

***16-350***

***Planning Techniques for Robotics***

***Multi-Robot Planning***

*Maxim Likhachev*

*Robotics Institute*

*Carnegie Mellon University*

# Different Categorizations of Multi-Robot Planning

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- Centralized vs. Decentralized
  - **Centralized:** one central control of (planning for) all the robots
  - **Decentralized:** each robot decides/plans what to do on its own

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*Robust to losing some robots in the team*

*Computationally more scalable*

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*Computationally more scalable*

*Challenges with decentralized planning?*

*How to guarantee that the overall team accomplishes its goal?*

# Different Categorizations of Multi-Robot Planning

- Multi-robot Path Planning vs. Multi-robot Cooperative Task Planning
  - **Multi-robot Path Planning:** how to plan paths for  $N$  robots so that they don't collide with each other during execution
  - **Multi-robot Cooperative Task Planning:** how to compute plans for  $N$  robots so that they achieve the overall goal that may require cooperation

# Different Categorizations of Multi-Robot Planning

- Small teams vs. large teams (swarms) of robots
  - **Planning for small teams:** Compute plans for  $N$  (potentially heterogeneous) robots, where  $N$  is typically 2-10
  - **Planning for (control of) swarms of robots:** how to control a swarm of  $N$  (usually homogeneous) robots, where  $N$  is typically 10-1000



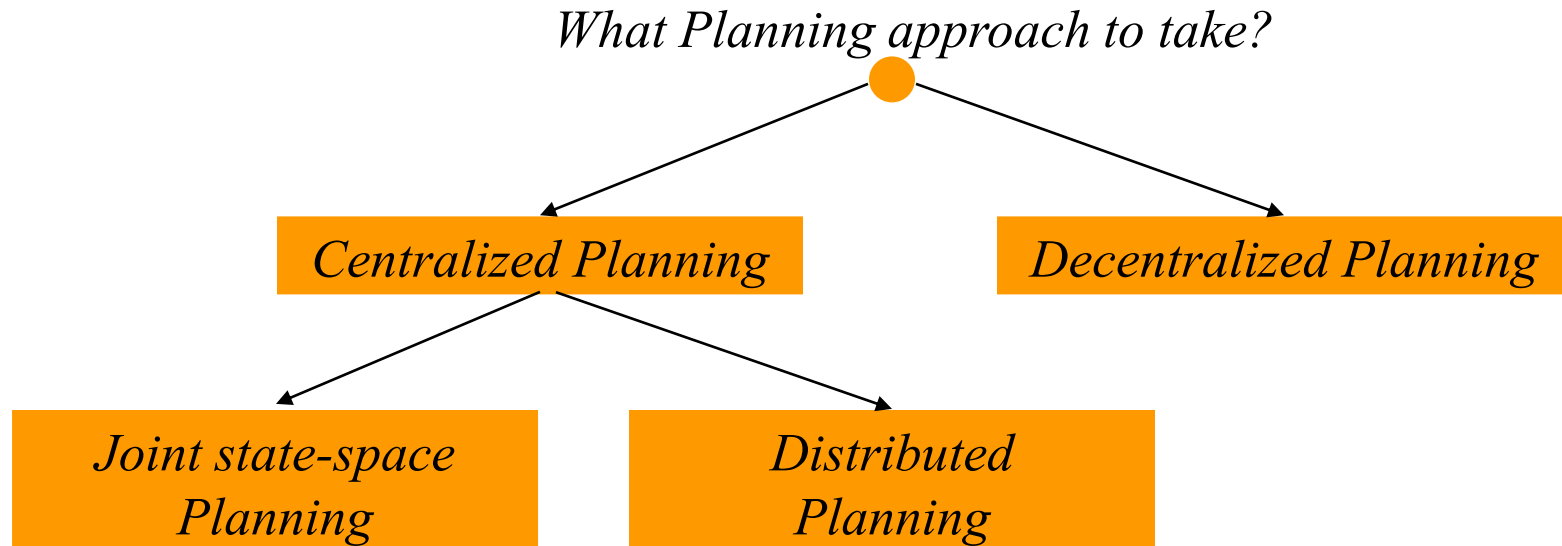
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*Control of swarms is typically decentralized*

