Object Reorientation with Finger Gaiting

Li Han and Jeffrey C. Trinkle *
lihan, trink@cs.tamu.edu
Dept. of Computer Science
Texas A&M University

Abstract

Coordinated object manipulation is a fundamental problem in the study of multi-fingered robotic hands. The conventional study has been focused on deriving control algorithms for finger joints to implement a specified object trajectory and recently more effort has been put into the study of automatic object trajectory planning. This paper discusses how to plan large-scale object reorientation with fingers of limited workspace. The relevant theories of contact kinematics, nonholonomic motion planning, grasp stability and finger gaiting are incorporated to develop a general framework of object reorientation with rolling contacts and finger gaiting. Our approach is illustrated by an example of reorienting a sphere with three hemi-spherical fingertips. The simulation results are presented.

1 Introduction

Given an object grasped by a robotic hand, the main tasks of coordinated object manipulation include: (a) manipulation planning: generate the finger joint trajectories, so that through the effects of contact constraints, the object can be transferred to a goal configuration; (b) control implementation: derive control algorithm to implement planned trajectories. One special case of object manipulation is object reorientation, for which only the orientation of the object need to be changed to obtain the object goal configuration. The conventional study on object manipulation has been focused on control implementation of a known trajectory for the object. Many algorithms [10, 4, 5] have been proposed and reported achieving good simulation and experimental results. On the other hand, there is relatively few work[17, 3, 1] addressing the manipulation planning problem and the general problem remains open.

One difficulty in the manipulation planning problem is the workspace limits of the robot fingers, arising from the mechanical and electrical limits of the finger joints and the constraints on the contactable regions on the fingertips(e.g., only the region covered by tactile sensors). If some finger reaches its limit(call such finger a *limiting finger*) while the hand manipulates an object, then the limiting finger need to be removed from the grasp and a new "desirable" grasp consisted of fingers all lying within their workspaces need to be formed before the object can be further moved to achieve large-scale motion. Currently a force-closure grasp (FC grasp)[14, 16] is considered to be desirable, since FC grasp can move the object in any direction which makes it possible to decouple the motion planning from force planning. It is possible to relax the force closure condition and require that the grasp can generate force that can move the object in planned directions.

This paper discusses how to achieve large-scale object reorientation with fingers of limited workspace. The relevant theories of contact kinematics[12], nonholonomic motion planning[9, 13], grasp stability [14, 16], and finger gaiting [7] are naturally incorporated to develop a general framework of object reorientation with *rolling contacts* and *finger gaiting.* Our approach is applied to the problem of re-orienting a sphere with three hemi-spherical fingertips. The simulation results are presented.

Remarks:(a) The restriction to object reorientation is to simplify the presentation and our planning methodology can be applied to general object manipulation planning problem.

(b) Also for simplifying the presentation, this paper only considers the desired trajectories of the fingertips. The corresponding finger joint trajectories can be obtained from finger Jacobians.

2 Large-Scale Object Reorientation

We make the following assumptions:each body in the hand-object system is rigid; the geometry of each body is known; only the fingertips can contact the object and contacts are point contacts; each finger has six degrees of freedom and its workspace is known. The workspace mentioned here is the dextrous workspace of the fingers, i.e. any point within

*Research supported in part by NSF grant no. IRI-9304734, Texas Advanced Research Program grant no. 999903-078 and Texas Advanced Technology Program, grant no. IRI-9619850

the workspace can be reached by the fingertip with any orientation.

As discussed in the previous section, some fingers with limited workspace may reach their limits while manipulating the object to achieve a large-scale reorientation. Then a new grasp need to be formed before further moving the object. If all fingers need to be used to form a grasp during the grasp adjustment procedure, then only rolling or sliding can be used to get the fingers relocated. Generating a rolling or a sliding path usually involves detailed computation of the geometry of the object and fingers. Also the requirement of maintaining the grasp to be force closure complicates the problem. It is more convenient if the fingers that need to be moved is not needed to form a grasp and thus can be lifted up from the object and move toward its new location. For this, other fingers need to form a FC grasp.

Our general methodology to implement large-scale object reorientation tasks is as the following: if the initial grasp uses all fingers, use rolling to get a FC grasp with at least 1 finger not used to form the grasp. Then rotate the object until some finger reaches its limit. Use finger gaiting to get a new FC grasp composed of fingers all lying within their workspace limits and move the object again. Repeat the above procedure if it is needed.

