Computational Foundations for ML

10-607

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AND is associative

1. assume $(a \wedge b) \wedge c$ — assumption

indent

2. c — $\wedge$-elim from 1
3. $a \wedge b$ — " " "
4. a — " " 3
5. b — " " "
6. $b \wedge c$ — $\wedge$-intro 5, 2
7. $a \wedge (b \wedge c)$ — " 4, 6 — conclusion

$(a \wedge b) \wedge c \rightarrow a \wedge (b \wedge c)$ — lemma

Exercise

Show that AND is commutative

assume

$a \wedge b$

show

$b \wedge a$

Assume something

Assume something else

→ intro (lemma 1)

lemma 1 available here

→ intro (second lemma 2)

lemma 2 available

1. Assume $PB \wedge J \rightarrow Sandwich$
2. Assume PB
3. Assume J.
4. Conclude $PB \wedge J$ $\wedge$-intro 2, 3
5. Conclude $Sandwich$ m.p. 1, 4
6. Conclude $J \rightarrow Sandwich$ $\rightarrow$ intro, 3-5
7. Conclude $PB \rightarrow (J \rightarrow Sandwich)$ $\rightarrow$ intro, 2-6
8. Conclude $[PB \wedge J \rightarrow Sandwich]$

$\rightarrow [PB \rightarrow (J \rightarrow Sandwich)]$

$\neg x \equiv x \to F$

from $\neg\neg\phi$ conclude ϕ DNE

excluded middle

$\phi \vee \neg\phi$

$\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$

$\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$

De Morgan

$((\phi \to \psi) \to \phi) \to \phi$ Pearce

Resolution

$$a \vee b \vee \neg c \vee d \vee \neg e \qquad b \vee \neg d \vee f$$

$$a \vee b \vee \neg c \quad \vee \neg e \vee \cancel{b} \vee f$$

$$\frac{\phi \vee \psi \qquad \neg \phi \vee \chi}{\psi \vee \chi}$$

$$(\phi \vee \psi) \wedge (\neg \phi \vee \chi) \rightarrow (\psi \vee \chi)$$

Types $\mathbb{N}$ int char

$\mathbb{N}(7) = T$

$\text{int}(3.2) = F$

$+ : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$

$(3, \text{Spot}) : \mathbb{N} \times \text{Dog}$

$\lambda x, y, z.\ (x+y) \times z$

$+(3,7) \mapsto 10$

$(\text{int} \mid \text{char})\ 7$

$(f \mid g)\ x : \text{dog}$; $x \to \text{int} \mid \text{char}$

$f : \text{int} \to \text{dog}$

$g : \text{char} \to \text{dog}$

any none

type $\mathbb{N}$ $\qquad$ $0 : \mathbb{N}$

$=$ $\qquad$ $a = b$ (T or F)

$a \neq b$

$\phi = \phi$

$\phi = \psi \equiv \psi = \phi$

$$\frac{\phi = \psi \quad \psi = \chi}{\phi = \chi}$$

$$\frac{(3 + a) \times 5 \geq 10 \quad a = b}{(3 + b) \times 5 \geq 10}$$

$S : \mathbb{N} \to \mathbb{N}$ $\qquad$ $\phi = \psi \equiv S(\phi) = S(\psi)$

$S(\phi) \neq 0$

induction

p a predicate

fresh term

$$\frac{p(0) \qquad p(x) \to p(s(x))}{p(\phi)}$$

$+ : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$

$$\phi + 0 = \phi$$

$$\phi + s(\psi) = s(\phi + \psi)$$

$$\begin{aligned} 3+2 &= 3 + s(1) \\ &= s(3+1) \\ &= s(3+s(0)) \\ &= ss(3+0) \\ &= ss3 \\ &= 5 \end{aligned}$$

$p(a) \longleftrightarrow a + 0 = 0 + a$

$p(0) \longleftrightarrow 0 + 0 = 0 + 0$ ✓ reflexive

assume $p(x)$, prove $P(s(x))$

$x + 0 = 0 + x$

$s(x+0) = s(0+x)$

$s(x) = s(0+x)$

$= s(0+x)$

$s(x) + 0$

$= 0 + s(x)$