



# 10-601 Introduction to Machine Learning

Machine Learning Department  
School of Computer Science  
Carnegie Mellon University

## Ensemble Methods + Recommender Systems

Matt Gormley  
Lecture 26  
Apr. 30, 2021

# Reminders

- **Homework 7: Graphical Models**
  - Out: Mon, Apr. 19
  - Due: Fri, Apr. 30 at 11:59pm
- **Homework 8: Learning Paradigms**
  - Out: Fri, Apr. 30
  - Due: Fri, May. 7 at 11:59pm
- **Exam 3:**
  - Mock Exam 3: Fri, May 7 during classtime
  - Exam 3: Fri, May 14 (final exam period)

# **EXAM LOGISTICS**

# Final Exam

- **Time / Location**

- **Time:** Registrar-scheduled Exam  
**Fri, May 14 at 9:30 – 11:30am**
- **Online Exam:** Same format as Midterm Exam 2
- Please watch Piazza carefully for announcements logistics

- **Logistics**

- Distribution of Topics: Lectures 18 – 27 (95%), Lectures 1 – 17 (5%)
- Format of questions:
  - Multiple choice
  - True / False (with justification)
  - Derivations
  - Short answers
  - Interpreting figures
  - Implementing algorithms on paper
- ~~You are encouraged to **bring** one 8½ x 11 sheet of notes (front and back)~~
- Open book according to our definition on Piazza

# Final Exam

- **How to Prepare**

- Attend (or watch) this final exam review session
- Review Practice Problems: Exam 3
  - Disclaimer: the practice problems are somewhere between homework-style problems and exam-style problems
- Review this year's **homework problems**
- Review the **poll questions** from each lecture
- Consider whether you have achieved the **learning objectives** for each lecture / section

# Final Exam

- **Advice (for during the exam)**
  - Solve the easy problems first (e.g. multiple choice before derivations)
    - if a problem seems extremely complicated you're likely missing something
  - Don't leave any answer blank!
  - If you make an assumption, write it down
  - If you look at a question and don't know the answer:
    - we probably haven't told you the answer
    - but we've told you enough to work it out
    - imagine arguing for some answer and see if you like it

# Topics for Midterm 1

- Foundations
  - Probability, Linear Algebra, Geometry, Calculus
  - Optimization
- Important Concepts
  - Overfitting
  - Experimental Design
- Classification
  - Decision Tree
  - KNN
  - Perceptron
- Regression
  - Linear Regression

# Topics for Midterm 2

- Classification
  - Binary Logistic Regression
  - Multinomial Logistic Regression
- Important Concepts
  - Stochastic Gradient Descent
  - Regularization
  - Feature Engineering
- Feature Learning
  - Neural Networks
  - Basic NN Architectures
  - Backpropagation
- Reinforcement Learning
  - Value Iteration
  - Policy Iteration
  - Q-Learning
  - Deep Q-Learning



# Topics for Final Exam

- Generative Models
  - Generative vs. Discriminative
  - MLE / MAP
  - Naïve Bayes
- Graphical Models
  - Representation
  - Bayesian Networks
  - Learning and Inference
  - EM and Clustering
- Learning Theory
  - PAC Learning
- Other Learning Paradigms
  - Ensemble Methods
  - Recommender Systems
  - Dimensionality Reduction

The Big Picture

# **CLASSIFICATION AND REGRESSION**

# ML Big Picture

## Learning Paradigms:

*What data is available and when? What form of prediction?*

- supervised learning
- unsupervised learning
- semi-supervised learning
- reinforcement learning
- active learning
- imitation learning
- domain adaptation
- online learning
- density estimation
- recommender systems
- feature learning
- manifold learning
- dimensionality reduction
- ensemble learning
- distant supervision
- hyperparameter optimization

## Theoretical Foundations:

*What principles guide learning?*

- probabilistic
- information theoretic
- evolutionary search
- ML as optimization

