16-350
Planning Techniques for Robotics

Multi-Robot Planning

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Different Categorizations of Multi-Robot Planning

- Centralized vs. Decentralized
  - **Centralized**: one central control of (planning for) all the robots
  - **Decentralized**: each robot decides/plans what to do on its own

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Robust to loosing some robots in the team
Computationally more scalable

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Challenges with decentralized planning?

How to guarantee that the overall team accomplishes its goal?

Different Categorizations of Multi-Robot Planning

- Multi-robot Path Planning vs. Multi-robot Cooperative Task Planning

  – **Multi-robot Path Planning**: how to plan paths for N robots so that they don't collide with each other during execution

  – **Multi-robot Cooperative Task Planning**: how to compute plans for N robots so that they achieve the overall goal that may require cooperation

Different Categorizations of Multi-Robot Planning

- Small teams vs. large teams (swarms) of robots
  - **Planning for small teams**: Compute plans for N (potentially heterogeneous) robots, where N is typically 2-10
  - **Planning for (control of) swarms of robots**: how to control a swarm of N (usually homogeneous) robots, where N is typically 10-1000

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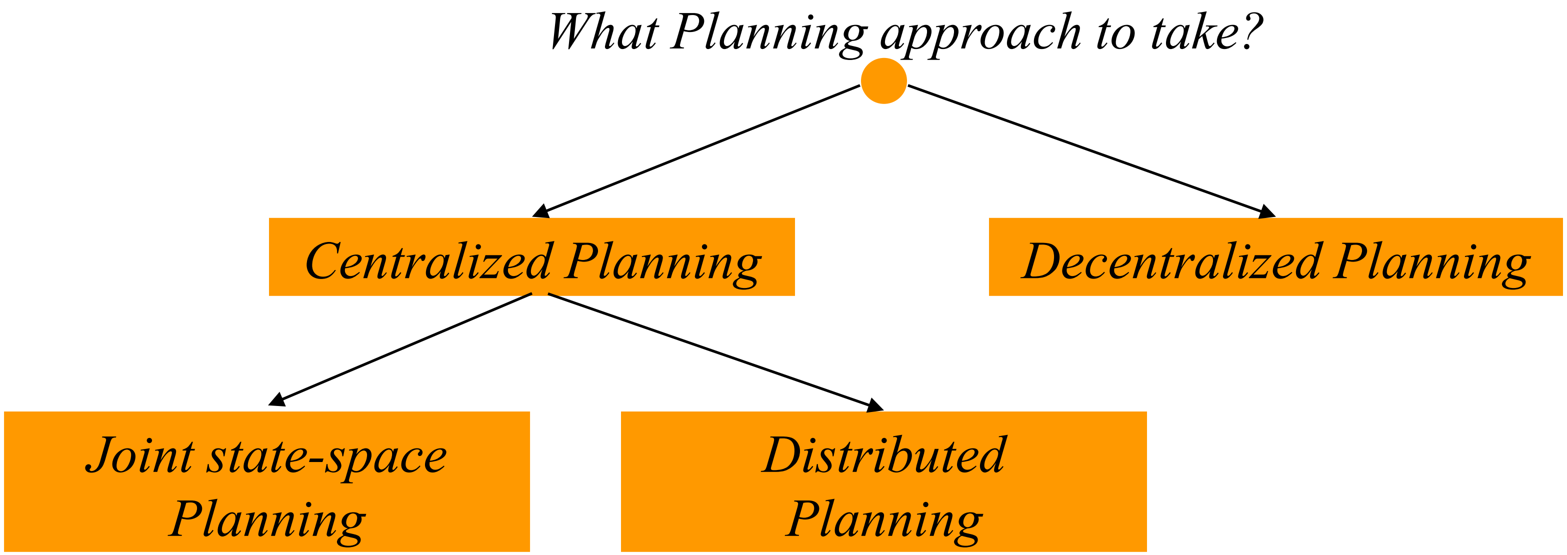
Control of swarms is typically decentralized

Different Categorizations of Multi-Robot Planning

- Joint state-space vs. distributed planning (within centralized)
  - **Joint state-space planning**: Planning for *N* robots in a state-space that represents joint configurations of robots
  - **Distributed planning**: Planning is split into *N* individual planners that share their results (and potentially re-plan) to obtain a final plan for all *N* robots



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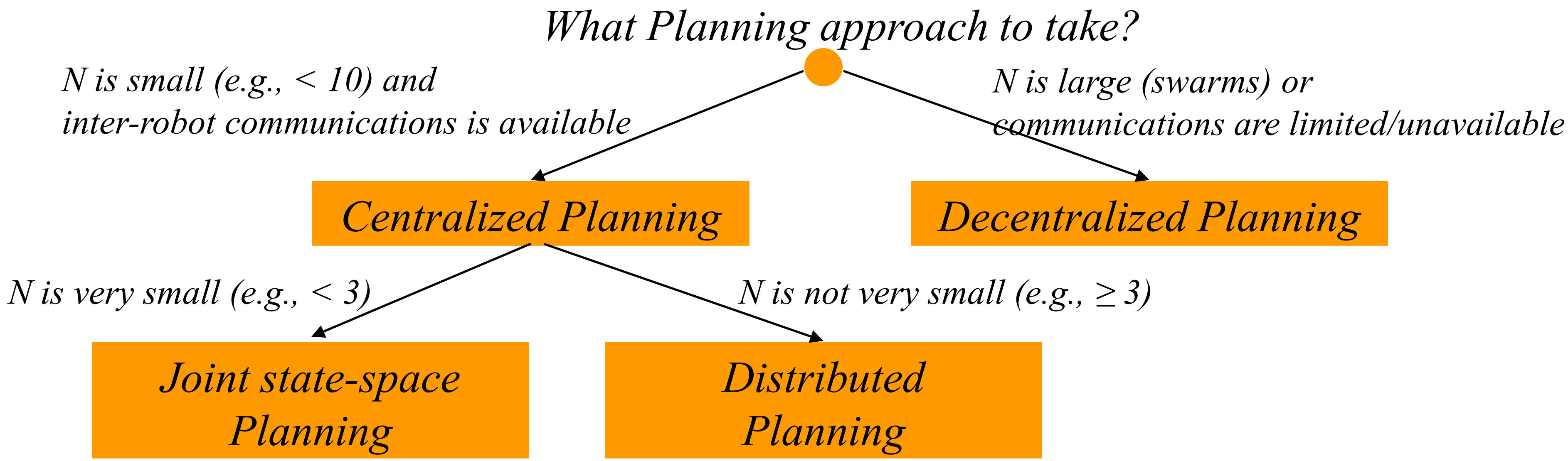
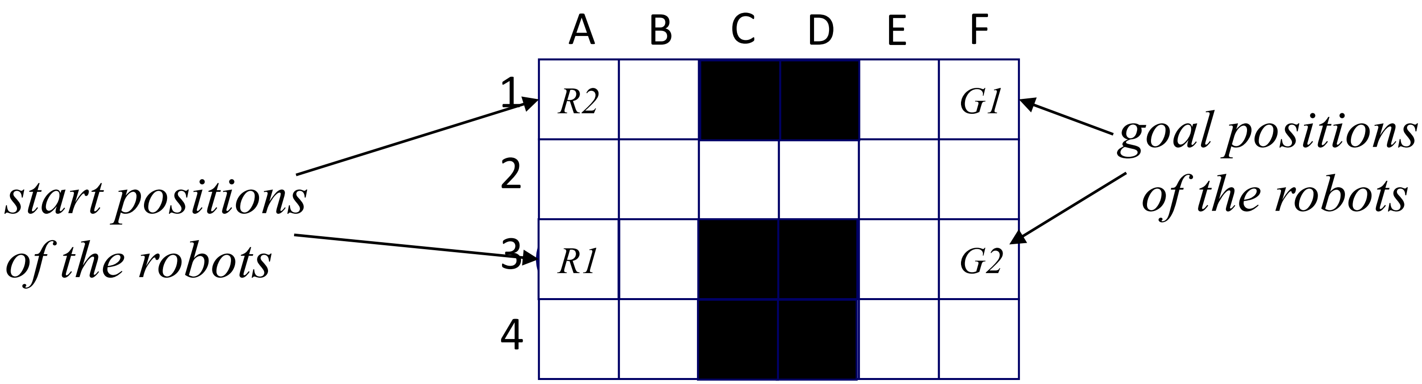


