Linear Load Model for Robust Power System Analysis

Marko Jereminov[1]*, Amritanshu Pandey[1]*, Hyun Ah Song[2]*, Bryan Hooi[2]*, Christos Faloutsos[2], Larry Pileggi[1]

[1]Dept. of Electrical and Computer Engineering
Carnegie Mellon University
Pittsburgh, PA

[2]School of Computer Science
Carnegie Mellon University
Pittsburgh, PA

Abstract — Extension of constant power (PQ) load models to more accurately represent the electric load behavior in the grid has produced models (e.g. ZIP) that have been shown to improve the accuracy of load characterization, but like PQ load models, they introduce nonlinearities in the power flow formulation that make it more susceptible to divergence or convergence to a non-physical solution. In this paper a first-order load model (*BIG*) based on equivalent circuit principles is proposed that offers accuracy comparable to a ZIP model, but unlike ZIP, is linear and captures angle information when used in a current-voltage based power flow formulation. Advanced machine learning techniques are applied to fit parameters for the *BIG* model and traditional models using time series measurement data from our university campus and from µPMUs installed at Lawrence Berkeley National Laboratory. The results show that the linear *BIG* model characterizes the load behavior far better than PQ load models while having a similar fit to that of more complex non-linear ZIP load model while offering complexity and modeling benefits.

Index Terms— **aggregated load model, equivalent circuit formulation, linear formulation, machine learning, powerflow analysis.**

I. INTRODUCTION

Power flow analysis is a special case of nonlinear AC frequency domain analysis wherein the power system is modeled and analyzed at a single frequency. For a traditional power flow formulation, the electric load and generation models are connected via an impedance network and the flow of power is formulated to solve for the complex voltages in polar coordinates. This formulated system of nonlinear equations is solved via Newton methods that can sometimes diverge or converge to a non-physical solution.

Interestingly, some of the challenges with convergence are related to the models that are used for aggregated load and generation in the system (PQ and PV buses respectively), which are based on non-physical real and reactive power (P and Q) representations. For instance, the constant PQ load model represents the aggregated electric load behavior in a manner that does not match what is observed in the field. Consider the B.C. Hydro system wherein it was shown that decreasing the substation voltage by 1% decreased the active and reactive power demand by 1.5% and 3.4%, respectively [1]. The loads based on constant power variables are independent of the complex voltage magnitude or angle, therefore, the decrease in complex voltage must have a corresponding increase in complex current (to maintain constant power), a behavior not exhibited by most individual electric loads. Improvements to these load models (e.g. ZIP model, exponential) can better characterize the load model by incorporating the voltage magnitude dependency; however, similar to that of PQ load model, they introduce non-linearities in the formulation. Furthermore, it has been shown that ZIP and exponential load models cannot characterize load characteristics on constant voltage node in the system (e.g. load connected to a generator node) [2] due to non-dependency on voltage angle at the node.

To derive an appropriate model for an aggregated load, we begin with consideration of the behavior of actual loads. Referring to the measurement data for a randomly chosen 48-hour period for the Carnegie Mellon University (CMU) campus in Fig. 1, the load current variation (I_R and I_I) can be attributed to two factors: 1) system voltage variation and 2) variation in actual load demand (i.e. devices turning on and off). We therefore seek a model form that can accurately capture the voltage dependency (magnitude and angle) of the system load, which is clearly not the case with PQ or ZIP models. Such a load model template can then be characterized using machine learning algorithms that can predict system load variability based on the time series data and identify the time interval breakpoints for which a new set of model parameters are required.

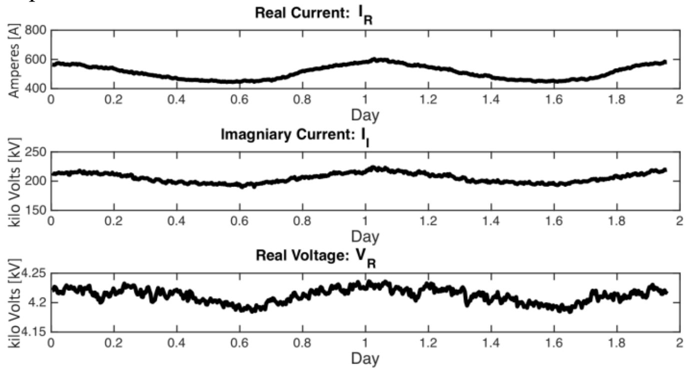

Fig. 1: CMU Dataset - current (real and imaginary), and voltage over time (2 days)

