# Math Foundations for ML

## 10-606

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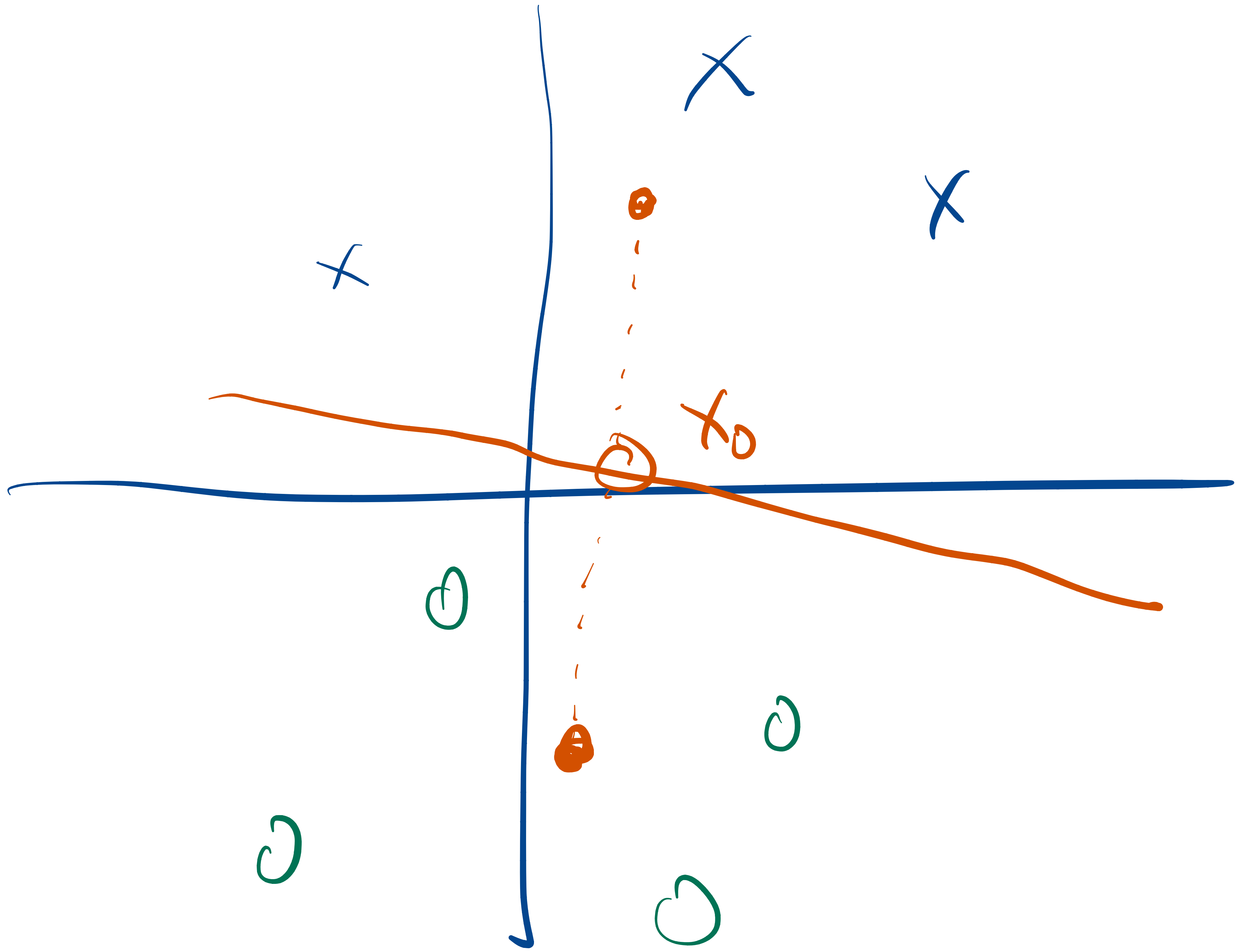


