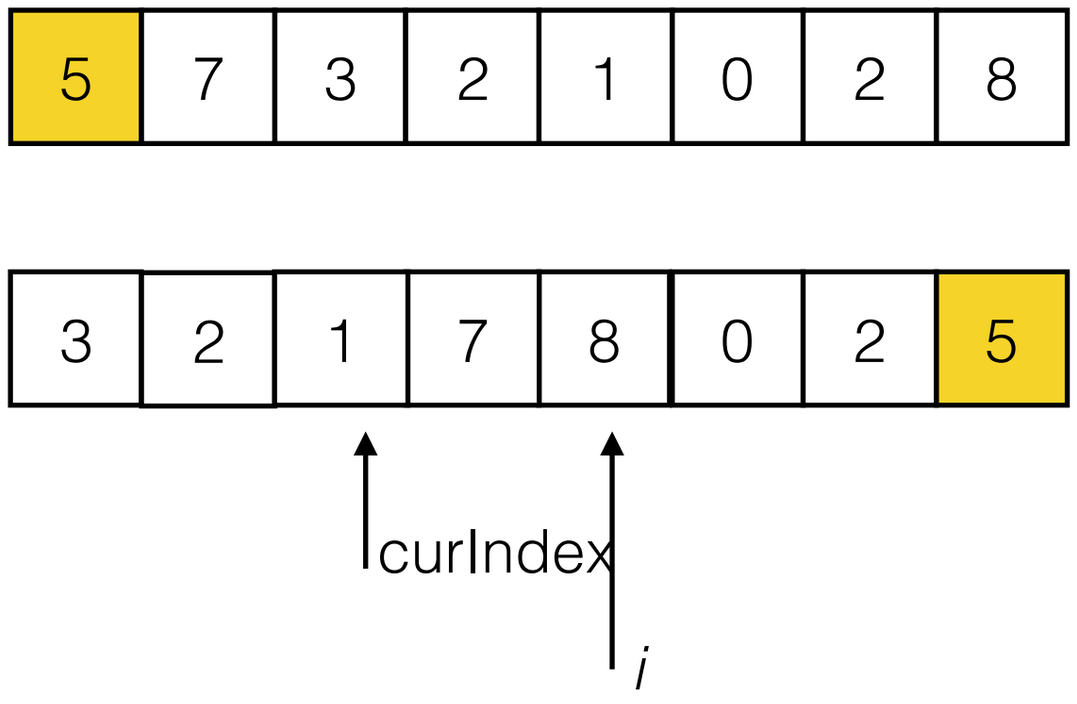

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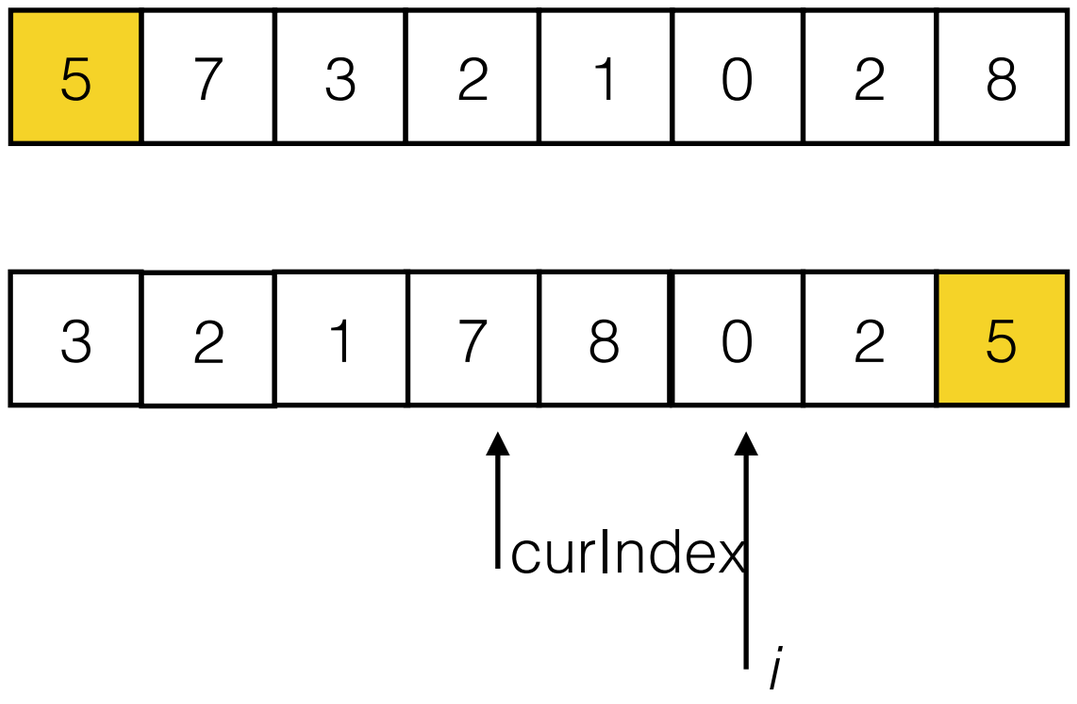

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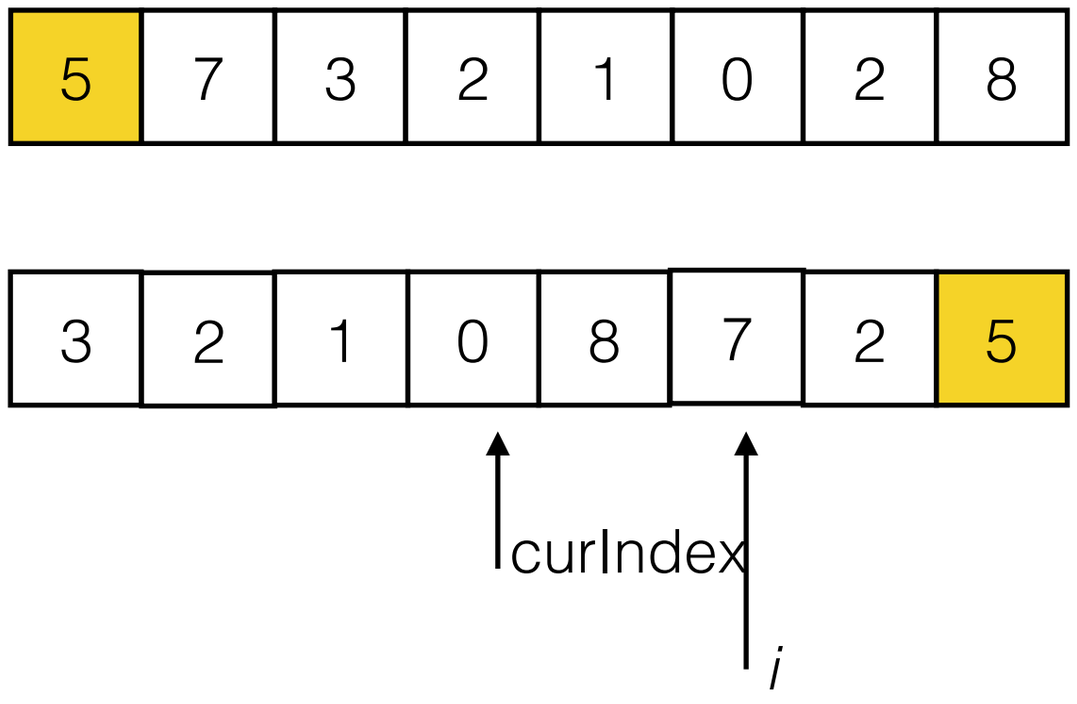

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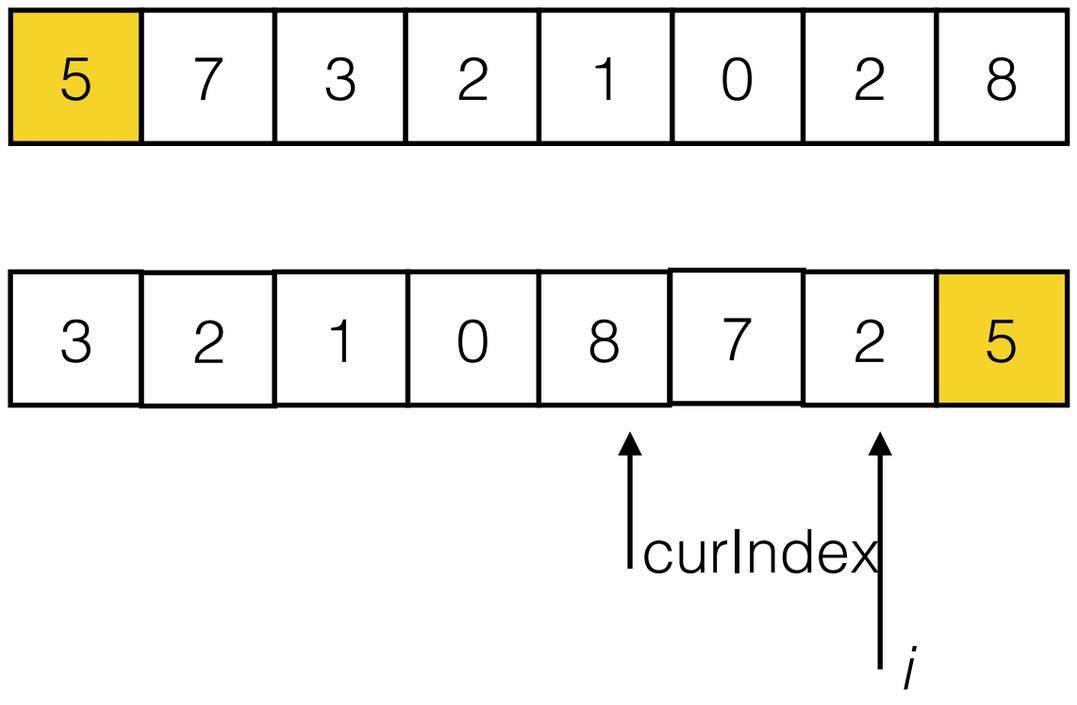

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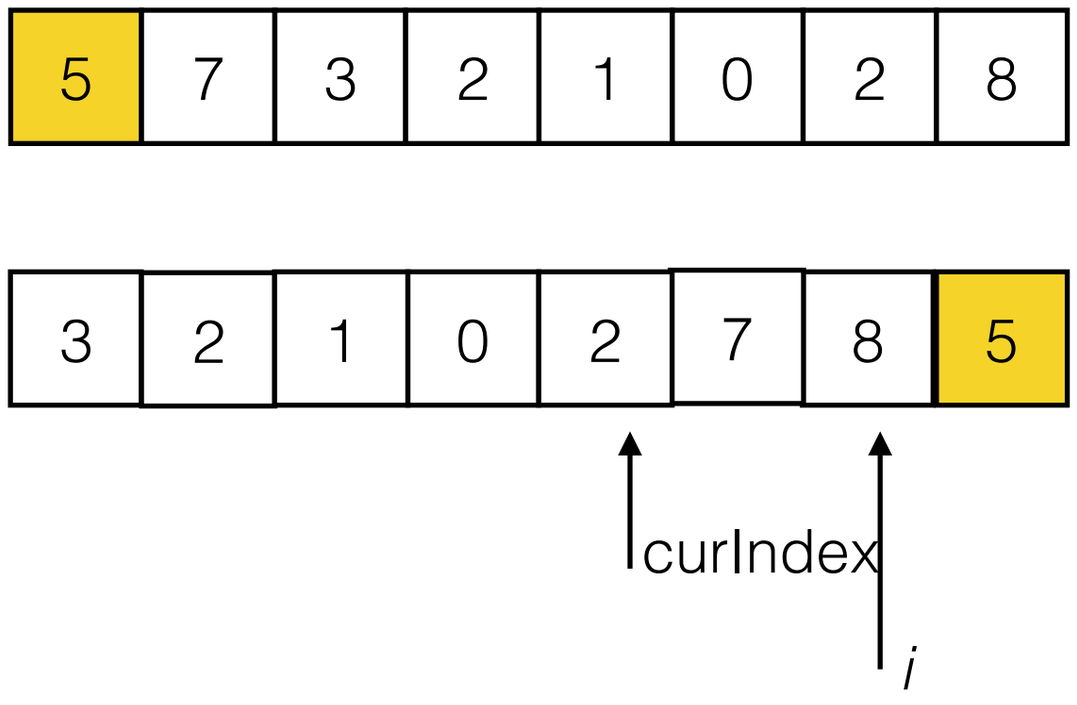