Toward this goal, we propose an equivalent circuit load model that can capture the fundamental behavior of any aggregated load. For a single frequency, the mathematical relationship between the voltage and current phasors in terms of an equivalent circuit requires only a first order model that includes a conductance (G) and susceptance (B) in parallel. At nominal voltage, this model can be uniquely characterized to match the nominal P and Q, yet still represent the potential or variation of power with voltage. For example, if G and B are linear, and G is positive, this first order model represents a quadratic drop in real load power with decrease in voltage. To enable complete representation of voltage change sensitivity, a complex current source (I) is added in parallel with B and G,

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such that the signs of G and B can be now used to provide a first order (linear) approximation of load behavior as a function of voltage. To model loads over a larger expected range of voltage magnitude, a nonlinear G and/or B can be used as demonstrated in [2]. Importantly, circuit theory provides a framework for these models to be derived reliably and efficiently. In this paper, we will show how circuit methods can be used to derive model templates and uniquely fit the B, I, and G parameter values to best match the load behavior.

Importantly, when the linear first order *BIG* model is utilized within the equivalent circuit formulation for power flow that was introduced in [3]-[7], it dramatically decreases the complexity of the power flow problem while concurrently increasing the accuracy of the solution. For example, a linear *BIG* model can be used to represent the load more accurately than a PQ model and similar to that of ZIP model, yet result in a linear set of equations for that load bus within the power flow formulation. Furthermore, the *BIG* load model can capture the voltage angle information since the equivalent circuit formulation represents the load model as a function of complex voltage variables (V_R, V_I). As previously stated, this is in contrast to ZIP models and exponential models that only capture voltage magnitude information [2].

An important facet of accurately modeling the electric load behavior using the proposed equivalent circuit model is the estimation of the *BIG* parameters from measurement data or forecasting techniques. Bulk installations of phasor measurement units (PMUs) on the transmission system and of remote terminal units (RTUs) on the distribution system are now providing a tremendous amount of actual electric load and generation data. Machine learning techniques can be used to process this collected times series data to calculate accurate *BIG* parameters.

We developed a novel machine learning algorithm, *PowerFit*, that is based on the most advanced machine learning methods. We evaluate *PowerFit* using real electric load measurement data that was collected from the Carnegie Mellon University campus (Fig. 1) and from the µPMUs installed at Lawrence Berkeley National Laboratories (LBNL) [8]. *PowerFit* automatically determines the time points where the load model (*BIG* parameter values) should transition to a new set of parameters using an iterative merging process. The model error and model complexity are balanced based on the Bayesian Information Criterion (BIC) measure [9]. The results show that the *BIG* model with the proposed machine-learning algorithm can fit the measured data from CMU and LBNL accurately within error measure of less than 2.0 % with less than 4 segments per day. Furthermore, it is shown that the linear *BIG* model characterizes the LBNL load behavior far better than the non-linear PQ load model and similar to that of more complex non-linear ZIP load model, yet offers dramatically simplified power flow complexity and capturing of angle information.

II. Circuit Theoretic Load Model

A first-order equivalent circuit can be used to represent any phase and magnitude relationship between current and voltage phasors at a single frequency. A first-order load impedance can be represented as an equivalent circuit model with a conductance (G) and susceptance (B) in series or parallel, and as such, would capture the load behavior wherein the current flowing into the load bus is directly proportional to the voltage across it. However, the aggregated loads will sometimes behave contrary to the aforementioned behavior; for example, consider an aggregated load with a large percentage of induction motors that run to maintain a constant mechanical torque. Such loads are likely to exhibit a behavior wherein the current flowing into the load bus is inversely proportional to the applied voltage. This behavior is similar to that which is represented by a constant PQ load model, where the increase in voltage has no influence on the constant power P and would conceptually correspond to a decrease in current.

To begin from a circuit modeling perspective, we consider the driving point load model [10]-[11] for a generalized power flow load that could capture both of the positive and negative correlation between the current and voltage characteristics. We first consider the governing complex circuit equations [3]-[7] that would be representative of the PQ load model:

$$I_{R,PQ} + jI_{I,PQ} = \frac{PV_R + QV_I}{V_R^2 + V_I^2} + j\frac{PV_I - QV_R}{V_R^2 + V_I^2} \tag{1}$$

where P and Q are known constant coefficients of real and reactive powers respectively, and V_R, V_I, $I_{R,PQ}$ and $I_{I,PQ}$ are the real and imaginary voltages and current of the PQ load model.

