Written Assignment #3 Solution

15-462 Graphics I, Fall 2003
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Due: Tuesday, November 18, 2003 (**before lecture**)
80 POINTS

November 27, 2003

- The work must be all your own.
- The assignment is due before lecture on Tuesday, November 18.
- Be explicit, define your symbols, and explain your steps. This will make it a lot easier for us to assign partial credit.
- Use geometric intuition *together* with trigonometry and linear algebra.
- Verify whether your answer is meaningful with a simple example.

1 Angel, Chapter 7 (Discrete Techniques), Exercise 7.8

Suppose that we have two translucent surfaces characterized by opacities α_1 and α_2. What is the opacity of the translucent material that we create by using the two in series? Given an expression for the transparency of the combined material.

7.8 If a surface has opacity α, a fraction $1-\alpha$ of the light from behind it will pass through it and will be seen from the front. Now consider two surfaces with opacities α_1 and α_2. A fraction $1-\alpha_1$ will pass through the first and this amount will be reduced by $1-\alpha_2$ by passing through the second, leaving the same amount of light as would a surface with transparency $(1-\alpha_1)(1-\alpha_2)$ or equivalently the amount of light passing through a surface of opacity $1-(1-\alpha_1)(1-\alpha_2)$.

2 Angel, Chapter 7 (Discrete Techniques), Exercise 7.10

In Section 7.9 we used $1-\alpha$ and α for the destination and source blending factors, respectively. What would be the visual difference if we used 1 for the destination factor, and kept α for the source factor?

7.10 There are two issues. First, if we use α and $1-\alpha$ we avoid problems of having colors and opacities exceeding 1 and being clipped. However, by using this choice, the order in which we composite multiple surfaces matters which would not make a difference if we used α and 1.

3 Angel, Chapter 7 (Discrete Techniques), Exercise 7.14

When we supersample a scene using jitter, why shoulud we use a random jitter pattern?

7.14 Whenever we use regular patterns, we risk creating beat patterns or Moire effects. Random jitter avoids the problem. Note that in a mathematical sense we are no better off using jitter. However, our visual systems are very sensitive to regular patterns so that jittering makes the images appear to be better.

4 Angel, Chapter 8 (Implementation of a Renderer), Exercise 8.12

Devise a method for testing whether one planar polygon is fully on one side of another planar polygon, i.e., so that there exists a separating plane.

The original question was ambiguous about whether the answer was 3D or 2D, e.g., separating plane vs separating line. Answering either case is sufficient, and involves similar algebra, e.g., testing using an oriented half plane (3D), or an oriented half line (2D).

8.12 We can from the equation of the plane of one the polygons from any three of its vertices. We can then successively test the values of all the vertices of the other in the this equation. If we get the same sign for each vertex, then the second polygon does not intersect the first. Note that we must also test if the first polygon intersects the second by forming the equation of the plane from three vertices of the second.
We can also test the sign of the determinant

$$\begin{vmatrix} x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_i & y_i & z_i & 1 \end{vmatrix},$$

where the first three lines are from three vertices of the first (second) polygon and the last vertex is each vertex of the second (first) polygon.

5 Angel, Chapter 13 (Advanced Rendering), Exercise 13.3

Derive an implicit equation for a torus whose center is at the origin. (You can derive the equation by noting that a plane that cuts through the torus reveals two circles on the same radius.)

13.3 Consider two identical circles of radius r centered at $(a, 0)$ and $(-a, 0)$. We can describe them through the single implicit equation

$$((x-a)^2 + y^2 - r^2)((x+a)^2 + y^2 - r^2),$$

by simply multiplying together their individual implicit equations. We can form the torus by rotation these circles about the y axis which is equivalent to replacing x^2 by $x^2 + z^2$.

6 Angel, Chapter 13 (Advanced Rendering), Exercise 13.4

Using your previous result (from 13.3), show that you can ray trace a torus using the quadratic equation to find the required intersections.

The key here is that the equation for the torus is factorable into two quadratic parts, and therefore the roots of intersection can be analytically determined using the roots of the quadratic polynomials.

13.4 If we make the usual substitutions

$$x = x_0 + td_x,$$

$$y = y_0 + td_y,$$

$$z = z_0 + td_z,$$

in the equation for the torus, we obtain an equation whose highest power in t is 4. The roots of this equation can be obtained analytically.

7 Angel, Chapter 13 (Advanced Rendering), Exercise 13.12

Suppose that you have an algebraic function in which the highest term is $x^i y^j z^k$. What is the degree of the polynomial that we need to solve for the intersection of a ray with the surface defined by this function. Provide a derivation.

Substituting the equation for a ray, $\mathbf{r} = \mathbf{r_0} + \mathbf{dt}$, into the polynomial, we see that the highest term determines the degree of the polynomial that must be solved,

$$\begin{aligned} x^i y^j z^k &= (x_0 + d_x t)^i (y_0 + d_y t)^j (z_0 + d_z t)^k && (1) \\ &= d_x d_y d_z t^{i+j+k} + \mathcal{O}(t^{i+j+k-1}) && (2) \end{aligned}$$

and therefore the degree of the polynomial that must be solved is $(i + j + k)$.

8 Angel, Chapter 13 (Advanced Rendering), Exercise 13.13

Consider again an algebraic function in which the highest term is $x^i y^j z^k$. If $i = j = k$, how many terms are in the polynomial that is created when we intersect the surface with a parametric ray?

If the highest degree of the polynomial is $(i + j + k)$, and $i = j = k$, then the highest degree is $3i$, and therefore there can only be as many as $3i + 1$ terms in the polynomial.