Multi-Robot Path Planning

- Path planning for *N* robots to get to their goals w/o collisions

simple example for two omnidirectional point-size robots



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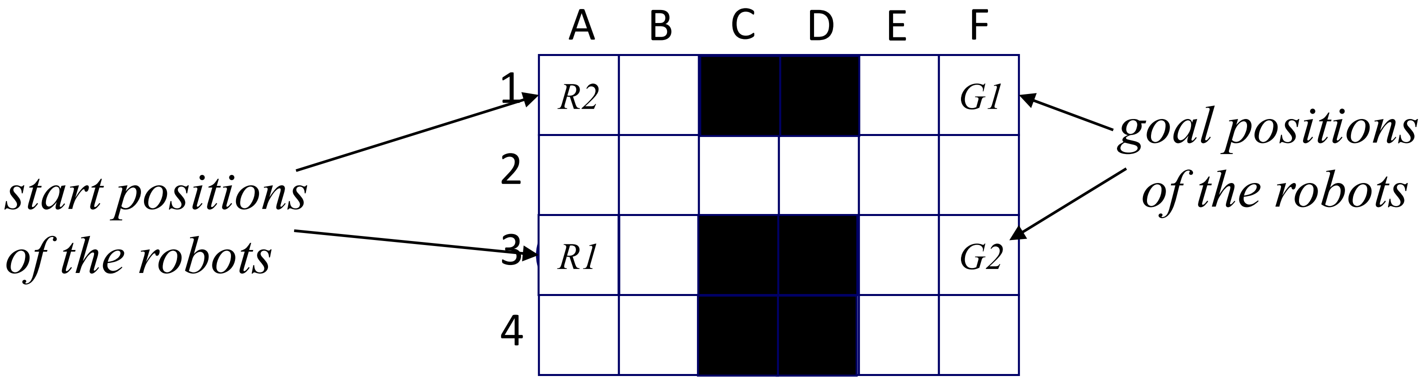

Any examples of this in industry?

Multi-Robot Path Planning

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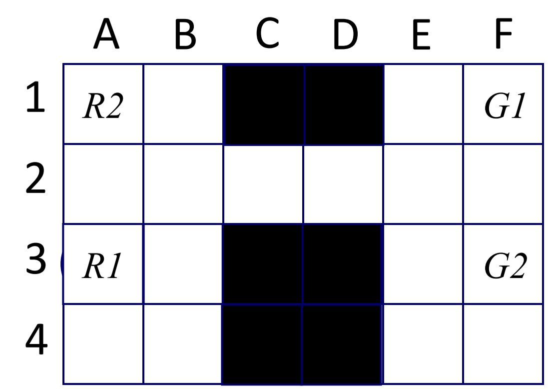
Joint state-space planning



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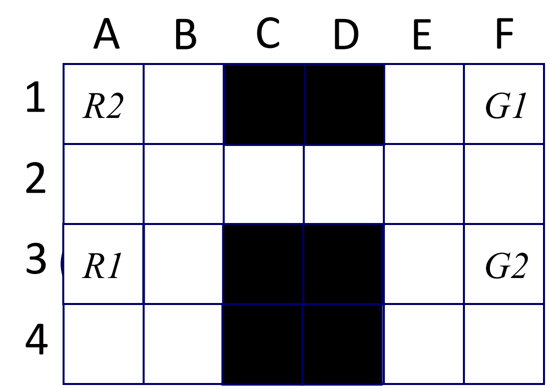

The simplest approach: construct and search a graph, where each state encodes positions of all the robots and each action encodes all possible movements

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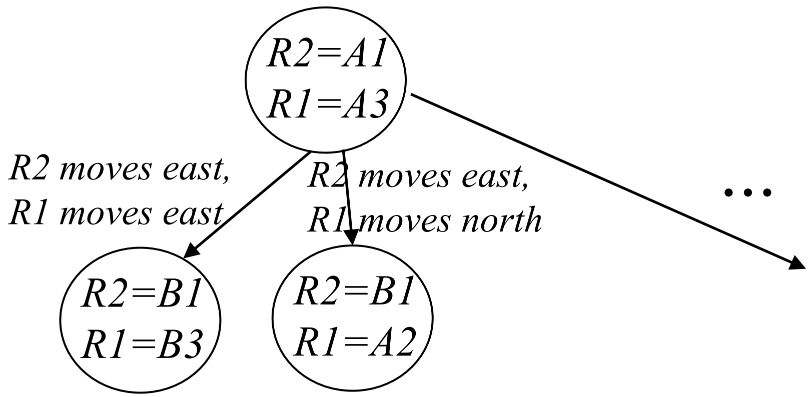