We can further split the complex current function from (1) and linearize it to obtain:

$${I_{R,PQ}}^{k+1} = 2{I_{R,PQ}}^{k} + \frac{\partial I_{R,PQ}}{\partial V_R}V_R^{k+1} + \frac{\partial I_{R,PQ}}{\partial V_I}V_I^{k+1} \tag{2}$$

$${I_{I,PQ}}^{k+1} = 2{I_{I,PQ}}^{k} + \frac{\partial I_{I,PQ}}{\partial V_R}V_R^{k+1} + \frac{\partial I_{I,PQ}}{\partial V_I}V_I^{k+1} \tag{3}$$

where the constant terms represent the values of real and imaginary currents known from k^{th} iteration and are represented by a constant current source. Note that partial derivatives for which the real and imaginary currents are directly proportional to the voltages across the respective split circuit models, i.e. real and imaginary, are represented as a conductance (G), while the partial derivatives for which real and imaginary currents are directly proportional to the voltages of other sub circuits are represented by a voltage controlled current source.

Furthermore, it can be shown that the respective partial derivatives defined in (2) and (3) have the following properties:

$$\frac{\partial I_{R,PQ}}{\partial V_R} = \frac{\partial I_{I,PQ}}{\partial V_I} \equiv G < 0 \tag{4}$$

$$\left|\frac{\partial I_{R,PQ}}{\partial V_I}\right| = \left|\frac{\partial I_{I,PQ}}{\partial V_R}\right| \equiv B \tag{5}$$

From (4) and (5) we can observe that the governing equations of a PQ load model, i.e. (1)-(3), can be translated to an equivalent circuit corresponding to a constant current source in parallel with the susceptance and a negative conductance that compensates for the inverse relationship between the current and voltage of the load. With this model, as the voltage across the load increases, the current will decrease and vice versa. This model is, however, unable to capture the voltage sensitivity for both of the aforementioned aggregated load types (both positive and negative correlation between the complex voltage and current variables).

Next, we propose a circuit theoretic load model that is able to capture both load type sensitivities with respect to voltage. The complex governing equation of the generalized load current is given by:

$$I_R + jI_I = \alpha_R + j\alpha_I + (V_R + jV_I)(G + jB) \tag{6}$$

where the complex admittance ($G + jB$) with positive G captures the constant impedance load behavior and is directly proportional to the voltage across the load, and the combined impedances capture the voltage sensitivities. Specifically, a negative conductance in conjunction with complex current

$(\alpha_R + j\alpha_I)$ will mimic the inverse current/voltage sensitivity relationship and positive conductance will represent the other. Both the positive and negative impedances capture the change in load with voltage with respect to the portion of the load that is modeled by the current source.

The complex equivalent circuit and the split-circuit of the proposed susceptance (B), current source (I), and conductance (G) load model, *BIG*, defined by equations (7) - (8), is shown in Fig. 2:

$$I_R = \alpha_R + V_R G - V_I B \quad (7)$$
$$I_I = \alpha_I + V_I G + V_R B \quad (8)$$

It is worth noting that the *BIG* model is equivalent to the ZIP load model with the real power coefficient set to zero and a different "fixed complex current" term. Most importantly, the *BIG* load model is linear in a current/voltage formulation, while the ZIP model is nonlinear in both current/voltage and traditional PQV formulations. In addition, the *BIG* model is capable of capturing angle information and compatible with constant voltage magnitude devices, whereas the ZIP and PQ models are not [2].

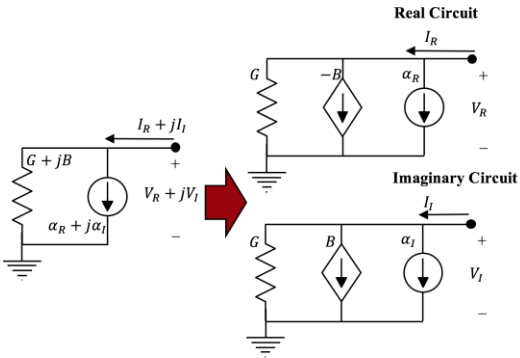

Fig. 2: BIG Load Model.

