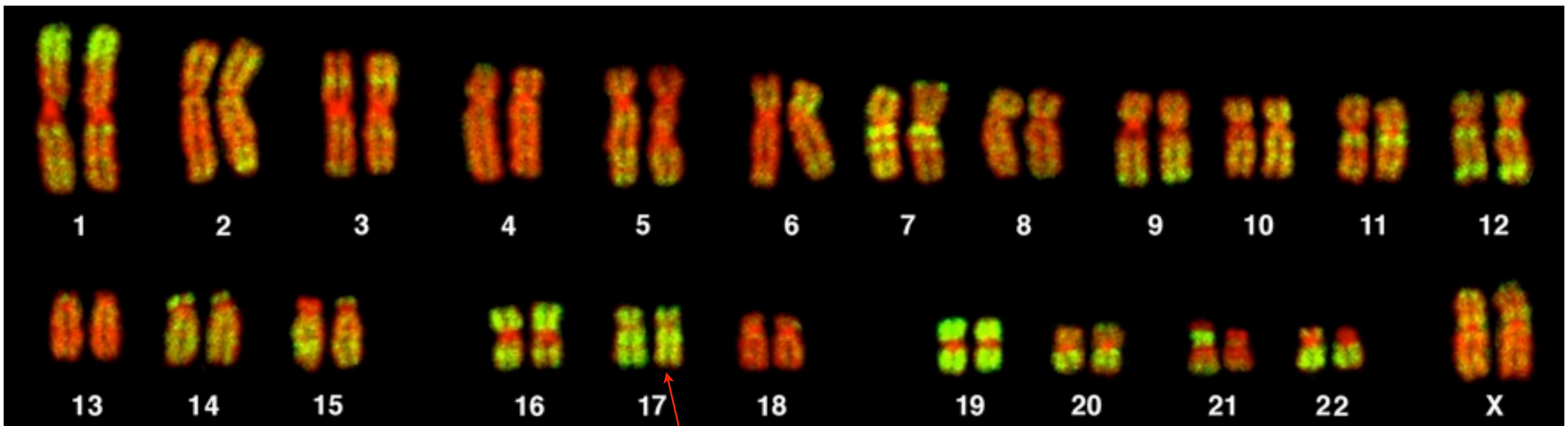
String Matching Z

(Following Gusfield Chapter 2)

Exact String Search

Microscopy of chromosomes of a human female (karyotype):



Bolzer et al, PLoS Biol. 2005

ggccgggccctgtgaccacagtccacatcacaccaggacacagaggaagggccgggccctgtgaccacagtccacatcacaccaggacacagaggaagggccgggcctcatgaccacagt

gtccacatcaca

Where does this string occur in the genome?

Exact String Matching

Exact String Matching Problem. Given a (long) string T and a shorter string P, find all occurrences of P in T. Occurrences of P are allowed to overlap.

- Motivation is obvious:
  - search for words in long documents, webpages, etc.
  - find subsequences of DNA, proteins that are known to be important.

- We'll see 4 efficient algorithms for this problem.

The Simple (Slow) Algorithm

```
SimpMatch(T, P):
  for i = 1..|T|:
    j = 1
    while j ≤ |P| and T[i+j-1] == P[j]:
      j += 1
    if j == |P|+1: print "Occurs at", i
```

- Runs in $O(|T| \times |P|)$ time.
- Information gathered in **while** loop at iteration i is ignored in iteration $i+1$.
- Key idea for speeding it up: use what we learned about T in the **while** loop to increment i by more than 1 in the outer loop.

