Lecture 14
Shape

ch. 9, sec. 1-8, 12-14 of *Machine Vision* by Wesley E. Snyder & Hairong Qi

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16-725 (CMU RI) : BioE 2630 (Pitt)

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Shape Analysis

- Image analysis requires quantification of image contents
  - We desire a relatively small number of highly meaningful image descriptors.
- But, segmentation gives us *lots* of data.
- We need a way to derive meaningful measures from a segmentation.

Shape Analysis

Segmentation
(Pixel labeling from object differentiation)

Image Understanding
(By means of shape analysis)

- How can I quantify the shape of this object?
- What, physically, is this segmented object?
- Does it look normal?

Shape Analysis & Linear Transformations

- We want to identify objects…
  - Based on numerical shape descriptors.
- But:
  - Changing the the zoom (size), position, or orientation of an object (or the “camera”) changes the contents of the resulting image.
- We often need…
  - Shape descriptors that evaluate to the same (vector or scalar) value for all sizes, positions, and/or orientations of any given shape

Shape Analysis & Linear Transformations

- Most shape descriptors are not invariant to all linear transforms.
- Many are not even invariant to similarity transformations
- Similarity transforms (i.e. *pose* transforms):
  - Translation and/or rotation only
  - Do not change the “shape” of an object

A digression into transformations

- Linear transforms can be implemented as a matrix that multiplies the vector coordinates of each pixel in an object
  - Example of rotating shape S about the z-axis (2D in-plane rotation):

$$S' = R_Z S = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 4 & 1 & 3 & 2 \\ 3 & 7 & 9 & 8 \end{bmatrix}$$

Coordinates of point 1 in shape S

Coordinates of point 3 in shape S

- Several types:
  - Rotation
  - Translation
  - Zoom
  - Affine
    - skew
    - different scaling in different directions
  - Perspective
    - lines stay straight, but not parallel

Homogeneous coordinates

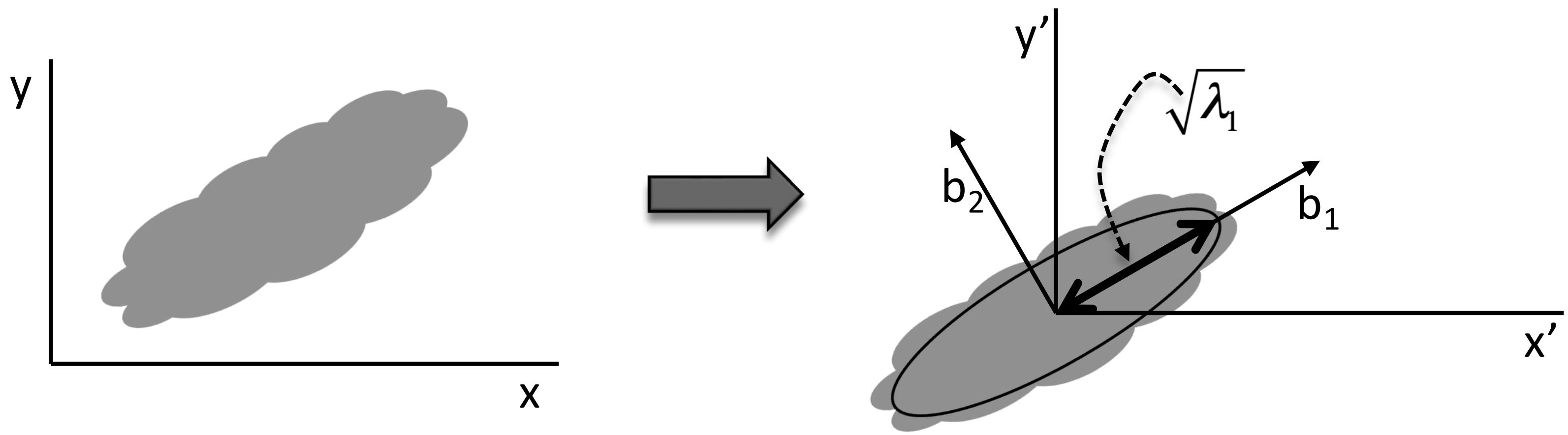
- What:
  - A slick way to implement translation via matrix multiplication
- How:
  - Add the “dummy” coordinate of 1 to the end of every coordinate vector:

$$X' = \begin{bmatrix} \cos\theta & -\sin\theta & dx \\ \sin\theta & \cos\theta & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Transformations for Medical Imaging

- In medical imaging, we usually don’t have optical perspective.
  - So, we usually don’t want or need invariance to perspective transformations.
  - We often don’t even need affine transforms.
- In medical imaging, we know the size of each voxel.
  - So, in some cases, we don’t want or need invariance to scale/zoom either.