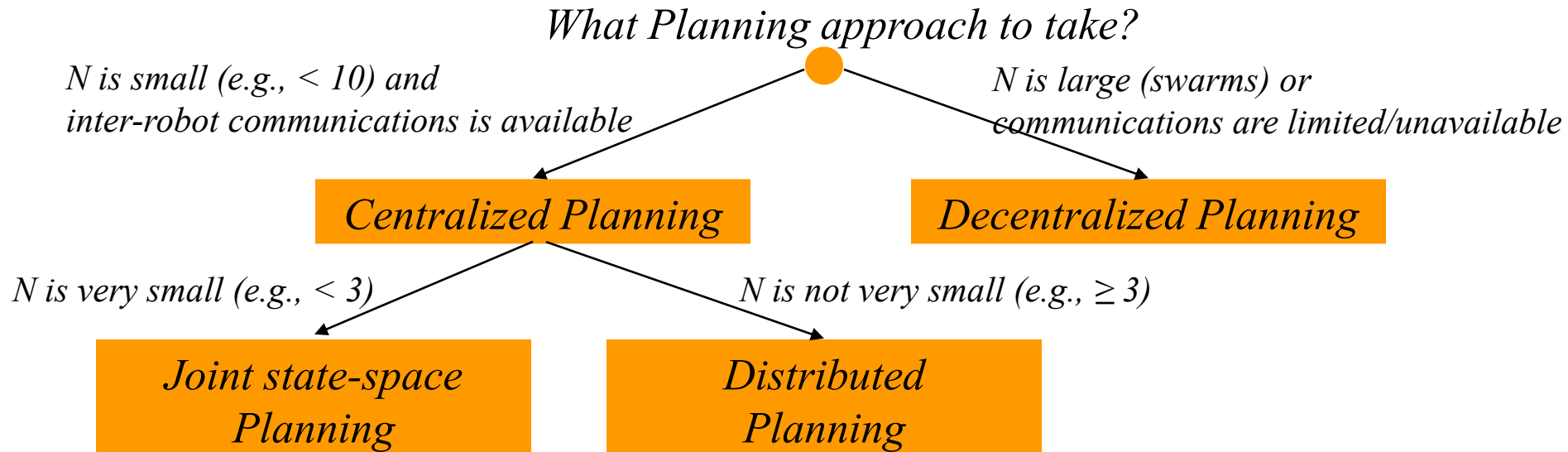
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- Joint state-space vs. distributed planning (within centralized)
  - **Joint state-space planning:** Planning for  $N$  robots in a state-space that represents joint configurations of robots
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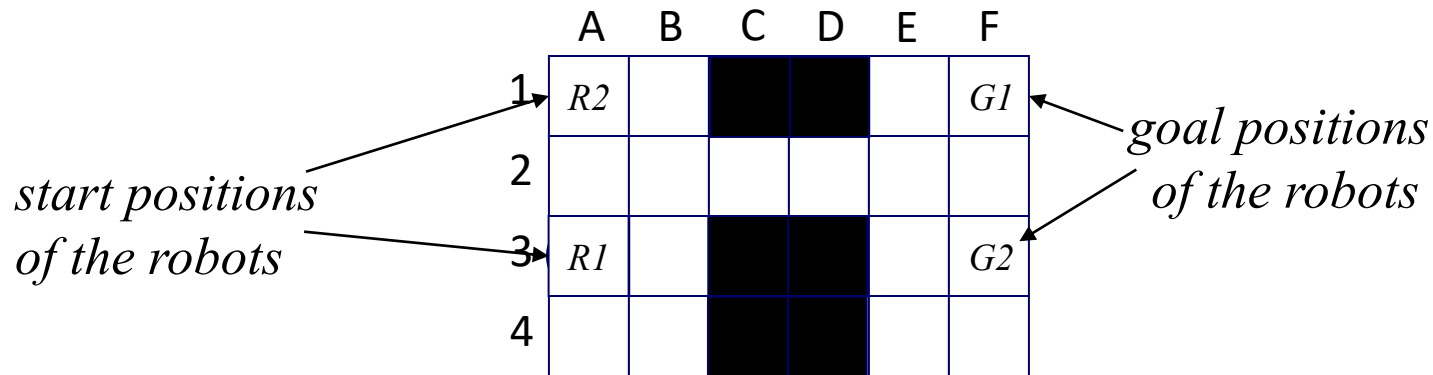
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# Multi-Robot Path Planning

- Path planning for  $N$  robots to get to their goals w/o collisions

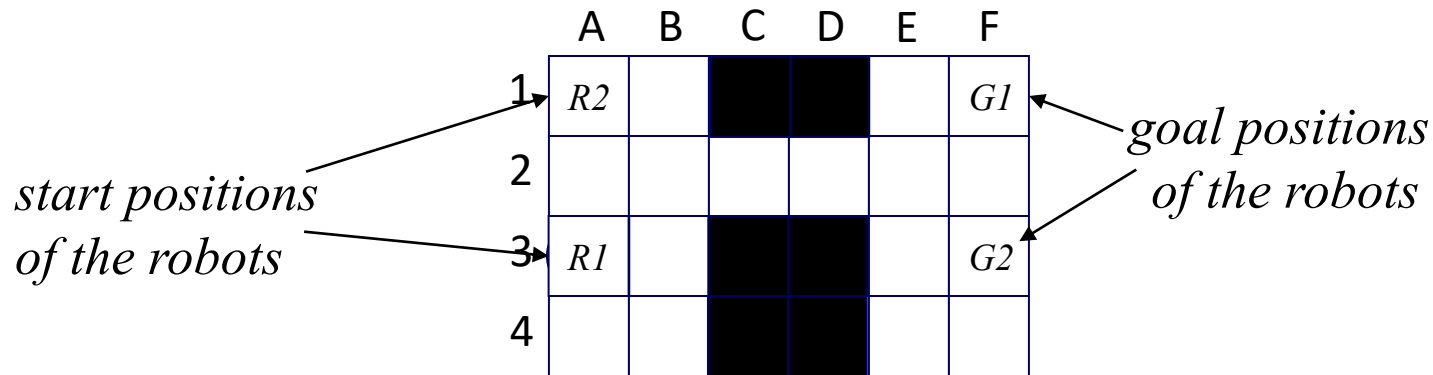
*simple example for two omnidirectional point-size robots*



# Multi-Robot Path Planning

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*simple example for two omnidirectional point-size robots*



*Any examples of this in industry?*

# Multi-Robot Path Planning

- Path planning for  $N$  robots to get to their goals w/o collisions

*Joint state-space planning*

	A	B	C	D	E	F
1	$R2$					$G1$
2						
3	$R1$					$G2$
4						

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*Joint state-space planning*