Sliding is not used in our current planner since control of sliding is very tricky. However, note that sliding can usually simplify the motion planning of the contacts.

2.1 Finger Gaiting

Finger gaiting is defined as a periodic movements of fingers that can form a new grasp consisted of fingers lying within their workspace limits. In the following, the fingers used in a grasp will be called as *grasping fingers*, and the others as *free fingers*. Considering the fact that several research robotic hands are composed of three fingers, we will study three-finger gaitings in more detail. The contact model for three-finger grasp is *hard-finger* contact and the model for two-finger grasp is *soft-finger* contact[13].

To implement finger gaiting with three fingers, two fingers need to form a FC grasp. Then a necessary condition for using gaiting with three fingers is that at least one of the grasping fingers can form a force closure grasp with other two fingers of the hand.

Theory 1 *For a hand with three fingers, suppose two fingers f_i and f_j form a FC grasp. A necessary condition for using a finger gaiting to form a new grasp is that at least one of the grasping fingers, f_i and f_j, can form a FC grasp with the third finger f_k.*

Clearly if neither of the grasping fingers can form a FC grasp with the third finger, then none of them can be lifted and rolling must be used to relocate the finger(s).

Assuming the above necessary condition is satisfied, we identify two finger gaiting primitives: finger rewind and finger substitution.

The three-finger gaiting proposed in paper [7] relocates the limiting fingers back to their workspace limits. The scenario is as the following: suppose fingers f_1 and f_2 form a grasp but reach their limits. Suppose f_3 can form a grasp with f_1, then f_2 will be relocated back to a new position within its workspace which also forms a FC grasp with f_3, then f_1 can be relocated back to its workspace while simultaneously forms a FC grasp with f_2. Then fingers f_1 and f_2 form a new valid FC grasp and the object will be moved again. We call such a finger gaiting as *finger rewind* since the basic procedure is to rewind the limiting fingers back to their workspace.

Another useful finger gaiting primitive is *finger substitution*, which works as the following:

Assume f_1 and f_2 form a FC grasp. Suppose only f_1 reaches its limit and f_2 is in a position that can also form a FC grasp with f_3. Then use f_3 to form a FC grasp with f_2. If the new location of f_3 is not at the boundary of its workspace, then the grasp of f_2 and f_3 can be used to further move the object and the limiting finger f_1 can be lifted up and becomes free finger. In this case, f_1 is substituted by f_3 and there will be no need to rewind f_1 back to form a grasp with f_2.

Finger substitution is an easy way to remove the limiting finger(s) from the grasp.

For a general object reorientation problem, a sequence of finger gaiting need to be planned to achieve a new grasp, which usually involves detailed analysis of the workspace of the fingers and the geometry of the object as well as the fingers to determine the existence of grasps and the connectivity between the grasps. As an example, the results of a sphere manipulated by three spherical fingertips will be given in next section.

3 One Example: Reorientation of A Sphere

This section applies our general strategy for large-scale object orientation to the problem of re-orienting a sphere with three hemi-spherical fingertips: first use rolling to change an three-finger FC grasp to two-finger FC grasp; then use finger gaits to achieve large amount motion of the sphere. The simulation results will be given in next section.

3.1 FC Grasps of a Sphere

This section gives several results of FC grasps of spheres. Due to the limited paper space, we don't give proofs here and interested readers please refer to our technique report [6].

Denote a sphere with radius r and center o as $sphere(o, r)$. An orthogonal coordinate chart for the sphere can be obtained by using spherical coordinates (u, v):

$$f(u, v) = [rcos(u)cos(v), -rcos(u)sin(v), rsin(u)] \quad (1)$$

with $U = \{(u, v) : -\pi/2 < u < \pi/2, -\pi < v < \pi\}$. A sphere can be covered by two such coordinate charts.

The 'longitudes' and 'latitudes' in such a coordinate system are curves with constant v and u respectively. The curve $u = 0$ is the 'equator'.

Suppose $p(u, v)$ represents a point p on the sphere with local coordinates(u, v). $-p$ is the opposite point of p. Note for point $p(u, v)$, $-p$ has coordinate$(-u, v - \pi)$ if $v > 0$ or $(-u, v + \pi)$ if $v < 0$.