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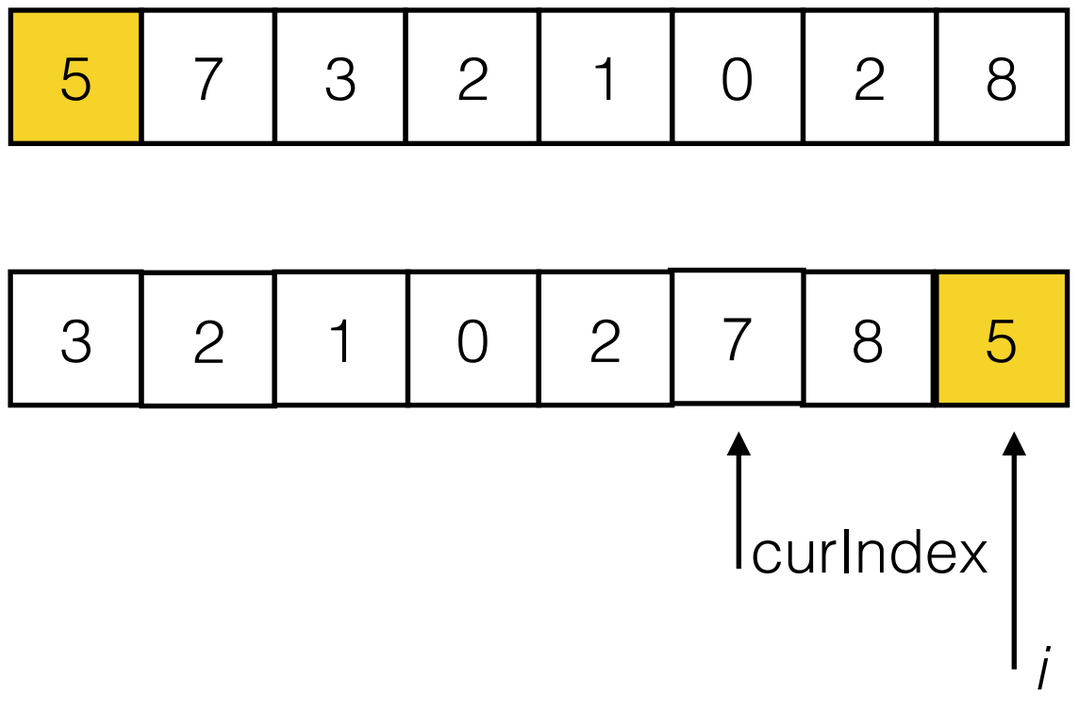
```

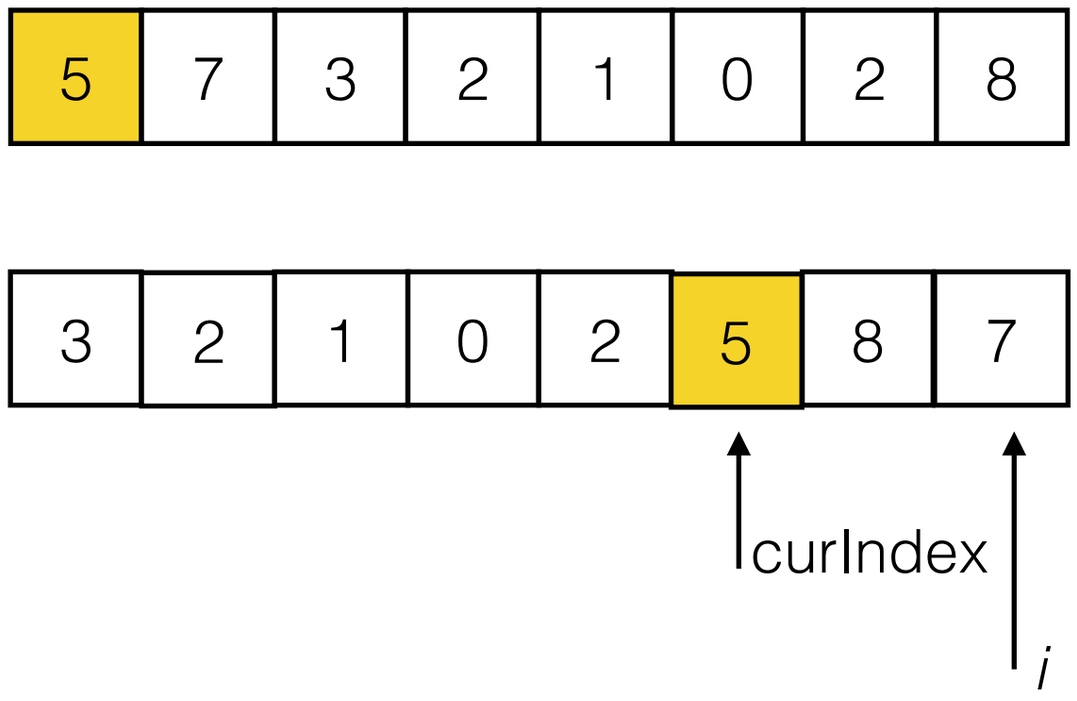


Partition

```
func partition(inList []int) int {
    pivot := inList[0]
    lastPos := len(inList)-1

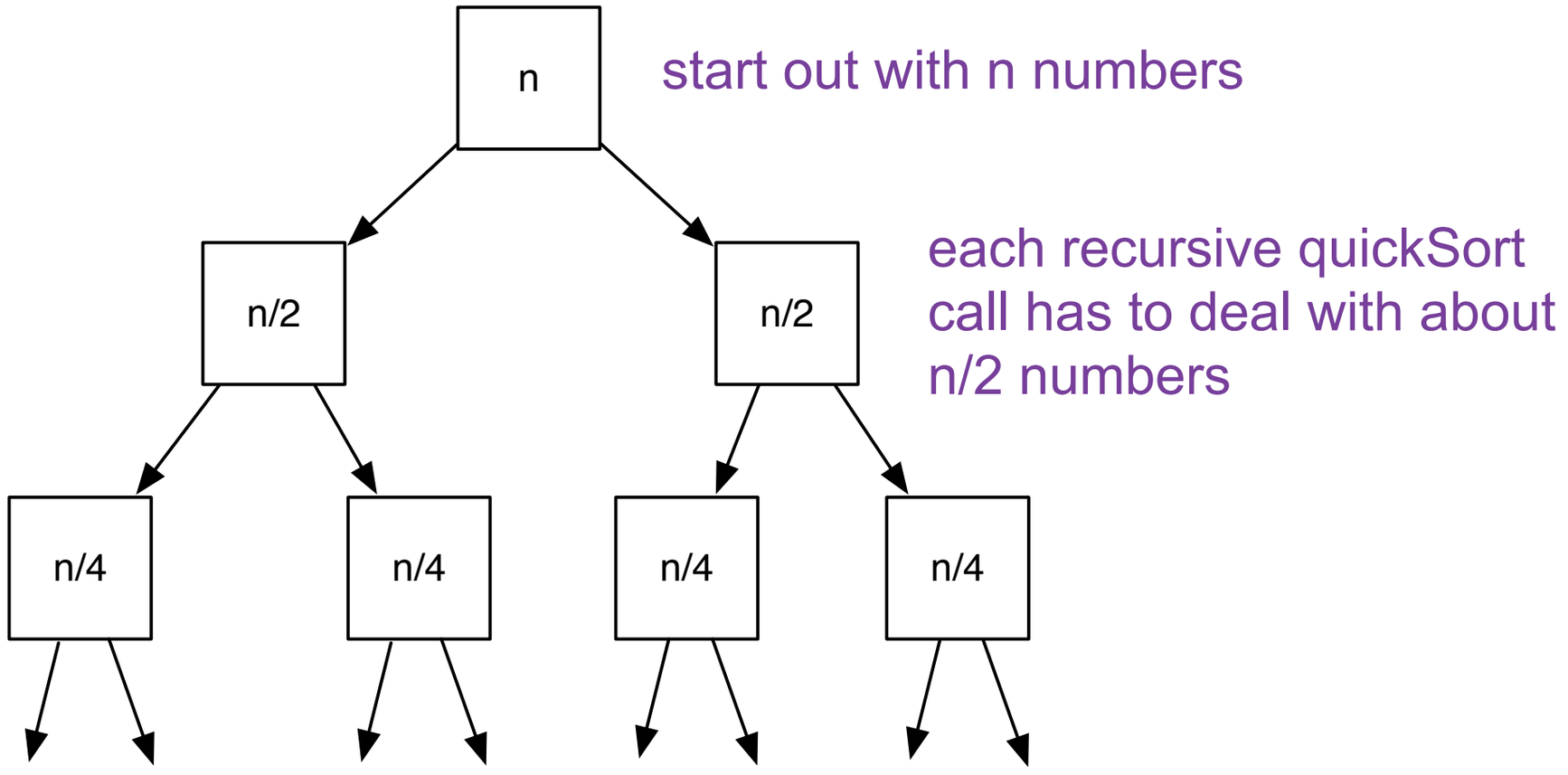
    // swap the first and list items
    inList[0], inList[lastPos] = inList[lastPos], inList[0]

    curIndex := 0
    for i := 0; i < lastPos; i++ {