$-1, 2 \rightarrow -1$

$-1, -1 \rightarrow +1$

$+1, 3 \rightarrow -1$

$+1, -3 \rightarrow +1$

$\mu_x$: $0, 2.5$

$\mu_o$: $0, -2$

$$w \cdot x_0 = b$$

$$(\mu_{+1} + \mu_{-1})/2$$

$$x_0 = \frac{\mu_x + \mu_o}{2}$$

$$= (0, 0.25)$$

$$\mu_{+1} - \mu_{-1}$$

$$w = \mu_o - \mu_x$$

$$\boxed{= (0, -4.5)}$$

$$\begin{pmatrix}0\\-4.5\end{pmatrix} \cdot \begin{pmatrix}0\\0.25\end{pmatrix} = b$$

$$\boxed{= -1.125}$$

$$\begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ \alpha_{ij} & & 1 & \\ & & & 1 \end{pmatrix}$$

take $\alpha_{ij}$ times row $j$
add to row $i$

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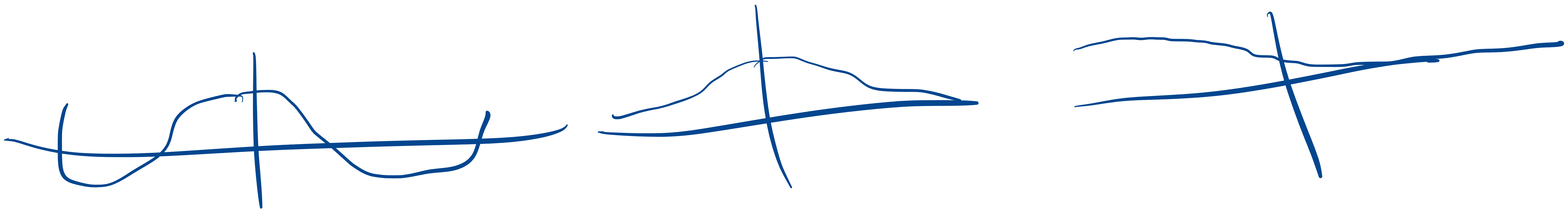
$$\operatorname{span}\left(\cos(x),\ e^{-x^2},\ 1/(1+e^x)\right) = V$$



$f \in V$ $\quad f(0) = 3 \quad f(1) = 2$
$\quad f(2) = 5$

$$a \cos(0) + b e^{-0^2} + c \, \frac{1}{1+e^0} = 3$$

$$a + b + \frac{1}{2} c = 3$$

$$A = \begin{pmatrix} 3 & 1 & 7 \\ 2 & 4 & -1 \\ -1 & 0 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 3 & 1 & 7 \\ 2 & 4 & -1 \\ 0 & 1/3 & 5\frac{1}{3} \end{pmatrix} = R_1 A$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/3 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/3 & 0 & 1 \end{pmatrix}$$

$$R_1^{-1} \qquad\qquad R_1$$

$$\begin{pmatrix} 3 & 1 & 7 \\ 0 & 3\frac{1}{3} & -5\frac{2}{3} \\ 0 & 1/3 & 5\frac{1}{3} \end{pmatrix} = R_2 R_1 A$$

$$R_2$$

$$R_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ +2/3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2/3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\hat{U} = R_2 R_1 A$$

$$R_2^{-1} \hat{U} = R_1 A$$

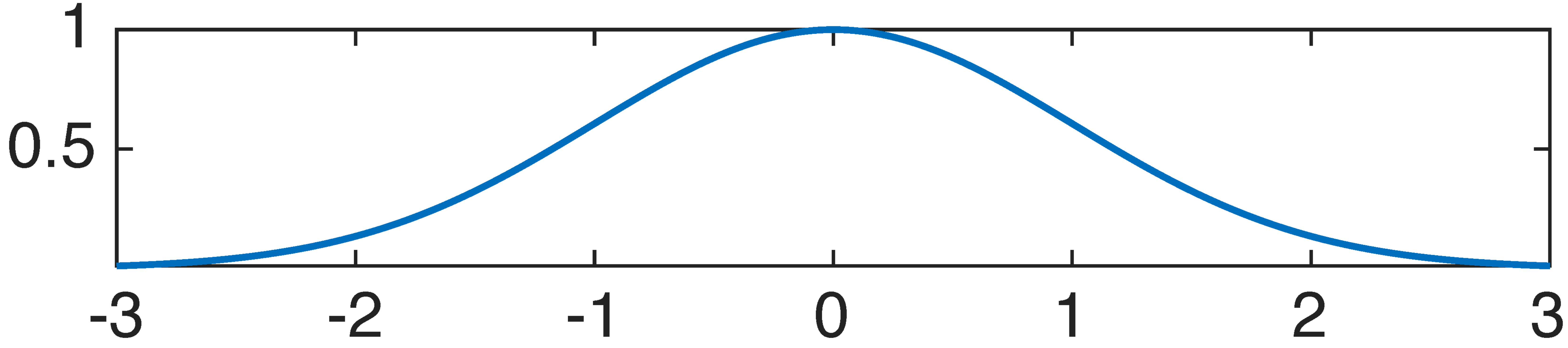
$$\left(R_1^{-1} R_2^{-1}\right) \hat{U} = A$$