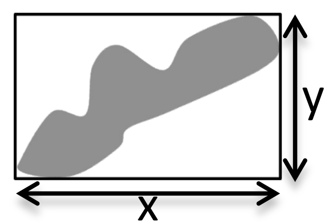
PCA (K-L Expansion)



- **Big Picture:** Fitting a hyper-ellipsoid & then (typically) reducing dimensionality by flattening the shortest axes
- Same as fitting an (N+1)-dimensional multivariate Gaussian, and then taking the level set corresponding to one standard deviation
- Mathematically, PCA reduces the dimensionality of data by mapping it to the first n eigenvectors (*principal components*) of the data's covariance matrix
- The first principal component is the eigenvector with the largest eigenvalue and corresponds to the longest axis of the ellipsoid
- The variance along an eigenvector is exactly the eigenvector's eigenvalue
- This is VERY important and VERY useful. Any questions?

Basic Shape Descriptors

- Trivial to compute—O(n) with a *small* coefficient:
  - Average, max, and min *intensity*
  - *Area* (A) and *perimeter** (P)
  - *Thinness / compactness / isoperimetric measure* (T), if based on P^2/A
  - *Center of mass (i.e. center of gravity)* → $\mathbf{m} = \frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix} x_i \\ y_i \end{bmatrix}$
  - *X-Y Aspect Ratio* →

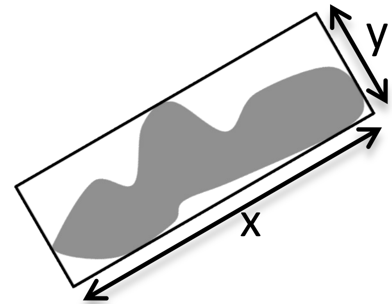


- Easy to compute:
  - *Number of holes*
  - *Triangle similarity* (ratio of side lengths to P)

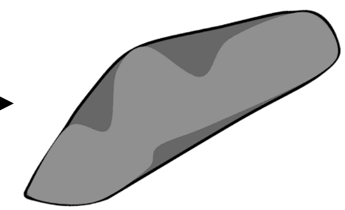
* Perimeter has several definitions; some are trivial to compute

Basic Shape Descriptors

- Requires PCA first, which itself is O(D^3+$D^2 n$):
  - Approximate *minimum aspect ratio*
  - Approximate *diameter* (D)
  - *Thinness / compactness / isoperimetric measure* (T), if based on D/A



- O(n log n):
  - Convex discrepancy



- Difficult to compute:
  - Exact *diameter* = absolute max chord
  - Exact *minimum aspect ratio*
  - *Symmetry*, mirror or rotational

* Perimeter has several definitions; some are difficult to compute

Shape Analysis in (Simple)ITK

- SimpleITK's LabelShapeStatisticsImageFilter:
  - http://www.itk.org/SimpleITKDoxygen/html/classitk_1_1simple_1_1LabelShapeStatisticsImageFilter.html
- Underlying ITK Filter & Data Classes:
  - http://www.itk.org/Doxygen/html/classitk_1_1LabelImageToShapeLabelMapFilter.html
  - http://www.itk.org/Doxygen/html/classitk_1_1ShapeLabelObject.html
- C++ ITK Example:
  - http://www.itk.org/Doxygen/html/WikiExamples_2ImageProcessing_2ShapeAttributes_8cxx-example.html