(p_1, p_2) represents the vector from p_1 to p_2. $d(p_1, p_2)$ is the the Euclidean length of the vector (p_1, p_2).$GC(p_1, p_2)$ represents the great circle passing p_1 and p_2. When p_1 and p_2 are not identical or opposite to each other , the great circle is unique, otherwise there are infinitely many. In our case, it will become clear that any great circle passing identical/opposite points can be used for our purpose. Therefore $GC(p_1, p_2)$ is always determined in our context.

$Cone(p_1, p_2, \alpha)$ denotes a cone that originates at point p_1, has (p_1, p_2) as the axis, and has half angle α. The opposite cone of $Cone(p_1, p_2, \alpha)$ is $Cone(p_1, -p_2, \alpha)$. Assume the coefficient of friction between the ball and fingers are $\mu = \tan(\theta)$. The FC region of a point p on the $sphere(o, r)$ is the intersection of the sphere with the $cone(p, o, \theta)$

Due to the symmetric geometry of the sphere, we have:

Fact 1
The intersection of $sphere(o, r)$ with $cone(p, o, \theta)$ is the same as the intersection of the $sphere(o, r)$ with $cone(o, -p, 2\theta)$.

.

Fact 2 *The maximum independent region of FC grasps on the sphere are the intersections of the sphere with two opposite cones, each with half angle θ.*

.

From two theories [15, 16] developed by previous researchers, the force-closure conditions on the sphere are :

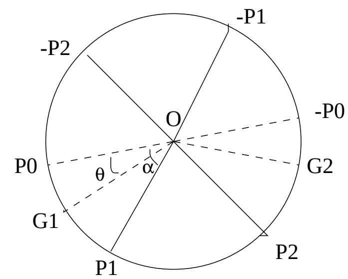

Figure 1: Shortest Path for 2 pts to form FC grasp

Fact 3 *Two soft-finger contacts, p_1 and p_2, on a sphere with radius r form a force-closure grasp if and only if $d(p_1, p_2) \geq 2rcos(\theta)$*

Fact 4 *A sufficient condition for three hard-finger contacts on a sphere to form a force-closure grasp is that the triangles formed by the three contact points on the sphere is acute and the distance from the center of the sphere to the triangle is less than $r\sin(\theta)$.*

Note that a result of differential geometry[2] indicates that the great circles are the only geodesics of a sphere and realize the distance between any two points lying on the same semi-circle. Refer to figure(1), suppose two points P_1 and P_2 don't form a FC grasp . Then the shortest path for them to form a FC grasp is to move along the great circle $GC(P_1, P_2)$ toward each other's opposite points with angle $\alpha - \theta$, i.e. move P_1 and P_2 to G_1 and G_2, where α is the half angle between (o, P_1) and $(o, -P_2)$, which is also the the half angle between $(o, -P_1)$ and (o, P_2). Also note that moving P_1 and P_2 with angle α will make them reach opposite points, p_0 and $-p_0$.

3.2 Reorientate a Sphere with Rolling Contacts and Finger Gaits

A workspace of a robot finger is determined by its joint limits, which are caused by mechanical and electrical constraints of the parts used to build the joints. The cross product of all finger workspaces is the workspace of the hand. When there exist obstacles in the hand workspace, a hand may be separate from/ contact/collide with the obstacles. The set of all configurations of the hand that are separate from/contact/collide with the obstacles forms the free/contact/collision space of the robot hand. Clearly, only the free space and contact space may be used to generate valid hand trajectory. Note that for a given hand, the partition of its configuration space into free/contact/collision space is determined by the obstacle geometries and configurations.

Besides workspace limits of the fingers, there also exist limits for the maximum rotation of fingertips.

Recall that the first step of our manipulation strategy is to roll the fingers to form a new FC grasp with at least 1 finger free. While only the contact space is useful for the rolling movement, it is not straight forward to find a path connecting two points in the contact space with fingers of limited rotation capability. The following example of rolling a sphere(finger) on a sphere(object) exhibits the complexity of the problem.