## Problem Formulation:

*What is the structure of our output prediction?*

boolean	Binary Classification
categorical	Multiclass Classification
ordinal	Ordinal Classification
real	Regression
ordering	Ranking
multiple discrete	Structured Prediction
multiple continuous	(e.g. dynamical systems)
both discrete & cont.	(e.g. mixed graphical models)

## Facets of Building ML Systems:

*How to build systems that are robust, efficient, adaptive, effective?*

1. Data prep
2. Model selection
3. Training (optimization / search)
4. Hyperparameter tuning on validation data
5. (Blind) Assessment on test data

## Big Ideas in ML:

*Which are the ideas driving development of the field?*

- inductive bias
- generalization / overfitting
- bias-variance decomposition
- generative vs. discriminative
- deep nets, graphical models
- PAC learning
- distant rewards

## Application Areas

*Key challenges?*

NLP, Speech, Computer Vision, Robotics, Medicine, Search

# Classification and Regression: The Big Picture

## Recipe for Machine Learning

1. Given data  $\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^N$
2. (a) Choose a decision function  $h_{\boldsymbol{\theta}}(\mathbf{x}) = \dots$   
(parameterized by  $\boldsymbol{\theta}$ )  
(b) Choose an objective function  $J_{\mathcal{D}}(\boldsymbol{\theta}) = \dots$   
(relies on data)
3. Learn by choosing parameters that optimize the objective  $J_{\mathcal{D}}(\boldsymbol{\theta})$

$$\hat{\boldsymbol{\theta}} \approx \underset{\boldsymbol{\theta}}{\operatorname{argmin}} J_{\mathcal{D}}(\boldsymbol{\theta})$$

4. Predict on new test example  $\mathbf{x}_{\text{new}}$  using  $h_{\boldsymbol{\theta}}(\cdot)$

$$\hat{y} = h_{\boldsymbol{\theta}}(\mathbf{x}_{\text{new}})$$

# Classification and Regression: The Big Picture

## Recipe for Machine Learning

1. Given
2. (a)
- (b)
3. Learn
4. Pred

### Decision Functions

- Perceptron:  $h_{\theta}(\mathbf{x}) = \text{sign}(\theta^T \mathbf{x})$
- Linear Regression:  $h_{\theta}(\mathbf{x}) = \theta^T \mathbf{x}$
- Discriminative Models:  $h_{\theta}(\mathbf{x}) = \underset{y}{\text{argmax}} p_{\theta}(y | \mathbf{x})$ 
  - Logistic Regression:  $p_{\theta}(y = 1 | \mathbf{x}) = \sigma(\theta^T \mathbf{x})$
  - Neural Net (classification):  
 $p_{\theta}(y = 1 | \mathbf{x}) = \sigma((\mathbf{W}^{(2)})^T \sigma((\mathbf{W}^{(1)})^T \mathbf{x} + \mathbf{b}^{(1)}) + \mathbf{b}^{(2)})$
- Generative Models:  $h_{\theta}(\mathbf{x}) = \underset{y}{\text{argmax}} p_{\theta}(\mathbf{x}, y)$ 
  - Naive Bayes:  $p_{\theta}(\mathbf{x}, y) = p_{\theta}(y) \prod_{m=1}^M p_{\theta}(x_m | y)$

# Classification and Regression: The Big Picture

## Recipe for Machine Learning

## Decision Functions

### Objective Function

- MLE:  $J(\boldsymbol{\theta}) = - \sum_{i=1}^N \log p(\mathbf{x}^{(i)}, \mathbf{y}^{(i)})$

- MCLE:  $J(\boldsymbol{\theta}) = - \sum_{i=1}^N \log p(\mathbf{y}^{(i)} | \mathbf{x}^{(i)})$

- L2 Regularized:  $J'(\boldsymbol{\theta}) = J(\boldsymbol{\theta}) + \lambda \|\boldsymbol{\theta}\|_2^2$   
(same as Gaussian prior  $p(\boldsymbol{\theta})$  over parameters)

