Deformable / Non-Rigid Registration

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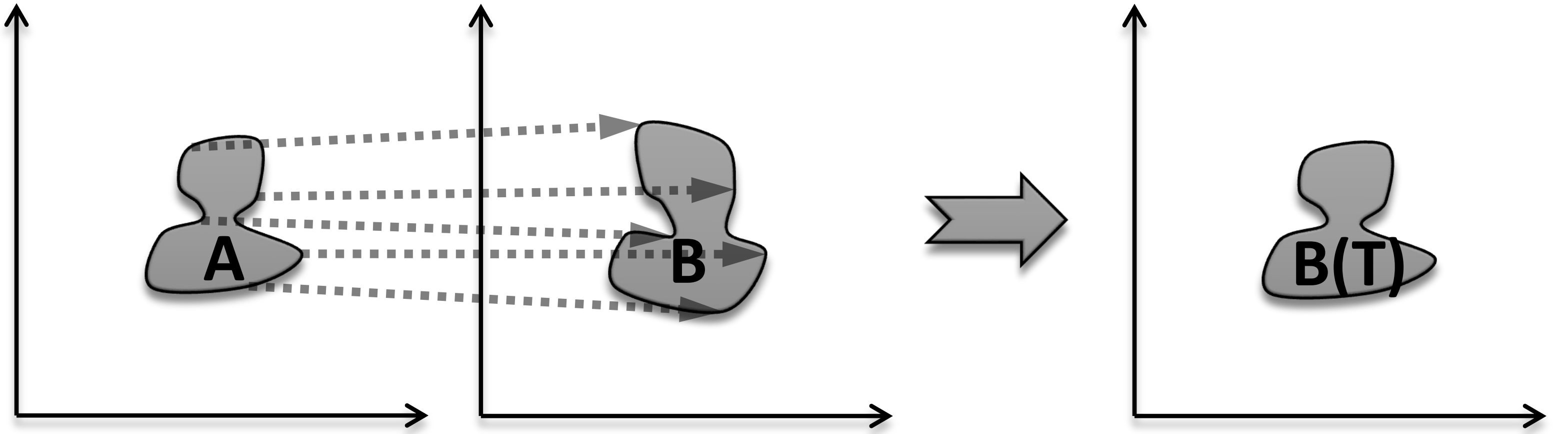
Registration: “Rigid” vs. Deformable

- Rigid Registration:
  - Uses a simple transform, *uniformly* applied
  - Rotations, translations, etc.
- Deformable Registration:
  - Allows a non-uniform mapping between images
  - Measure and/or correct small, varying discrepancies by deforming one image to match the other
  - Usually only tractable for deformations of small spatial extent!

Deformable, i.e. Non-Rigid, Registration (NRR)



- Vector field (aka deformation field) T is computed from A to B
- Inverse warp transforms B into A's coordinate system
- Not only do we get correspondences, but…
- We also get shape differences (from T)

NRR Clinical Background

- Internal organs are non-rigid
- The body can change posture
  - Even skeletal arrangement can change
- Single-patient variations:
  - Normal
  - Pathological
  - Treatment-related
- Inter-subject mapping: People are different!
  - Atlas-based segmentation typically requires NRR

More Clinical Examples

- Physical brain deformation during neurosurgery
- Normal squishing, shifting and emptying of abdominal/pelvic organs and soft tissues
  - Digestion, excretion, heart-beat, breathing, etc.
- Lung motion during respiration can be huge!
- Patient motion during image scanning

Optical Flow

- Traditionally for determining motion in video—assumes 2 sequential images
- Detects small shifts of small intensity patterns from one image to the next
- Output is a vector field, one vector for each small image patch/intensity pattern
- Basic gradient-based formulation assumes intensity values are conserved over time