III. Machine Learning for Load Model Fitting

Our objective is to take time series measurement data for actual aggregated loads and fit *BIG*, ZIP and PQ models to represent them. This requires determination of breakpoints in the data due to changes in demand and fitting of the model parameters in some best-fit sort of way. For our approach, we apply machine learning techniques to estimate the parameters of the *BIG* aggregated load model in Fig. 2. One of the most popular machine learning tools used for time series analysis is autoregressive integrated moving average (ARIMA) model [12]. ARIMA predicts the forthcoming values using linear combination of the past values and noise. ARIMA models are used for time series forecasting as well as time series segmentation where the cut points in a time series is determined by the change in ARIMA models.

Prediction of power consumption using a variant of the ARIMA model has been proposed in [13], where the authors use the ARIMA model optimized with Akaike Information Criterion (AIC) to predict the power consumption. Although machine learning techniques have proven their effectiveness for these problems, these approaches to date have not considered for fitting to a circuit theoretic load model form.

We developed an algorithm and implementation, *PowerFit*, to provide a fast segmentation and summarization algorithm for current and voltage time sequences. We apply a philosophy of "Occam's razor" for our model fitting, that the simplest explanation/model for our dataset is the best, and develop a machine learning approach to characterize the *BIG* model and determine how many cut points are required to represent the load time-sequences with sufficiently small error. Table 1 summarizes our notation:

TABLE 1: SYMBOLS

Symbols	*Definition*
T	Number of time-ticks for a time series
ns	Number of segments
x	A time series of length T
$\hat{x}$	Model approximation of x
$\mathcal{S}$	A set of segments of x
$\hat{\mathcal{S}}$	A set of model approximations of segments
$\mathcal{C}$	Model parameters
k	Number of parameters of the models

The *PowerFit* pseudocode is shown in Algorithm 1, and can be further broken down into two parts: 1) *cut point search*, and 2) *optimization*.

A. Cut point search (bottom-up approach)

We first describe our approach for finding the parameters of the *BIG* model. Similar steps can be applied for ZIP, PQ and other load model forms. Starting with the *BIG* load model template, we approximate the conductance (G), susceptance (B) and current (α_R and α_I) parameters as piece-wise constant for a given time segment. The proposed *PowerFit* algorithm then tries to find the cut-points where the measured load behavior drastically changes, and hence assign a new set of load parameters that is required to accurately capture the new load behavior.

Finding the optimal cut points for a given modeling template is an $O(T^{ns-1})$ problem, which is computationally expensive or even intractable for large T. Hence, we propose a greedy heuristic that finds cut points in the time series, which aims to minimize reconstruction error, while keeping the model succinct (with as few segments as possible). For a given time series of length T, we start with the equi-spaced, finest resolution of a segment, and keep merging adjacent segments in a greedy manner until the model fit cannot be longer improved based on the Bayesian Information Criterion (BIC) measure in (10).

Since the model parameters are assumed to be piece-wise constant within each segment, the *LinFit* function is used to fit the parameters $c = (G, B, \alpha_R$, and $\alpha_I)$ using linear regression:

$$\begin{aligned} \hat{I}_R &\sim GV_R - BV_I + \alpha_R \\ \hat{I}_I &\sim BV_R + GV_I + \alpha_I \end{aligned} \quad (9)$$

At each iteration, we consider merging each pair of adjacent segments. The proposed *PowerFit* algorithm starts by computing the change in the BIC measure if each pair of adjacent segments is merged. It then merges the two segments which result in the largest improvement of the BIC measure. The algorithm continues merging segments in this way, until the BIC measure cannot be reduced by any further merging. The final number of segments is denoted as ns.

B. Parameter estimation

Determining the number of segments is a challenging optimization problem and represents a tradeoff between complexity and accuracy. For instance, if a new model is assigned for each time-tick, we may have zero modeling error, however, the model complexity will be at maximum. In contrast, if a single model is determined for the whole time series of measured data, we may have minimum model complexity, but the maximum model error. To tackle this problem, we use Bayesian Information Criterion (BIC) [9] as an optimization measure for selecting a model with a good trade-off between the model error and model complexity; i.e. the count of parameters we need for our *BIG* model and cut-points. Assuming the modeling errors are Gaussian, independent and identically distributed (i.i.d.), penalizing the negative log likelihood and the number of parameters in the model, the BIC measure is calculated as:

$$BIC(\hat{s}_i, s_i, k, T) = \sum_{t=1}^{|s_i|} (\hat{s}_{it} - s_{it})^2 + k \ln(T) \quad (10)$$

where s_i refers to the values of both I_R and I_I in the i th segment, $\hat{s}_i$ is the model approximation of s_i, T is the total number of time-ticks and k is the number of model parameters. In our case $k = 4 \times ns$, since we have four parameters (G, B, α_R, α_I) for each of ns models (or segments).