- L1 Regularized:  $J'(\boldsymbol{\theta}) = J(\boldsymbol{\theta}) + \lambda \|\boldsymbol{\theta}\|_1$   
(same as Laplace prior  $p(\boldsymbol{\theta})$  over parameters)

# Classification and Regression: The Big Picture

## Recipe for Machine Learning

1. Given data  $\mathcal{D} = \{(\mathbf{x}^{(i)}, y^{(i)})\}_{i=1}^N$
2. (a) Choose a model  $f(\mathbf{x}; \boldsymbol{\theta})$  (parameters  $\boldsymbol{\theta}$ )  
(b) Choose a loss function  $J(\boldsymbol{\theta})$  (reliability)
3. Learn by choosing  $\boldsymbol{\theta}$
4. Predict on new data

## Optimization Method

- Gradient Descent:  $\boldsymbol{\theta} \rightarrow \boldsymbol{\theta} - \gamma \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$

- SGD:  $\boldsymbol{\theta} \rightarrow \boldsymbol{\theta} - \gamma \nabla_{\boldsymbol{\theta}} J^{(i)}(\boldsymbol{\theta})$   
for  $i \sim \text{Uniform}(1, \dots, N)$

where  $J(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i=1}^N J^{(i)}(\boldsymbol{\theta})$

- mini-batch SGD

- closed form

1. compute partial derivatives
2. set equal to zero and solve

## Decision Functions

Decision function  $f(\mathbf{x}; \boldsymbol{\theta}) = \text{argmax}_y p_{\boldsymbol{\theta}}(y | \mathbf{x})$

$\Gamma_{\mathbf{x}}$

$$= \text{argmax}_y p_{\boldsymbol{\theta}}(y | \mathbf{x})$$

$$p_{\boldsymbol{\theta}}(y = 1 | \mathbf{x}) = \sigma(\boldsymbol{\theta}^T \mathbf{x})$$

);

$$\boldsymbol{\theta}^T \sigma((\mathbf{W}^{(1)})^T \mathbf{x} + \mathbf{b}^{(1)}) + \mathbf{b}^{(2)})$$

$$\text{argmax}_y p_{\boldsymbol{\theta}}(\mathbf{x}, y)$$

$$p_{\boldsymbol{\theta}}(y) \prod_{m=1}^M p_{\boldsymbol{\theta}}(x_m | y)$$

$y^{(i)}$

$| \mathbf{x}^{(i)}$

$$+ \lambda \|\boldsymbol{\theta}\|_2^2$$

(regularizer over parameters)

$$+ \lambda \|\boldsymbol{\theta}\|_1$$

(regularizer over parameters)

# Classification and Regression: The Big Picture

## Recipe for Machine Learning

1. Given data  $\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^N$
2. (a) Choose a decision function  $h_{\theta}(\mathbf{x}) = \dots$   
(parameterized by  $\theta$ )  
(b) Choose an objective function  $J_{\mathcal{D}}(\theta) = \dots$   
(relies on data)
3. Learn by choosing parameters that optimize the objective  $J_{\mathcal{D}}(\theta)$

$$\hat{\theta} \approx \underset{\theta}{\operatorname{argmin}} J_{\mathcal{D}}(\theta)$$

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$$\hat{y} = h_{\theta}(\mathbf{x}_{\text{new}})$$