        if inList[i] < pivot {
            inList[i], inList[curIndex] = inList[curIndex], inList[i]
            curIndex++
        }
    }
    inList[curIndex], inList[lastPos] = inList[lastPos], inList[curIndex]
    return curIndex
}
```



Runtime of Quicksort

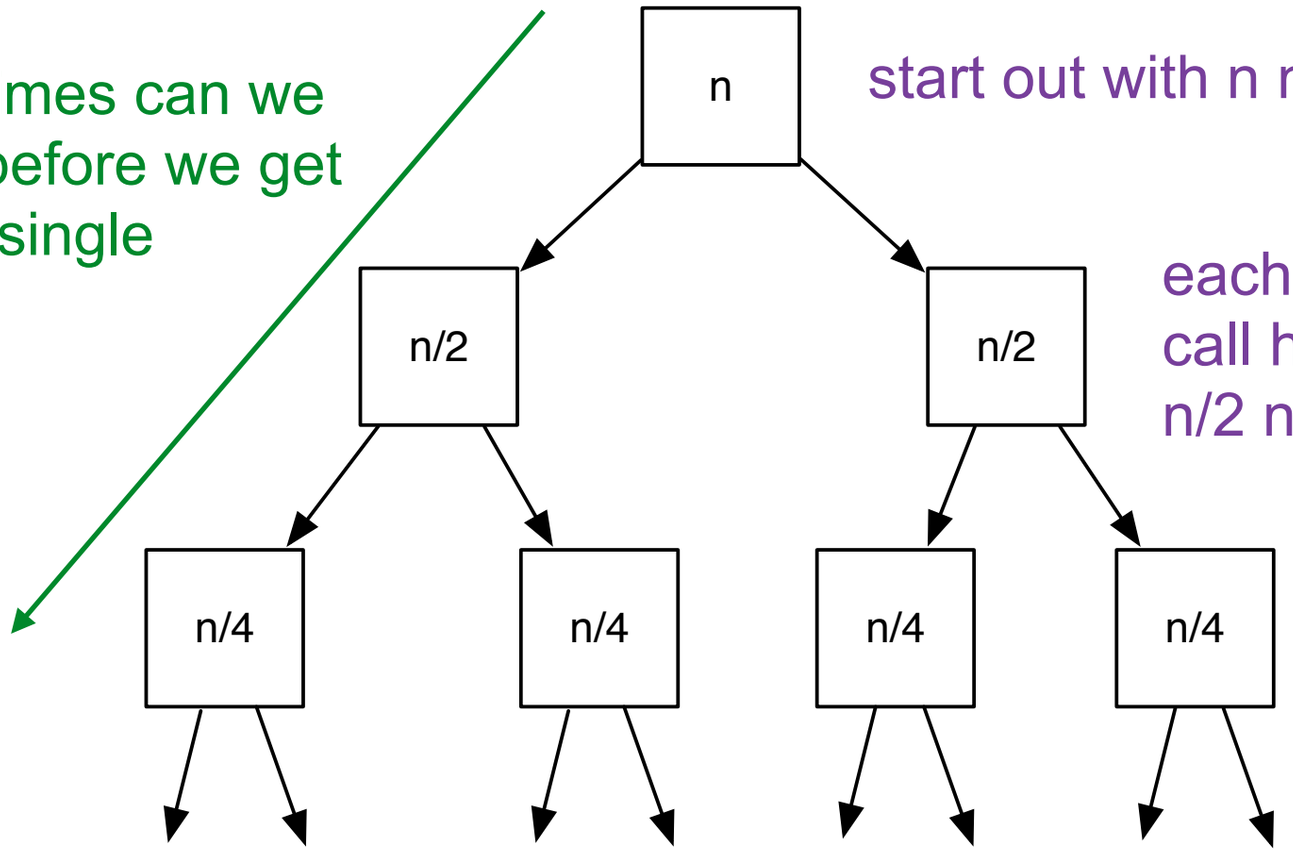
- What if luckily inList[0] was always the median of the remaining numbers?



Runtime of Quicksort

