15-829 **Lec 7: Little's Law, Open Systems, Closed Systems (Chpts 2.6, 2.7, 6)**

OUTLINE

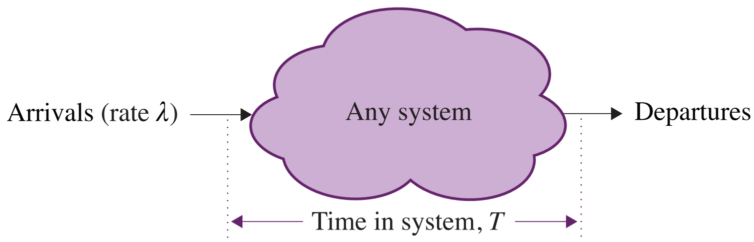
1. Little's Law for Open Systems

2. Examples applying Little's Law to Open Systems

3. Review of Closed Systems

4. Examples applying Little's Law to Closed Systems

Little's Law



Little's Law is a relationship between $\mathbf{E}[T]$ and $\mathbf{E}[N]$.

- $\mathbf{E}[T]$ = mean response time across jobs
- $\mathbf{E}[N]$ = mean number of jobs in system

Question: How do you think $\mathbf{E}[T]$ and $\mathbf{E}[N]$ are related? Think of a queue.

Question: Given that Law needs to hold for ANY system, what's a better approach?

Important: Little's Law holds for any system or any portion of a system or any subset of jobs. Little's Law holds regardless of arrival process, job sizes, number of queues/servers, scheduling policy!
(see you textbook for proof)

Warmup Example

A professor practices the following strategy with respect to taking on new students. On the even-numbered years, she takes on 2 new PhD students. On the odd-numbered years, she takes on 1 new PhD student. Assume the average time to graduate is 6 years.

Question: How many PhD students on average will the professor have?

Example: Utilization Law

Recall the formula $\rho = \frac{\lambda}{\mu}$ which applies to a single server.

Question: How can Little's Law be used to prove this?

Example: Little's Law for Time in Queue

Shashank proposed the following variation of Little's Law:

$$\mathbf{E}[N_Q] = \lambda \mathbf{E}[T_Q] \ .$$

Question: Is Shashank's variation true for a single-server queue?

Question: Is Shashank's variation true for a general system with many queues?

Example: Little's Law for Red Jobs

Shashank proposes another variation of Little's Law:

$$\mathbf{E}\left[N_{red}\right] = \lambda_{red}\mathbf{E}\left[T_{red}\right] \ .$$

Question: Explain what Shashank's Law is saying.

Question: Is this "Little's Law for Red Jobs" valid?

Another Simple Example

Kunhe's system consists of a single-server queue. Based on Kunhe's Measurements:

- The average arrival rate is $\lambda = 5$ jobs/sec
- The average job size is $\mathbf{E}[S] = 0.1$ sec
- The average number of jobs in the system is $\mathbf{E}[N] = 10.5$ jobs.

Question: What fraction of time is Kunhe's server busy?

Question: What is $\mathbf{E}[T_Q]$, the avg fraction of time that jobs queue?
(2 Ways!)

Example: Little's Law for Queue with Bounded Buffer

Suppose your queue has room for only a finite number of jobs.

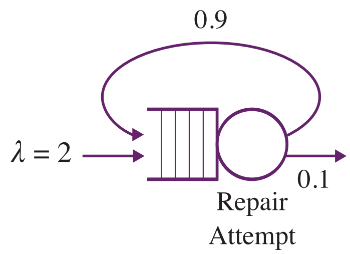
Any job that arrives and finds the buffer full is dropped.

Suppose that on average 1% of jobs are dropped.

Question: What is Little's Law for this system?