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# Learning Paradigms

Paradigm	Data
Supervised	$\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^N \quad \mathbf{x} \sim p^*(\cdot) \text{ and } y = c^*(\cdot)$
↔ Regression	$y^{(i)} \in \mathbb{R}$
↔ Classification	$y^{(i)} \in \{1, \dots, K\}$
↔ Binary classification	$y^{(i)} \in \{+1, -1\}$
↔ Structured Prediction	$\mathbf{y}^{(i)}$ is a vector
Unsupervised	$\mathcal{D} = \{\mathbf{x}^{(i)}\}_{i=1}^N \quad \mathbf{x} \sim p^*(\cdot)$
↔ Clustering	predict $\{z^{(i)}\}_{i=1}^N$ where $z^{(i)} \in \{1, \dots, K\}$
↔ Dimensionality Reduction	convert each $\mathbf{x}^{(i)} \in \mathbb{R}^M$ to $\mathbf{u}^{(i)} \in \mathbb{R}^K$ with $K \ll M$
Semi-supervised	$\mathcal{D} = \{\mathbf{x}^{(i)}, y^{(i)}\}_{i=1}^{N_1} \cup \{\mathbf{x}^{(j)}\}_{j=1}^{N_2}$
Online	$\mathcal{D} = \{(\mathbf{x}^{(1)}, y^{(1)}), (\mathbf{x}^{(2)}, y^{(2)}), (\mathbf{x}^{(3)}, y^{(3)}), \dots\}$
Active Learning	$\mathcal{D} = \{\mathbf{x}^{(i)}\}_{i=1}^N$ and can query $y^{(i)} = c^*(\cdot)$ at a cost
Imitation Learning	$\mathcal{D} = \{(s^{(1)}, a^{(1)}), (s^{(2)}, a^{(2)}), \dots\}$
Reinforcement Learning	$\mathcal{D} = \{(s^{(1)}, a^{(1)}, r^{(1)}), (s^{(2)}, a^{(2)}, r^{(2)}), \dots\}$

# ML Big Picture

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# Outline for Today

We'll talk about two distinct topics:

1. **Ensemble Methods:** combine or learn multiple classifiers into one  
(i.e. a family of algorithms)
2. **Recommender Systems:** produce recommendations of what a user will like  
(i.e. the solution to a particular type of task)

We'll use a prominent example of a recommender systems (the Netflix Prize) to motivate both topics...

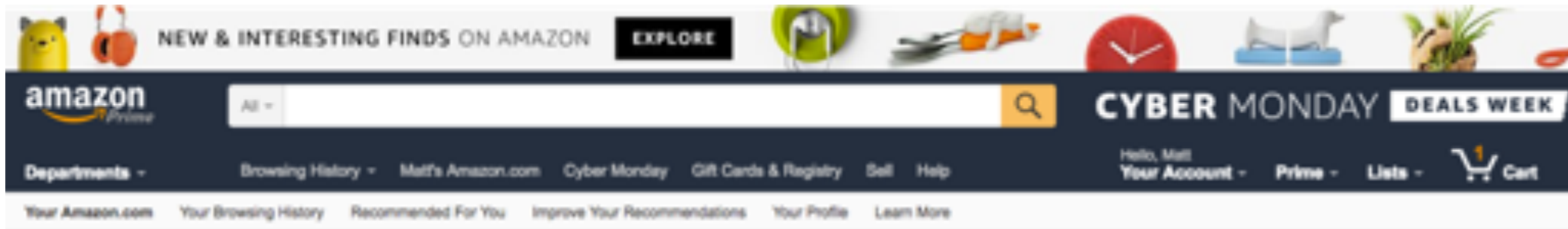
# **RECOMMENDER SYSTEMS**

# Recommender Systems

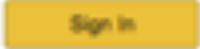
## A Common Challenge:

- Assume you're a company selling **items** of some sort: movies, songs, products, etc.
- Company collects millions of **ratings** from **users** of their **items**
- To maximize profit / user happiness, you want to **recommend** items that users are likely to want

# Recommender Systems



You could be seeing useful stuff here!  
Sign in to get your order status, balances and rewards.



## Recommended for you, Matt

Buy It Again in Grocery  
14 ITEMS

This block displays a collection of grocery items. At the top left is a box of Jif cereal. Next to it is a large jug of maple syrup. Below these are two jars of Jif creamy peanut butter and a bag of Jif cereal. The text 'Buy It Again in Grocery' and '14 ITEMS' is centered below the images.

Buy It Again in Pets  
6 ITEMS

This block displays pet-related products. At the top left is a box of Advantage II flea treatment. Next to it is a bag of Purina World's Best cat food. Below these are a small green pet accessory and a bag of Purina cat food. The text 'Buy It Again in Pets' and '6 ITEMS' is centered below the images.