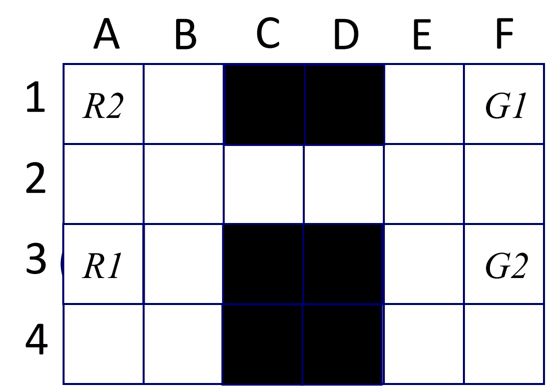

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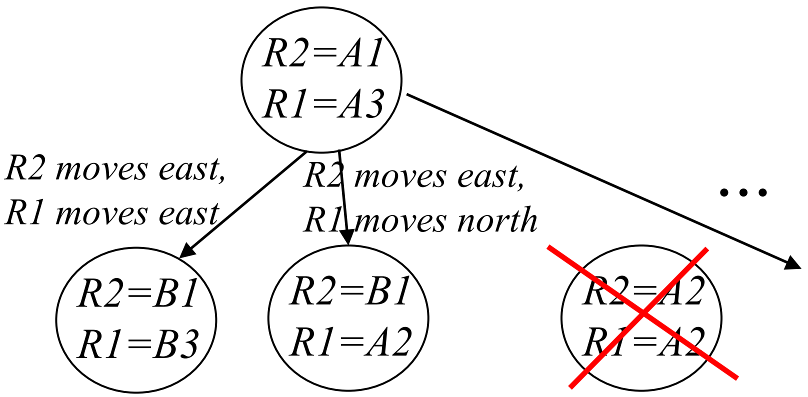

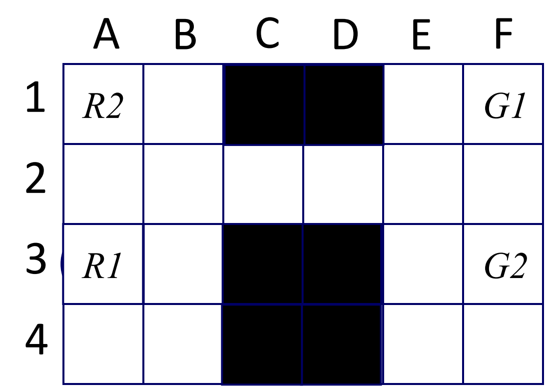

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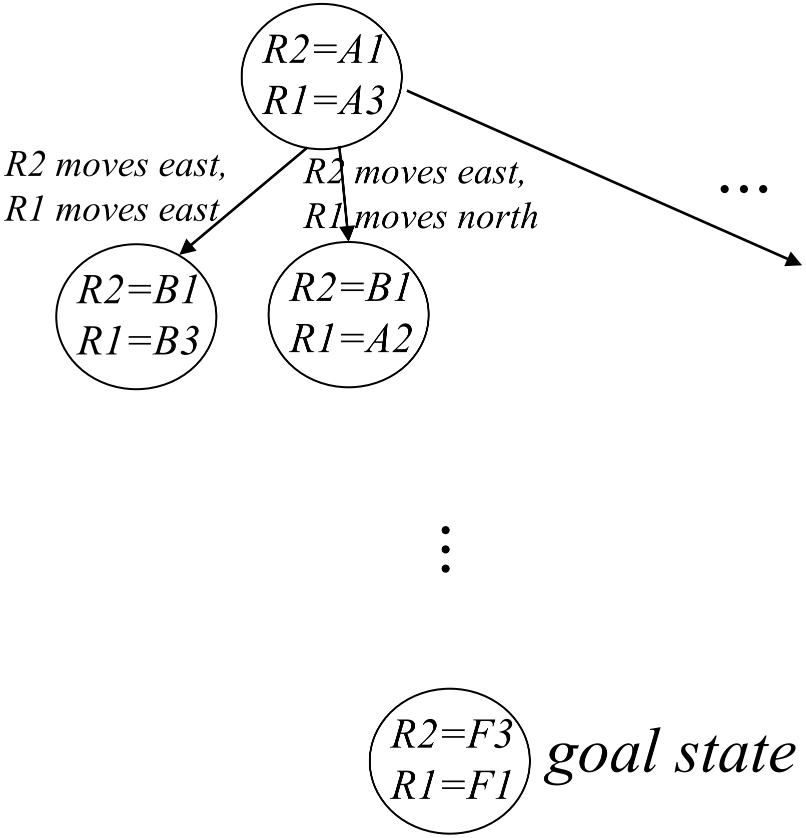
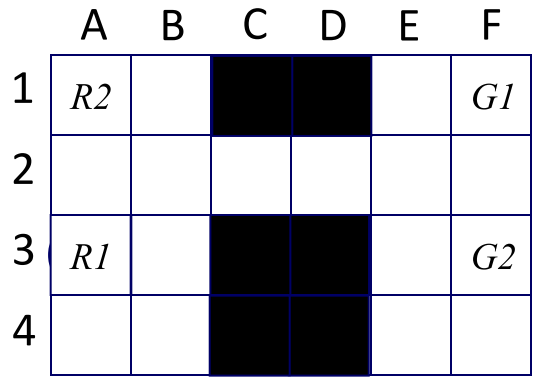


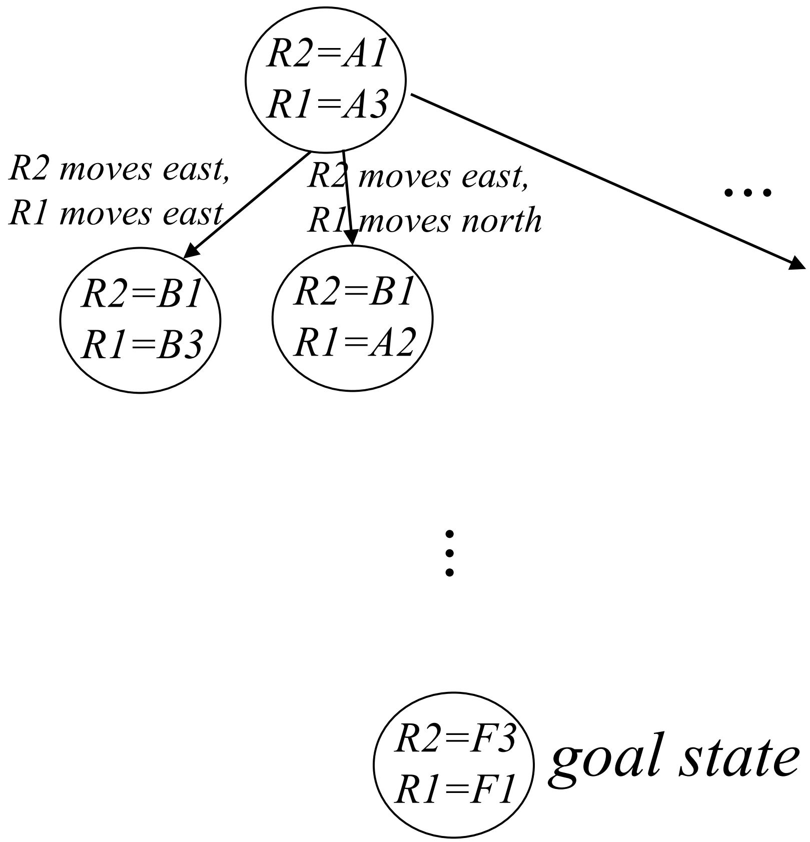


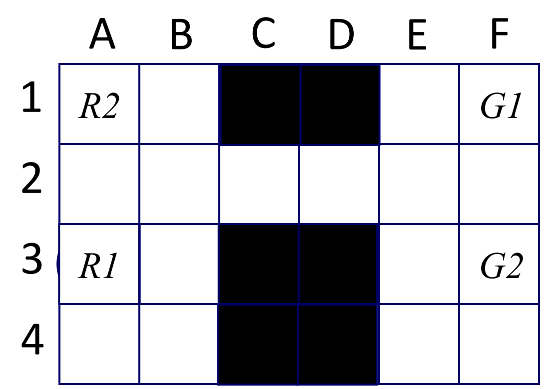
Multi-Robot Path Planning

- Path planning for *N* robots to get to their goals w/o collisions

Assuming 4-connected grid,
what is the maximum branching factor
(how many actions/successors)?