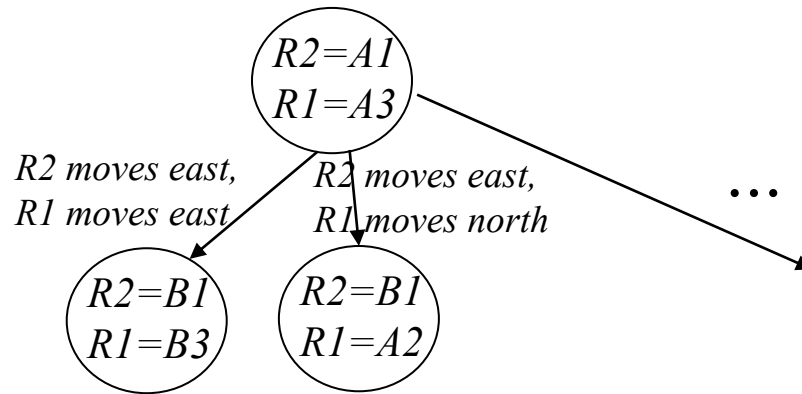
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*The simplest approach: construct and search a graph, where each state encodes positions of all the robots and each action encodes all possible movements*

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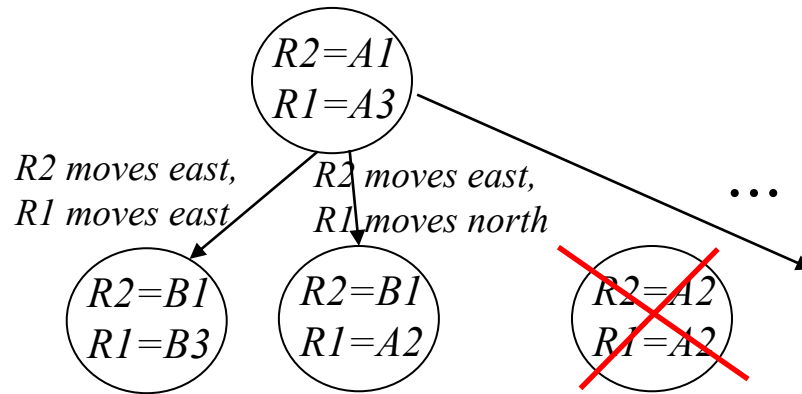
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# Multi-Robot Path Planning

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## *Joint state-space planning*



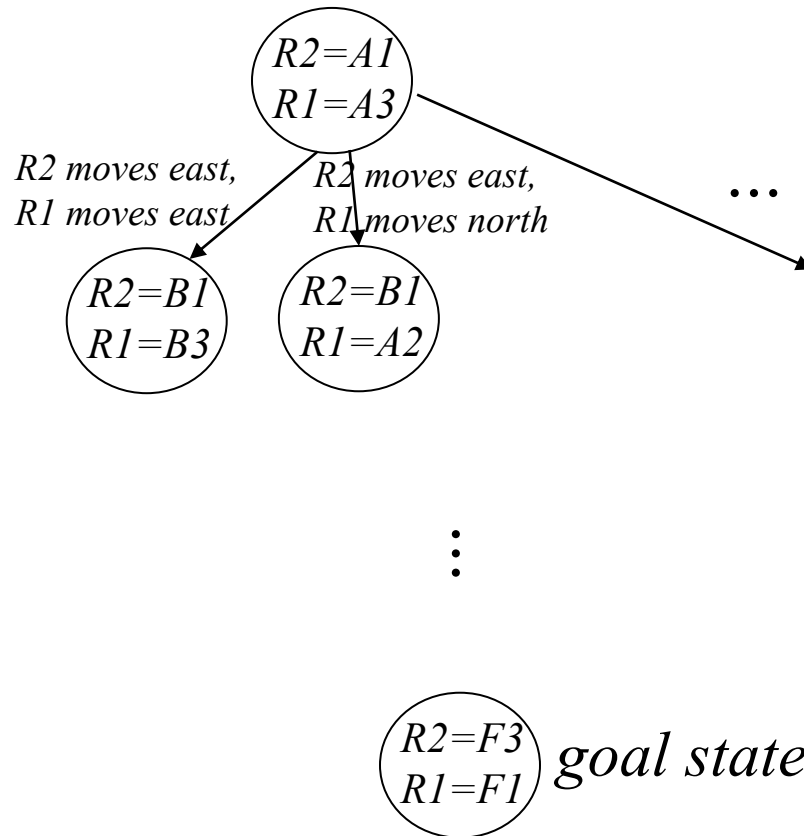
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*Joint state-space planning*