- What if luckily inList[0] was always the median of the remaining numbers?

How many times can we divide by 2 before we get to a list of a single number?



start out with n numbers

each recursive quickSort call has to deal with about n/2 numbers

Runtime of Quicksort

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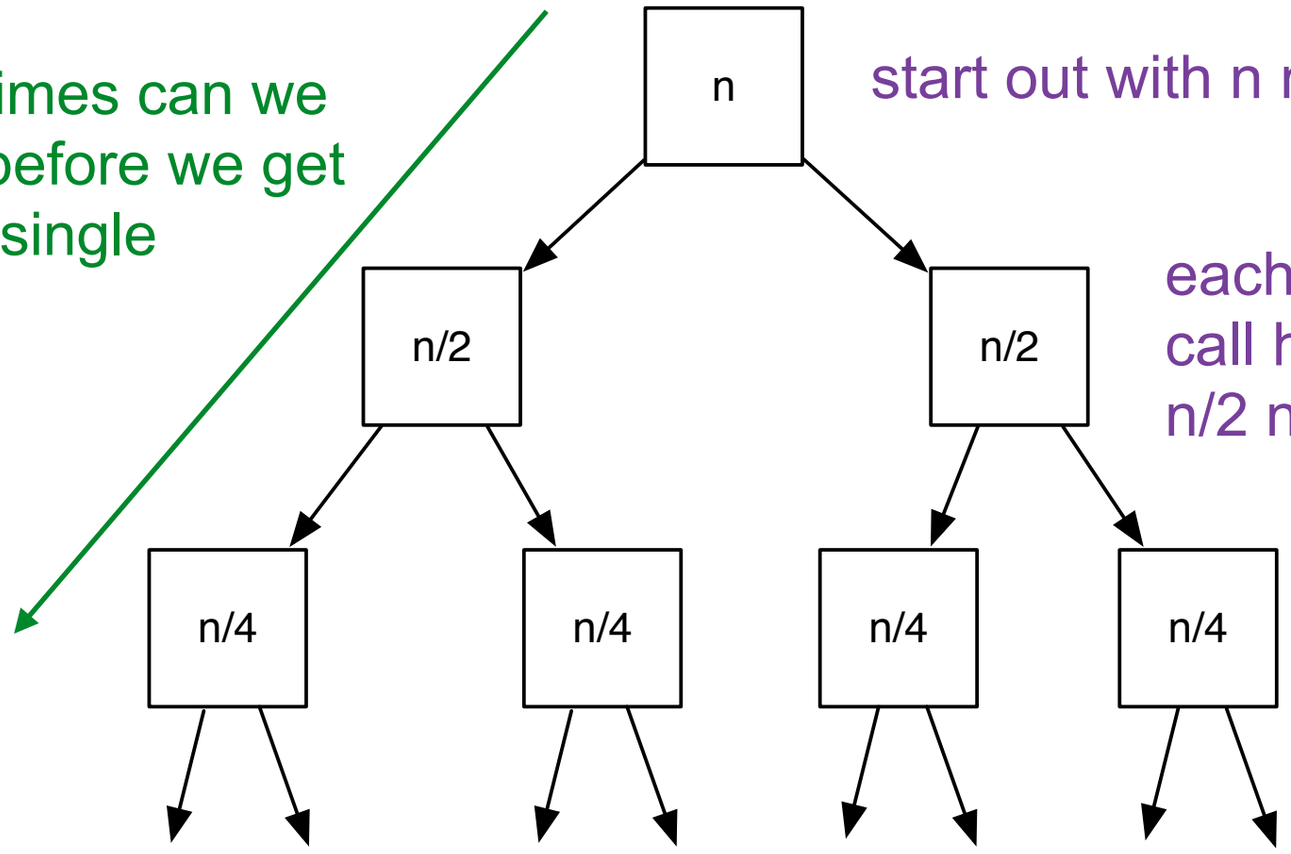


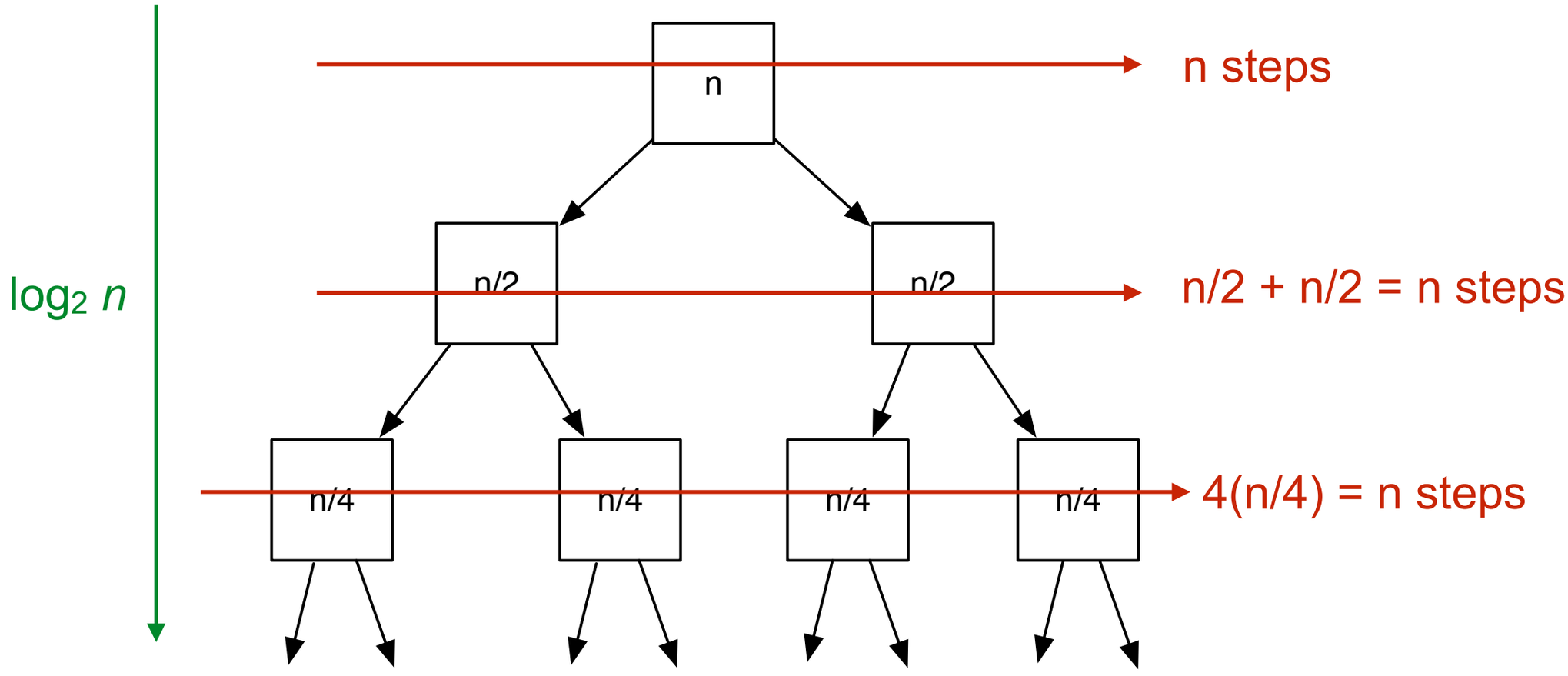
start out with n numbers