Joint state-space planning





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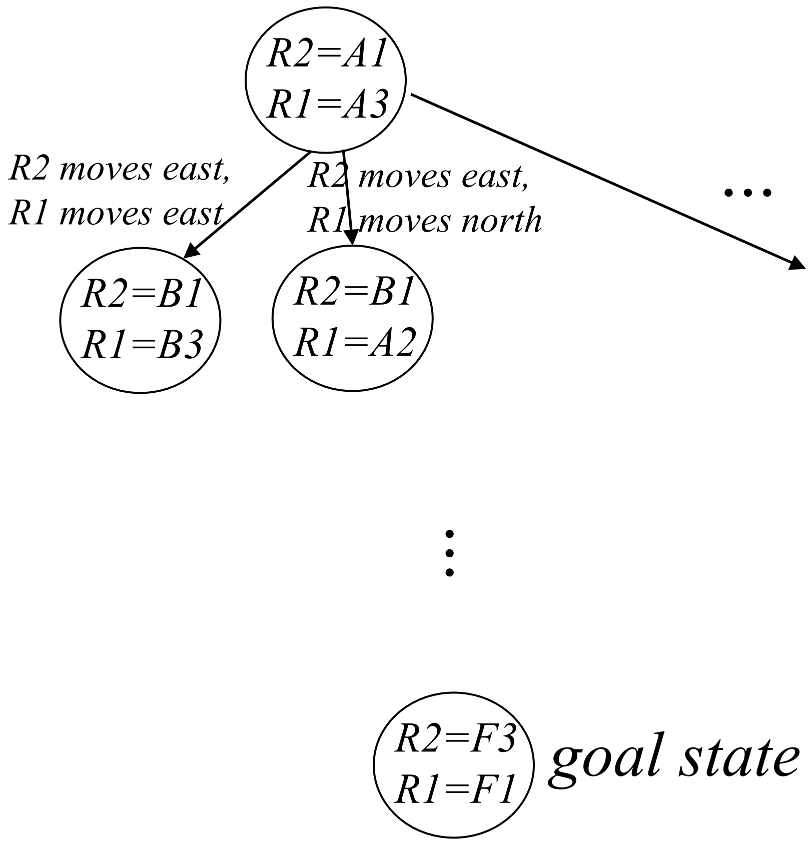
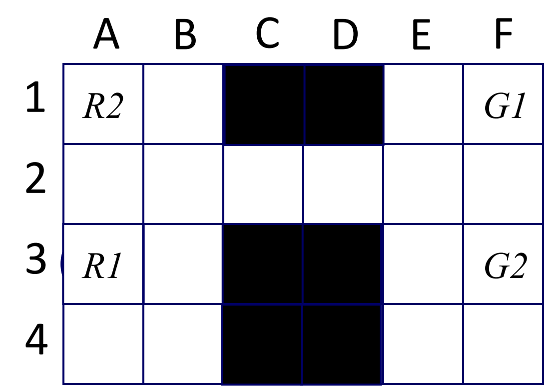
What is the size of the graph?

Joint state-space planning





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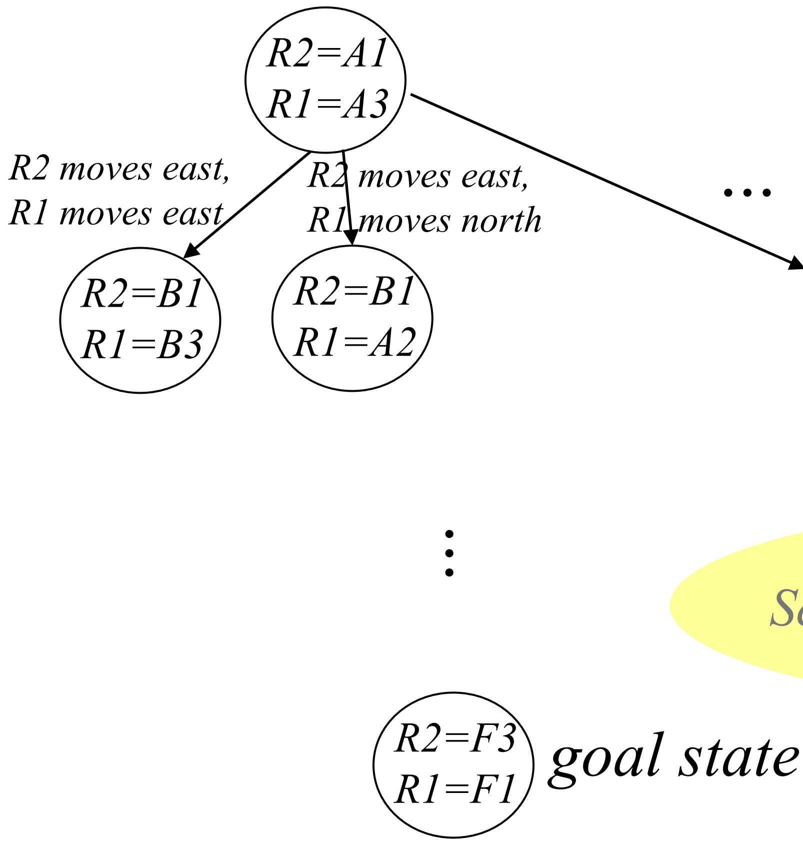
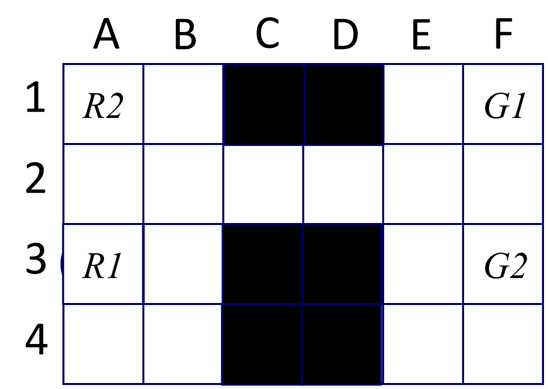


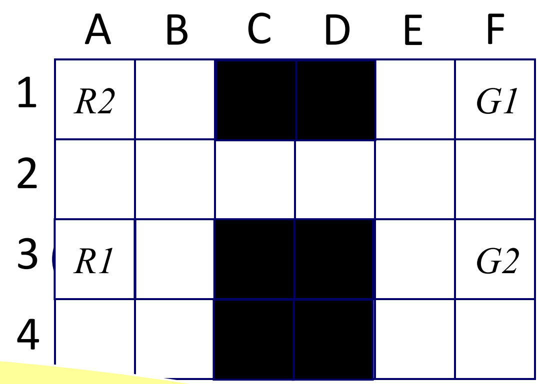


Scalability w.r.t. N is clearly an issue!

