Computational Foundations for ML

10-607

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Notes and reminders

- HW1 out
- Schedule is posted through Quiz I (on M 4/11)

$$(\lambda x.\ x+2)(17) \rightarrow 17+2$$

$$(\lambda x.\ x(x+2))(3) \rightarrow 3(3+2)$$

$$(\lambda x.\ \underbrace{(\lambda x.\ x+2)})(3x) \rightarrow \lambda x.\ x+2$$

$$(\lambda x.\ (x,\ \lambda x.\ x+2))(3x) \rightarrow (3x,\ \lambda x.\ x+2)$$

$$\lambda y.\ y+2$$

$$\cancel{\rightarrow}\ (3x,\ \lambda x.\ \cancel{3x}+2)$$

$$[\lambda x.\ (\lambda x.\ x+2)(3x)](7)$$

$$\hookrightarrow (\lambda x.\ x+2)(3\cdot 7) \rightarrow 3\cdot 7+2$$

$p = happy(x)$ $\quad p[x \to 17]$ $\quad happy(17)$

$first(3, 7) \to 3$

$((\lambda x{:}int.\ T) \mid (\lambda x{:}char.\ F))\ (y)$

if $y =$ 'q' we get F

if $y = 17$ we get T

$\hookrightarrow (\lambda x{:}char.\ F)$ 'q'

$\hookrightarrow F$

pair $= \lambda x, y. \lambda f. f(x, y)$

first $= \lambda x, y. x$

second $= \lambda x, y. y$

$(\text{pair}(3, 7))(\text{first})$

$\hookrightarrow (\lambda f. f(3, 7))(\text{first})$

$\hookrightarrow \text{first}(3, 7)$

$\hookrightarrow 3$

$E(\varnothing)$

$O(S\varnothing)$

$E(\psi) \to E(SS\psi)$

$O(\psi) \to O(SS\psi)$

prove: $E(x) \vee O(x)$ $x : \mathbb{N}$

$E(\varnothing) \vee O(\varnothing)$

$\hookrightarrow$ $\vee$-intro

$E(x) \vee O(x) \overset{?}{\to} E(Sx) \vee O(Sx)$

this doesn't work

IH: $(E(x) \vee O(x)) \wedge (E(Sx) \vee O(Sx))$

base: $(E(\varnothing) \vee O(\varnothing)) \wedge (E(S\varnothing) \vee O(S\varnothing))$

ind: prove $(E(Sx) \vee O(Sx)) \wedge (E(SSx) \vee O(SSx))$

cases: $E(x) \to E(SSx) \to (E(SSx) \vee O(SSx))$

$$O(x) \Rightarrow O(SSx) \Rightarrow \underbrace{(E(SSx) \vee O(SSx))}$$

By assumption $O(Sx)$

$\vee$-intro $E(Sx) \vee O(Sx)$

$\exists x.\ dog(x)$

$\forall x.\ dog(x) \to good(x)$

$\exists x.\ (dog(x) \vee \forall y\ friends(x, y))$

$\exists x.\ (dog(x) \wedge \exists x.\ cat(x))$

$N(x)$ ... $\forall x.\ N(x)$

$\forall x : T.\ p(x)$

$\exists x : T.\ p(x)$

$\forall x.\ p(x)$ ... $p(\text{Fred})$

$p(\text{father}(y))$

$(\forall y.\ p(y))$

$p(\text{Fred})$ $\exists x.\ P(x)$

$\exists x.\ p(x)$

$p(\text{foo})$

$$\frac{a \vee b \vee \neg c \qquad\qquad c \vee d}{a \vee b \vee d}$$

$\forall z \qquad \neg q(Fred, f(z)) \qquad\qquad q(x, f(Bob))$

$x = Fred \quad z = Bob$

$a \wedge (b \vee c) \longrightarrow (a \wedge b) \vee (a \wedge c)$

a

$b \vee c$

$::\quad b$

$\quad a \wedge b$

$\quad (a \wedge b) \vee (a \wedge c)$

$b \to (a \wedge b) \vee (a \wedge c)$

$::\quad c$

$\quad a \wedge c$

$\quad (a \wedge b) \vee (a \wedge c)$

$c \to (\quad - \; - \; - \quad)$

from a question after class: how to show distributivity