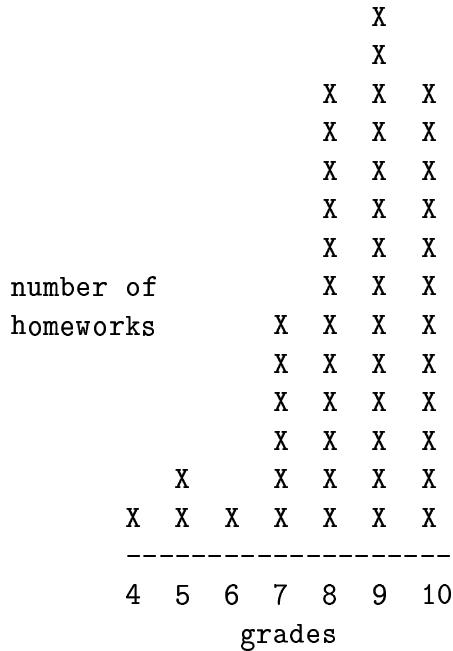


# Analysis of Algorithms: Solutions 2



The histogram shows the distribution of grades, from 0 to 10.

## Problem 1

Suppose that  $A[1..n]$  is an integer array, and some integers occur several times in this array. Write an algorithm for counting integers that occur several times; note that each of them must be counted only once.

For each element  $A[j]$ , we increment the counter if we find a duplicate in the  $A[(j + 1)..n]$  segment and no duplicate in  $A[1..(j - 1)]$ . The DUPLICATES function first checks for duplicates of  $A[j]$  in  $A[1..(j - 1)]$ , and then looks for duplicates in  $A[(j + 1)..n]$ .

COUNT-DUPLICATES( $A, n$ )	<i>cost</i>	<i>times</i>
<i>counter</i> $\leftarrow 0$	$c_1$	1
<b>for</b> $j \leftarrow 1$ <b>to</b> $n$	$c_2$	$n + 1$
<b>do if</b> DUPLICATES( $A, j, n$ )	see below	
<b>then</b> <i>counter</i> $\leftarrow$ <i>counter</i> + 1	$c_3$	$\leq n$
<b>return</b> <i>counter</i>	$c_4$	1

<b>DUPPLICATES</b> ( $A, j, n$ )	<i>cost</i>	<i>times</i>
<b>for</b> $i \leftarrow 1$ <b>to</b> $j - 1$	$c_5$	$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$
<b>do if</b> $A[i] = A[j]$	$c_6$	$0 + 1 + 2 + \dots + (n - 1) = \frac{n(n-1)}{2}$
<b>then return</b> FALSE	$c_7$	$\leq 0 + 1 + 2 + \dots + (n - 1) = \frac{n(n-1)}{2}$
<b>for</b> $i \leftarrow j + 1$ <b>to</b> $n$	$c_8$	$\leq n + (n - 1) + (n - 2) + \dots + 1 = \frac{n(n+1)}{2}$
<b>do if</b> $A[i] = A[j]$	$c_9$	$\leq (n - 1) + (n - 2) + (n - 3) + \dots + 0 = \frac{n(n-1)}{2}$
<b>then return</b> TRUE	$c_{10}$	$\leq (n - 1) + (n - 2) + (n - 3) + \dots + 0 = \frac{n(n-1)}{2}$
<b>return</b> FALSE	$c_{11}$	$\leq n$

**Problem 2**

Estimate the worst-case running time of your algorithm.

$$\begin{aligned} T(n) &\leq c_1 + c_2(n+1) + c_3n + c_4 + c_5 \frac{n(n+1)}{2} + c_6 \frac{n(n-1)}{2} + c_7 \frac{n(n-1)}{2} \\ &\quad + c_8 \frac{n(n+1)}{2} + c_9 \frac{n(n-1)}{2} + c_{10} \frac{n(n-1)}{2} + c_{11}n \\ &= \left( \frac{c_5}{2} + \frac{c_6}{2} + \frac{c_7}{2} + \frac{c_8}{2} + \frac{c_9}{2} + \frac{c_{10}}{2} \right) n^2 \\ &\quad + \left( c_2 + c_3 + \frac{c_5}{2} - \frac{c_6}{2} - \frac{c_7}{2} + \frac{c_8}{2} - \frac{c_9}{2} - \frac{c_{10}}{2} + c_{11} \right) n \\ &\quad + \left( c_1 + c_2 + c_4 \right) \\ &= \Theta(n^2) \end{aligned}$$