Tractable Quantified Constraints Satisfaction Problems over Positive Temporal Templates

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Outline

1. Quantified Constraint Satisfaction Problems

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2. Complete Classification Theorems

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1. Quantified Constraint Satisfaction Problems
2. Complete Classification Theorems
3. Quantified Classification for Positive Temporal Languages
4. Techniques used
   - Algebraic Approach to (Q)CSP
   - Filter Representation of Temporal Relations

Outline

Conjunctive Positive Formula

Definition

Let Σ be some set of relational symbols (signature).
Conjunctive positive formula over Σ is of the form:

$$Q_1 x_1 \dots Q_k x_k \bigwedge_{i=1}^{n} R_i(v_1^i, \dots, v_k^i),$$

where $Q_i \in \{\forall, \exists\}$, and $R_i \in \Sigma$ for all $1 \leq i \leq n$.

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Example

$\exists x \exists z \forall y \; x \leq y \wedge y \leq z$ over signature $\{\leq\}$

QCSP is a Computational Problem

QCSP(Γ)

Let $\langle \Gamma, \mathcal{D} \rangle$ be a constraint language of some signature Σ.

Instance: a positive conjunctive formula ψ without free variables over Σ.

Question: is ψ true in Γ?

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Example

Consider a constraint language $\langle \{x_1 \leq x_2\}, \mathbb{Q} \rangle$
Instance: $\exists x \exists z \forall y \; x \leq y \wedge y \leq z$
Answer: This is not true in $\langle \mathbb{Q}, \leq \rangle$.

The variable y may be always set to some value less than x or greater than z.

Conjunctive Positive Definitions.

Definition

A constraint language Γ expresses an n-ary relation R if there exists a conjunctive positive formula $\phi(x_1, \ldots, x_n)$ over Σ such that $t \in R$ iff t satisfies ϕ.

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Replace each occurrence of $(x_1 \geq x_2)$ *in the instance of* **QCSP**$(\{x_1 \geq x_2\})$ *with its definition in* $\{(x_1 \geq x_2 \vee x_2 \geq x_3)\}$.

Scheme of Complete Classification Theorems

A sometimes challenging question.

Given: a set of relations Γ over domain $\mathcal{D}$ and of some signature Σ.
Final form: a complexity characterization of $\textbf{QCSP}(\Gamma')$ for each (finite) Γ' such that $\Gamma' \subseteq \Gamma$.

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Examples of complete characterization theorems about CSP:

- If Γ — all relations over two element domain, then **CSP**(Γ') is either in *P* or is *NP*-complete, (Shaefer, STOC 1978).
- If Γ — all temporal relations (FO-definable over $\langle \mathbb{Q}, < \rangle$), then **CSP**$(\Gamma')$ is either in *P* or is *NP*-complete, (Bodirsky and Kára, STOC 2008).

Quantified Characterization of Equality Languages

Theorem

Γ — all relations FO-definable using $=$ only over some countable domain. Then for each $\Gamma' \subseteq \Gamma$ holds exactly one of the following.

- *Negative languages. Relations of Γ' may be defined as:*

$$\bigwedge_{i=1}^{n}(x_i = y_i) \wedge \bigwedge_{i=1}^{k}(z_1^i \neq v_1^i \vee \ldots \vee z_{k_i}^i \neq v_{k_i}^i),$$

  and then **QCSP**(Γ') *is in LOGSPACE.*

Theorem of Bodirsky and Chen, LICS 2007

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- *Positive languages. Relations of Γ' may be defined as:*
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- *In any other case,* **QCSP**(Γ') *is PSPACE-complete.*

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Our classification concerns relations definable using:

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- $R_2(x_1, x_2, x_3) \equiv ((x_1 \leq x_2 \vee x_1 \leq x_3) \wedge x_2 \leq x_1 \wedge x_3 \leq x_1)$ **QCSP**($\{R_2\}$) is P-complete

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Although we don't have negation ($\neg$), the classification is not trivial.

Positive Temporal Languages

Complete Quantified Classification Theorem

Theorem

Γ — *all relations positive definable over* $\langle \mathbb{Q}, \leq \rangle$.
For each $\Gamma' \subseteq \Gamma$ *holds exactly one of the following.*

1. *Relations of* Γ' *are definable as:* $\bigwedge_{i=1}^{n} x_i = y_i$ *and then* **QCSP**(Γ') *is in LOGSPACE.*

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3. *Definable by: $\bigwedge_{i=1}^{n} (x_{i_1} \leq x_{i_2} \vee \ldots \vee x_{i_1} \leq x_{i_k})$ and then* **QCSP**(Γ') *is P-complete.*

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4. $\bigwedge_{i=1}^{n} (x_{i_2} \leq x_{i_1} \vee \ldots \vee x_{i_k} \leq x_{i_1})$ *— P-complete.*

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5. *Positive equality languages — NP-complete.*
6. *The problem* **QCSP**(Γ') *is PSPACE-complete.*

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- Constraints definable as $\bigwedge_{i=1}^{n}(x_{i_1} \leq x_{i_2} \vee \ldots \vee x_{i_1} \leq x_{i_k})$ or $\bigwedge_{i=1}^{n}(x_{i_2} \leq x_{i_1} \vee \ldots \vee x_{i_k} \leq x_{i_1})$ are closely related to AND/OR precedence constraints used in scheduling.