Buy It Again in Baby Products  
5 ITEMS

This block displays baby products. At the top left is a box of 10 crayons. Next to it is a box of Crayola 64 baby wipes. Below these are two baby teethers and a baby toy. The text 'Buy It Again in Baby Products' and '5 ITEMS' is centered below the images.

Engineering Books  
86 ITEMS

This block displays the cover of the book 'Probabilistic Graphical Models: Principles and Techniques' by Daphne Koller and Nir Friedman. The cover features a blue background with a network diagram. The text 'Engineering Books' and '86 ITEMS' is centered below the book cover.

# Recommender Systems

**NETFLIX**

## Netfix Prize

**COMPLETED**

Home Rules Leaderboard Update

### Congratulations!

The Netflix Prize sought to substantially improve the accuracy of predictions about how much someone is going to enjoy a movie based on their movie preferences.

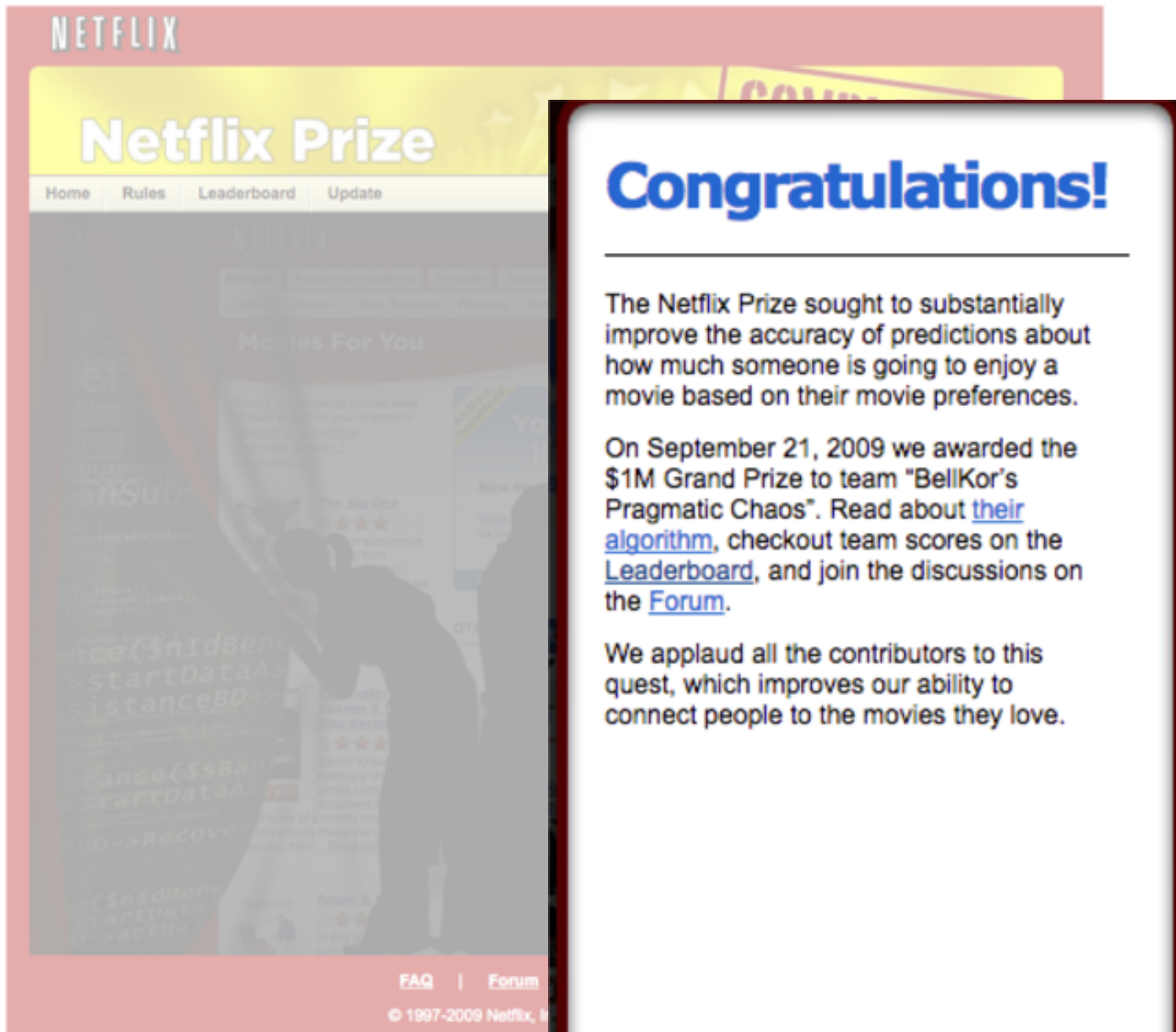
On September 21, 2009 we awarded the \$1M Grand Prize to team "BellKor's Pragmatic Chaos". Read about [their algorithm](#), checkout team scores on the [Leaderboard](#), and join the discussions on the [Forum](#).