each recursive quickSort call has to deal with about n/2 numbers

About $\log_2 n$ by the same reasoning as with power()

Runtime of Quicksort, 2

Each quickSort call calls partition() which does work proportional to the size of the remaining list.



$\log_2 n$ levels, each with about n steps = $n \log n$ total steps

WORST-CASE Runtime of Quicksort

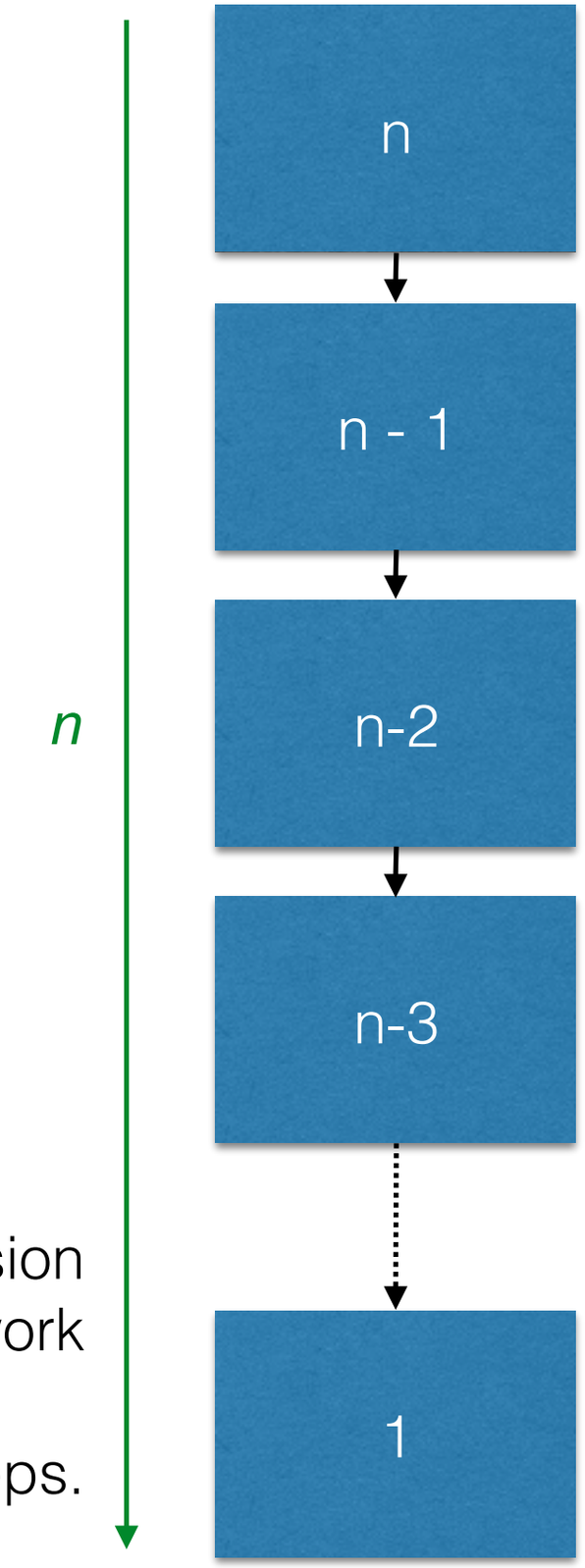
- What if we didn't get lucky?
