# Automatic SAT-Compilation of Protocol Insecurity Problems via Reduction to Planning<sup>\*</sup>

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#### Introduction

• **Context:** Dramatic speed-up of SAT solvers in the last decade: problems with thousands of variables are now solved routinely in milliseconds.

This has led to breakthroughs in planning and hardware verification.

• **Approach:** In this work we have investigated if similar results can be obtained by applying SAT-based model-checking to security protocols.

# Roadmap

- Example.
- The Model and Protocol Insecurity Problems.
- Encoding Protocol Insecurity Problems into SAT.
- Implementation and Experimental Results.
- Conclusions and Perspectives.

#### An example

Consider the following simple protocol:

1. 
$$
A \rightarrow B
$$
:  $\{N_A\}_{K_{AB}}$   
2.  $B \rightarrow A$ :  $\{f(N_A)\}_{K_{AB}}$ 

#### An example

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This protocol is flawed. In fact, by executing

1.1. 
$$
A \to I(B)
$$
:  $\{N_A\}_{K_{AB}}$   
\n2.1  $I(B) \to A$ :  $\{N_A\}_{K_{AB}}$   
\n2.2  $A \to I(B)$ :  $\{f(N_A)\}_{K_{AB}}$   
\n1.2  $I(B) \to A$ :  $\{f(N_A)\}_{K_{AB}}$ 

A believes to speak with  $B$  in the first session, but  $A$  is really speaking with  $I$ .

# **Modeling**

- Perfect cryptography: an encrypted message can be neither altered nor read without the appropriate key.
- The Dolev-Yao attacker:
	- controls all the traffic in the network.
	- can compose and send fraudulent messages from the knowledge he can glean from the observed traffic and his own initial knowledge.

# Protocol Specification Language

We use an expressive formalism based on first-order multiset rewriting, called IF, (see Jacquemard, Rusinowitch, and Vigneron, "Compiling and Verifying Security Protocols" in LPAR 2000) that

- supports the specification of different protocol models and properties,
- has a clear, well-defined semantics, and
- results from automatic compilation of high-level protocol specifications in the "Alice&Bob"-style notation.

States represented by multisets of terms of the form:

•  $m(j, s, r, t)$  means that sender s has (supposedly) sent message t to principal  $r$  at protocol step  $j$ .

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	- knows the terms stored in the lists  $ak$  (acquired knowledge) and  $ik$  (initial knowledge), and
	- is waiting for a message from s (if  $j \neq 0$ ).

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- $\bullet$   $i(t)$  means that the intruder knows t.
- $fresh(n)$  means that n has not been used yet.

#### • Initial State:

 $w(0,alice,alice,[], [alice, bob, kab], 1) w(1,alice, bob,[], [bob,alice, kab], 1) \ldots$  $fresh(nc(na, 1))$ *i*(alice)*i*(bob)

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• Protocol Rules:  $w(0, xA, xA,[], [xA, xB, xK], xC)$  f $resh(nc(na, xC))$  $\Rightarrow$  $m(1, xA, xB, \{nc(na, xC)\}_{xK})$  $w(2, xB, xA, [nc(na, xC)], [xA, xB, xK], xC)$ 

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- Intruder Rules:

 $m(xj, xs, xr, xmsg) \Rightarrow i(xs) i(xr) i(xmsg)$  $i({\{xmsg\}_x}\)$ i( $xK$ ) $\Rightarrow$ i( $xmsg$ )i( ${\{xmsg\}_x}\$ i( $xK$ )

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• Bad States:  $w(0,alice,alice,[], [alice, bob, kab], s(1))$ .  $w(1,alice, bob, [], [bob,alice, kab], 1)$ 

# Protocol Insecurity Problems

A Protocol Insecurity Problem is a tuple  $\langle S, \mathcal{L}, \mathcal{R}, \mathcal{I}, \mathcal{B} \rangle$  where:

- $S$  is a set of atomic formulae of a sorted first-order language called *facts*;
- $\bullet$   $\mathcal{L}$  is a set of function symbols called *rule labels*;
- $R$  is a set of (deterministic) labelled rewrite rules of the form L  $\stackrel{l}{\rightarrow} R$ , where  $L,R\subseteq\mathcal{S}$ , and  $l\in\mathcal{L}$ ;
- $I$  and  $B$  are the initial state and a set of bad states.

A solution to a Protocol Insecurity Problem is a sequence  $S_1$  $\stackrel{l_1}{\longrightarrow} S_2 \stackrel{l_2}{\longrightarrow} \cdots \stackrel{l_{n-1}}{\longrightarrow} S_n$ , where  $l_i \in \mathcal{L}$ ,  $\mathcal{I} \equiv S_1$ , and there exists  $S_{\mathcal{B}} \in \mathcal{B}$  such that  $S_{\mathcal{B}} \subseteq S_n$ .