Multi-Robot Path Planning

- Path planning for *N* robots to get to their goals w/o collisions

Distributed planning



One popular approach: Prioritized Planning

For i = 1:N

Compute path for robot R_i that avoids collisions with paths for robots $R_1..R_{i-1}$

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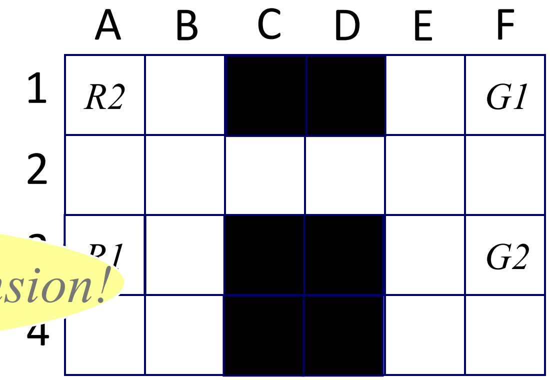

Each planning needs to include time as a dimension!

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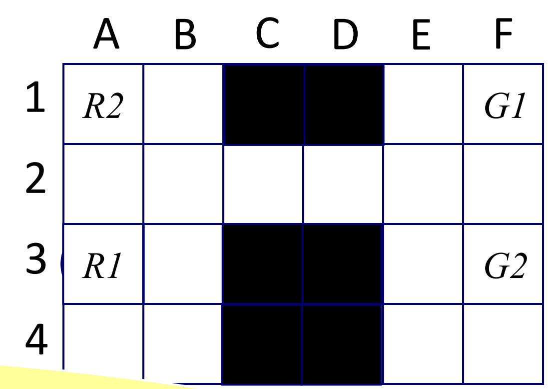

What will be the plan returned by Prioritized Planning?

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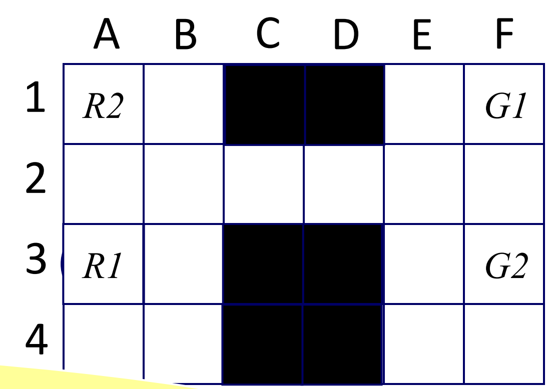
Is it complete?

Is it optimal?

What is the complexity of Prioritized Planning?



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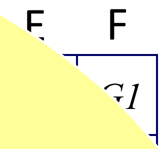
Distributed planning

***complete & optimal** approach*

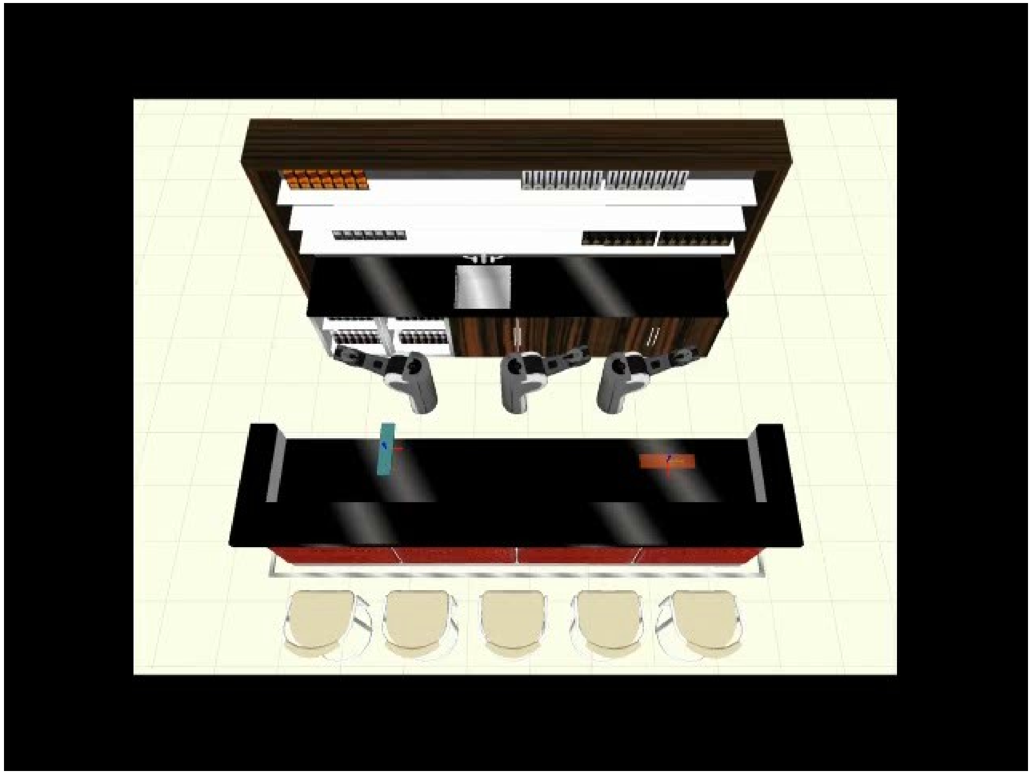
Conflict-based Search (CBS)

```
C(Sstart) = {}; //no constraints on paths at the start state
OPEN = {Sstart}; g(Sstart) = 0;
while (OPEN ≠ 0)
    remove S with the smallest g-value from OPEN
    if no collisions between paths associated with S, then return them as the overall plan
    for each (or at least one) collision of robots Ri and Rj at vertex v at time t
        C1 = C(S) U {Ri(t) ≠ v} //Ri can't be at vertex v at time t
        compute N paths that satisfy C1 constraints (w/o collision-checking
                between paths); let the overall cost be PLANCOST1
        C2 = C(S) U {Rj(t) ≠ v} //Rj can't be at vertex v at time t
        compute N paths that satisfy C2 constraints (w/o collision-checking
                between paths); let the overall cost be PLANCOST2
        insert state S1 with C(S1) = C1 and g(S1) = PLANCOST1
        insert state S2 with C(S2) = C2 and g(S2) = PLANCOST2
```



Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for N robotic arms to move an object



[Cohen et al., '14]
(performs joint state-space planning)

Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for N robotic arms to move an object

(planning is distributed: plan on Roman platform first, then on PR2)

Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for multi-robot exploration/mapping

N robots need to explore and build a map of unknown environment

One approach: Distributed Greedy Mapping

For i = 1:N

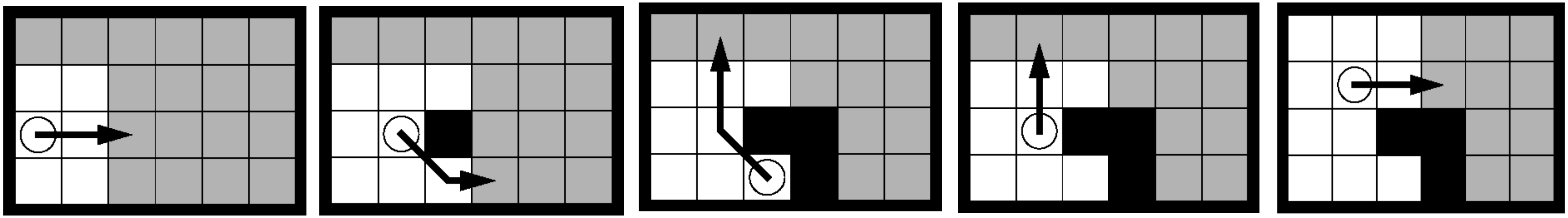
Compute a path using Greedy Mapping approach for robot R_i taking into account what paths were computed for $R_1..R_{i-1}$ (and what cells they would see)

Multi-Robot Cooperative Planning/Task Allocation

- Example: planning for multi-robot exploration/mapping

N robots need to explore and build a map of unknown environment

- Greedy Mapping for a single robot:
  - always move the robot on a shortest path to the closest unobserved (or unvisited) cell
  - it always achieves a gain in information.
  - thus, it is guaranteed to map the environment that is reachable (assuming all moves are reversible)



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[Butzke et al., '11]

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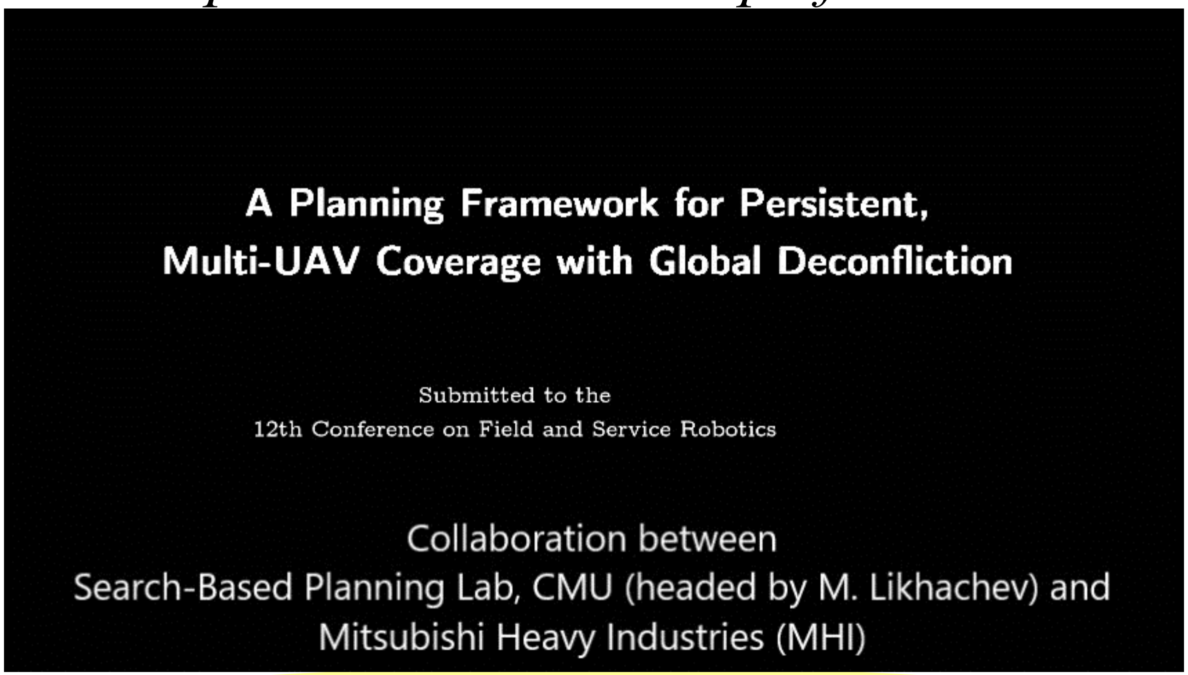

[Kusner et al., '19]

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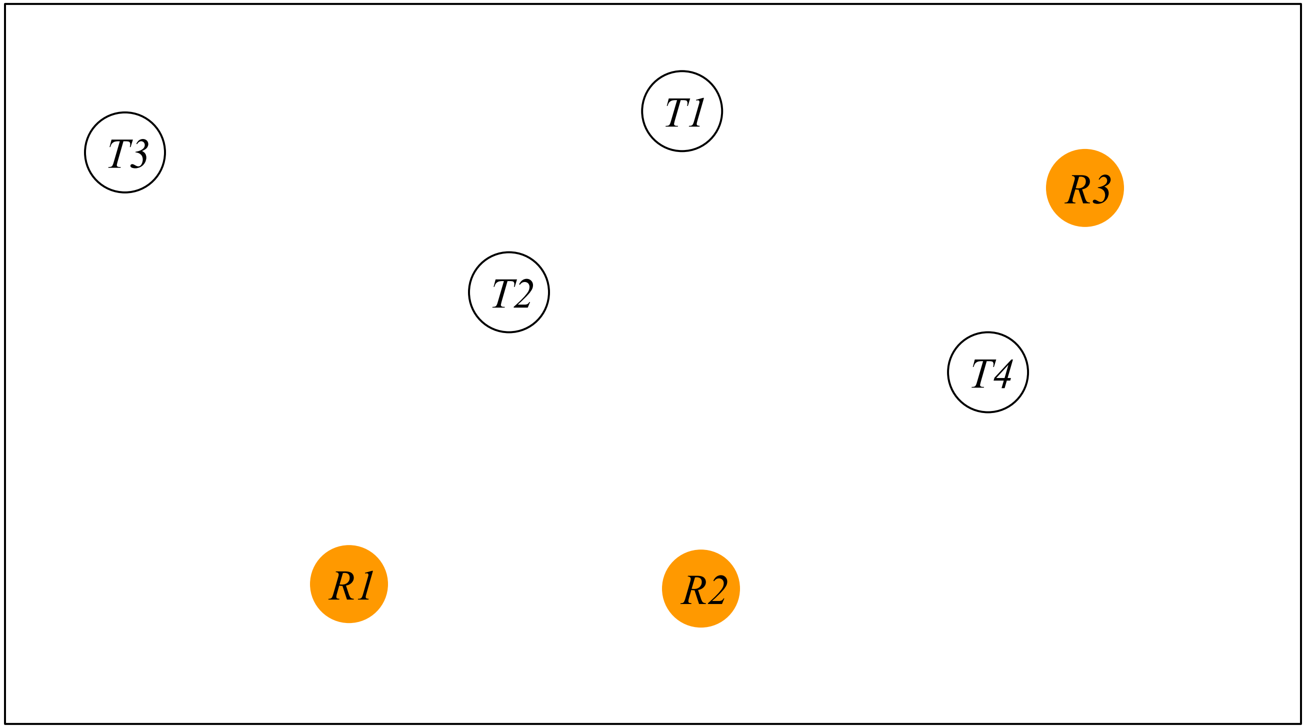
Multi-Robot Cooperative Planning/Task Allocation