  What would the worst pattern of partitions be?

WORST-CASE Runtime of Quicksort

- What if we didn't get lucky?
  What would the worst pattern of partitions be?

When the partitions are not balanced (say empty vs. everything else)

Say: when the input is already sorted.



n levels of recursion
each doing about *n* work

= n^2 steps.

Binary Search

Searching for an item in a sorted list

- Let S be a sorted slice
- How would we find the item with value *k* ?

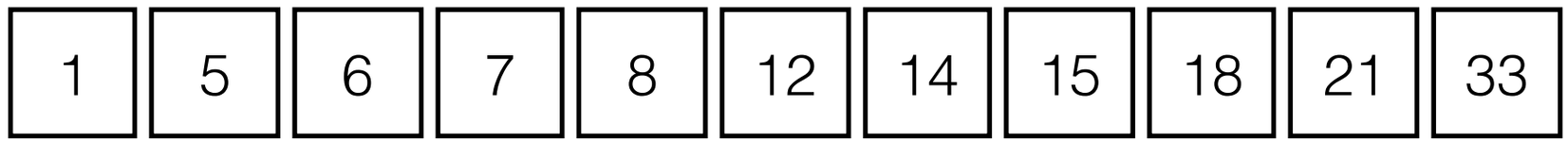
Option 1: Start at the beginning of S and walk through it until you find k:

```
for i, x := range S {
   if x == k {
      return i
   }
}
```

Option 2: the phone book algorithm: open the phone book at the middle, if the item you're looking for is in the first half, go to the middle of the first half, and so on:

Binary Search: The Phone Book Algorithm

Find 8:



At each step:

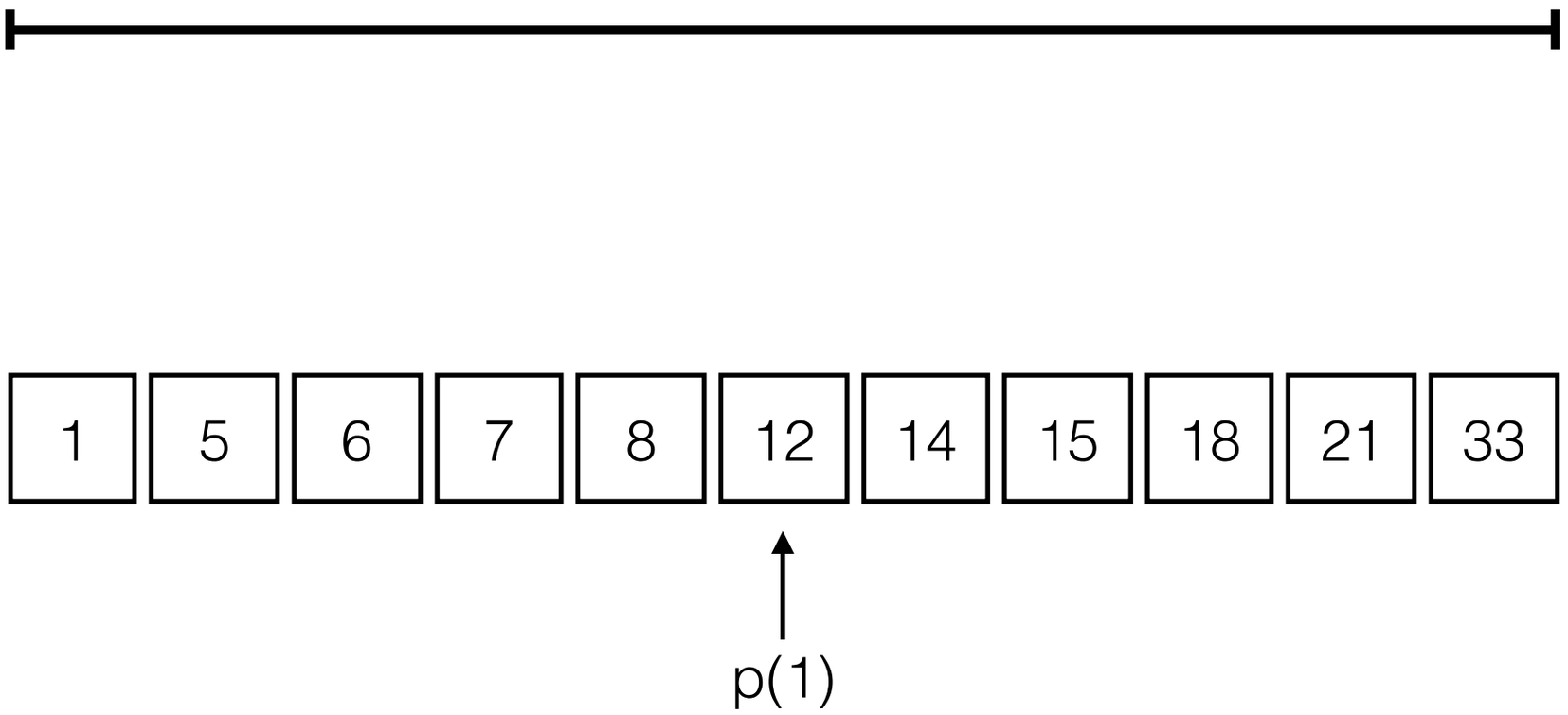
If *p == item, report it

if *p > item, look to the left half

if *p < item, look to the right half

Binary Search: The Phone Book Algorithm