It was proven in paper[8] that the rolling constraints between two spheres with different radius are completely nonholonomic. Paper[8] also presents an geometric algorithm to roll a sphere on a plane to reach any contact configurations. As for rolling a sphere on a sphere, first note the following result:

Theory 2 *For two spheres rolling on each other, the trajectory of contact points of one sphere is a great circle if and only if the contacting trajectory of the other sphere is a great circle.*

Proof: First derive the surface parameters of the sphere. Note that the spherical charts mentioned in the previous section determines a Guass frame for a point on the sphere:

$$g_{oc} = \left[\begin{array}{ccc|c} C_{u_o} & 0 & S_{u_o} & \rho_o S_{u_o} \\ -S_{u_o}S_{v_o} & C_{v_o} & C_{u_o}S_{v_o} & \rho_o C_{u_o}S_{v_o} \\ -S_{u_o}C_{v_o} & -S_{v_o} & C_{u_o}C_{v_o} & \rho_o C_{u_o}C_{v_o} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad (2)$$

$$f_2: \quad U_2 \subset \Re^2 \rightarrow \Re^3; \quad (u_o, v_o) \mapsto f_2(u_o, v_o)$$

$$U_2 = \left\{(u_o, v_o) \middle| -\frac{\pi}{2} < u_o < \frac{\pi}{2}, \quad -\pi < v_o < \pi\right\}$$

Two charts are needed to cover a sphere and the Guass frame introduced by the other chart has similar form.

The metric and curvature tensors and torsion form in these two charts are:

$$K_o = \begin{bmatrix} \frac{1}{\rho_o} & 0 \\ 0 & \frac{1}{\rho_o} \end{bmatrix}, \quad M_o = \begin{bmatrix} \rho_o & 0 \\ 0 & \rho_o C(u_o) \end{bmatrix},$$

$$T_o = \begin{bmatrix} 0 & -\frac{1}{\rho_o}t(u_o) \end{bmatrix}$$

where S, C, t stand for mathematic function sin, cos and tan, respectively.

Montana's kinematic equation of rolling contacts[12] relates the rate of change of contact coordinates to the contact velocity:

$$\begin{cases} \dot{\alpha}_f = M_f^{-1}(K_f + \tilde{K}_o)^{-1} \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} \\ \dot{\alpha}_o = M_o^{-1}R_\psi(K_f + \tilde{K}_o)^{-1} \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} \\ \dot{\psi} = T_f M_f \dot{\alpha}_f + T_o M_o \dot{\alpha}_o \end{cases} \quad (3)$$

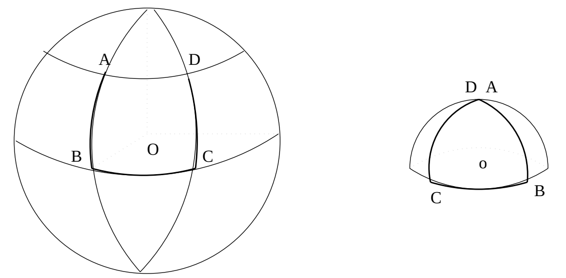

Figure 2: Rolling Path

where

$$R_\psi = \begin{bmatrix} cos\psi & -sin\psi \\ -sin\psi & -cos\psi \end{bmatrix}$$

and $\tilde{K}_o = R_\psi K_o R_\psi$ is the curvature of O seen in the local frame of F.

The above equation can be further developed to get the relation between the movement of the contact coordinates of the finger to the movement of the contact coordinates of the object:

$$\dot{\alpha}_f = (M_f^{-1}R_\psi M_o)\dot{\alpha}_o \quad (4)$$

To prove the theory, note that any great circle can be made to be the equator of some coordinate chart. Therefore, we just need to prove for two spheres initially contacting at their equators, rolling on one equator will also roll on the other equator. Note a point $p(u, v)$ is on the equator if and only if $u = 0$.

Without lose of the generality, we can assume the initial contact configuration to be $(0, 0, 0, 0, 0)$. Suppose the radius for two balls are R and r, and the contact coordinate of ball r changes to $(0, \Delta V_r)$.

Then from equation(4), it can be computed that the contact coordinate of ball R changes to $(0, -\frac{r}{R}\Delta V_r)$, which means the contact point remains on the equator. QED.

Based on theory 2 and motivated by the geometric algorithm of rolling a sphere on a plane[8], we propose an algorithm to roll a sphere on a sphere to reach a contact point which is beyond its rotation limit by using 'longitudes' and 'equator', which is shown in figure(2) and briefly explained below.