# Gaussian radial basis function

## also called squared exponential function

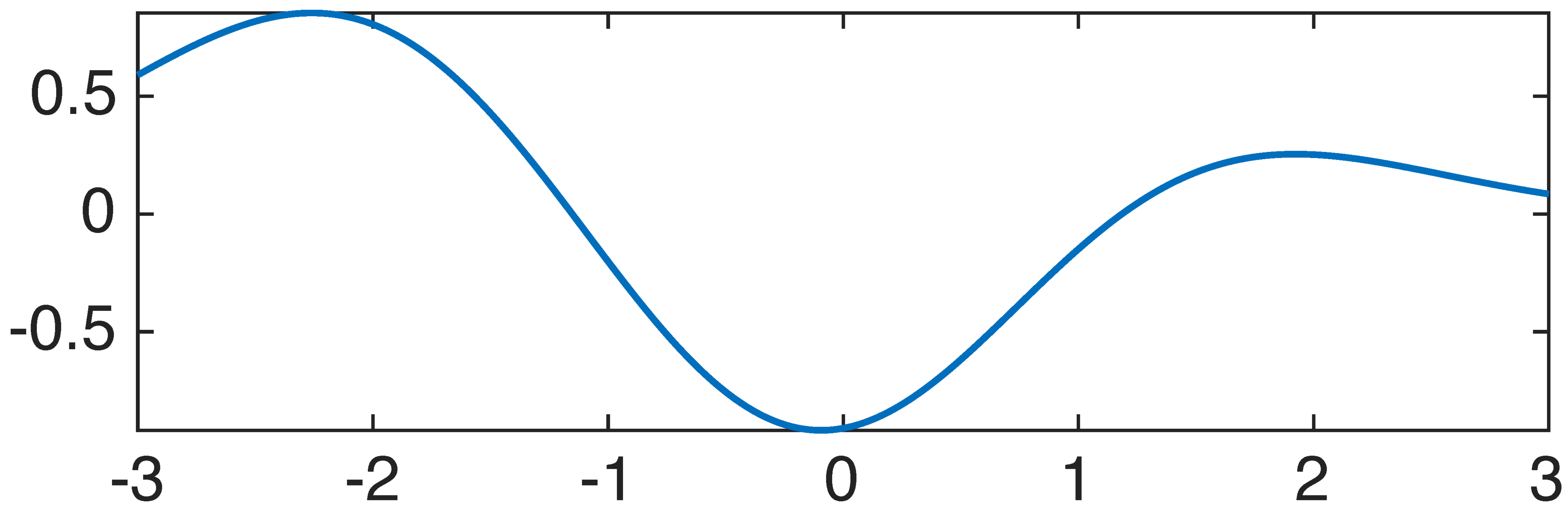
$e^{-\frac{1}{2}(x-x_0)^2}$

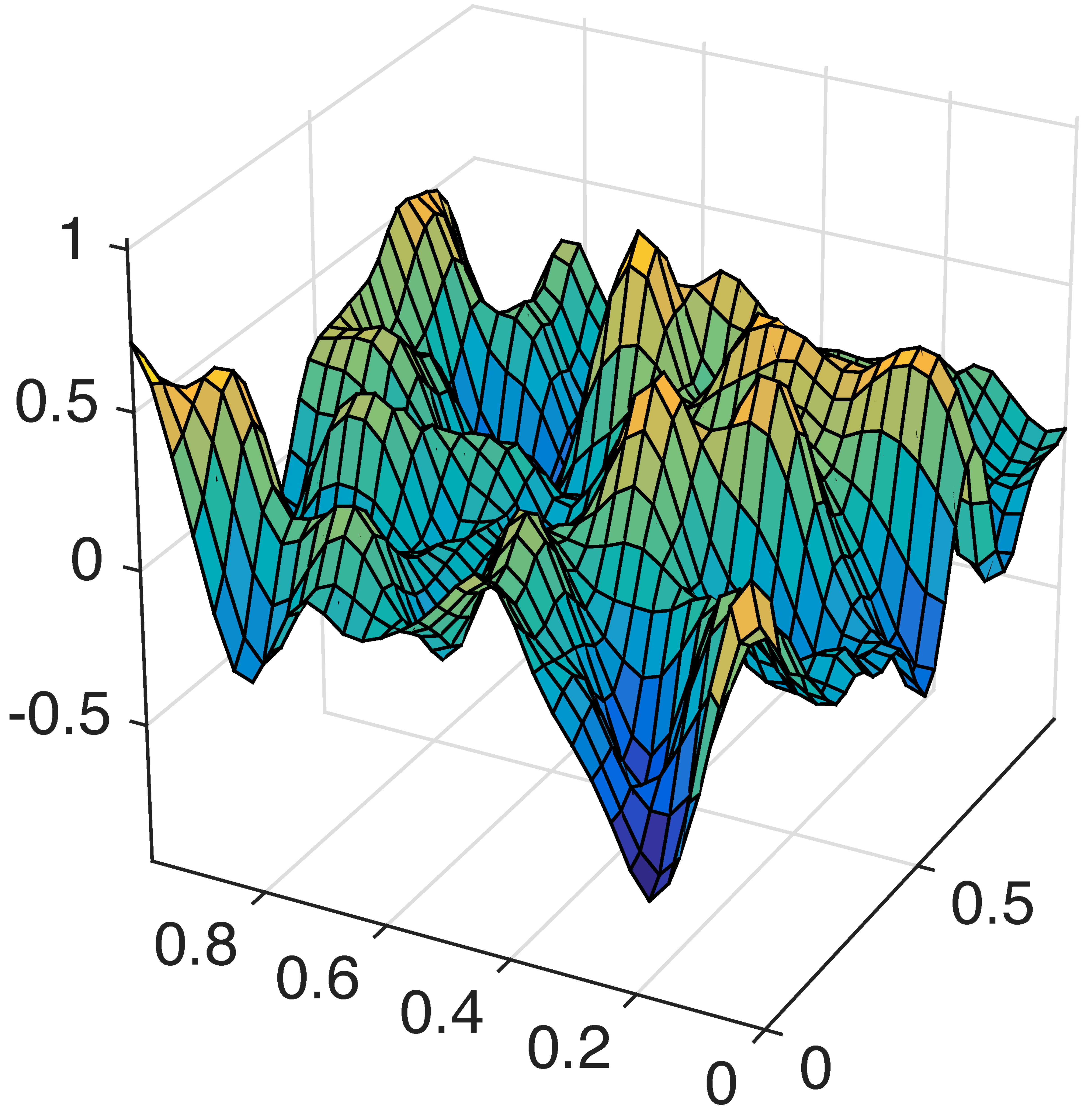
$e^{-\frac{1}{2}\|x-x_0\|^2}$



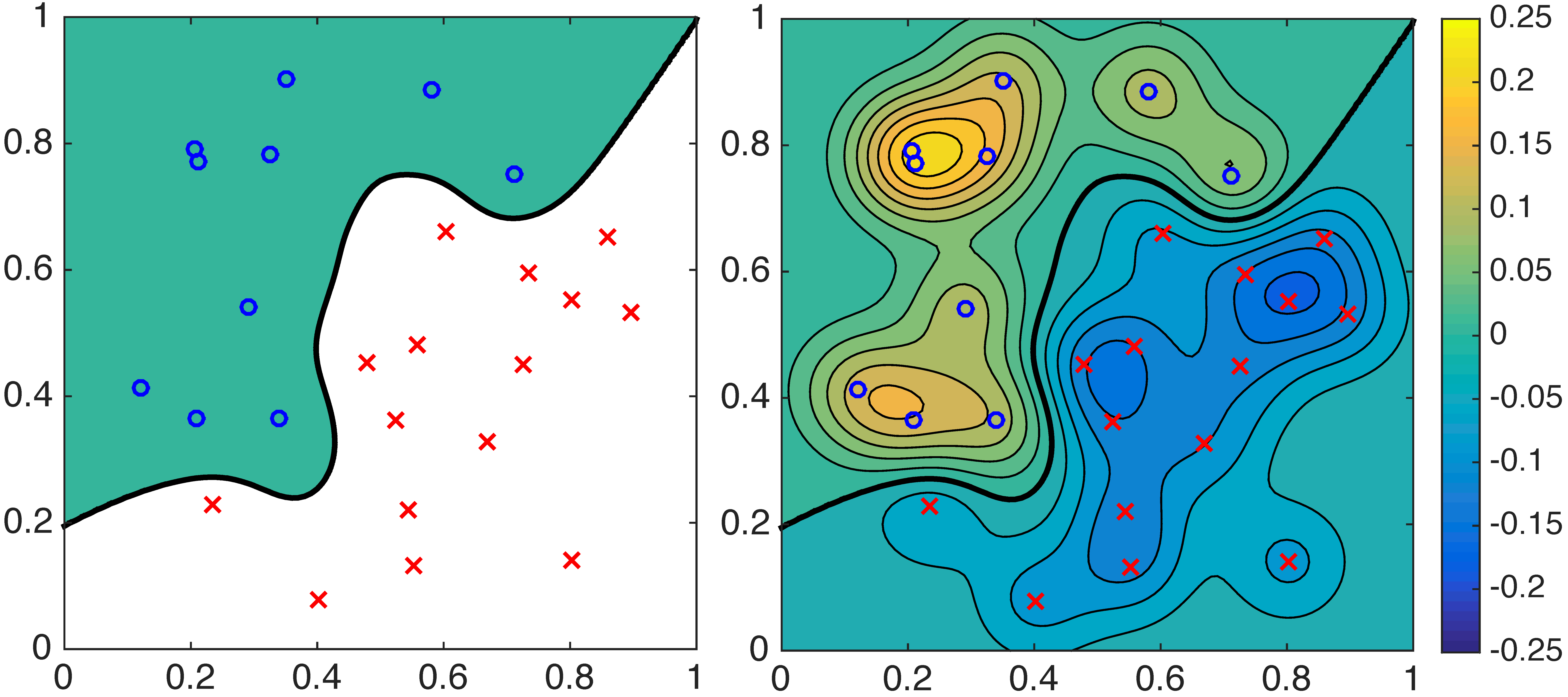

$x_0 = 0$

# GRBF vector examples

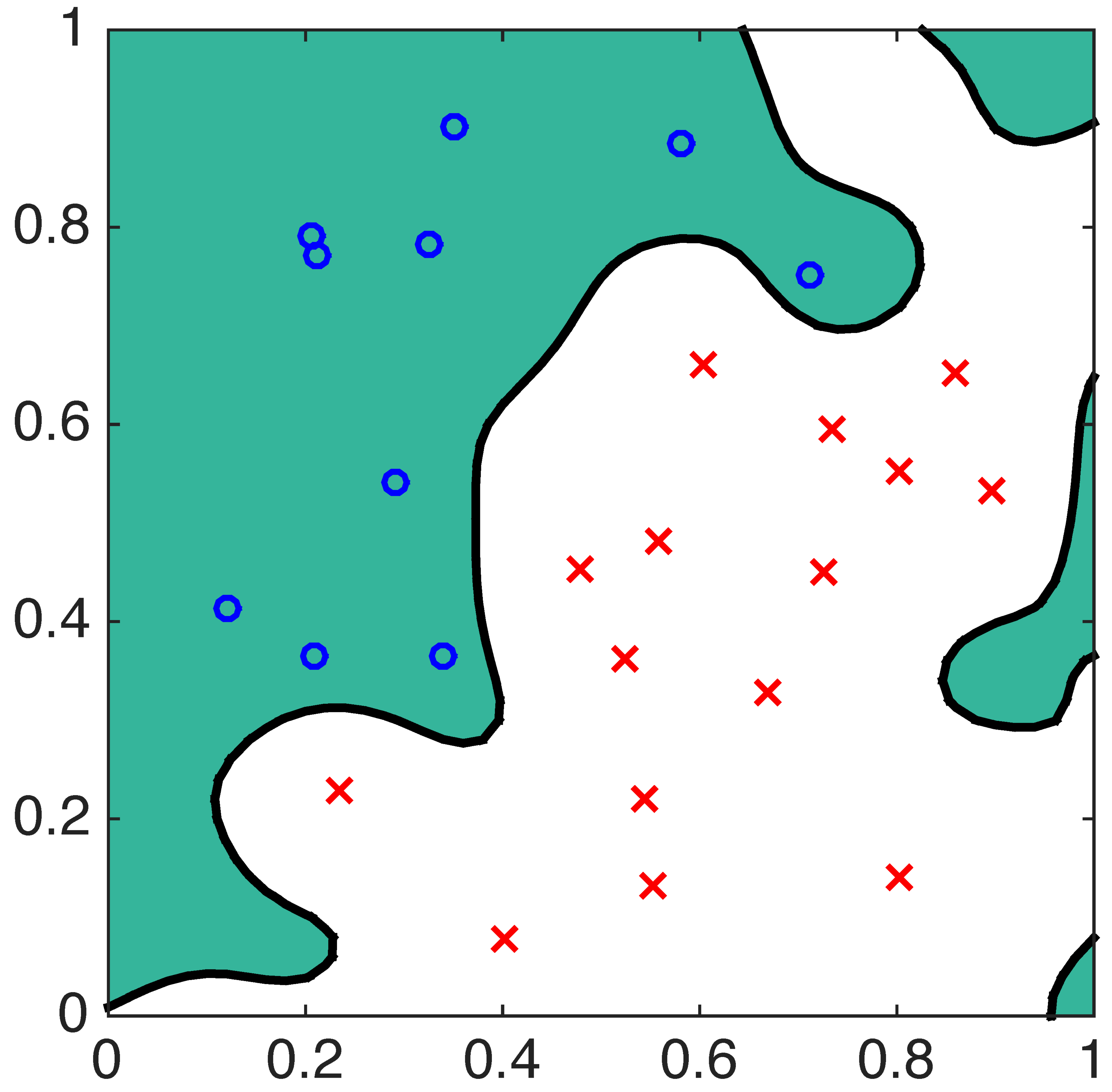


