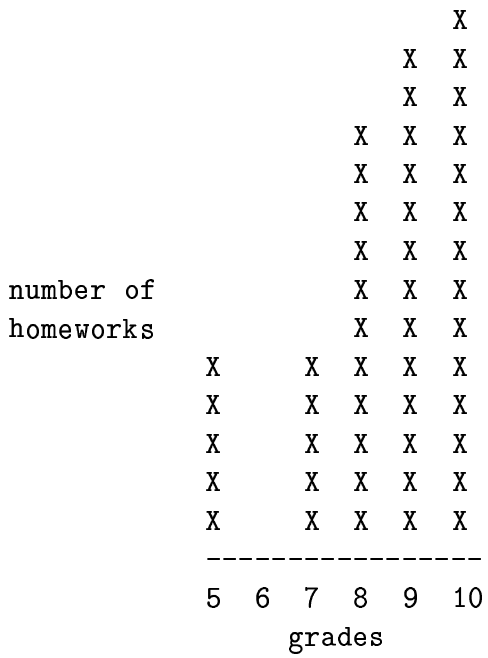
Analysis of Algorithms: Solutions 3



The histogram shows the distribution of grades, from 0 to 10.

Problem 1
Write pseudocode for the MERGE(A, p, q, r) procedure.

We use an auxiliary array $B[p..r]$, for storing the result of merging $A[p..q]$ and $A[q+1..r]$. After the merge, we copy the contents of $B[p..r]$ into $A[p..r]$. The running time is $\Theta(r-p+1)$.

```
MERGE(A, p, q, r)
i ← p        ▷ index in A[p..q]
j ← q + 1    ▷ index in A[q + 1..r]
k ← p        ▷ index in B[p..r]
while i ≤ q or j ≤ r      ▷ merge A[p..q] and A[q + 1..r]
    do if j > r
          then B[k] ← A[i]
               i ← i + 1
       else if i > q
          then B[k] ← A[j]
               j ← j + 1
       else if A[i] ≤ A[j]
          then B[k] ← A[i]
               i ← i + 1
       else B[k] ← A[j]
            j ← j + 1
       k ← k + 1
for k ← p to r      ▷ copy the merged array to A[p..r]
    do A[k] ← B[k]
```

Problem 2
Write an algorithm that combines INSERTION-SORT and MERGE-SORT.

The following algorithm calls INSERTION-SORT for array segments whose length is at most k; the running time of this algorithm is $\Theta(n \cdot k + n \cdot \lg(n/k))$.

```
INSERTION-SORT(A, p, r)
for j ← p + 1 to r
    do key ← A[j]
       i ← j − 1
       while i ≥ p and A[i] > key
           do A[i + 1] ← A[i]
              i ← i − 1
       A[i + 1] ← key
```

```
COMBINED-SORT(A, p, r, k)
if r − p < k
    then INSERTION-SORT(A, p, r)
    else q ← ⌊(p+r)/2⌋
         COMBINED-SORT(A, p, q, k)
         COMBINED-SORT(A, q + 1, r, k)
         MERGE(A, p, q, r)
```