Constructive Logic (15-317), Fall 2022
Assignment 8: Inversion

Constructive Logic Staff
(Instructor: Karl Crary)

Due: Wednesday, November 2, 2022, 11:59 pm

This assignment consists of a written portion only. Please submit a file name "`hw.pdf`" to Gradescope, to the assignment labelled "Homework 8."

We recommend that you typeset your written solutions. Most students use LaTeX, but other software is acceptable. (Please put each task on its own page to speed up grading.) If you choose not to typeset your solutions, be aware that you are answerable for your handwriting. Any that the grader has difficulty reading (in the sole judgement of the grader), will be marked wrong.

1 Inversion

For each of the following rules, indicate whether it is invertible or not. If it is invertible, prove it. If not, give an instance of the rule (*i.e.,* fill in the metavariables) so that conclusion of the rule is derivable, but at least one of the premises is not derivable. If there is more that one premise, indicate which one is not derivable. You do not need to give a derivation of the conclusion, nor do you need to prove the premise is not derivable.

In your proofs pertaining to sequent calculus, feel free to use cut admissibility, identity, weakening, and/or contraction, as necessary.

Task 1 (10 points).
$$\frac{\Gamma \vdash A \text{ true} \quad \Gamma \vdash B \text{ true}}{\Gamma \vdash A \wedge B \text{ true}} \wedge I$$

Task 2 (10 points).
$$\frac{\Gamma \vdash A \text{ true}}{\Gamma \vdash A \vee B \text{ true}} \vee I1$$

Task 3 (10 points).
$$\frac{}{\Gamma, A \text{ true} \vdash A \text{ true}} \ hyp$$

Task 4 (10 points).
$$\frac{A \text{ true} \in \Gamma}{\Gamma \vdash A \text{ true}} \ hyp$$

Task 5 (10 points).
$$\frac{\Delta, A \longrightarrow B}{\Delta \longrightarrow A \supset B} \supset R$$

Task 6 (10 points).

$$\frac{\Delta, A \supset B \longrightarrow A \quad \Delta, B \longrightarrow C}{\Delta, A \supset B \longrightarrow C} \supset L$$