We applaud all the contributors to this quest, which improves our ability to connect people to the movies they love.

FAQ | Forum | Netflix Home

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# Recommender Systems



The image shows a screenshot of the Netflix Prize website. The top navigation bar includes 'Home', 'Rules', 'Leaderboard', and 'Update'. The main content area is titled 'Movies For You' and features a grid of movie recommendations. A large, semi-transparent overlay on the right side of the page contains a congratulatory message. The message is titled 'Congratulations!' and describes the Netflix Prize competition, mentioning the \$1M Grand Prize awarded to the team 'BellKor's Pragmatic Chaos' on September 21, 2009. It includes links to 'their algorithm', the 'Leaderboard', and the 'Forum'. The footer of the page contains 'FAQ | Forum' and '© 1997-2009 Netflix, Inc.'.

## NETFLIX

### Netflix Prize

Home Rules Leaderboard Update

#### Congratulations!

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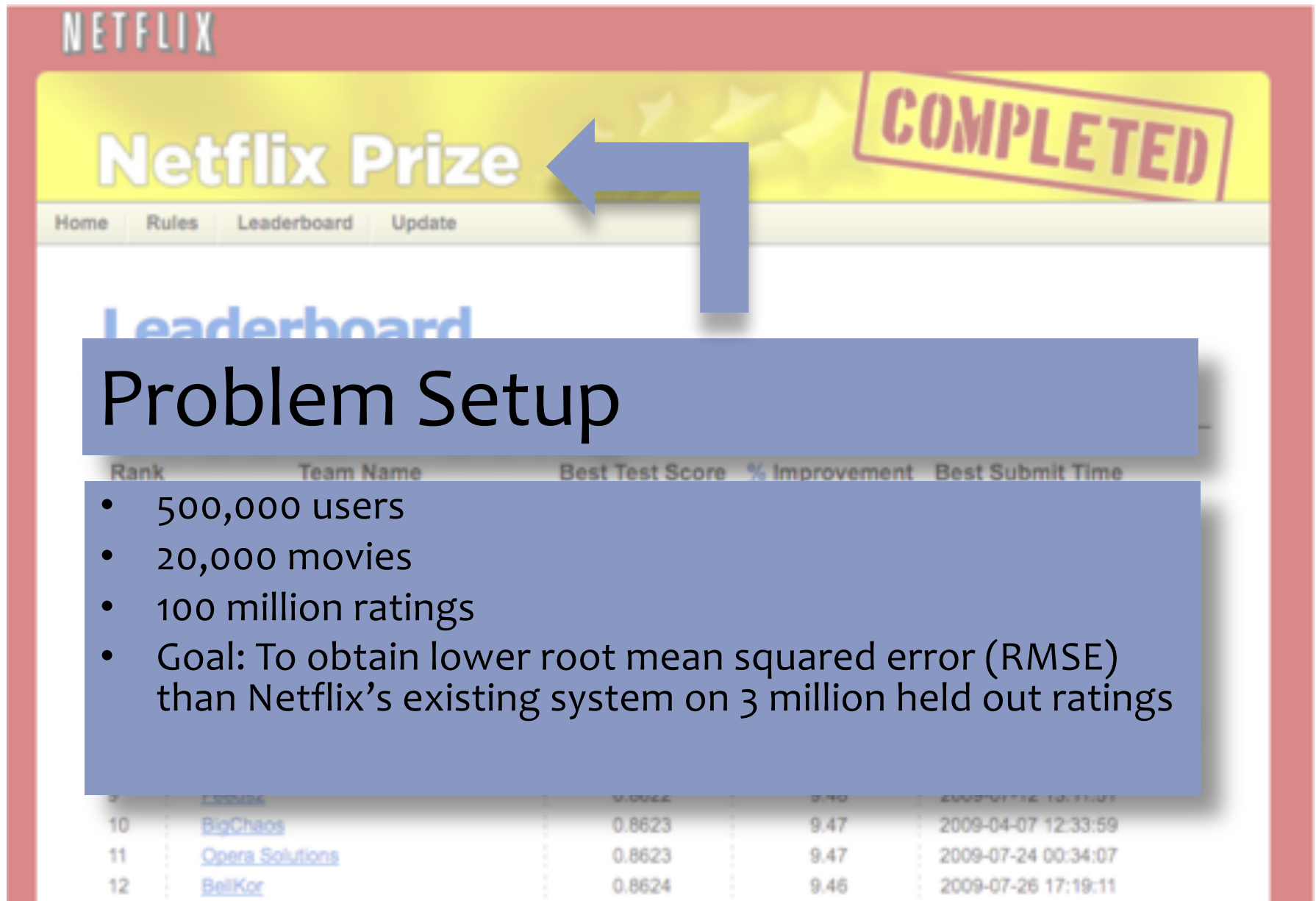
We applaud all the contributors to this quest, which improves our ability to connect people to the movies they love.

FAQ | Forum

© 1997-2009 Netflix, Inc.



# Recommender Systems



**Netflix Prize** ← **COMPLETED**

Home Rules Leaderboard Update

## Leaderboard

### Problem Setup

- 500,000 users
- 20,000 movies
- 100 million ratings
- Goal: To obtain lower root mean squared error (RMSE) than Netflix's existing system on 3 million held out ratings

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
10	<a href="#">BigChaos</a>	0.8623	9.47	2009-04-07 12:33:59
11	<a href="#">Opera Solutions</a>	0.8623	9.47	2009-07-24 00:34:07
12	<a href="#">BellKor</a>	0.8624	9.46	2009-07-26 17:19:11

# Recommender Systems

**NETFLIX**

## Netflix Prize

**COMPLETED**

Home Rules Leaderboard Update

### Leaderboard

Showing Test Score. [Click here to show quiz score](#)