Figure(2) shows a path for rolling a small sphere, with radius r and rotation limit γ, on a big sphere with radius R, The contact point on the big sphere changes from A to D while the small sphere has 'north pole' as contact points in both ends. The corresponding contact points on both sphere are labeled with same alphabets. The path consists of rolling from A to B, B to C and then C to D. The arc-lengths of AB and CD are $r\pi/2$, and the arc-length of BC is γr.

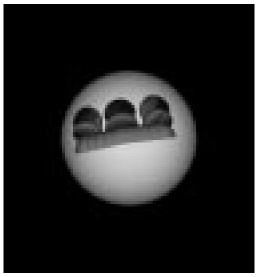

Figure 3: Simulated Rolling Path

The small sphere reaches its rotation limit at point D. However, it can rotate itself back to its operating region and use the above path again to further roll on the big sphere. Figure(3) shows a simulated rolling path which uses the local path twice to achieve large-scale contact adjustments. Note the 'longitude' and 'equator' here are not restricted for one specific coordinate system of the sphere. Any two great circles that are perpendicular to each other can be made into a longitude and an equator, through some coordinate transformation[6].

If the initial grasp is only 3-finger FC but not 2-finger FC, the rolling path need be used to adjust the contact points to get a 2-finger FC grasp. Theoretically, a 3-finger FC grasp may be changed to a 2-finger FC grasp by choosing the pair of contacts with largest straight line distance, moving them along the great circle toward each other and moving the 3rd finger 'accordingly' to maintain the grasp to be FC. However, it requires detailed computation to maintain the 3 fingers to form a FC grasp before two fingers reach antepodal positions. Borrow the concept [11] of task-oriented optimal grasp, the initial grasps which are or close to 2-finger FC grasp are preferred for dextrous manipulation tasks with three fingers.

4 Simulation Results

Figure(4) shows a simulation result of a sphere re-orientated by three spherical fingertips with finger substitution. The figure frames go across instead of down and are numbered starting from 1.

The radii of the object and fingertips are 0.5 and 0.1, respectively. The coefficient of friction is $\tan(20^o) = 0.364$. The concerned workspaces of the fingers include (a)the contactable areas on the sphere, modeled as the intersection of the $sphere(O, R)$ and a $cone(O, FO^i, 60^o), i = 1, 2, 3$, where FO_is are 120^o apart on the equator; (b) the surrounding free space, modeled as the space between the contactable area and the intersection of a grown $sphere(O, R + \delta R)$ with the $cone(O, FO^i, 60^o), i = 1, 2, 3$, where δR is some positive number.