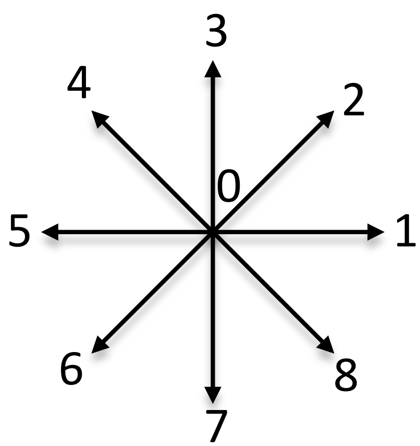
Method of Normalization

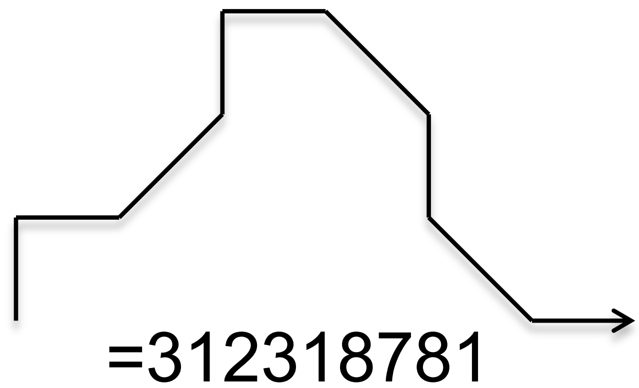
- Idea: Transform each shape’s image region into a canonical frame before attempting to identify shapes
- Simple, but common, example:
  - Move origin to the center of gravity (CG) of the current shape
  - Used by central moments (next slide)
- Complex example:
  - Attempt to compute and apply an affine transform to each object such that all right-angle-triangle objects appear *identical*

Moments

- Easy to calculate
- Sequence of derivation:
  - Moments: $m_{pq} = \sum x^p y^q f(x,y)$
  - Central moments: μ_{pq} (origin @ CG)
  - Normalized central moments: η_{pq}
    - Invariant to translation & scale
  - **Invariant moments: φ_n**
    - **Invariant to translation, rotation, & scale**
    - Only 7 of them in 2D
    - Equations are in the text
- Problem: Sensitive to quantization & sampling

Chain codes



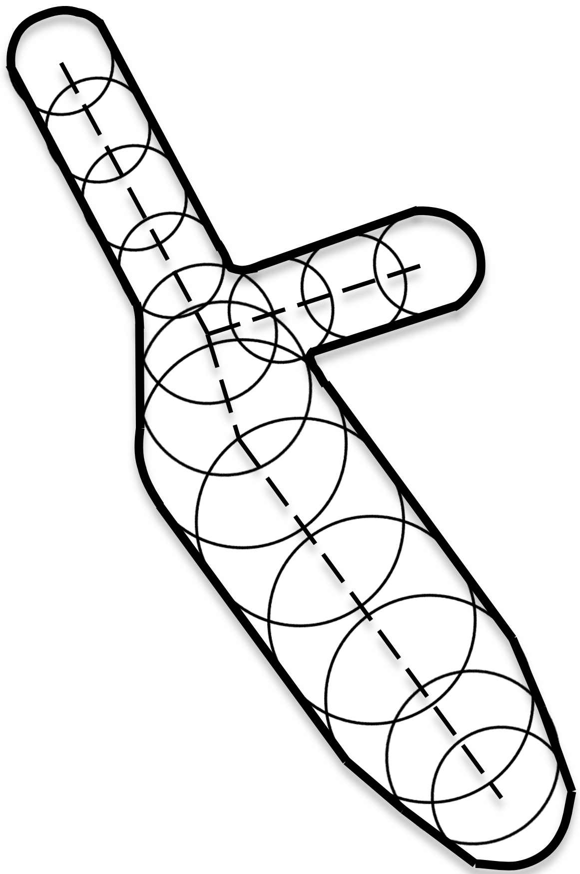


- Describe the boundary as a sequence of steps
  - Typically in 2D each step direction is coded with a number
- Conventionally, traverse the boundary in the counter-clockwise direction
- Useful for many things, including syntactic pattern recognition

Fourier Descriptors

- Traverse the boundary
  - Like for chain codes
- But, take the FT of the sequence of boundary-point coordinates
  - In 2D, use regular FT with i = *y-axis*
- Equivalences make invariance "easy":
  - Translation = DC term
  - Scale = multiplication by a constant
  - Rotation about origin = phase shift
- Problem: Quantization error

Medial Axis



- I may revisit this in another lecture (if time allows)
- For now:
  - Locus of the centers of the maximal bi-tangent circles/spheres/…

Deformable Templates

- Represent a shape by the active contour that segments it
  - Deforming the contour deforms the shape
- Two shapes are considered similar if the boundary of one can be “easily” deformed into the boundary of the other.
  - E.g., “easy” = small strain on the deformed curve and low energy required to deform the curve

Generalized Cylinders (GCs)

- Fit a GC to a shape
  - This can be challenging
- Get two descriptive functions:
  - Axis of the GC
    - A vector-valued function
  - Radius along the axis
    - Typically a scalar-valued function