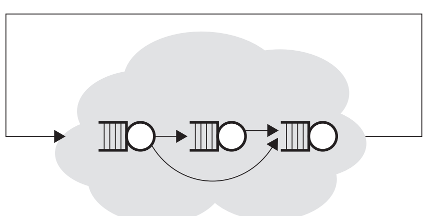
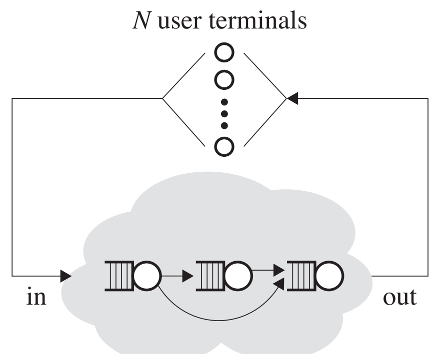
Review: Little's Law for Open Systems

- $\mathbf{E}[N] =$ ____________________
- $\mathbf{E}[N_Q] =$ ____________________
- $\mathbf{E}[N_{red}] =$ ____________________

Review: Little's Law for Closed Systems

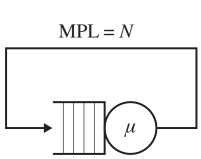




$N =$ ________________________ $N =$ ________________________

Question: When $\mathbf{E}[R]$ goes down, what happens to X? ____________________

Question: What is $\mathbf{E}[T]$ for this simple case?



More Operational Laws

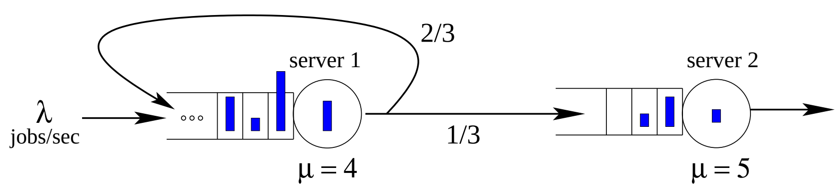
Utilization Law (applies to single device i):

- $\rho_i =$ ____________________

Forced Flow Law:

- X = system throughput
- X_i = throughput at device i
- V_i = number of visits to device i per job

- Forced Flow Law: ____________________



$X =$ ____________ $X_1 =$ ____________ $X_2 =$ ____________

Example

Suppose we have an interactive system with the following characteristics:

- 25 terminals ($N = 25$)
- 18 seconds average think time ($\mathbf{E}[Z] = 18$)
- 20 visits to a specific disk per interaction on average ($\mathbf{E}[V_{disk}] = 20$)
- 30% utilization of that disk ($\rho_{disk} = .3$)
- .025 sec average service time per visit to that disk ($\mathbf{E}[S_{disk}] = .025$)

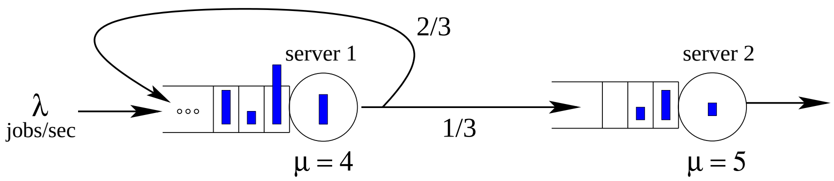
Question: What is the mean residence time, $\mathbf{E}[R]$?

Device Demands

<u>Defn</u>: D_i = total demand on device i for all visits of a single arriving job.

$D_i =$ ________________________ $\mathbf{E}[D_i] =$ ________________________

EXAMPLE:



$\mathbf{E}[D_1] =$ ________________________ $\mathbf{E}[D_2] =$ ________________________

Bottleneck Law

Ques: How can we express ρ_i in terms of $\mathbf{E}[D_i]$? Called "Bottleneck Law"

PROOF OF BOTTLENECK LAW:

Device Demands, cont.

Imagine that your system has m devices. (Can be open or closed)

Let

$$D_{max} = \max_i \{\mathbf{E}[D_i]\}$$

Question: What is the meaning of D_{max}?
How is D_{max} related to λ, in open system or X in closed?

Let

$$D = \sum_{i=1}^{m} \mathbf{E}[D_i]$$

Question: What is the meaning of D?
How is D related to $\mathbf{E}[T]$ in open system or $\mathbf{E}[R]$ in closed?

REST OF LECTURE :

- Closed Systems Only

- We will see how to estimate X and $\mathbf{E}[R]$ as a function of N for ANY closed system
  – Only bounds, but super useful!

- "Modification analysis" or "What-If analysis"
  – Work of Systems Consultant
  – Which devices should I change to improve $\mathbf{E}[R]$, X?

EVERYTHING WE'LL SAY IS DISTRIBUTION-INDPT!!

