02-713 Homework #11: NP-completeness

Due: May 5 by 9:30am

Don't look up solutions in the web. The point of these homeworks is to prepare for the exams. You may talk with your classmates about these problems, but you **must write up your solutions independently**, without using common notes or worksheets. You must indicate at the top of your homework with whom you worked. Your write up should be clear and concise. It should be submitted via Autolab (`https://autolab.cs.cmu.edu/02713-s14/`) as a typeset PDF.

For the following problems, you can assume that Independent Set, 3-SAT, Graph Coloring, and Hamiltonian Path (and its variants) are **NP**-complete.

1. The Clique problem is the following: given an undirected graph $G = (V, E)$ and a positive integer $k \leq |V|$, does G contain a set C of $\geq k$ vertices such that every two vertices $u, v \in C$ are connected by an edge?

   Prove Clique is **NP**-complete.

2. Suppose $G = (V, E)$ is an undirected graph. A *strongly independent set* is a subset S of vertices such that for any two vertices $u, v \in S$ there is no path of length ≤ 2 between u and v.

   Consider the following Strongly Independent Set (SIS) problem:

   > Given an undirected graph $G = (V, E)$ and an integer k, does G have a strongly independent set of size k?

   Prove SIS is **NP**-complete.

3. The Longest Path problem: given an undirected graph $G = (V, E)$, and a positive integer $K \leq |V|$, does G contain a simple path (a path visiting no vertex more than once) with K or more edges?

   Prove that Longest Path is **NP**-complete.

4. Let Integer Linear Programming be the decision problem asking whether a given maximization integer linear program has a solution of objective value $\geq$ a given k. Prove Integer Linear Programming is **NP**-complete.