Constructive Logic (15-317), Spring 2023
Recitation 5

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1 Rule Induction and Programming Languages Dynamics

Task 1. In lecture on Thursday, we discussed the dynamic of programming languages and how there is some difference between logic and these dynamics. Several theorems were brought up that are important to prove about programming languages. Here, we look at progress. The statement of the theorem is as follows:

If $M : A$ then either $M \longrightarrow M'$ or $M\ value$ (but not both).

While doing the entire proof would take a long time, lets look at the progress proof involving implication elimination to get practice with rule induction. As a reminder, the dynamics for application are the following:

$$\frac{M \longrightarrow M'}{M\ N \longrightarrow M'\ N} \qquad \frac{M\ value}{M\ N \longrightarrow M\ N'}$$

$$\frac{N\ value}{(\lambda x.M)N \longrightarrow [N/x]M}$$

While the typing rule for application (aka statics) is:

$$\frac{M : A \supset B \quad N : A}{M\ N : B}\ \supset E$$

2 Primitive Recursion

In lecture, several recursive definitions were given. For example, pred was given as

$$pred(n) \triangleq R(n, 0, x.r.x)$$

plus was given as

$$plus(n) \triangleq R(n, \lambda k.k, x.r.\lambda k.s(r\ k))$$

mult was given as

$$mult(n) \triangleq R(n, \lambda k.0, x.r.\lambda k.plus(r\ k)\ k)$$

Here we aim to construct a few more.
Task 2. Define a recursive definition for factorial being allowed to use any of the definitions above.
Task 3. Define a recursive definition for subtraction being allowed to use any of the definitions above.