Analysis of Algorithms: Assignment 5

Due date: October 21 (Thursday)

Problem 1 (3 points)
Give a recursive version of the TREE-INSERT procedure.

Problem 2 (2 points)
Suppose we apply the CONNECTED-COMPONENTS algorithm to an undirected graph G, with vertices $G[V] = \{a, b, c, d, e, f, g, h, i, j, k\}$, and its edges $E[G]$ are processed in the following order: $(d, i), (f, k), (g, i), (b, g), (c, e), (i, j), (d, k), (b, j), (d, f), (g, j), (a, e)$. Using Figure 22.1 in the textbook as a model, illustrate the steps of CONNECTED-COMPONENTS on this graph.

Problem 3 (5 points)
Write pseudocode for MAKE-SET, FIND-SET, and UNION, using the linked-list representation of disjoint sets. Your UNION algorithm must always append the shorter list to the longer one. For every linked list, you will need to store its size, a pointer to the first element, and a pointer to the last element.

Problem 4 (bonus)
This is an optional problem, inherited from Exam 1; if you solve it, then you get 2 bonus points toward your final grade. You cannot submit this bonus problem after the deadline.

Suppose that you are using a programming language that allows only integer numbers and supports three operations on them: addition, subtraction, and multiplication. Write an *efficient* algorithm DIVIDE(n, m) that computes $\lfloor n/m \rfloor$, where n and m are positive integers, and give the time complexity of your algorithm.

Hint: An efficient algorithm runs in $\Theta(\lg\lceil n/m \rceil + 1)$ time and requires $\Theta(\lg\lceil n/m \rceil + 1)$ space. Alternatively, you may develop a somewhat slower algorithm that takes $\Theta(1)$ space.