- Market-based approach (very popular distributed approach)
  - Consider planning the allocation of tasks to *N* robots
  - General scheme: *robots auction out their tasks to their teammates with the goal of increasing their own revenue*

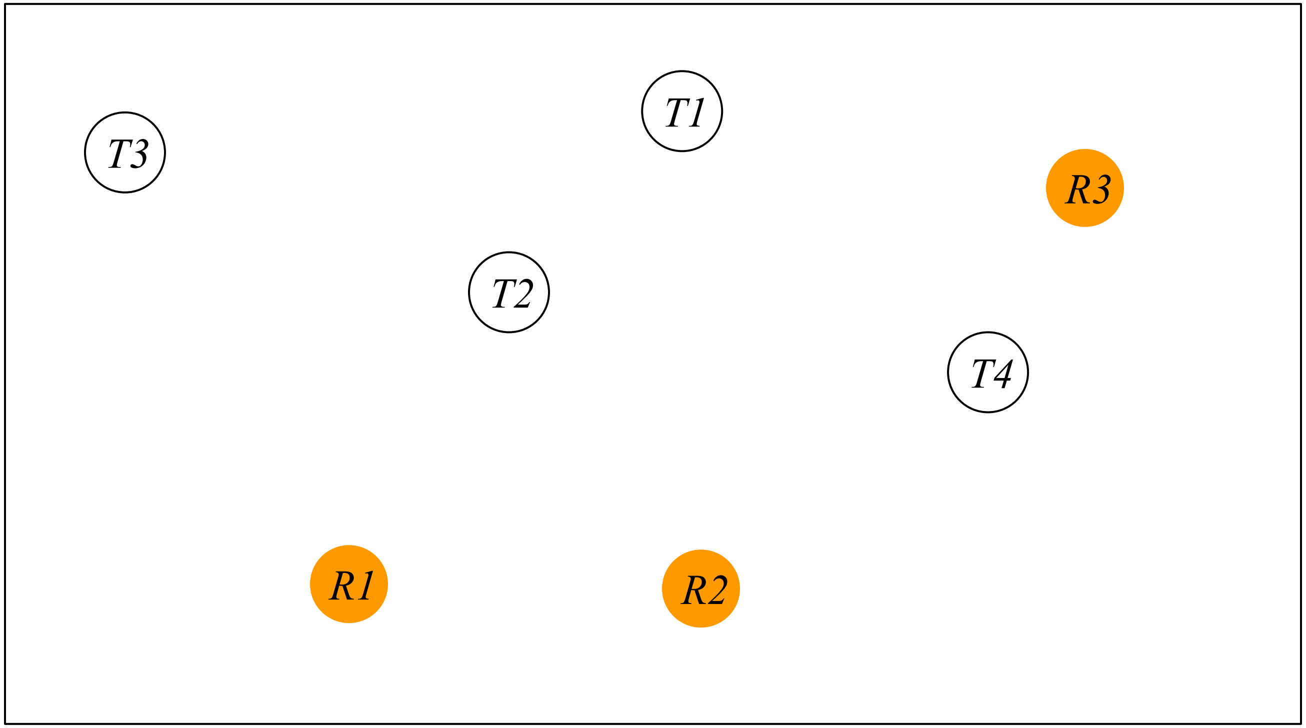


Market-based Approach

Given N robots $R_1 \ldots R_N$, M tasks $T_1 \ldots T_M$, and C_i^{Rj} – cost of executing task i by robot R_j (cost may depend on other tasks executed by this robot)

Planner needs to decide: Which task gets executed by which robot?

Find a plan(mapping) $\pi^*: T_i \rightarrow R_j$ *such that* $\pi^* = \text{argmin} \sum C_i^{\pi(Ti)}$



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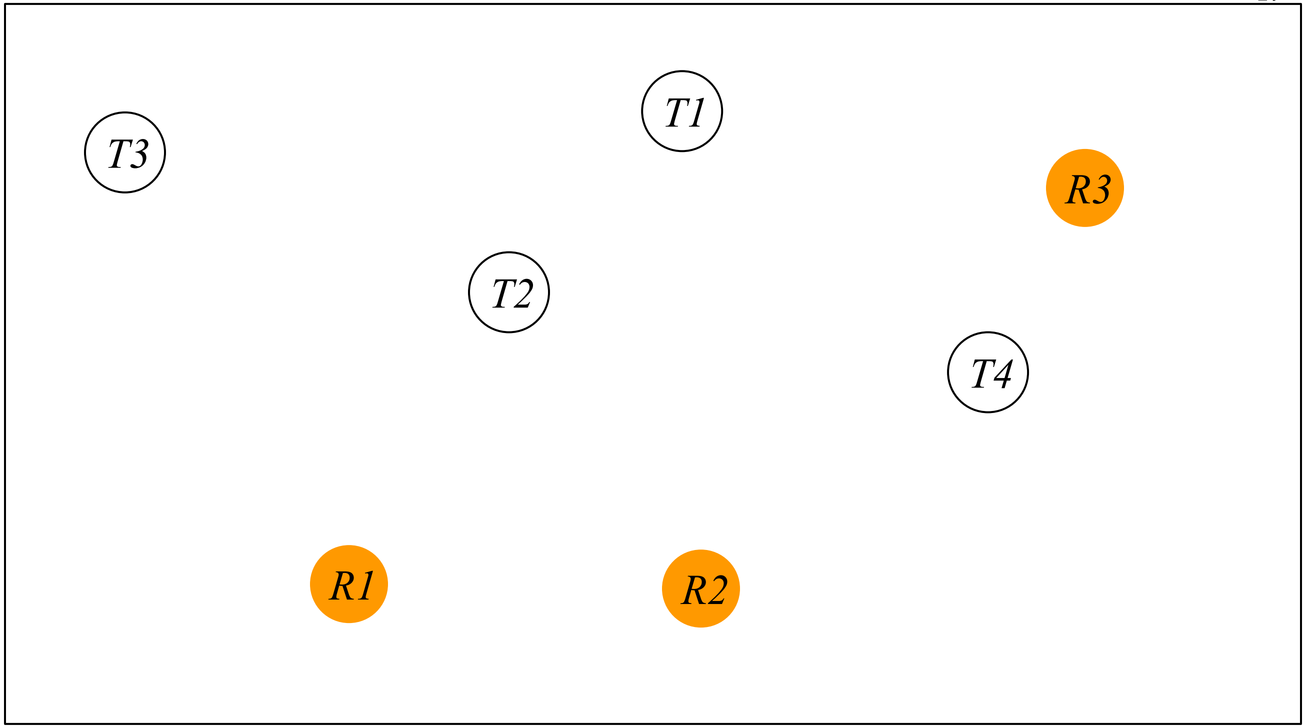
Iterate over steps 1-4 until convergence or planning time expires