Asymptotic bounds theorem

Theorem: Given any closed interactive system with N users and m devices. Let

$$D = \sum_{i=1}^{m} \mathbf{E}[D_i] \qquad\qquad D_{max} = \max_i \{\mathbf{E}[D_i]\}$$

Then

$$X \;\leq\; \min\left(\frac{N}{D + \mathbf{E}[Z]} \;,\; \frac{1}{D_{max}}\right)$$

$$\mathbf{E}[R] \;\geq\; \max\left(D \;,\; ND_{max} - \mathbf{E}[Z]\right)$$

where the first term in each is an asymptote for small N, and the second term in each is an asymptote for large N.

PROOF FOR LARGE N ASYMPTOTE:

Asymptotic bounds theorem

Theorem: Given any closed interactive system with N users and m devices. Let

$$D = \sum_{i=1}^{m} \mathbf{E}[D_i] \qquad\qquad D_{max} = \max_i \{\mathbf{E}[D_i]\}$$

Then

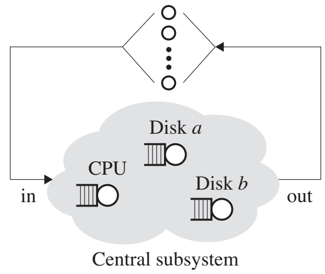
$$X \;\leq\; \min\left(\frac{N}{D + \mathbf{E}[Z]} \;,\; \frac{1}{D_{max}}\right)$$

$$\mathbf{E}[R] \;\geq\; \max\left(D \;,\; ND_{max} - \mathbf{E}[Z]\right)$$

where the first term in each is an asymptote for small N,
and the second term in each is an asymptote for large N.

PROOF FOR SMALL N ASYMPTOTE:

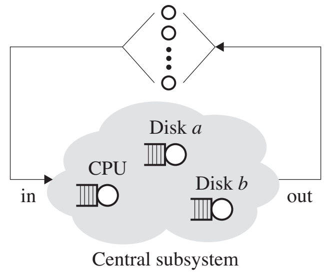
Bounds Example



$$\mathbf{E}[Z] = 18s \qquad \mathbf{E}[D_{cpu}] = 5s \qquad \mathbf{E}[D_A] = 4s \qquad \mathbf{E}[D_B] = 3s \ .$$

<u>Determine X and $\mathbf{E}[R]$ as a function of N:</u>

Bounds Example, cont.



$\mathbf{E}[Z] = 18s \qquad \mathbf{E}[D_{cpu}] = 5s \qquad \mathbf{E}[D_A] = 4s \qquad \mathbf{E}[D_B] = 3s$.

Ques: Where is the "knee", N^*, of these curves?

Ques: What does N^* represent?

Ques: If $N > N^*$, what do we need to do to increase X?

Ques: What if we instead decrease $D_{\text{next-to-max}}$?

Ques: How does this change when $\mathbf{E}[Z] = 0$ (batch system)?

Modification Analysis Example

The following measurements were obtained for an interactive system over some time:[1]

- $B_{\text{cpu}} = 400$ seconds
- $B_{\text{slowdisk}} = 100$ seconds
- $B_{\text{fastdisk}} = 600$ seconds
- $C = C_{\text{cpu}} = 200$ jobs
- $C_{\text{slowdisk}} = 2,000$ jobs
- $C_{\text{fastdisk}} = 20,000$ jobs
- $\mathbf{E}[Z] = 15$ seconds
- $N = 20$ users

Your job is to examine four possible improvements (modifications):

1. **Faster CPU:** Replace the CPU with one that's twice as fast.
2. **Balancing slow and fast disks:** Shift some files from the fast disk to the slow disk, balancing their demand.
3. **Second fast disk:** Buy a second fast disk to handle half the load of the busier existing fast disk.
4. **Balancing among three disks plus faster CPU:** Make all three improvements together: Buy a second fast disk, balance the load across all three disks, and also replace the CPU with a faster one.

[1] Just as most fairy tales start with "once upon a time," most performance analysis problems begin with "the following measurements were obtained."

Start by filling in these quantities:

- $D_{\text{cpu}} =$
- $D_{\text{slowdisk}} =$
- $D_{\text{fastdisk}} =$
- $\mathbf{E}\left[V_{\text{cpu}}\right] =$
- $\mathbf{E}\left[V_{\text{slowdisk}}\right] =$
- $\mathbf{E}\left[V_{\text{fastdisk}}\right] =$
- $\mathbf{E}\left[S_{\text{cpu}}\right] =$
- $\mathbf{E}\left[S_{\text{slowdisk}}\right] =$
- $\mathbf{E}\left[S_{\text{fastdisk}}\right] =$
- $N^* =$

Now evaluate each of the 4 potential modifications:

Why we like modification analysis

1. Distribution independent

2. Bounds only, but sufficient to make decisions in all closed systems