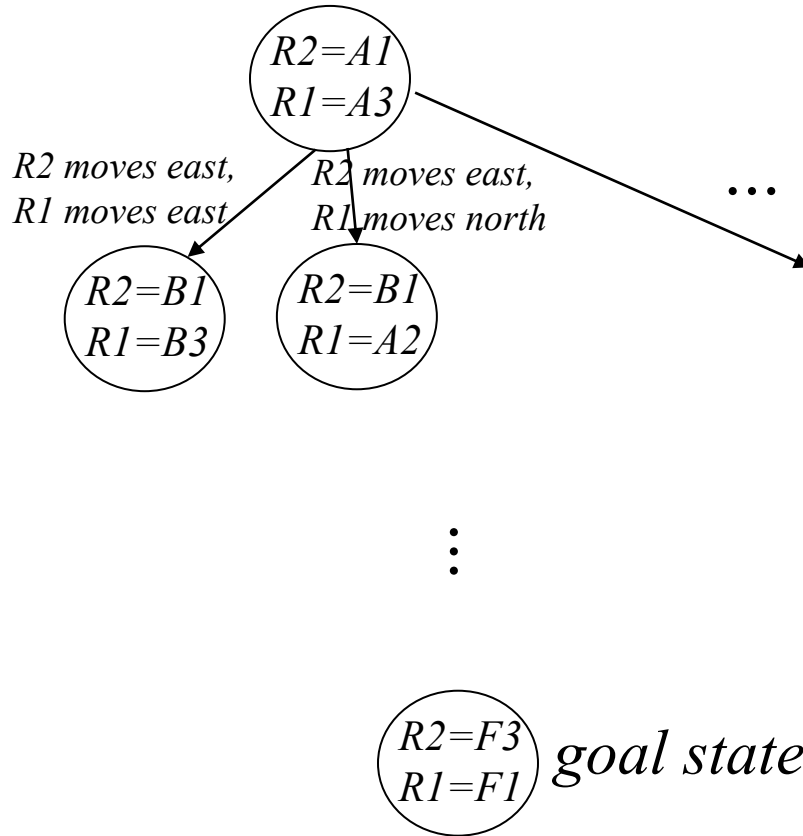
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# Multi-Robot Path Planning

- Path planning for  $N$  robots to get to their goals w/o collisions

Assuming 4-connected grid,  
what is the maximum branching factor  
(how many actions/successors)?

*Joint state-space planning*



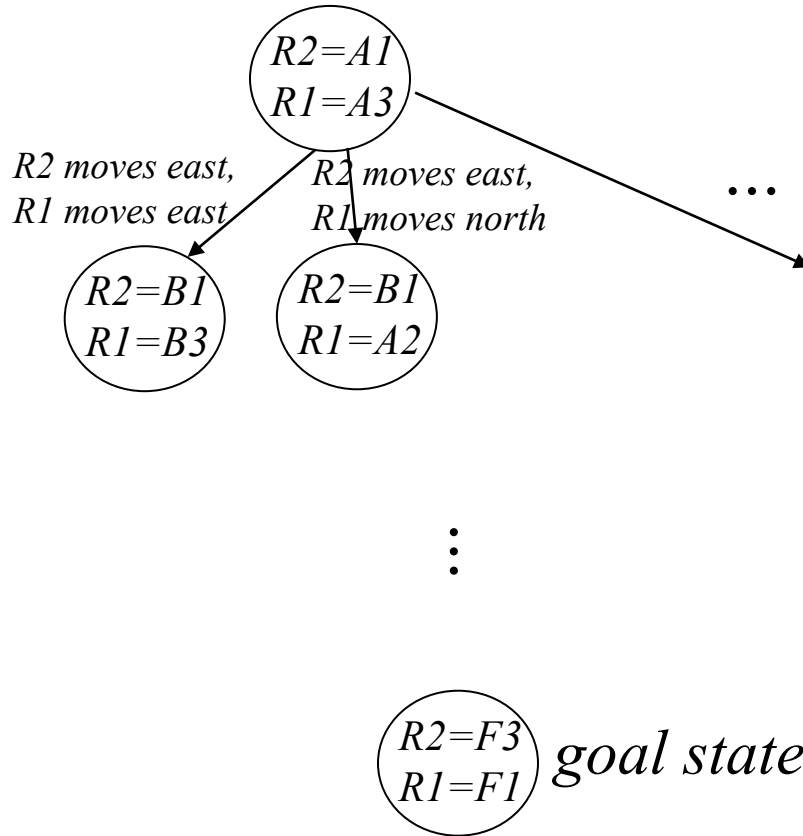
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What is the size of the graph?

*Joint state-space planning*

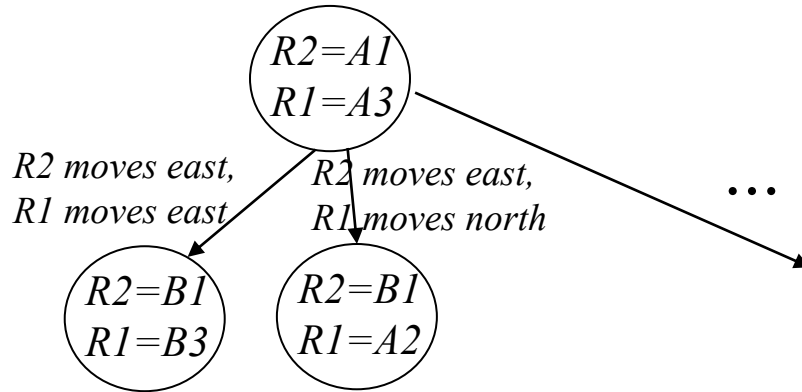


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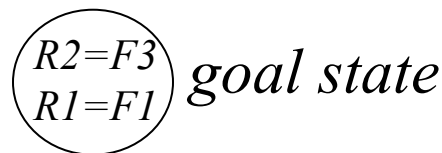


*What is the size of the graph?*

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⋮

*Scalability w.r.t.  $N$  is clearly an issue!*



# Multi-Robot Path Planning

- Path planning for  $N$  robots to get to their goals w/o collisions

*Distributed planning*

	A	B	C	D	E	F
1	$R2$		■	■		$G1$
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*One popular approach: Prioritized Planning*