Step 1: start with an arbitrary plan π

Step 2: all robots offer their tasks T_i *at auction at the max. price of* $C_{Ti}^{\pi(Ti)} - \epsilon$

Step 3: all robots R_j *bid on the offered tasks* T_i *with the bid* $= C_i^{Rj} + \epsilon$

Step 4: robots sell to the lowest bidders if they are below max. price and get profit: $C_{Ti}^{\pi(Ti)} - C_i^{Rj} - \epsilon$



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When does it converge in one iteration?

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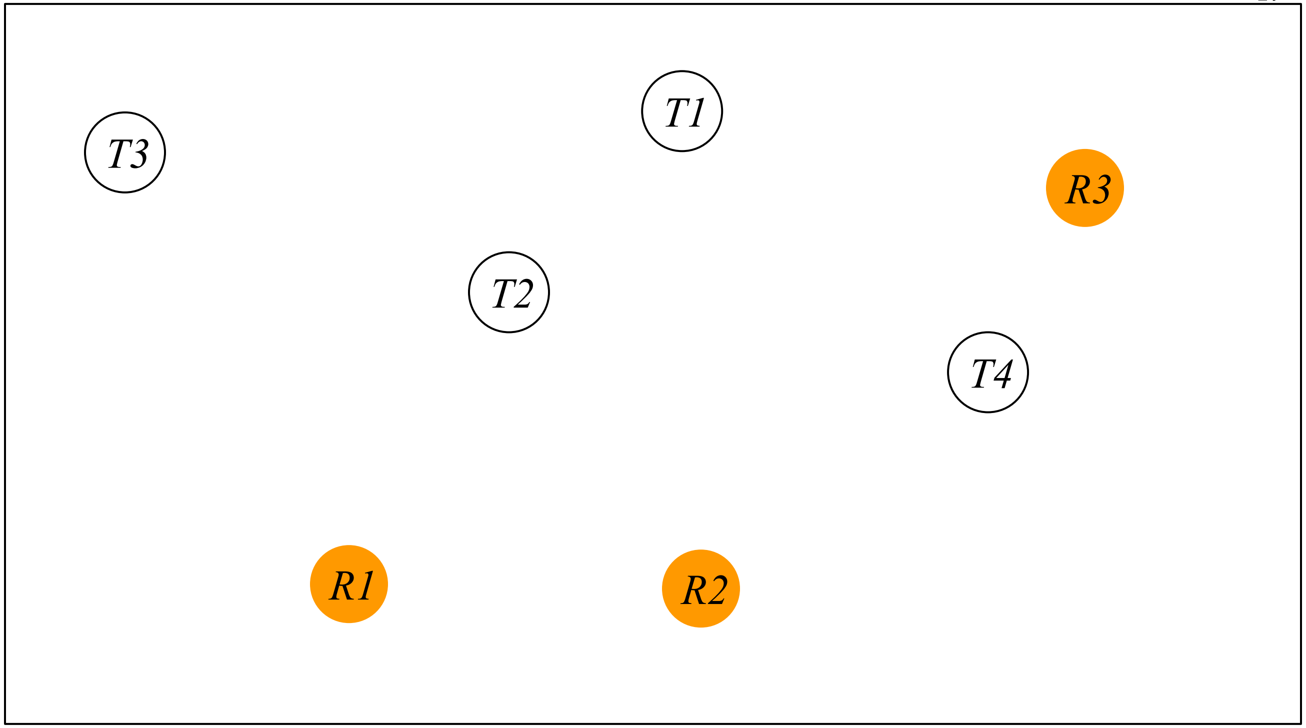
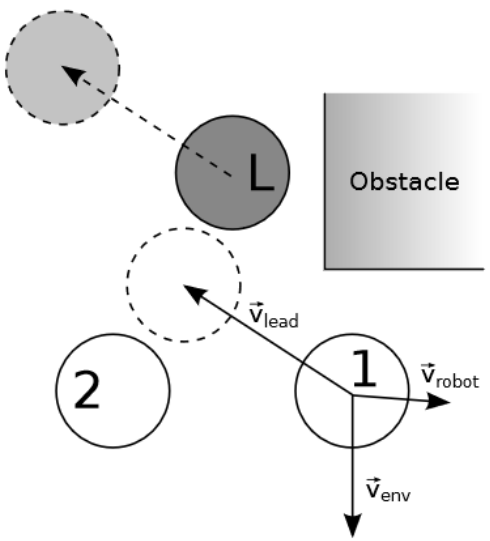


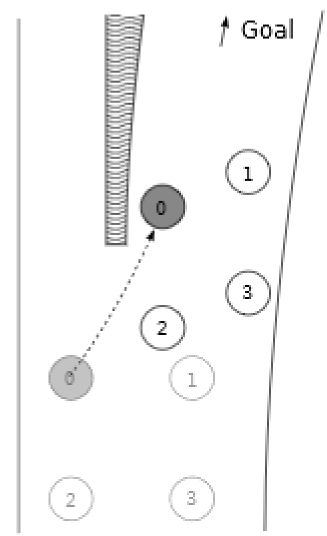
Planning for Leader-based Coordination

• **Fully decentralized approach** (doesn't rely on the presence of communication between robots)

• Plan for the "leader" robot (sometimes leader can be just a centroid of the team or some other reference point)

• All other robots execute either "follow the leader" or "follow neighbors within field-of-view" behaviors while avoiding collisions





What You Should Know…

- Different styles of multi-robot planning
  - Centralized vs. decentralized
  - Joint state-space planning vs. distributed planning
  - Multi-robot path planning vs. cooperative task planning

- Prioritized Multi-robot Path Planning

- Market-based Approach to multi-robot planning