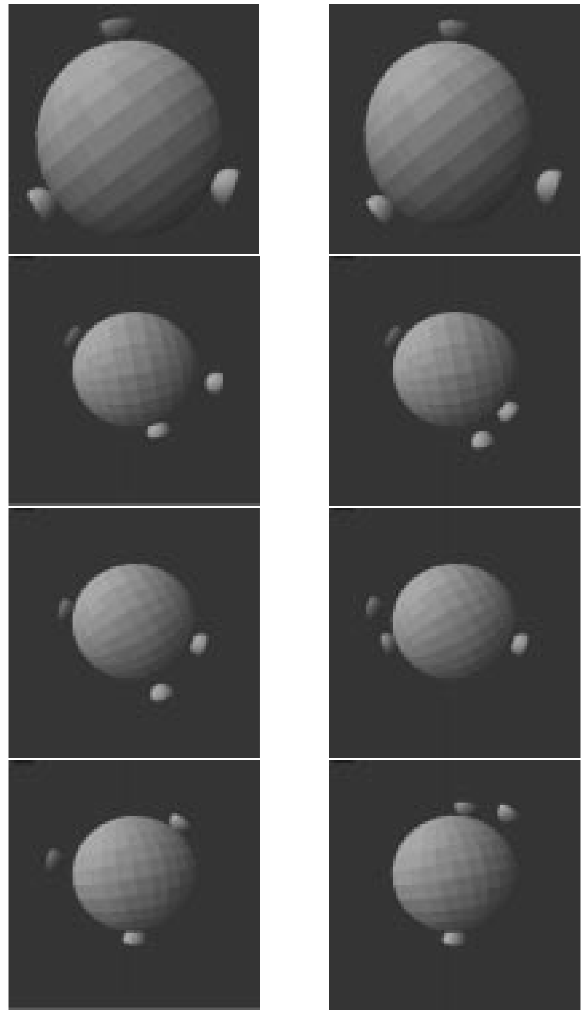

Figure 4: Reorientation with finger substitution

Initially three fingers are 120^o apart on a great circle. Such a grasp is three-finger force closure but none of the pair of the fingers form a 2-finger FC grasp. Therefore none of the finger can be lifted and rolling need to be used to move fingers to form a 2-finger force closure grasp,shown in frame 2. Then a sequence of finger substitution may be used to achieve arbitrary amount of sphere re-orientation. Note finger rewind can achieve same effect.

5 Conclusion

This paper integrates the relevant theory of contact kinematics, nonholonomic motion planning, finger gaiting and grasp stability to develop a general framework for large-scale object reorientation with rolling contacts and finger gaiting, taking into account of the hand workspace limits. Currently we are working on incorporating the finger chains into the system. More simulation and experiments of general object manipulation planning problem are conducted.

[**Acknowledgement**] We would like to thank Prof. Peter Stiller of Texas A&M Univ. Prof. Zexiang Li of Hongkong Univ. of Science and Technology, and Prof. John Canny of UC Berkeley for the valuable discussions on this work. The first author also wants to thank Wookho Son for the help with the rendering tool.

References

[1] A. Bicchi, A. Marigo, and Y. Chitour. Manipulation of polyheral parts by rolling. In *Proc. of IEEE Intl. Conf. on Robotics and Automation*, 1997.

[2] W. Boothby. *An Introduction to Differentiable Manifolds and Riemannian Geometry.* Academic Press, 1975.

[3] M. Cherif and K. Gupta. Planning quasi-static motions for reconfiguration objects with a multi-fingered robotic hand. In *Proc. of IEEE Intl. Conf. on Robotics and Automation*, 1997.

[4] A. Cole, J. Hauser, and S. Sastry. Kinematics and control of a multifingered robot hand with rolling contact. *IEEE Transaction on Automatic Control*, 34(4), 1989.

[5] A. Cole, P. Hsu, and S. Sastry. Kinematics and manipulation of multifingered robot hand with sliding contact. In *Proc. of IEEE Intl. Conf. on Robotics and Automation*, 1989.

[6] L. Han and J.C. Trinkle. Dextrous manipulation with rolling contacts and finger gaitings. Technical report, Computer Science, Texas A & M Univ., 1997.

[7] J.W. Hong, G. Lafferriere, B. Mishra, and X.L. Tang. Fine manipulation with multifinger hand. In *Proc. of IEEE Intl. Conf. on Robotics and Automation*, pages 1568–1573, 1990.

[8] Z. Li and J. Canny. Motion of two rigid bodies with rolling constraint. *IEEE Trans. on R. & A.*, RA2-06:62–72, 1990.

[9] Z. X. Li and J. Canny, editors. *Nonholonomic Motion Planning.* Kluwer Academic Publisher, 1993.

[10] Z.X. Li, P. Hsu, and S. Sastry. On grasping and coordinated manipulation by a multifingered robot hand. *IJRR*, 8:4, 1989.

[11] Z.X. Li and S. Sastry. Task oriented optimal grasping by multifingered robot hand. *IEEE Trans. on Robotics and Automation*, 4(1):32–44, 1988.

[12] D. Montana. The kinematics of contact and grasp. *IJRR*, 7(3), 1988.

[13] R. Murray, Z.X. Li, and S. Sastry. *A Mathematical Introduction to Robotic Manipulation.* CRC Press, 1994.

[14] V. Nguyen. The synthesis of force-closure grasps. Master's thesis, Department of Electrical Engineering and Computer Science, MIT, 1986.

[15] V.-D. Nguyen. Constructing force-closure grasps. *IJRR*, 7(3), 1988.

[16] Jean Ponce and Bernard Faverjon. On computing three-finger force-closure grasps of polygonal objects. *IEEE Trans. on R. & A.*, 11(6), 12/1995.

[17] D. Rus. *Fine Manipulation Planning for Dextrous Manipulation.* PhD thesis, Dept. of Computer Science, Cornell Univ., 1992.