*For  $i = 1:N$*

*Compute path for robot  $R_i$  that avoids collisions with paths for robots  $R_1..R_{i-1}$*

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*Each planning needs to include time as a dimension!*

*One popular approach: Prioritized Planning*

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*What will be the plan returned by Prioritized Planning?*

*One popular approach: Prioritized Planning*

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# Multi-Robot Path Planning

- Path planning for  $N$  robots to get to their goals w/o collisions

*Distributed planning*

*Is it complete?*

*Is it optimal?*

*What is the complexity of Prioritized Planning?*

*One popular approach: Prioritized Planning*

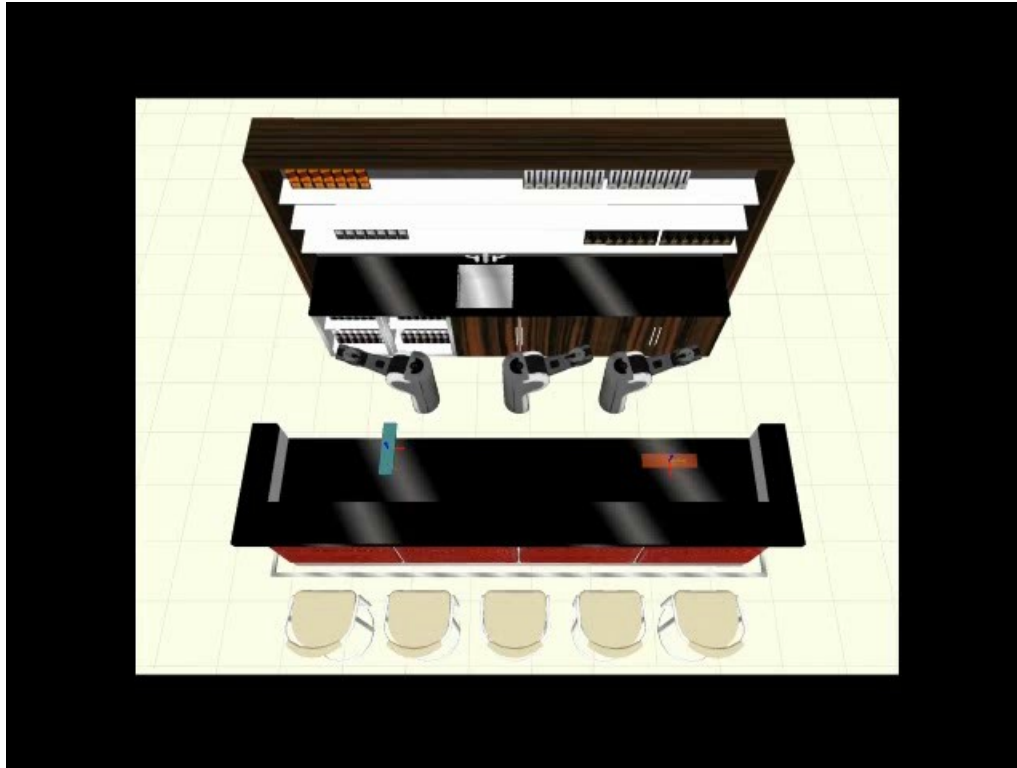
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# Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for  $N$  robotic arms to move an object



*[Cohen et al., '14]  
(performs joint state-space planning)*

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Search-Based Planning Lab

Master of Science  
Robotic Systems Development | Carnegie Mellon University  
The Robotics Institute

## Collaborative Manipulation

Dr. Maxim Likhachev

Ishani Chatterjee	Clare Cui
Andrew Dornbush	Brad Factor
Sung Kyun Kim	Maitreya Naik
Tae-Hyung Kim	Angad Sidhu
Venkatraman Narayanan	Logan Wan

*(planning is distributed: plan on Roman platform first, then on PR2)*

# Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for multi-robot exploration/mapping

*N robots need to explore and build a map of unknown environment*

*One approach: Distributed Greedy Mapping*

*For  $i = 1:N$*

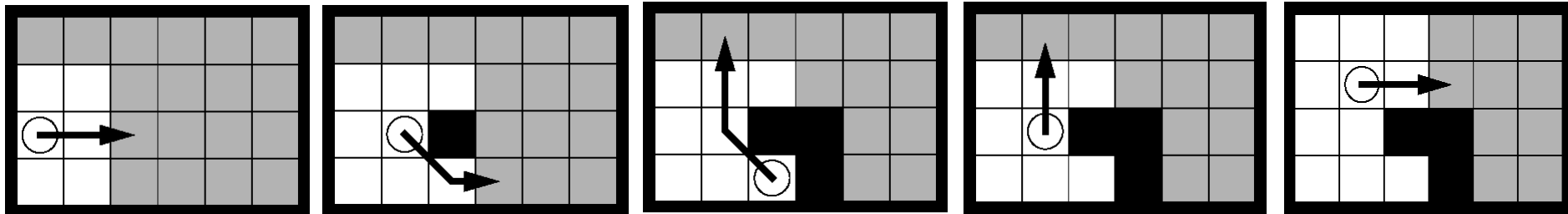
*Compute a path using Greedy Mapping approach for robot  $R_i$  taking into account what paths were computed for  $R_1..R_{i-1}$  (and what cells they would see)*

# Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for multi-robot exploration/mapping

