Constructive Logic (15-317), Fall 2022
Assignment 4: Quantifiers and Arithmetic

Constructive Logic Staff
(Instructor: Karl Crary)

Due: Wednesday, September 28, 2022, 11:59 pm

This assignment is coding only, using Dcheck. Please submit a file named "`hw.deriv`" to "Homework 4."

You can find documentation on Dcheck at `cs.cmu.edu/~crary/dcheck/dcheck.pdf` and a sample file at `cs.cmu.edu/~crary/dcheck/example.deriv`. (Be aware that the sample file uses several logics that we have not seen yet in class.)

1 Quantifiers

Using Dcheck, give derivations of the following judgements, if they are derivable. For the ones that are not derivable, simply put:

`deriv` ⟨*name*⟩ `= omitted`

Use system "AR" (even though you are not using arithmetic in this section), and name your derivations `task1`, `task2`, etc. Note: to avoid revealing the answer, the autograder will not provide feedback on any problem for which you answer `omitted`.

Distributivity

Task 1 (6 points). $(\exists x{:}\tau.\, A(x) \vee B(x)) \supset (\exists x{:}\tau.A(x)) \vee (\exists x{:}\tau.B(x))$ true

Task 2 (6 points). $(\exists x{:}\tau.A(x)) \vee (\exists x{:}\tau.B(x)) \supset (\exists x{:}\tau.\, A(x) \vee B(x))$ true

DeMorgan

These judgements are true in classical logic, but may or may not be true constructively.

Task 3 (4 points). $\neg(\forall x{:}\tau.\, A(x)) \supset \exists x{:}\tau.\neg A(x)$ true

Task 4 (4 points). $\neg(\exists x{:}\tau.\, A(x)) \supset \forall x{:}\tau.\neg A(x)$ true

Forall/Exists

Task 5 (3 points). $(\forall x{:}\tau.\, A(x)) \supset (\exists x{:}\tau.\, A(x))$ true

Task 6 (5 points). $(\forall x{:}\tau.\, A(x)) \wedge (\exists x{:}\tau.\, \mathsf{T}) \supset (\exists x{:}\tau.\, A(x))$ true

2 Arithmetic

Task 7 (12 points). $\forall x{:}\mathsf{nat}.\, \forall y{:}\mathsf{nat}.\, \forall z{:}\mathsf{nat}.\, x = y \supset y = z \supset x = z\ \mathsf{true}$

Use system "AR" and name your derivation `task7`. (Caution: Many students find this proof challenging, so we recommend you start early.)

3 Context Mastery

Task 8 (4 points). Using Dcheck, give a derivation of the following judgement using contexts (*i.e.,* using the `NDC` system):

$$\vdash \neg A \vee \neg B \supset \neg(A \wedge B)\ \mathsf{true}$$

Name your derivation `task8`. **Instant feedback is turned off for this task, so be extra careful.**

4 Verifications and Uses Mastery

Task 9 (4 points). Using Dcheck, give a derivation of the following judgement:

$$\neg P \vee \neg Q \supset \neg(P \wedge Q)\uparrow$$

Name your derivation `task9`. (Remember that Dcheck takes the propositions `P` and `Q` to be atomic.) **Instant feedback is turned off for this task, so be extra careful.**