Input: A time series x of length T;
Output: A set of model approximations for ns segments $\hat{\mathcal{S}}$ and the model parameters $\mathcal{C}$ for each segment;

Initialize the segments:
$\mathcal{S} = \left\{s_i: s_i = x_{(i-1)\times 2+1:(i-1)\times 2+2}, \forall i = 1, \cdots, \frac{T}{2}\right\}$;
$\mathcal{C} = \{\}$ (a set of model parameters);
$\hat{\mathcal{S}} = \{\}$ (a set of model approximations for segments);
Cut point search (*bottom-up*);
for $s_i \in \mathcal{S}$ **do**
  Fit a model to each segment;
  $[\hat{s}_1, c_1] = LinFit(s_i)$;
  $[\hat{s}_2, c_2] = LinFit(s_{i+1})$;
  $[\hat{s}_3, c_3] = LinFit([s_i\ s_{i+1}])$;
  Compute the change in BIC if s_i and s_{i+1} are merged;
  $\delta_i = BIC(\hat{s}_3, [s_i\ s_{i+1}], k, T) - BIC(\hat{s}_1, s_i, k, T)$
  $-BIC(\hat{s}_2, s_{i+1}, k, T)$
repeat until $\min_k \delta_k > 0$:
  Perform the merge that decreases BIC the most;
  Find the smallest delta value, $\delta_i = \min_k \delta_k$
  Update $\mathcal{S}$ by replacing s_i, s_{i+1} with $[s_i\ s_{i+1}]$;
  $[\hat{s}_3, c_3] = LinFit([s_j\ s_{j+1}])$;
  Add $\{c_3\}$ to $\mathcal{C}$;
  Add $\{\hat{s}_3\}$ to $\hat{\mathcal{S}}$;
  Update δ_i and δ_{i-1} based on the same formula applied to the current $\mathcal{S}$

Algorithm 1 *PowerFit*: Fast segmentation and summarization algorithm for current and voltage time sequences.

IV. Results and Comparison

We applied the *PowerFit* algorithm to *BIG*, PQ and ZIP load models. The efficacy of the model is determined by the fit of the load behavior with a small approximation error. For all three model types, the *PowerFit* algorithm automatically determines the minimum number of segments needed to represent the load behavior over the time range of interest with sufficiently small error. The results show that the algorithm is able to identify the cut points corresponding to the daily activities and patterns that would be expected during a typical day, or week, etc.

A. PowerFit validation with BIG model

1) Description of dataset

We applied the proposed *PowerFit* algorithm to actual current and voltage measurements from the Carnegie Mellon University (CMU) campus and from micro-phasor measurement units (μPMUs) from the Lawrence Berkeley National Laboratory (LBNL) Open μPMU project [8].

The CMU dataset consists of the current and voltage data, measured from August 23, 2016 11:10 am to August 25, 2016 11:00 am in intervals of 5 minutes, which represents 575 time-samples. The available data for the campus meters included the real voltage (V_R), real current (I_R), and imaginary current (I_I), as shown in Fig. 1. Imaginary voltage was not available through these meters and, therefore, it is set to zero for our model fitting. This causes our resulting model to be less accurate than it could be if voltage angle information was available.

The LBNL data consists of both the current and voltage data for the period of approximately 3 months of which 12 days, from September 30 to October 11 2015, were used for this analysis and is shown in Fig. 3. The available data from the μPMUs included the real voltage (V_R), real current (I_R), imaginary voltage (V_I), and imaginary current (I_I), at 120Hz. We preprocess the data to reduce noise by smoothing it via moving average, and perform further averaging to reduce the data to a total of 500 time ticks.