*N robots need to explore and build a map of unknown environment*

- Greedy Mapping for a single robot:
  - always move the robot on a shortest path to the closest unobserved (or unvisited) cell
  - it always achieves a gain in information.
  - thus, it is guaranteed to map the environment that is reachable (assuming all moves are reversible)



*One approach: Distributed Greedy Mapping*

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*[Butzke et al., '11]*

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## A Planning Framework for Persistent, Multi-UAV Coverage with Global Deconfliction

Submitted to the  
12th Conference on Field and Service Robotics

Collaboration between  
Search-Based Planning Lab, CMU (headed by M. Likhachev) and  
Mitsubishi Heavy Industries (MHI)

*[Kusner et al., '19]*

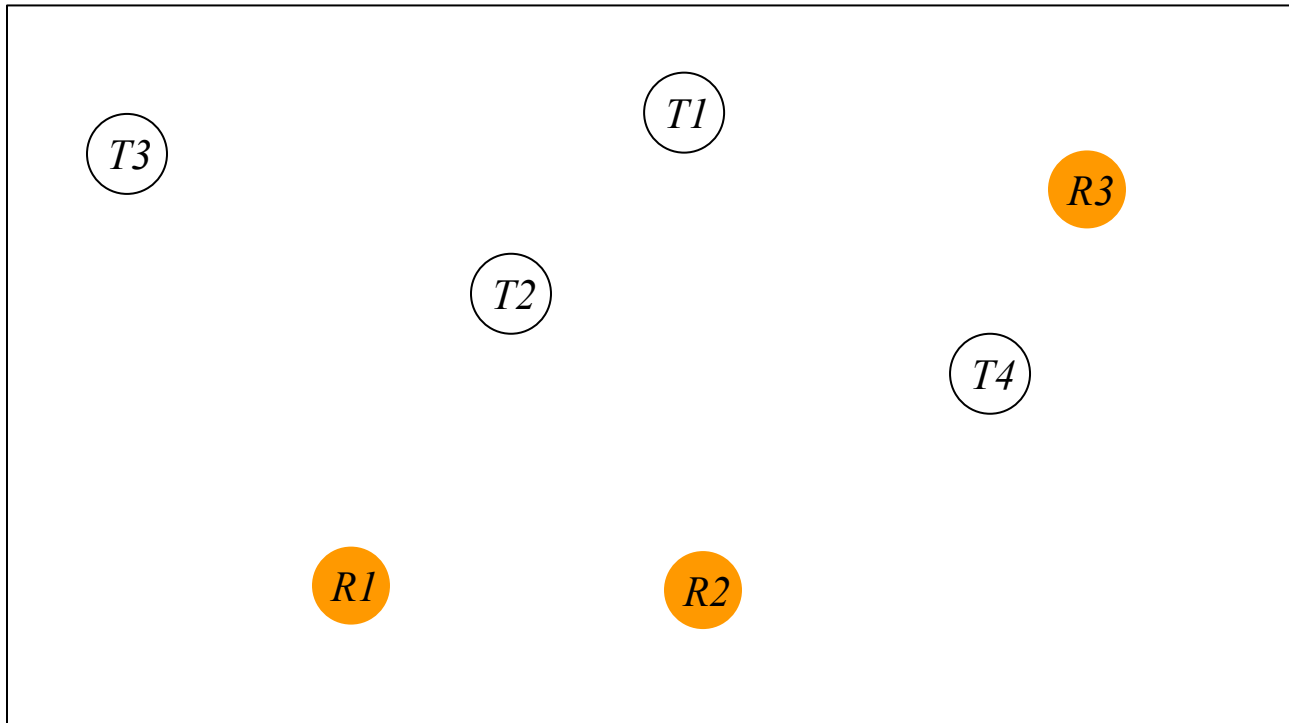
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# Multi-Robot Cooperative Planning/Task Allocation

- Market-based approach (very popular distributed approach)
  - Consider planning the allocation of tasks to  $N$  robots
  - General scheme: *robots auction out their tasks to their teammates with the goal of increasing their own revenue*