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- The theorem looks quite nice, doesn't it?

Polymorphisms of Temporal Relations

Definition of a polymorphism

R — temporal relation of arity n.

Function $f : \mathbb{Q}^m \to \mathbb{Q}$ is polymorphism of R if:

$$\begin{bmatrix} (a_1^1, & \dots & , a_n^1) & \in R \\ \vdots & & \vdots & \\ (a_1^m & \dots & , a_n^m) & \in R \end{bmatrix} \textit{for all tuples } a^1, \dots, a^m \in R$$

then

$$(f(a_1^1, \dots, a_1^m), \quad \dots \quad , f(a_n^1, \dots, a_n^m)) \in R$$

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Example

$R \equiv (x_1 \leq x_2)$ is closed under $f(x) = x$ but
it is not closed under $g(x) = -x$
$(x_1 = 3 \leq x_2 = 4) \in R$ but $(x_1 = -3 > x_2 = -4) \notin R$

Why do we like surjective polymorphisms?
Surjective polymorphisms vs. quantified constraints

Surjective Preservation Theorem

Let Γ_1, Γ_2 be positive temporal languages. Then $sPol(\Gamma_2) \subseteq sPol(\Gamma_1)$ iff Γ_2 expresses each relation in Γ_1.

- $sPol(\Gamma)$ - surjective polymorphisms of Γ
- f preserves Γ if f preserves each $R \in \Gamma$

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Surjective preservation theorem holds for all ω-categorical structures. (Bodirsky, Chen, 2007)

That what we really prove in the LPAR paper.

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4. *Definable by: $\bigwedge_{i=1}^{n}(x_{i_2} \leq x_{i_1} \vee \ldots \vee x_{i_k} \leq x_{i_1})$.*
5. *The set Γ is closed under unary surjective polymorphisms only.*

How do we use Surjective preservation theorem.

By Bodirsky-Chen Theorem

All relations of a positive equality language Γ may be defined as:

$$\bigwedge_{i=1}^{n}(x_1^i = y_1^i \vee \ldots \vee x_{k_i}^i = y_{k_i}^i),$$

and then **QCSP**(Γ) is NP-complete. The set **sPol**(Γ) is equal to the set of all unary surjections.

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We prove: If Γ is not definable as in one of cases: 1,2,3,4; then Γ expresses some positive equality language.

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We prove: If Γ is not definable as in one of cases: 1,2,3,4; then Γ expresses some positive equality language.

By surjective preservation theorem, the language Γ *is closed under unary surjections only*.

The rest of the proof.

Quantified Positive Temporal Constraints (CSL 08)

- If Γ closed under unary polymorphisms only, then either NP-c or PSPACE-c.
- Complexity proof of the P-complete case.

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(Bodirsky, Chen, 07)

Complexity characterizations of LOGSPACE, NLOGSPACE-c, and NP-c cases.

Representing temporal relations using filters.

We define $R(x_1, x_2, x_3)$ *by saying which tuples are not in* R.

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We decide that relation $R \subseteq \mathbb{Q}^3$ will not contain tuples satisfying:

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$R(x_1, x_2, x_3)$ may be defined as $\neg(x_2 < x_3) \wedge \neg(x_3 < x_1 < x_2)$
or equivalently as $(x_2 \geq x_3) \wedge (x_3 \geq x_1 \vee x_1 \geq x_2)$.

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- From the shape of f it is easily seen that R has tuples satisfying:
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- In many proofs we use such a property to reason about relations expressible by a constraint language $\Gamma \supseteq \{R\}$.

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- We work in the algebraic approach but also use other techniques.

Conclusions and Future Work

- We gave the complete complexity characterization for QCSP over Positive Temporal Languages.
- It is a step towards complete characterization over all Temporal Languages.
- Our theorem generalizes former-known results and seems to have some applications.
- We work in the algebraic approach but also use other techniques.
- The natural continuation of our research would be a similar classification over all Temporal Languages.

The Last Slide

Thank you for your attention.