# Encoding Protocol Insecurity Problems into SAT

The reduction of Protocol Insecurity Problems to SAT is carried out in two phases:

- 1. the Protocol Insecurity Problem is translated into Planning Problem;
- 2. the Planning Problem is then encoded into SAT, using standard encoding techniques, with iterative deepening on the number of steps.

### Planning Problems

A planning problem is a tuple  $\Pi = \langle \mathcal{F}, \mathcal{A}, Ops, I, G \rangle$ , where:

- $F$  and  $A$  are sets of ground atomic formulas called fluents and actions respectively.
- $Ops$  is a set of expressions of the form

 $Pre \overset{Act}{\longrightarrow} Add ; Del,$ 

where  $Act \in \mathcal{A}$ , and  $Pre$ ,  $Add$ , and  $Del$  are finite sets of fluents such that  $Add \cap Del = \emptyset.$ 

• I and  $G$  are boolean combinations of fluents representing the initial state and the final states respectively.

A solution to a planning problem is a sequence of actions whose execution leads from the initial state to a final state and the preconditions of each action hold in the state to which it applies.

#### Protocol Insecurity Problems as Planning Problems

Map IF rewrite rules into "actions":



# Encoding Planning Problems into SAT (1)

Fact: Given,

- a planning problem  $\Pi = \langle \mathcal{F}, \mathcal{A}, Ops, I, G \rangle$ , and
- a positive integer  $n$ ,

then it is possible to build a propositional formula  $\Phi_{\Pi}^n$  such that any model of  $\Phi_{\Pi}^n$  corresponds to a partial-order plan of  $\Pi$ .

Intuition: add an additional time-index parameter to each action or fluent, to indicate the state at which the action begins or the fluent holds.

# Encoding Planning Problems into SAT (2)

The formula  $\Phi_{\Pi}^n$  is given by the conjunction of the following axioms:

**Universal Axioms:** for each action  $\alpha \in \mathcal{A}$  s.t.  $(Pre \stackrel{\alpha}{\longrightarrow} Add; Del) \in Ops$  and for each  $i = 0, \ldots, n - 1$ 

$$
\alpha_i \supset \bigwedge \{p_i \mid p \in Pre\}
$$
  
\n
$$
\alpha_i \supset \bigwedge \{p_{i+1} \mid p \in Add\}
$$
  
\n
$$
\alpha_i \supset \bigwedge \{\neg p_{i+1} \mid p \in Del\}
$$

**Cardinality:**  $O(n|\mathcal{A}||\mathcal{F}|)$ , but usually  $O(n|\mathcal{A}|r)$ , where r is the maximal number of fluents mentioned in an operator (usually a small number).

# Encoding Planning Problems into SAT (3)

**Explanatory Frame Axioms:** for all fluents  $p \in F$  and for each  $i = 1, \ldots, n - 1$ 

$$
(p_i \land \neg p_{i+1}) \supset \bigvee \left\{ \alpha_i \mid \alpha \in \mathcal{A}, Pre \xrightarrow{\alpha} Add; Del \in Ops, p \in Del \right\}
$$
  

$$
(\neg p_i \land p_{i+1}) \supset \bigvee \left\{ \alpha_i \mid \alpha \in \mathcal{A}, Pre \xrightarrow{\alpha} Add; Del \in Ops, p \in Add \right\}
$$

**Cardinality:**  $O(n|\mathcal{F}||\mathcal{A}|)$ , but usually  $O(n|\mathcal{F}|k)$ , where k is a small number.

# Encoding Planning Problems into SAT (4)

**Conflict Exclusion Axioms:** for each  $i = 0, \ldots, n - 1$ 

 $\neg(\alpha_i \wedge \alpha'_i)$  $\binom{l}{i}$ 

for all  $\alpha, \alpha' \in \mathcal{A}$  s.t.  $\alpha \neq \alpha'$ ,  $Pre \stackrel{\alpha}{\longrightarrow} Add; Del \in Ops$ ,  $Pre' \stackrel{\alpha'}{\longrightarrow} Add'; Del' \in Ops,$  and  $Pre \cap Del' \neq \emptyset$  or  $Pre' \cap Del \neq \emptyset.$ Cardinality:  $O(n|\mathcal{A}|^2)$ 

**Initial State Axioms:**  $I_0$ , i.e. the formula I in which each occurrence of a fluent is replaced by a fluent time-indexed with 0. **Cardinality:**  $O(|\mathcal{F}|)$ .