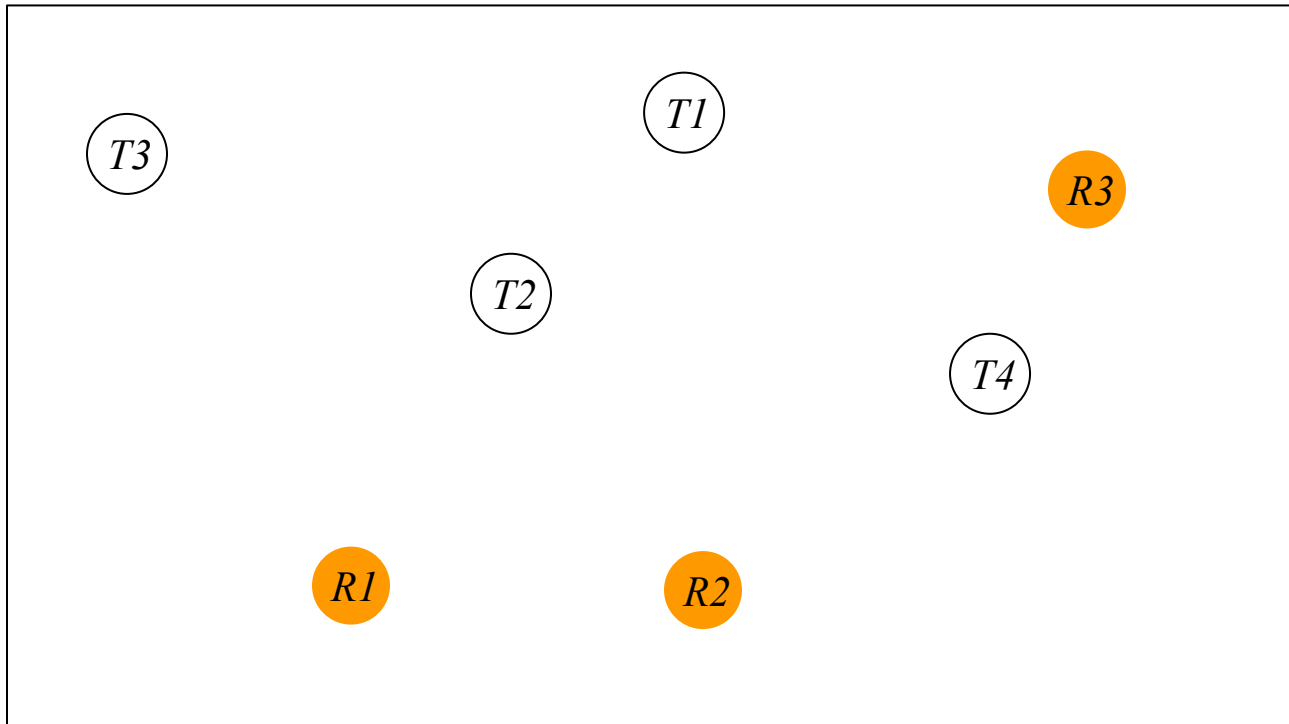


# Market-based Approach

Given  $N$  robots  $R_1 \dots R_N$ ,  $M$  tasks  $T_1 \dots T_M$ , and  $C_i^{R_j}$  – cost of executing task  $i$  by robot  $R_j$  (cost may depend on other tasks executed by this robot)

*Planner needs to decide: Which task gets executed by which robot?*

*Find a plan(mapping)  $\pi^*$ :  $T_i \rightarrow R_j$  such that  $\pi^* = \operatorname{argmin} \Sigma C_i^{\pi(T_i)}$*



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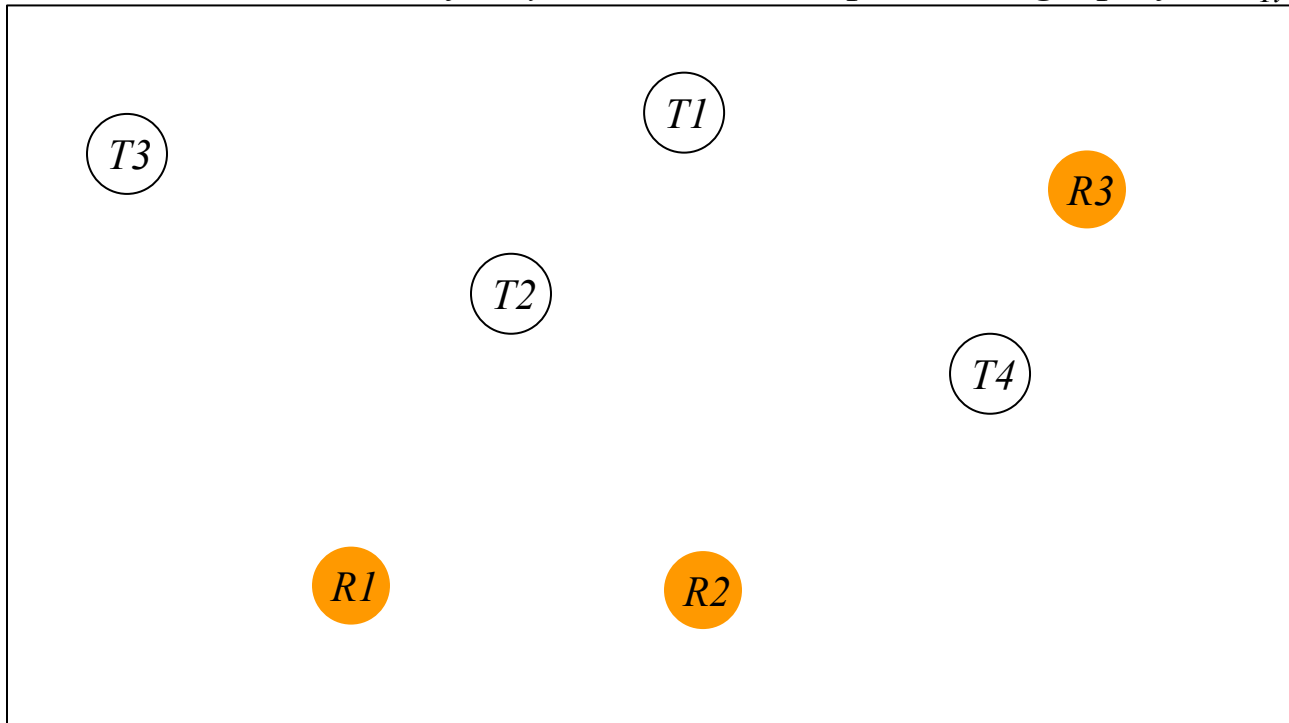
**Iterate over steps 1-4 until convergence or planning time expires**

*Step 1: start with an arbitrary plan  $\pi$*

*Step 2: all robots offer their tasks  $T_i$  at auction at the max. price of  $C_{T_i}^{\pi(T_i)} - \epsilon$*