Example: Repair Queue

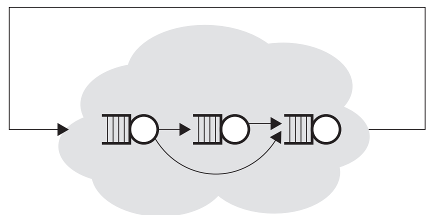
Repairs don't always work. In Jenny's repair center, every arriving item undergoes a "repair attempt," but with probability 0.9, the item needs to go in for another round. We say that the total time for repair, T, is the time from when the item first arrives until it is fully repaired. Based on Jenny's measurements: $\lambda = 2$ items/hour arrive to the repair center, the average repair attempt takes $\mathbf{E}[S] = \frac{1}{30}$ hours, and $\mathbf{E}[T] = 10$ hours.



Question: What fraction of time is the repair center busy?

Question: What is the expected number of items in the repair center, $\mathbf{E}[N]$? (2 ways!)

Closed system: Batch type

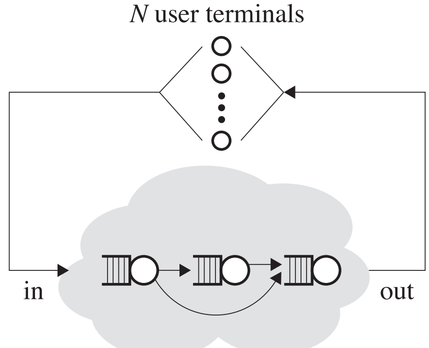


Central subsystem

Question: What does Little's Law say for closed batch systems?

Closed system: Interactive type

N user terminals

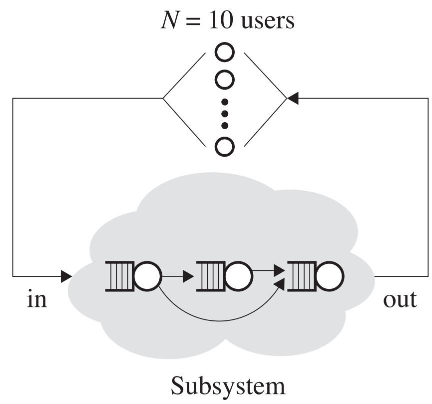


Central subsystem

Question: What does Little's Law say for closed interactive systems?

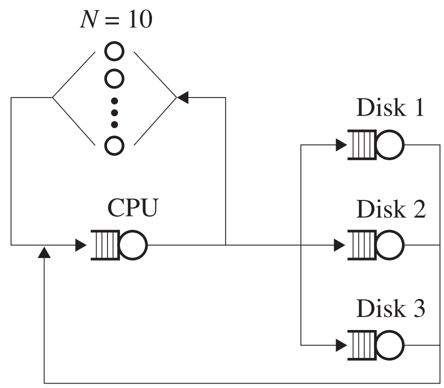
Example: Simple closed system

- $N = 10$ users
- $\mathbf{E}[Z] = 5$ seconds
- $\mathbf{E}[R] = 15$ seconds



Question: What is the throughput, X?

Example: More complex closed system



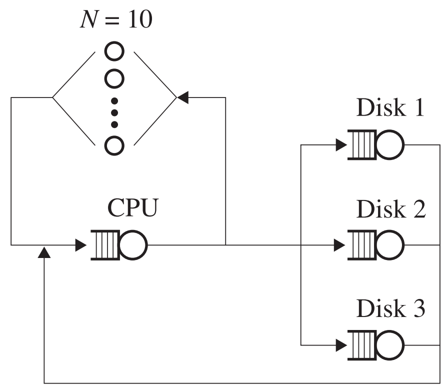
We're given the following info:

- $X_{\text{disk 3}} = 40$
- $\mathbf{E}\left[S_{\text{disk 3}}\right] = .0225$ seconds
- $\mathbf{E}\left[N_{\text{disk 3}}\right] = 4$ seconds

Question: What is the utilization of disk 3?

Question: What is the mean time spent queueing at disk 3, $\mathbf{E}\left[T_Q^{\text{disk 3}}\right]$? (2 ways!)

Example: More complex closed system, cont:



Suppose we're now additionally told:

- $\mathbf{E}$ [Number of ready users (not thinking)] = 7.5.
- $\mathbf{E}[Z] = \mathbf{E}$ [Think time] = 5 sec.

Question: What is the system throughput, X? Do this in 2 ways!