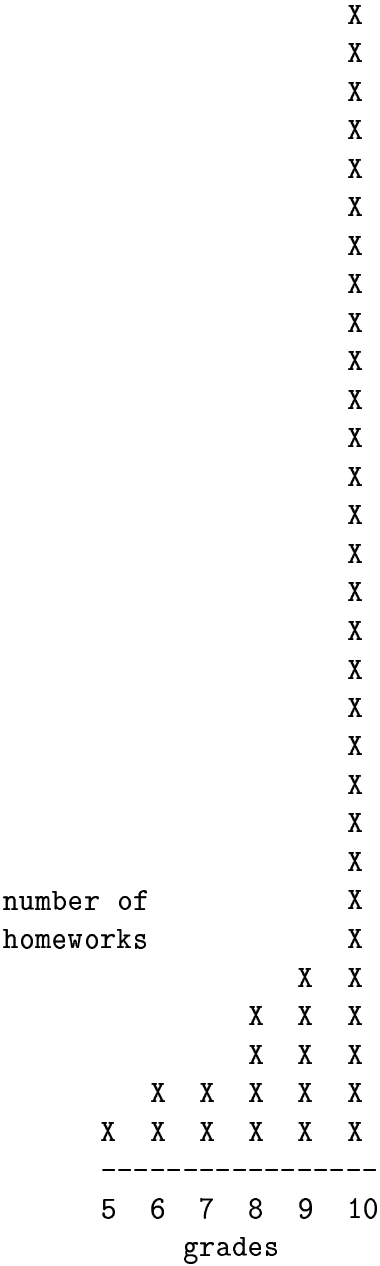


Analysis of Algorithms: Solutions 1



The histogram shows the distribution of grades, from 0 to 10.

Problem 1

Let $A[1..n]$ be a sorted array of n distinct numbers. Write an efficient algorithm `BINARY-SEARCH(A, n, k)` that finds a given value k in the array $A[1..n]$. It should return the index of the found element; if the array does not include k , the algorithm should return 0.

`BINARY-SEARCH(A, n, k)`

`$p \leftarrow 1$`

`$r \leftarrow n$`

while `$p < r$`

do `$q = \lfloor (p + r) / 2 \rfloor$`

if `$k \leq A[q]$`

then `$r \leftarrow q$`

else `$p \leftarrow q + 1$`

if `$k = A[p]$`

then return `p`

else return `0`

Problem 2

(a) $1 + 2 + 3 + 4 + \dots + n = \frac{n \cdot (n+1)}{2}$.

We use a proof by induction. Clearly, the equality holds for $n = 1$; we now show that, if it holds for n , then it also holds for $n + 1$:

$$\begin{aligned}
 1 + 2 + \dots + n + (n + 1) &= (1 + 2 + \dots + n) + (n + 1) \\
 &= \frac{n \cdot (n + 1)}{2} + (n + 1) \\
 &= \frac{n \cdot (n + 1) + 2 \cdot (n + 1)}{2} \\
 &= \frac{(n + 2) \cdot (n + 1)}{2} \\
 &= \frac{(n + 1) \cdot ((n + 1) + 1)}{2}
 \end{aligned}$$

(b) $1 + x + x^2 + x^3 + \dots + x^n = \frac{x^{n+1} - 1}{x - 1}$ (where $x \neq 1$).

We observe that the equality holds for $n = 1$, and apply induction to “step” from n to $n + 1$:

$$\begin{aligned}
 1 + x + \dots + x^n + x^{n+1} &= \frac{x^{n+1} - 1}{x - 1} + x^{n+1} \\
 &= \frac{x^{n+1} - 1 + x^{n+1} \cdot (x - 1)}{x - 1} \\
 &= \frac{x^{n+1} - 1 + x^{n+2} - x^{n+1}}{x - 1} \\
 &= \frac{x^{(n+1)+1} - 1}{x - 1}
 \end{aligned}$$