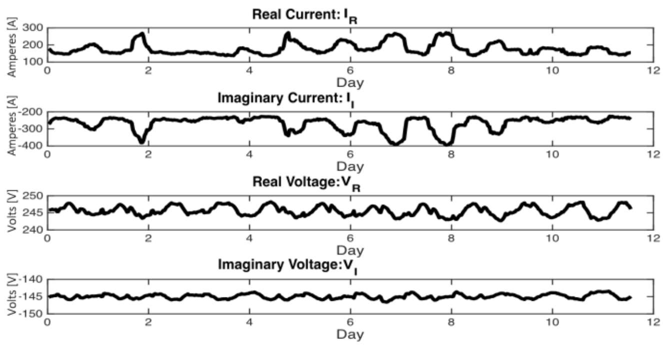

Fig. 3: LBNL Dataset – real and imaginary current and voltage over the period of 12 days.

2) Segmentation on CMU dataset

PowerFit was applied to the dataset in Fig. 1 to determine the cut points for the time sequences (I_R and I_I). For each detected segment, *PowerFit* assigns a distinct set of parameters to the load model, as shown in Table 2. The comparison between the measured and approximated real and imaginary currents ($\mathrm{I_R}$ and $\mathrm{I_I}$) is shown in Fig. 4. The black solid lines are the measured time sequences and the red dotted lines are the approximated load currents. The approximated currents are represented by eight distinct set of load parameters for the time-period of 48 hours. The cut points corresponding to a *BIG* model segment change are identified by the blue vertical lines in the Fig. 4. Note the close agreement between the real data and the model, and how *PowerFit* accurately identifies the load changes on campus, such as at 8:15AM, when people start coming on campus, and at approximately 5pm, when they leave campus, etc.

The approximation errors for I_R, and I_I are 1.91% and 1.36%, respectively based on the formula:

$$Error = \frac{\sqrt{\sum_{i=1}^{T}(x_i - \hat{x}_i)^2}}{\sqrt{\sum_{i=1}^{T}(x_i)^2}} \quad (11)$$

PowerFit trades off fitting error versus number of segments. For the results shown, the errors for both I_R, I_I were bounded by 2.0 %, and modeling the CMU campus required only 4 model segments per day (Table 2) to capture the large power demand changes.

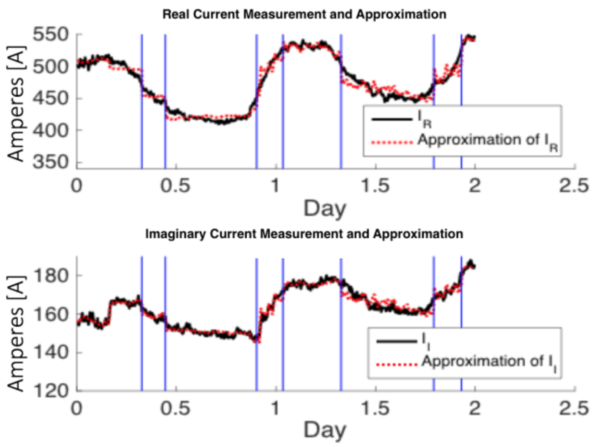

Fig. 4: ***Succinct, accurate*** *representation of the load current, by PowerFit: With very few parameters, our PowerFit (in red) matches measured data (in black); and also spots natural discontinuities (blue vertical lines).*

Table 2 has the model parameters for each segment of I_R and I_I for the 48-hour time-period. Note that, as expected, some of the G coefficients of the load model are negative to represent how the current changes with voltage magnitude in the normal operating range. Even with a negative G value, however, the load still consumes power since the G term captures only the offset of current in relation to the large current source in parallel with it.

TABLE 2: MODEL PARAMETERS FOR *BIG* MODEL

Seg.	1	2	3	4	5	6	7	8
G	-0.1440	0.2695	-0.2383	0.6350	-0.7207	0.7460	0.1198	-0.5656
B	0.1097	0.1673	0.0894	0.2522	0.1105	0.2679	0.2865	-0.0179
α_R	1108.8	-694.1	1431.5	-2187.3	3597.1	-2712.6	-4603.3	2950.3
α_I	-300.9	-553.3	-228.3	-904.2	-294.4	-975.2	-1045.9	260.7

3) Segmentation of LBNL Dataset

PowerFit was next applied to the LBNL dataset. Unlike the CMU dataset, the LBNL dataset included the voltage angle information. The results are plotted in Fig. 5 and the root mean square errors from (11) are 1.93% and 1.68% for I_R and I_I, respectively.