# Linear discriminant in function space



# Narrower RBFs



$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos t \\ \sin t \\ t \end{pmatrix}$
5
0
-5
2
0
-2
-4
-6
3
2
1
0
-1
2
0
-2
2
0
-2

$$f \in \mathbb{R}^n \to \mathbb{R}^m$$

$$(y - \hat{y}) = A\,(x - \hat{x})$$

$f(\hat{x})$

Jacobian

point where tangent

$$A_{ij} = \frac{\partial y_i}{\partial x_j}$$

$$\frac{dy}{dx}$$

$$dy = A\,dx$$

$$d f(x) = f'(x)\,dx$$

$$d(x^2 + 2x + 3) = (2x + 2)\,dx$$

$$d \ln \det A = \langle A^{-1}, dA \rangle$$

$$d \|x\| = \frac{x}{\|x\|} dx \qquad x \neq 0$$

$$d(af + bg) = a\,df + b\,dg$$

$$d(A^T X + 3Y) = A^T dX + 3\,dY$$

$$d(fg) = (df)g + f(dg)$$

if $f$ const, $= f\,dg$

$$d(fgh) = (df)gh + f(dg)h + fg(dh)$$

$df = f'(x)dx$

$\downarrow$

$\frac{df}{dx} = f'(x)$

$f = [\ldots \; x \; \ldots]$

$df = [\ldots \; x \; \ldots \; dx \ldots]$

$\uparrow$ linear in $dx$

$$df = \underbrace{[\;---\; x \;---\;]}_{\text{linear function}} dx$$

$\hookrightarrow f'(x)$

$$f(x) = u^3 + v^3$$

$$x = \begin{pmatrix} u \\ v \end{pmatrix}$$

$$df(x) = 3u^2 du + 3v^2 dv$$

$$= \underbrace{\begin{pmatrix} 3u^2 & 3v^2 \end{pmatrix}}_{\hookrightarrow f'(x)} \begin{pmatrix} du \\ dv \end{pmatrix}$$

$\hookrightarrow \dfrac{df(x)}{dx}$