EXPERIMENTAL UPPER BOUNDS FOR STAR QUALITY AS A FUNCTION OF POLYTOPE QUALITY

Alexandre Cunha and **Omar Ghattas**

Laboratory for Mechanics, Algorithms, and Computing
Carnegie Mellon University
5000 Forbes Avenue
Pittsburgh, PA 15213
cunha@cmu.edu, oghattas@cs.cmu.edu

Let v be a vertex of a simplicial mesh $\mathcal{M}$. The star $\mathcal{S}_v$ of v consists of all elements (triangles in 2D or tetrahedra in 3D) of which v is a vertex, $\mathcal{S}_v = \{t \,|\, v \in t\}$, and the polytope $\mathcal{P}_v$ (polygon or polyhedron) associated with v is formed by the boundary faces of $\mathcal{S}_v$. We offer experimental upper bounds for the quality of $\mathcal{S}_v$, $q(\mathcal{S}_v) = min\,\{q(t_i)\,|\,t_i \in \mathcal{S}_v\}$, as polynomial functions of the quality of $\mathcal{P}_v$, $q(\mathcal{P}_v)$, that we believe are tight. Analytical or empirical expressions for such quality bounds are nowhere to be found in the literature.

An upper bound for $\mathcal{S}_v$ can serve as a quality indicator for both mesh generation and mesh improvement. As an example, when performing the insertion of a new Steiner point in a cavity during Delaunay refinement, the insertion algorithm might decide, according to the upper bound value, to locate the point on an *optimal* position in the cavity based on its quality and not on the quality of a single element. In local smoothing, our target application, if the current quality of $\mathcal{S}_v$ is equal or very close to its upper bound, computed via $q(\mathcal{P}_v)$, we might decide not to reposition the free vertex v since the quality improvement would be none or minimal, thus avoiding indispensable computation.

Normalized *isoperimetric quotients* are used to assess the quality of polytopes, including triangles and tetrahedra. These are quantities involving the ratio between the volume, area, and edge lengths of the polytope. As an example, $\gamma = \frac{4\tan\left(\frac{\pi}{n}\right)\Delta}{\sum_{1\leqslant i\leqslant n} {l_i}^2}$, $\gamma \in [0,1]$, is one of the metrics we adopted for n-sided polygons, where tan is the trigonometric tangent function, Δ gives the polygon area, and l_i is the length of its $i-$th edge. Isoperimetric metrics are commonly employed to evaluate mesh quality.

We determine the upper bounds via exhaustive generation of random star shaped polytopes. For each random polytope $\mathcal{P}_v$ we compute the optimal location of the vertex v that maximizes the quality of $\mathcal{S}_v$ using numerical optimization. Thus, we solve an optimization problem that gives the best possible location for v inside $\mathcal{P}_v$.

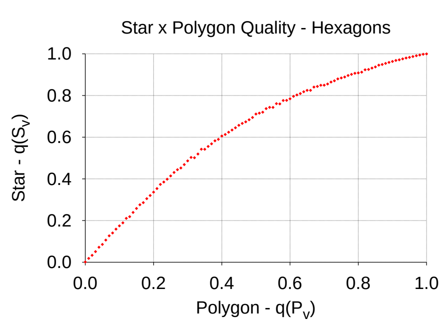

Figure 1: Star upper bound $q(\mathcal{S}_v)$ as a function of $q(\mathcal{P}_v)$ using quality metric γ.