Exploiting Patterns in P

```
        i
        ↓
All this happened, more or less.
         happy
          happy
```

- After comparing “happy” to “happe” at iteration i,
  - we know that $T[i...i+3]$ = “happ” = $P[1...4]$
  - we can deduce that there can be no match at $i+1$ because $T[i+1] = P[2]$ = “a” but $P[1]$ = “h”
  - in fact, since “h” does not appear in $T[i...i+3] = P[1...4]$, we could set $i = i + 4$

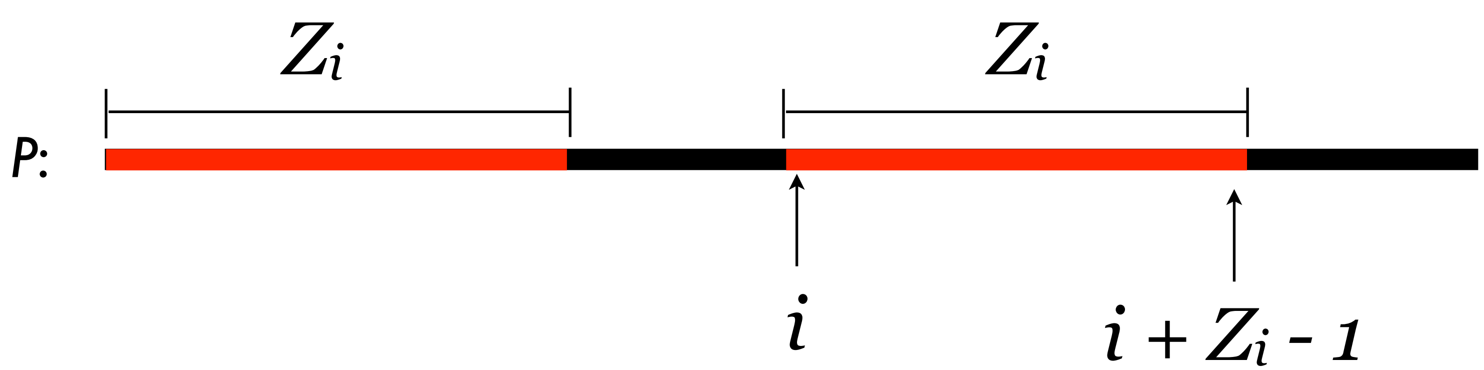
- Since T will have matched some part of P, it is the similarities between various parts of P that allow us to make these deductions.

$\Longrightarrow$ Preprocess P to find these similarities.

Z-Algorithm

Fundamental Preprocessing

Def. $Z_i(P)$ = the length of the longest substring of P that starts at $i > 1$ and matches a prefix of P.



- P = "aardvark": $Z_2 = 1$, $Z_6 = 1$
- P = "alfalfa": $Z_4 = 4$
- P = "photophosphorescent": $Z_6 = Z_{10} = 3$

String Search With Z_i

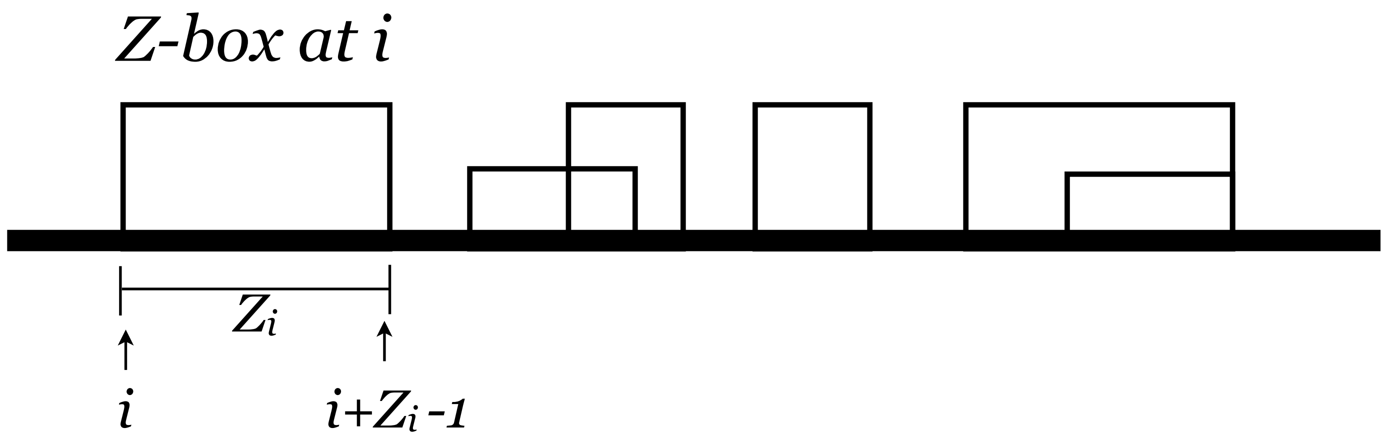
```
ZMatch(T, P):
  S = P$T
  Compute all Zi for S
  return all i-|P|-1 such that Zi = |P|
```

(map indices of S to indices of T)

Why does this work?