*Step 3: all robots  $R_j$  bid on the offered tasks  $T_i$  with the bid =  $C_i^{R_j} + \epsilon$*

*Step 4: robots sell to the lowest bidders if they are below max. price and get profit:  $C_{T_i}^{\pi(T_i)} - C_i^{R_j} - \epsilon$*



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*When does it converge in one iteration?*

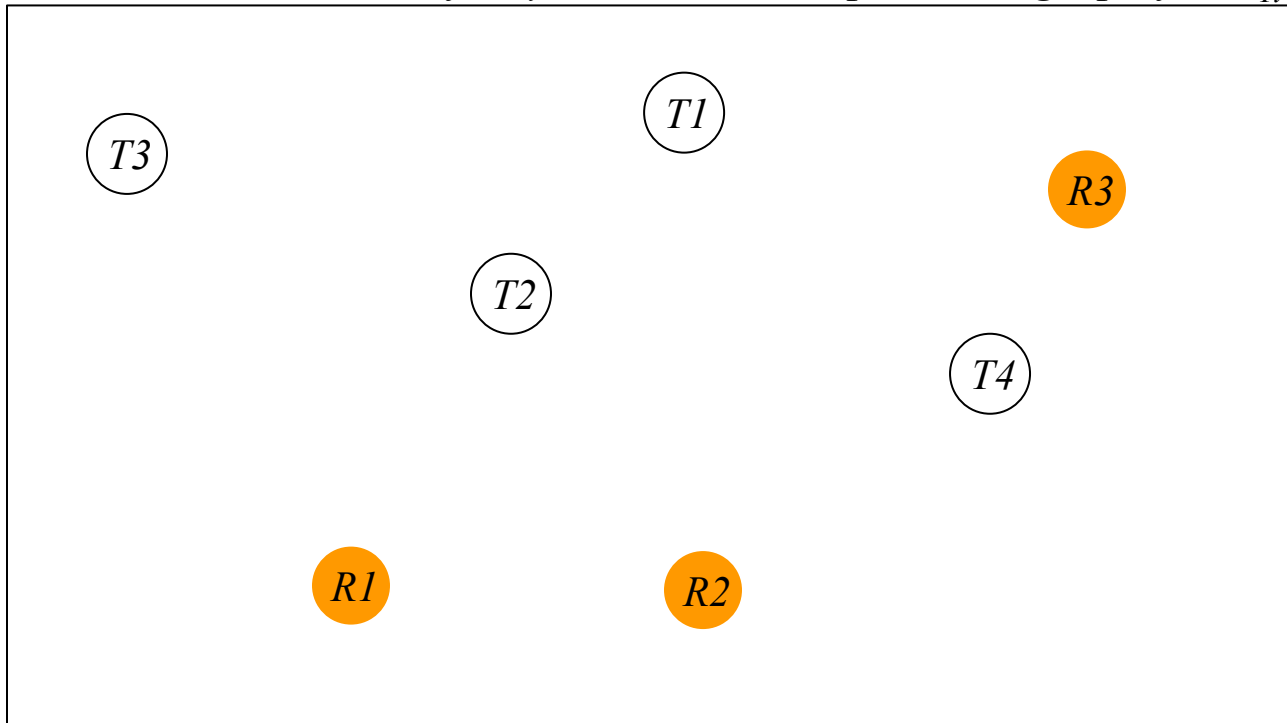
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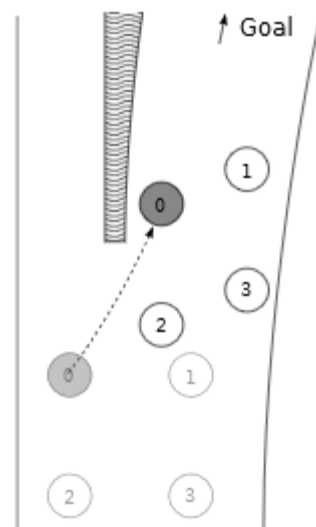
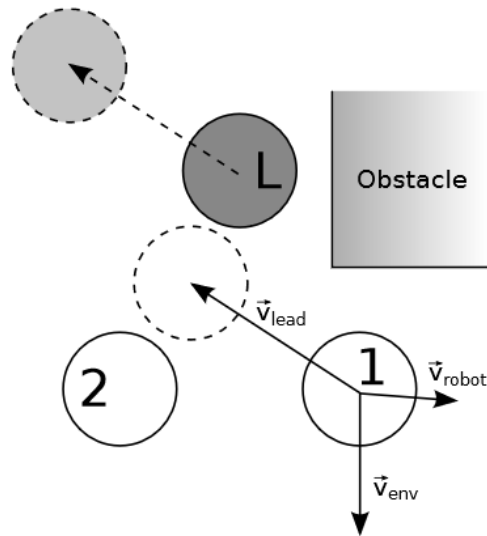
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# Planning for Leader-based Coordination

- **Fully decentralized approach** (doesn't rely on the presence of communication between robots)
- Plan for the “leader” robot (sometimes leader can be just a centroid of the team or some other reference point)
- All other robots execute either “follow the leader” or “follow neighbors within field-of-view” behaviors while avoiding collisions



# What You Should Know...

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- Different styles of multi-robot planning
  - Centralized vs. decentralized
  - Joint state-space planning vs. distributed planning
  - Multi-robot path planning vs. cooperative task planning
- Prioritized Multi-robot Path Planning
- Market-based Approach to multi-robot planning