Find 8:



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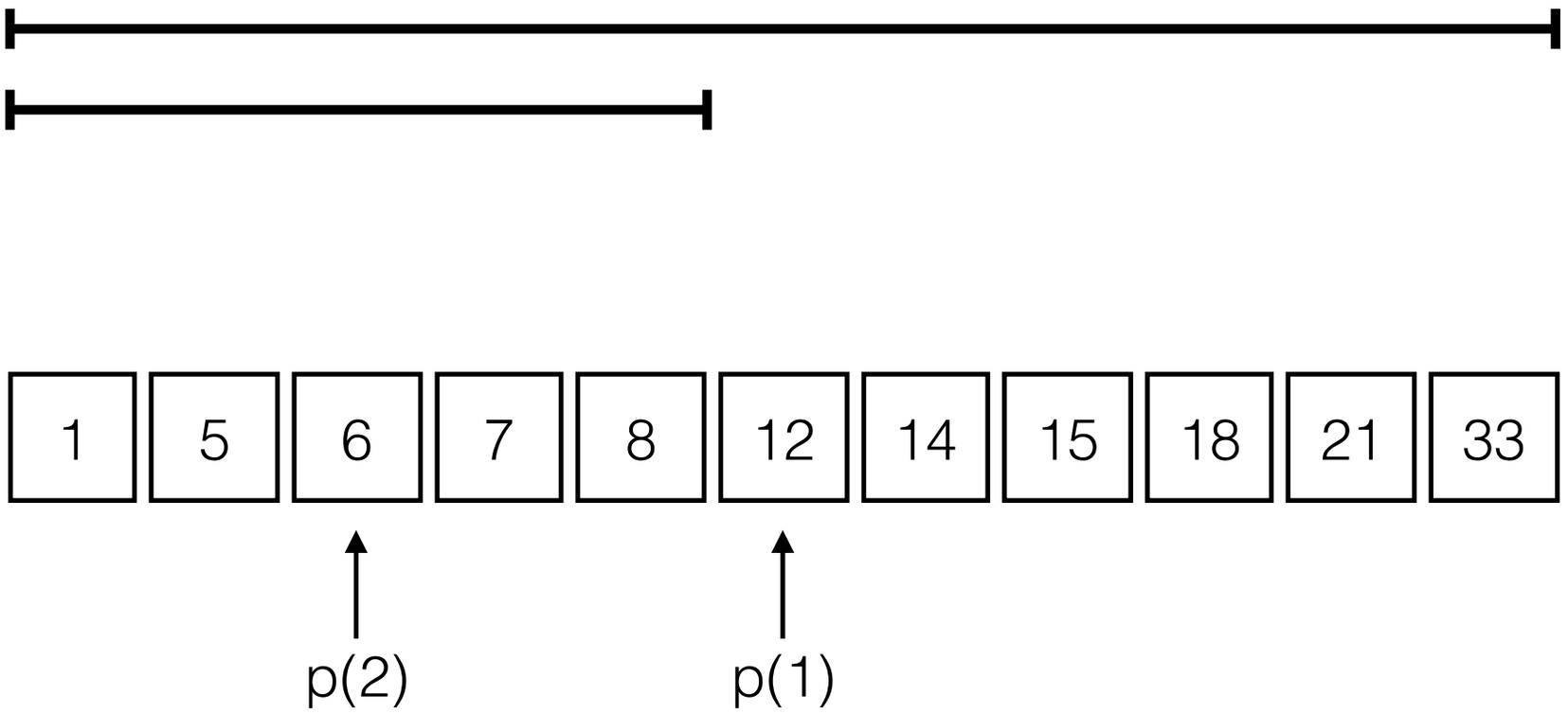

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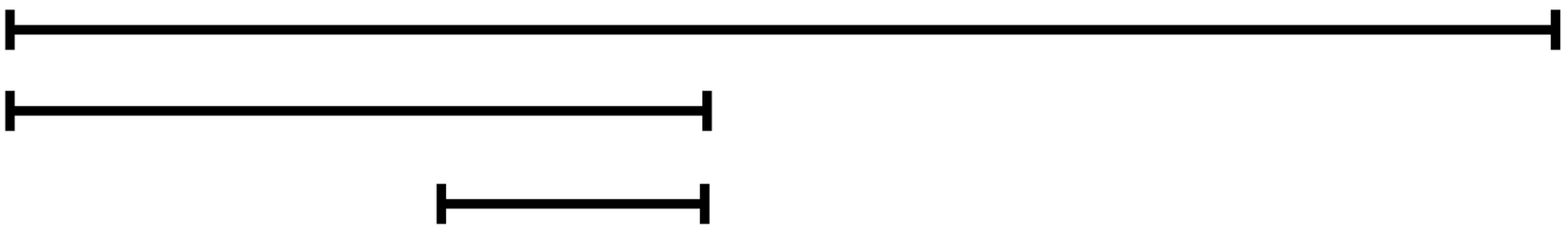

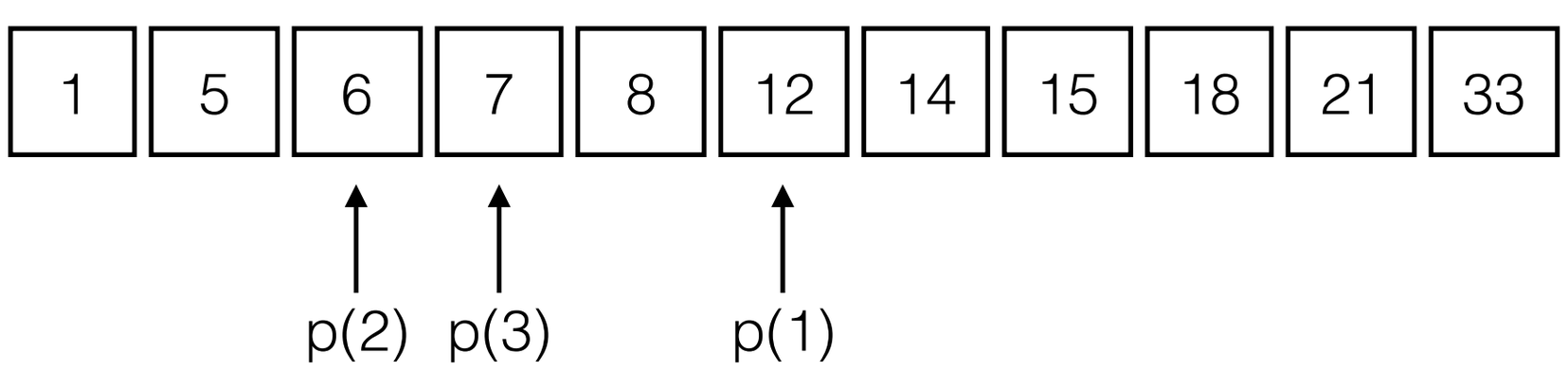

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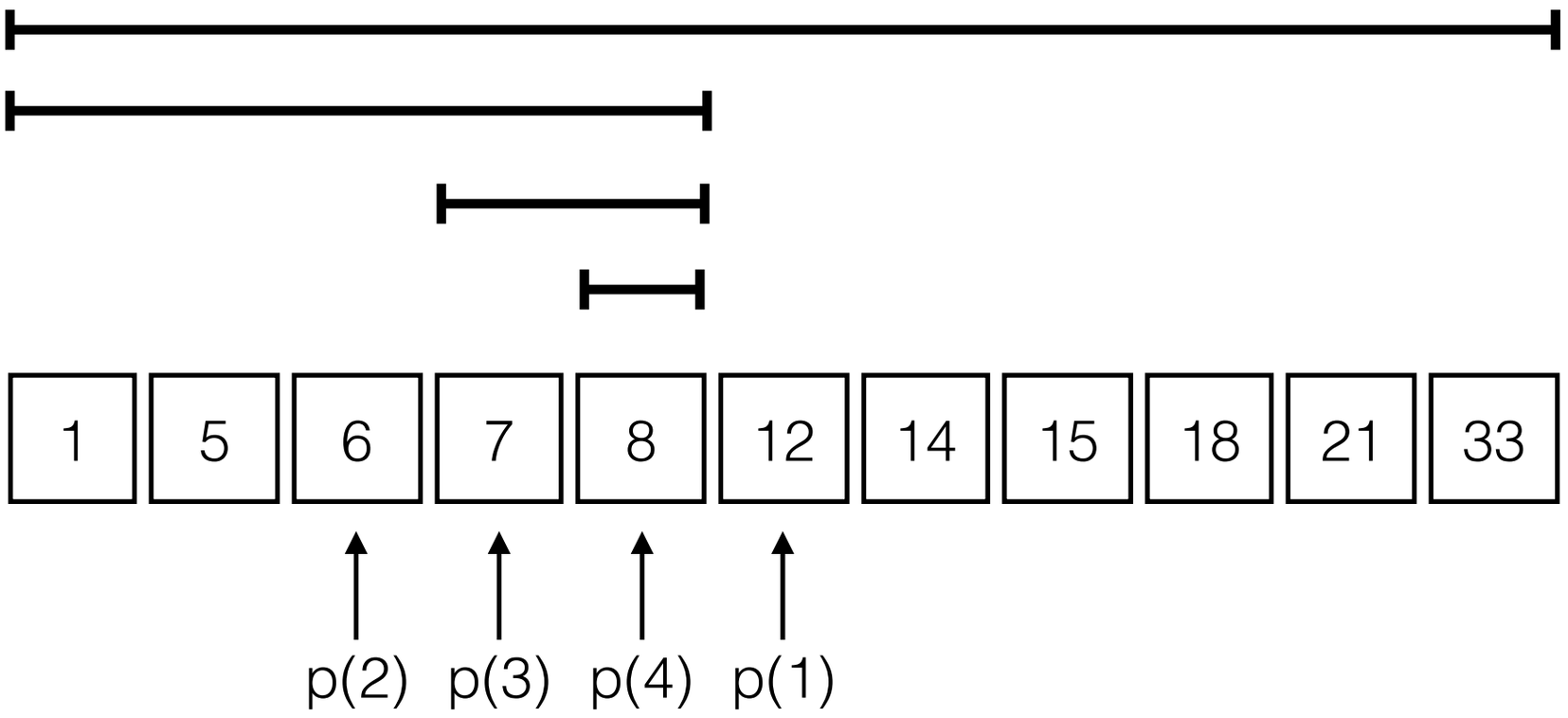

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if *p > item, look to the left half

if *p < item, look to the right half

Binary Search, Code

```
func binarySearch(inList []int, k int) (int, bool) {
    left, right := 0, len(inList)-1
    for left <= right {
        mid := (right + left) / 2
        if inList[mid] == k {
            return mid, true
        } else if inList[mid] > k {
            right = mid - 1
        } else if inList[mid] < k {
            left = mid + 1
        }
    }
    return 0, false
}
```

Binary Search, Recursive

```
func binarySearchRecur(inList []int, k int) (int, bool) {
    if len(inList) == 0 {
        return 0, false
    }
    mid := len(inList)/2
    if inList[mid] == k {
        return mid, true