- $Z_i(S) = |P|$ if and only if the string starting at i in S matches P.
- Running time is $O(|P| + |T| + Z_S)$, where Z_S is the time to compute the Z_i for S.
- **Next**: an $O(|P| + |T|)$ algorithm for computing the Z_i.

Z Boxes



Def. *Z-box at i* is the substring starting at i and continuing to $i+Z_i-1$. This is the substring that matches the prefix. There is no Z-box at i if $Z_i = 0$.

- Algorithm for computing Z_i will iteratively compute Z_k given:
  - $Z_2 \ldots Z_{k-1}$, and
  - the boundaries l, r of the rightmost Z-box found starting someplace in $2 \ldots k-1$.

Z Algorithm

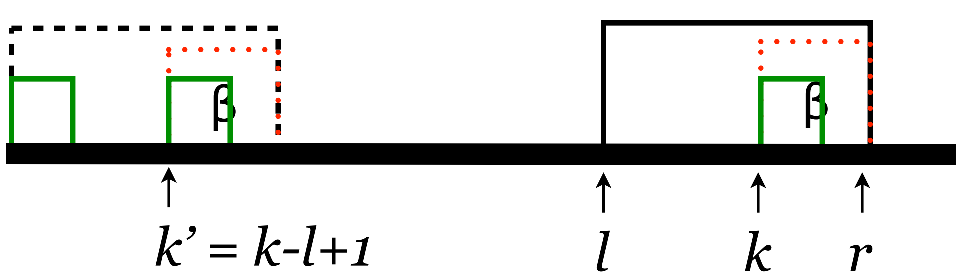
- Input: $Z_2 \ldots Z_{k-1}$, and the boundaries l, r of the rightmost Z-box found starting someplace in $2 \ldots k$-1.
- Output: Z_k, and updated l, r

1. If $k > r$, explicitly compute Z_k by comparing with prefix.
   If $Z_k > 0$: $l = k$ and $r = k + Z_k - 1$ (since this is a new farther right Z-box).

2. If $k \leq r$, this is the situation:

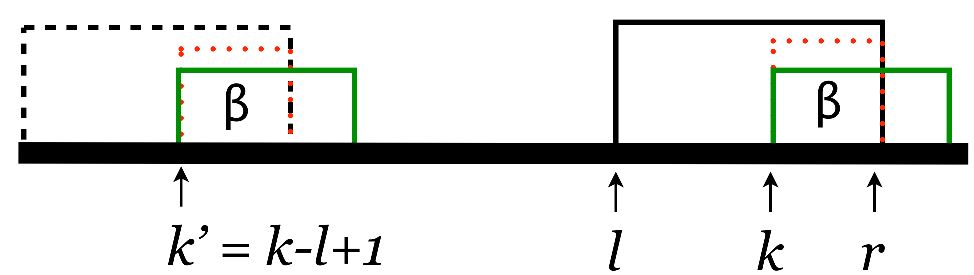
Two subcases:

$\underline{Z_{k'} < \beta}$:



Set $Z_k = Z_{k'}$ and leave l, r unchanged.

$\underline{Z_{k'} \geq \beta}$:



Explicitly compare <u>after r</u> to set Z_k.
$l = k$, r = point where comparison failed

Analysis

- Correctness follows by induction and the arguments we made in the description of the algorithm.
- Runs in O($|S|$) time:
  - only match characters covered by a Z-box once, so there are O($|S|$) matches.
  - every iteration contains at most one mismatch, so there are O($|S|$) mismatches.
- Immediately gives an O($|P| + |T|$)-time algorithm for string matching as described a few slides ago.
  - O($|P| + |T|$) is the best possible worst-case running time, since you might have to look at the whole input.
  - But better algorithms exist in practice that, for real instances, have expected sublinear runtime.