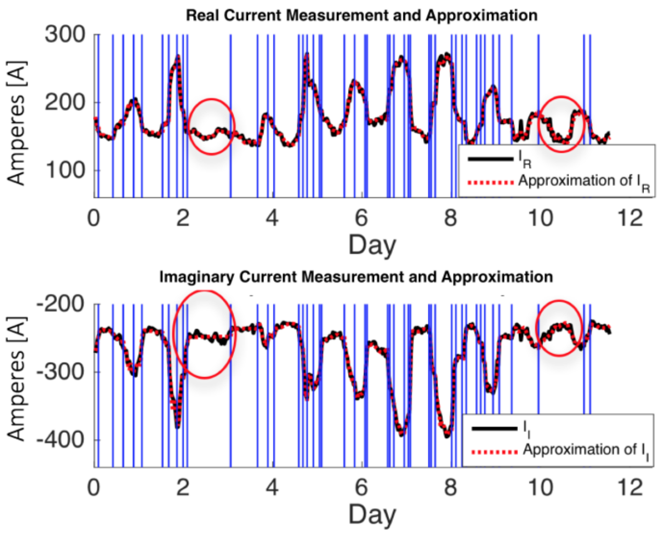

Fig. 5: BIG model representation of 12 days for the LBNL data; BIG model is clearly shown to capture the voltage variability of the load characteristics.

The results in Fig. 5 demonstrate excellent fit between the measured data and *BIG* model. Note how well the *BIG* model represents the voltage-change load characteristics (within a single segment) between days 2-3 and 10-11 as shown in the red circles in the Fig. 5.

B. Fitting and comparison for ZIP and PQ load models

The *PowerFit* algorithm was also used to characterize the CMU and LBNL load data for both PQ and ZIP load models. The segmentation of the CMU 48-hour-time-period data set with PQ model fitting is presented in Fig. 6.

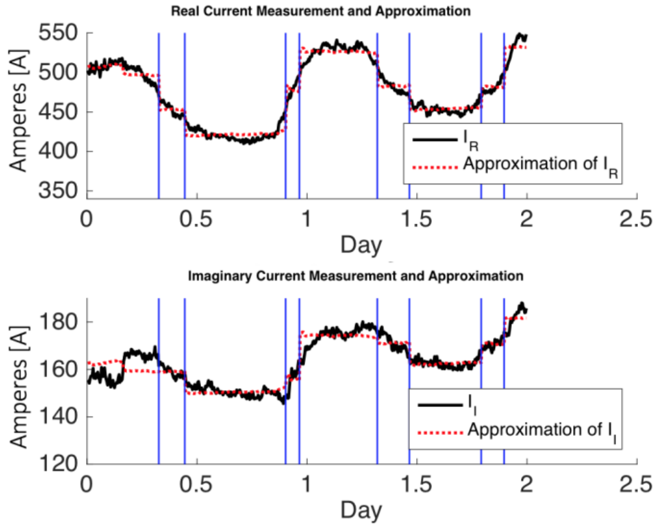

Fig. 6: PQ load model representation of the load currents.

As can be seen from Fig. 6, by using the same number of segments as used for the *BIG* load model, the measure error increases with the PQ model. Furthermore, the PQ load representation does not capture the voltage sensitivities of the load current, which impacts the power flow simulation accuracy if the load voltage operates below the nominal (measured) one.

Fig. 7 shows the segmentation of the LBNL data for the PQ model, where achieving a similar error measure as that of *BIG* (around 2%) requires significantly more segments (on average around 7 per day). This is partly due to the inability of the PQ load to accurately represent the voltage dependency of the load behavior resulting in higher number of segments. The ZIP model provided a fitting quite similar to that of the *BIG* model, requiring slightly fewer segments per day on average to achieve the same error.