    } else if inList[mid] > k {
        if mid == 0 {
            return 0, false
        }
        return binarySearchRecur(inList[:mid-1], k)
    } else {
        p, f := binarySearchRecur(inList[mid+1:], k)
        return mid+1+p, f
    }
}
```

If the middle of the list = k, we found it.

If k must be in the left half

check whether we're falling off the left end of the array

Binary Search Runtime

- What similar thing have see that could tell us the runtime?

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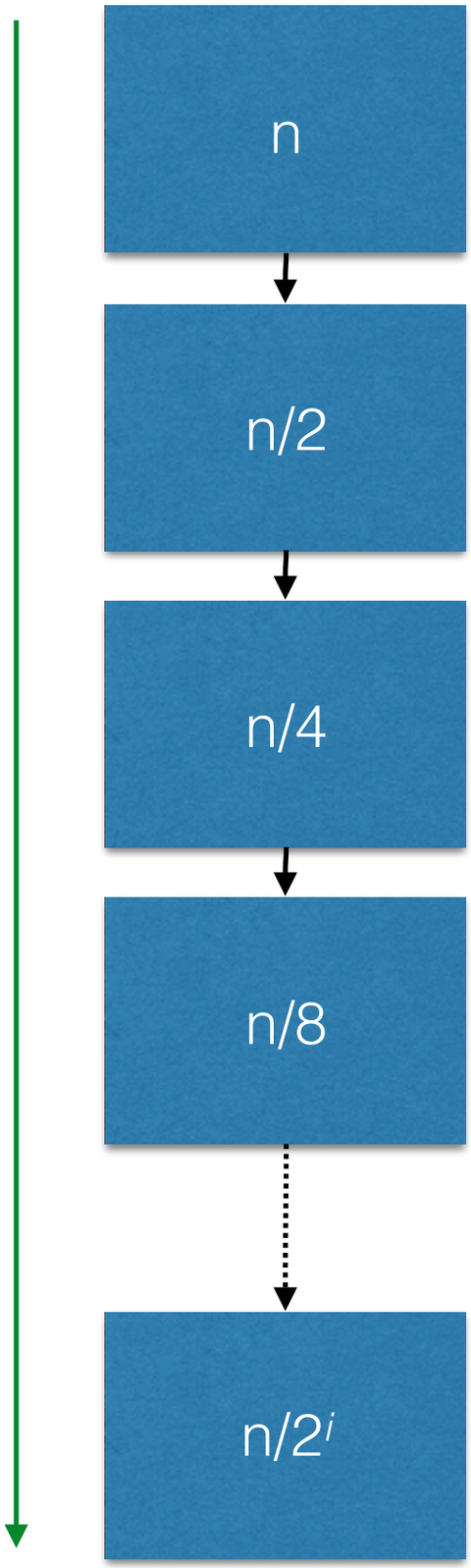

Will “recurse” $\log_2 n$ times.
At each level, we do a constant amount of work.

Runtime for binary search $\approx \log_2 n$

Summary

- Recursion is implemented in the same way as any other function call: the local variables are stored on the stack.

- Divide and conquer: good way to speed up algorithms (binary search, power, quicksort)

- Worst-case runtime for insertion sort is about n^2 steps

- Worst-case runtime for quick sort is about n^2 steps, but in practice, you expect around n log n steps.