

**Inversion****Antecedents and Succedents**

Antecedents, not left invertible, unordered  $\Gamma ::= A \supset B \mid P \mid \cdot \mid \Gamma_1, \Gamma_2$   
 Antecedents, ordered  $\Omega ::= A \cdot \Omega \mid \epsilon$   
 Succedent, not right invertible  $C ::= A \vee B \mid \perp \mid P$

**Judgments.**

Right inversion  $\Gamma ; \Omega \xrightarrow{R} A$   
 Left inversion  $\Gamma ; \Omega \xrightarrow{L} C$   
 Choice  $\Gamma ; \epsilon \xrightarrow{C} C$

**Rules.****Right Inversion.**

$$\frac{\Gamma ; \Omega \xrightarrow{R} A \quad \Gamma ; \Omega \xrightarrow{R} B}{\Gamma ; \Omega \xrightarrow{R} A \wedge B} \wedge R \qquad \frac{\Gamma ; A \cdot \Omega \xrightarrow{R} B}{\Gamma ; \Omega \xrightarrow{R} A \supset B} \supset R \qquad \frac{}{\Gamma ; \Omega \xrightarrow{R} \top} \top R$$

$$\frac{\Gamma ; \Omega \xrightarrow{L} A \vee B}{\Gamma ; \Omega \xrightarrow{R} A \vee B} LR \qquad \frac{\Gamma ; \Omega \xrightarrow{L} \perp}{\Gamma ; \Omega \xrightarrow{R} \perp} LR \qquad \frac{\Gamma ; \Omega \xrightarrow{L} P}{\Gamma ; \Omega \xrightarrow{R} P} LR$$

**Left Inversion.**

$$\frac{\Gamma ; A \cdot B \cdot \Omega \xrightarrow{L} C}{\Gamma ; (A \wedge B) \cdot \Omega \xrightarrow{L} C} \wedge L \qquad \frac{\Gamma ; A \cdot \Omega \xrightarrow{L} C \quad \Gamma ; B \cdot \Omega \xrightarrow{L} C}{\Gamma ; (A \vee B) \cdot \Omega \xrightarrow{L} C} \vee L$$

$$\frac{}{\Gamma ; \perp \cdot \Omega \xrightarrow{L} C} \perp L \qquad \frac{\Gamma ; \Omega \xrightarrow{L} C}{\Gamma ; \top \cdot \Omega \xrightarrow{L} C} \top L$$

$$\frac{\Gamma, A \supset B ; \Omega \xrightarrow{L} C}{\Gamma ; (A \supset B) \cdot \Omega \xrightarrow{L} C} LL \qquad \frac{\Gamma, P ; \Omega \xrightarrow{L} C}{\Gamma ; P \cdot \Omega \xrightarrow{L} C} LL$$

$$\frac{\Gamma ; \epsilon \xrightarrow{C} C}{\Gamma ; \epsilon \xrightarrow{L} C} CL$$

Choice.

$$\begin{array}{c}
 \frac{\Gamma; \epsilon \xrightarrow{R} A}{\Gamma; \epsilon \xrightarrow{C} A \vee B} \vee R_1 \qquad \frac{\Gamma; \epsilon \xrightarrow{R} B}{\Gamma; \epsilon \xrightarrow{C} A \vee B} \vee R_2 \\
 \\
 \frac{}{\Gamma, P; \epsilon \xrightarrow{C} P} \text{id}^* \qquad \frac{\Gamma, A \supset B; \epsilon \xrightarrow{R} A \quad \Gamma, [A \supset B]; B \xrightarrow{R} C}{\Gamma, A \supset B; \epsilon \xrightarrow{C} C} \supset L
 \end{array}$$

Loop Checking.

(fail, if  $\Gamma \subseteq \Gamma'$  and  $C = C'$ )

$$\begin{array}{c}
 \Gamma; \epsilon \xrightarrow{C} C \\
 \vdots \\
 \Gamma'; \epsilon \xrightarrow{C} C'
 \end{array}$$