Math Foundations for ML

10-606

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Course page

- https://www.cs.cmu.edu/~ggordon/10606s22/syllabus-and-lecture-outline.html

- Anyone with an Andrew ID can access all course materials starting from there

  - First step: click on signup link for Piazza

$$\langle x, y \rangle$$

$$\langle x, y, z, \omega \rangle \qquad x, y, z, \omega \in S$$

$$\text{flatten} \left(\begin{array}{c} \langle \langle 4,1,2 \rangle, \langle 5,5,5 \rangle, \langle 1,2,1,2 \rangle \rangle \\ \langle 4,1,2,5,5,5,1,2,1,2 \rangle \end{array} \right.$$

$$X \times Y = \{ \langle x, y \rangle \mid x \in X, y \in Y \}$$

$$X \times (Y \times Z)$$
$$= (X \times Y) \times Z \qquad [\text{auto flatten}]$$
$$= X \times Y \times Z$$

$$X^3 = X \times X \times X$$

Suppose $X = \phi = \{\}$

$Y = \{1, 2, 3\} \longrightarrow$ $\{1\} \subseteq Y$
$\{2, 3\} \subseteq Y$
$\phi \subseteq Y$

$1 \in Y$
$\{2, 3\} \notin Y$
$\phi \notin Y$

What is $X \times Y$?

set $\subseteq$ set

~~element $\in$ set~~

A. ~~$\{1, 2, 3\}$~~

B. ~~$\{(\phi, 1), (\phi, 2), (\phi, 3)\}$~~

$\{(x, y) \mid x \in X, y \in Y\}$

ϕ
$\llcorner$ size $0 \cdot 3 = 0$

can't
write $\uparrow$

C. element $\in$ set
$\llcorner$ ~~set $\in$ set~~
$\llcorner$ except if we choose
element type to
be sets

$\{1\}$ a set
$\mid$ an element

$3 \neq \{3\}$

set of subsets of
$\llcorner \{\phi, \{1\}, \{2\}, \{1, 2\}\}$
$\{1, 2\} \longrightarrow = \{2, 1\}$

```
union {
    int a;
    char *b
}                    → bit string 64 bits
                          T
                     ↳ bit string 64 bits ?

integers ∪ pointers

void main (int argc, char *argv[])    struct {
                                          int a;
                                          char *b
                                      }
```

LISP
cons
car
cdr

x.a
x.b

named
fn

float square (float x) {
    return x ** 2;
}

float $\rightarrow$ float

" $\mathbb{R} \rightarrow \mathbb{R}$ "

anonymous x^2 $\lambda x. x^2$
fn

                 $3x^2 + xy$ $\lambda x, y. 3x^2 + xy$

                               $\lambda y, x. 3x^2 + xy$

lambda x: x ** 2

                               $\lambda x: \mathbb{R}, y: \mathbb{R}. 3x^2 + xy : \mathbb{R}$

Exercise: anonymous functions

and comprehensions

`[ 2*x for x in range (3)]`

Suppose $f(x)$ $g(y)$

Define anonymous fn: takes x, y returns $f(x)g(y)$

Assign to variable t

Use in list comprehension

Make a 3×3 matrix as a list of lists

for $x, y \in \{0, 1, 2\}$