Optical Flow Assumptions

- Images are a function of space and time
- After short time dt, the image has moved $d\mathbf{x}$
- Velocity vector $\boldsymbol{v} = d\mathbf{x}/dt$ is the optical flow

$$I(\mathbf{x}, t) = I(\mathbf{x}+d\mathbf{x}, t+dt) = I(\mathbf{x}+\boldsymbol{v}\bullet dt, t+dt)$$

- Resulting optical flow constraint:

$$C_{of} = I_{\mathbf{x}}\bullet\boldsymbol{v} + I_t = 0$$

Image spatial gradient

Image temporal derivative

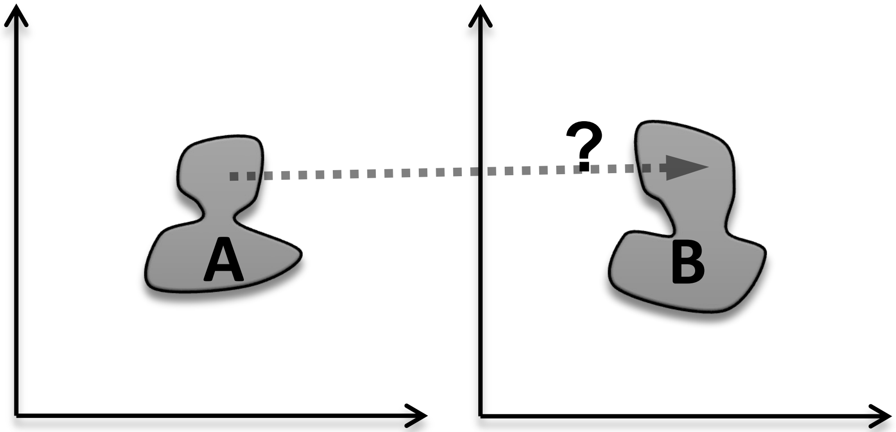
Optical Flow Constraint

- Optical flow constraint dictates that when an image patch is spatially shifted over time, that it will retain its intensity values
- Let image A = $I(\mathbf{x}, t=0)$ and let B = $I(\mathbf{x}, t=1)$
- Then I_t = A(T) – B

- This alone is not a sufficient constraint!

NRR Is Ill-Posed

- Review of well-posed problems:
  - A solution exists, is unique, and depends continuously on the data
  - Otherwise, a problem is ill-posed
- Ambiguity within homogenous regions:



Very Ill-Posed Problem

- NRR answer is not unique, and…
- NRR Search-space is often ∞-dimensional!

- Solution: Regularization
    - Adding a regularization term can provide provable uniqueness and a computable subspace
- Regularization usually based on continuum mechanics
    - T is restricted to be *physically admissible*
    - We're typically deforming *physical* anatomy, after all
    - Optimum T should deform "just enough" for alignment

NRR Regularization Methods

- Numerous continuum mechanical models available for regularization priors
  - Elastic
  - Diffusion
  - Viscous
  - Flow
  - Curvature
- Optimization is then physical simulation over time, t, of trying to deform one image shape to match another
- This optimization has 3 equivalent formulations:
  - Global potential energy minimization
  - Variational or weak form, as used in finite-element methods
  - Euler-Lagrangian (E-L) equations, as used in finite-difference techniques

Langrangian View

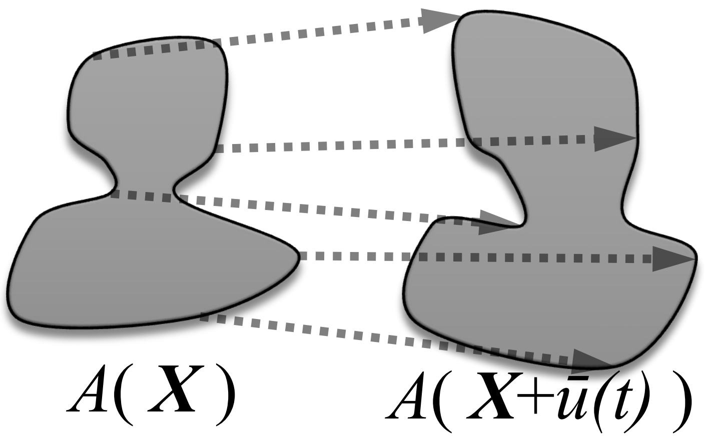
- Elastic physical model:
  - How much have we stretched, etc., from our *original* image coordinates?
  - Simulation calculates the physical model's resistance to deformation based on the *total* deformation from time t=0 to t=now.
- T is the final vector field $\bar{u}_f$:

$$\bar{u}_f = \bar{u}(\, t = t_{final} \,)$$

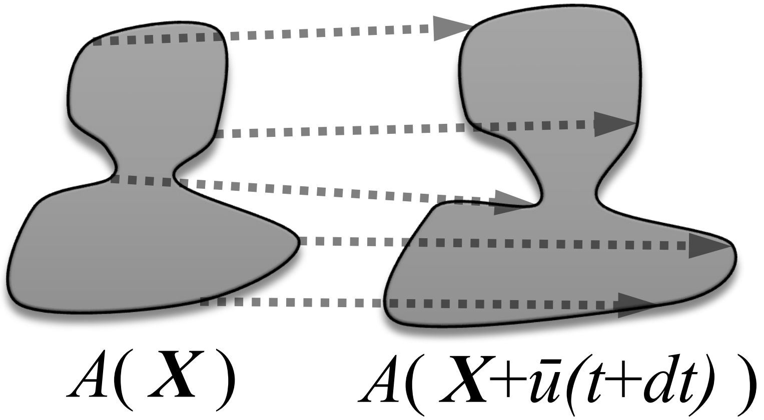
$$A(\boldsymbol{X} + \bar{u}_f) \sim B(\boldsymbol{x})$$

$$\boldsymbol{X} = \boldsymbol{x} - \bar{u}_f$$

- Deformation at time t:



- Deformation at time $t + dt$:



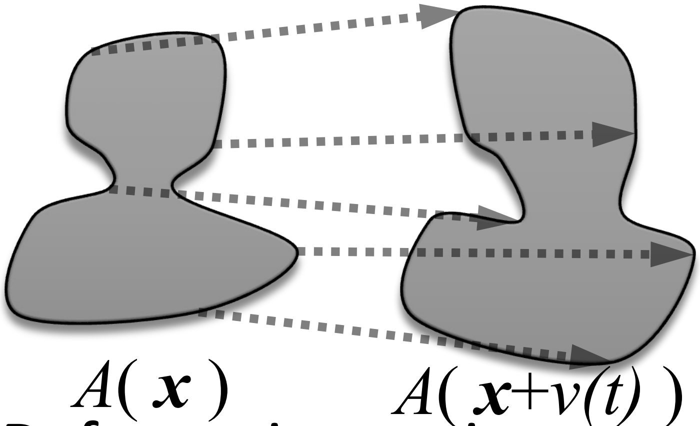
Eulerian View

- Viscous-flow physical model:
  - How much have we flowed from our *immediately previous* simulation state?
  - Simulation calculates the physical model's resistance to deformation based on the *incremental* deformation from time t=(now-1) to t=now.
- T is the aggregate flow of $\boldsymbol{x}(t)$, based on accumulated optical flow (i.e. velocity) $v(t)$:

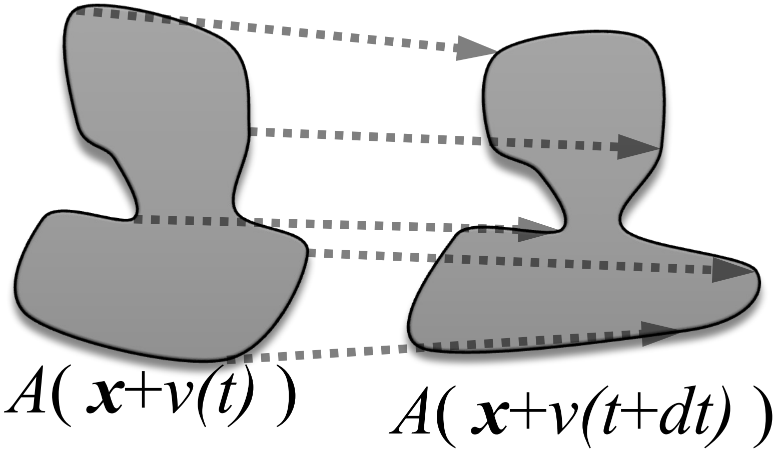
  $\boldsymbol{x}(t) = \boldsymbol{x} + v(t)$

  $A(\ \boldsymbol{x}(t=t_{final})\ ) \sim B(\boldsymbol{x})$

- Deformation at time t:



- Deformation at time $t + dt$:



Comparison of Regularization Reference Frames

- Langrangian
  - The entire deformation is regularized
    - Well constrained for “normal” physical deformation
    - Too constrained to achieve “large” deformations
  - Not ideal for many inter-subject mapping tasks
- Eulerian
  - Only the incremental updates are regularized
    - Underconstrained for “normal” physical deformation
    - Readily achieves large, inter-subject deformations
  - Unrealistic transformations can result

Transient Quadratic (TQ) Approach

- Enables better-constrained large deformations

- Uses Lagrangian regularization for specified time interval, followed by a re-gridding strategy
  - After an interval's deformation reaches a threshold, we begin a new interval for which the last deformation becomes the new starting point
  - TQ thus resets the coordinate system while permanently storing the past state of the algorithm
- Results in a hybrid E+L physical model, resembling soft, stretchable plastic
  - Maintains the elastic regularization for a given time then takes on a new shape until new stresses are applied

Optical Flow Regularized

$$E_D(v) = \int_{\Omega} \Phi\left(C_{of}\right) d\Omega + \int_{\Omega} \Psi(v)\, d\Omega$$

$$\text{e.g.,}\ \Phi\left(C_{of}\right) = C_{of}^2$$

$$\text{e.g.,}\ \int_{\Omega} \Psi(v)\, d\Omega = \|Lv\|^2$$

- Goal: Minimize global potential energy, E_D
- First term adjusts v to make the images match (wants $C_{of} = 0$ within the bounded domain Ω)
- Second term adds a stabilizing function Ψ, typically a regulator operator L applied to v

Optical Flow E-L Regularized

- After deriving the E-L equations & setting their derivative = 0, we find that the…
- Potential energy minimum will occur when:

$$I_x\left(I_x \cdot v + I_t\right) - v_{xx} = 0$$

- First term minimizes optical flow constraint
- Second term minimizes Laplacian (i.e. roughness) of velocity field v
- Note that this equation is evaluated *locally*
  - Allows for efficient implementation

Demons Algorithm: Math

- *Very* efficient gradient-descent NRR algorithm
- Originally conceived as having "demons" push image level sets around, but is also…
- Based on E-L regularized optical flow
- Alternates between minimizing each half of the previous equation:
  - Descent in optical flow direction, based on:

$$I_x\left(I_x \cdot v + I_t\right) = 0$$

  - Smoothing, which estimates v_{xx}=0 with a difference-of-Gaussian filter, by applying a Gaussian on each iteration

Demons Algorithm: Code

- Initialize solution (i.e. total vector field) = Identity
- Loop:
  - Estimate vector field update
    - Use (stabilized) optical flow
  - Add update to total vector field
  - Blur total vector field (for regularization)

- Allows much larger deformation fields than optical flow alone.
- **Langrangian registration**: blur the total vector field (as above)
- **Eulerian registration**: blur the individual vector-field updates

Choices & Details

- There are many more NRR algorithms available
- Almost all of them are slower than demons, but they may give you better results
- See the text for details, and lots of helpful pictures