

Constructive Logic (15-317), Spring 2023

Recitation 2

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1 Verifications & Uses

Consider the \heartsuit connective:¹

$$\frac{\frac{\overline{A \text{ true}}^v \quad \overline{A \text{ true}}^w}{\vdots} \quad \frac{\overline{B \text{ true}} \quad \overline{C \text{ true}}}{\vdots}}{\heartsuit(A, B, C) \text{ true}} \heartsuit I^{v,w}$$

$$\frac{\frac{\heartsuit(A, B, C) \text{ true} \quad \overline{B \text{ true}}^u \quad \overline{D \text{ true}}}{\vdots} \quad A \text{ true}}{D \text{ true}} \heartsuit E1^u \quad \frac{\frac{\heartsuit(A, B, C) \text{ true} \quad \overline{C \text{ true}}^u \quad \overline{D \text{ true}}}{\vdots} \quad A \text{ true}}{D \text{ true}} \heartsuit E2^u$$

Task 1. Give appropriate rules for \heartsuit in verifications & uses.

Task 2. Using Dcheck, give a derivation of the judgement:

$$(A \supset B \supset C) \supset ((A \wedge B) \supset C) \uparrow$$

¹in Latex: `\heartsuit`

2 Proof Terms/Proofs as Programs

Task 3. Consider the judgement in task 2. What would be the appropriate type signature for a program in SML? Write a proof term in SML for that expression.

Task 4. Consider again the \heartsuit connective from the previous section. Let us use the syntax $\text{hearti}(x.M, y.N)$ for the proof term for $\heartsuit I$, and the syntax $\text{heartel}(M, N, x.P)$ and $\text{hearter}(M, N, x.P)$ for the proof terms for $\heartsuit E1$ and $\heartsuit E2$.

Give proof term deduction rules corresponding to $\heartsuit I$, $\heartsuit E1$, and $\heartsuit E2$.