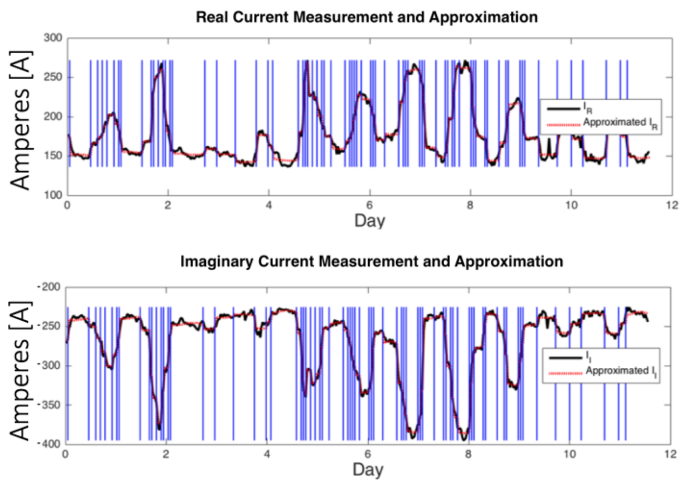

Fig. 7: Fit of PQ load models with PowerFit algorithm requires significant number of additional segments and results in higher error as compared with BIG model.

The error measure of all the three load models as a function of the average number of segments per day for both the LBNL and CMU data sets is displayed in Fig. 8, and Fig. 9 respectively. As expected, the linear *BIG* models perform far better than the PQ load model, while performing similar to the more complex ZIP load model. Importantly, in contrast to ZIP models, the *BIG* model results in linear constraints for our current-voltage power flow formulation, is compatible with FACTS devices that can correspond to fixed voltage magnitude buses, and can be used to capture voltage angle information.

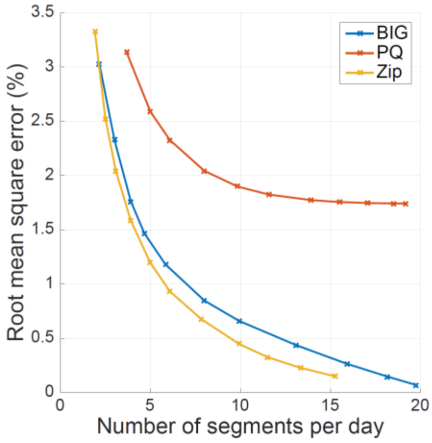

Fig. 8: Error measures for different load models using PowerFit algorithm for LBNL data set.

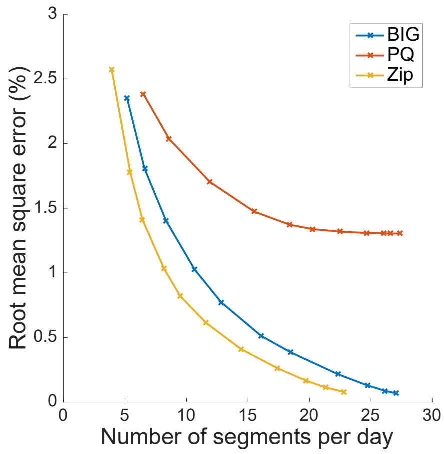

Fig. 9: Error measures for different load models using PowerFit algorithm for CMU data set.

Lastly, since the error measure is expected to be inversely related to the number of parameters used by the model, Fig. 10 shows a tradeoff curve, plotting error against number of parameters used by the model for both CMU (right) and LBNL (left) data sets. As we observe, the BIG model has better error tradeoff than the PQ and ZIP models over most of the range.

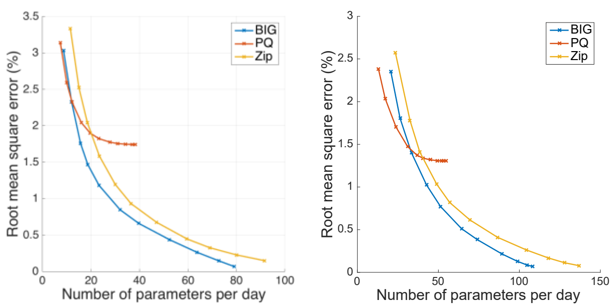

Fig. 10: Error vs. number of parameters tradeoff curve using PowerFit algorithm. The BIG model provides the best tradeoff for most of the range.

V. Conclusions

This paper proposes a new linear load model for the power flow and three-phase power flow problem based on circuit-theoretic representation. The load model can accurately capture the aggregated load in the grid in terms of a linear relationship with complex voltages and currents that when used in conjunction with our equivalent circuit formulation results in linear equalities for the load buses that can significantly reduce the complexity of power flow and three phase power flow problems. Furthermore, the proposed load model is more generic than the existing load models as it captures the complex voltage dependency (voltage magnitude and voltage angle) of the load. The machine learning techniques proposed in this paper can estimate the load parameters accurately by inherently recognizing natural cut-points (start-end of working day).

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