Analysis of Algorithms: Solutions 4

```
                          X
                          X
                          X  X
number of                 X  X
homeworks        X  X     X  X
                 X  X     X  X  X
              X  X  X     X  X  X
        X  X  X  X  X     X  X  X
        X  X  X  X  X  X  X  X  X
        X  X  X  X  X  X  X  X  X
       --------------------------
        2  3  4  5  6  7  8  9  10
                 grades
```

Problem 1
For each of the following functions, give an asymptotically tight bound (Θ-notation).

(a) $(2n^6 + 6n^2)^3 = (\Theta(n^6) + o(n^6))^3 = \Theta((n^6)^3) = \Theta(n^{18})$

(b) $(n+2)^2 \cdot (n+3)^3 \cdot (n+6)^6 = (\Theta(n))^2 \cdot (\Theta(n))^3 \cdot (\Theta(n))^6 = \Theta(n^2 \cdot n^3 \cdot n^6) = \Theta(n^{11})$

(c) $3^{2n} + 2^{3n} = (3^2)^n + (2^3)^n = 9^n + 8^n = 9^n + o(9^n) = \Theta(9^n)$

(d) $\sqrt{2n+2} \cdot \sqrt[3]{3n+3} \cdot \sqrt[6]{6n+6} = \Theta(n^{1/2} \cdot n^{1/3} \cdot n^{1/6}) = \Theta(n^{1/2+1/3+1/6}) = \Theta(n)$

(e) $3^{6n} + n! + \sqrt{n^n} = (3^6)^n + n! + n^{n/2} = o(n!) + n! + o(n!) = \Theta(n!)$

(f) $2^{\frac{\log_3 n}{\log_9 2}} = 2^{\frac{\lg n / \lg 3}{\lg 2 / \lg 9}} = 2^{\frac{\lg n / \lg 3}{\lg 2 / (2 \cdot \lg 3)}} = 2^{\frac{(2 \cdot \lg 3) \cdot \lg n}{\lg 3 \cdot \lg 2}} = 2^{2 \cdot \lg n} = \left(2^{\lg n}\right)^2 = n^2 = \Theta(n^2)$

Problem 2
Give an example of functions $f(n)$ and $g(n)$ that satisfy all of the following conditions:

$f(n) \neq \Theta(g(n))$
$f(n) \neq o(g(n))$
$f(n) \neq \omega(g(n))$

Consider the following two functions:

$$f(n) = \begin{cases} n & \text{if } n \text{ is even;} \\ 1 & \text{if } n \text{ is odd.} \end{cases}$$

$$g(n) = \begin{cases} 1 & \text{if } n \text{ is even;} \\ n & \text{if } n \text{ is odd.} \end{cases}$$

For even n, $f(n)$ grows asymptotically faster than $g(n)$. On the other hand, for odd n, $f(n)$ grows asymptotically slower. Therefore, $g(n)$ is neither asymptotically lower bound nor asymptotically upper bound for $f(n)$.