**Goal State Axioms:**  $G_n$ , i.e. the formula G in which each occurrence of a fluent is replaced by a fluent time-indexed with  $n$ . **Cardinality:** depending from the structure of  $G$ , typically small.

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# No!

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A number of optimizing transformations need to be done in order to get encodings of manageable size.

# **Optimizations**

- $\nabla$  Language Specialization. √
- **W** Fluent Splitting. √
- **W** Exploiting Static Fluents. √
- $\nabla$  Invariant-based Simplification. √
- W Reducing the Number of Conflict Exclusion Axioms. √
- Step-Compression (step\_compression). √
- Building Encryption Properties into the Encoding.
- Exploiting Mutual Exclusion of  $w$ -terms.
- $\Box$  Unit Propagation.

#### Optimizations: Fluent Splitting

- Since  $\langle j, s, r, c \rangle$  is a key for  $w(j, s, r, ak, ik, c)$ , replace  $w(j, s, r, ak, ik, c)$  with the conjunction of (new predicates)  $wk(j, s, r, ak, c)$  and  $inknw(j, s, r, ik, c)$ .
- Similar considerations allow us to simplify  $inknw(j, s, r, ik, c)$  to  $inknw(r, ik, c).$
- Replace  $wk(j, s, r, [ak_1, \ldots, ak_l], c)$  with:

 $wk(j,s,r,ak_1,1,c),\ldots, wk(j,s,r,ak_l,l,c)$ The number of  $wk$  terms reduces from  $O(|\texttt{text}|^l)$  to  $O(|l| * | \texttt{text}|).$ 

# Optimizations: Reducing the Number of Conflict Exclusion Axioms

Problem: The number of Conflict Exclusion Axioms grows quadratically in the number of actions.

Solution: exploit the monotonicity of the intruder knowledge. Since a monotonic fluent never appears in the delete list of some action, then it cannot be a cause of a conflict.

**Example:** IF rewrite rules of the form:

 $i(\langle xM1, xM2 \rangle) \Rightarrow i(xM1) \cdot i(xM2)$ 

are replaced by:

 $i(\langle xM1, xM2 \rangle) \Rightarrow i(xM1)$  $i(xM2)$  $i(\langle xM1, xM2 \rangle)$ 

## Optimizations: Step-Compression

Simple form of partial-order reduction: significant savings by compressing these rule triples.



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# Introduction of Bounds and Constraints

 $\nabla$  Bounding the Number of Session Runs (session\_repetitions). √

 $\nabla$  Constraining Variable Instantiation √ (constrain\_rule\_variables).

Note: These may introduce incompleteness, which can be overcome by searching (using iterative deepening) the parameter space.

#### Constraining Variable Instantiation

Let us consider part of the Kao-Chow protocol:

\n- $$
S \rightarrow B : \{A, B, Na, Kab\} Kas, \{A, B, Na, Kab\} Kbs
$$
\n- $B \rightarrow A : \{A, B, Na, Kab\} Kas, \{Na\} Kab, Nb$
\n

B cannot check that the occurrence of  $A$  in the first component is equal to that inside the second. As a matter of fact, we might have different terms at those positions.

This constraint imposes that the occurrences of  $A$  (as well as of  $B$ ,  $Na$ , and  $Kab$ ) in the first and in the second part of the message must coincide.

For instance, messages of the form

 $m(2, a, b, c({a, b, nc(na, s(1))}kas, {a, b, nc(nb, s(1))}kbs)))$ 

would be ruled out by the constraint.

# Implementation

• IF2SATE: a translator from the IF to SATE (a STRIPS-like language).

6,200 lines of Prolog code.

• SATE: a compiler from SATE to SAT (DIMACS format). 2,800 lines of Prolog code.

NOTE: state-of-the-art tools carrying out the SAT encoding of planning problems (Medic, BlackBox) are unable to handle STRIPS languages with complex term structure (only individual constants allowed).









## Experiments: Analysis

Over the Clark/Jacob library:

- Coverage: unsuited to detect type-flaws, but effective on the others.
- Effectiveness: it finds most of the known attacks.
- Performance: Encoding Time largely dominates Solving Time.

# Conclusions & Perspectives

- SATMC performs well on the Clark/Jacob library.
- Optimizations bring a lot! Some remain to be implemented. We expect orders of magnitude reduction in compilation time by:
	- building encryption properties into the encoding,
	- exploiting more sophisticated encoding (e.g. graphplan encoding), and
	- applying unit propation during the encoding phase.
- Approach is declarative and incremental: we can easily modify goal or initial state without recompilation.
- Symbolic representation trivially allows sets of initial states.