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	<a href="#">BellKor's Pragmatic Chaos</a>	0.8567	10.06	2009-07-26 18:18:28
2	<a href="#">The Ensemble</a>	0.8567	10.06	2009-07-26 18:38:22
3	<a href="#">Grand Prize Team</a>	0.8582	9.90	2009-07-10 21:24:40
4	<a href="#">Opera Solutions and Vandelay United</a>	0.8588	9.84	2009-07-10 01:12:31
5	<a href="#">Vandelay Industries I</a>	0.8591	9.81	2009-07-10 00:32:20
6	<a href="#">PragmaticTheory</a>	0.8594	9.77	2009-06-24 12:06:56
7	<a href="#">BellKor in BigChaos</a>	0.8601	9.70	2009-05-13 08:14:09
8	<a href="#">Dace_</a>	0.8612	9.59	2009-07-24 17:18:43
9	<a href="#">Feeds2</a>	0.8622	9.48	2009-07-12 13:11:51
10	<a href="#">BigChaos</a>	0.8623	9.47	2009-04-07 12:33:59
11	<a href="#">Opera Solutions</a>	0.8623	9.47	2009-07-24 00:34:07
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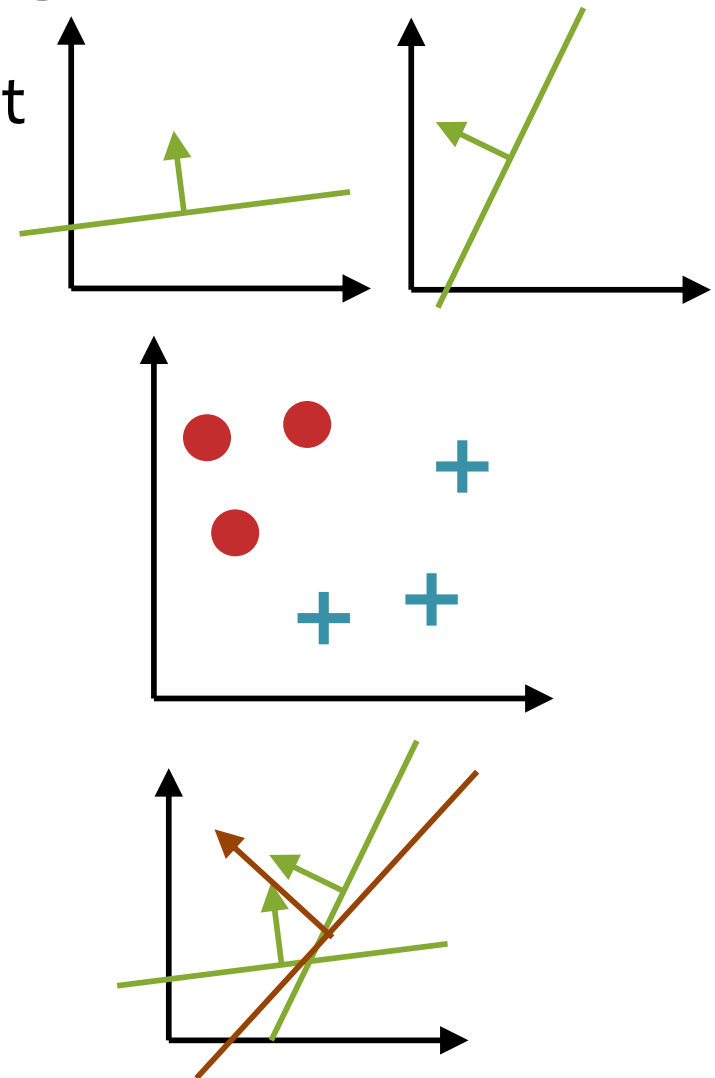
Top performing systems were ensembles

# ENSEMBLE METHODS

# Weighted Majority Algorithm

(Littlestone & Warmuth, 1994)

- **Given:** pool  $A$  of binary classifiers (that you know nothing about)
- **Data:** stream of examples (i.e. online learning setting)
- **Goal:** design a new learner that uses the predictions of the pool to make new predictions
- **Algorithm:**
  - Initially weight all classifiers equally
  - Receive a training example and predict the (weighted) majority vote of the classifiers in the pool
  - Down-weight classifiers that contribute to a mistake by a factor of  $\beta$



# Weighted Majority Algorithm

(Littlestone & Warmuth, 1994)

Suppose we have a pool of  $T$  binary classifiers  $\mathcal{A} = \{h_1, \dots, h_T\}$  where  $h_t : \mathbb{R}^M \rightarrow \{+1, -1\}$ . Let  $\alpha_t$  be the weight for classifier  $h_t$ .

---

## Algorithm 1 Weighted Majority Algorithm

---

- 1: **procedure** WEIGHTEDMAJORITY( $\mathcal{A}, \beta$ )
- 2:     Initialize classifier weights  $\alpha_t = 1, \forall t \in \{1, \dots, T\}$
- 3:     **for** each training example  $(\mathbf{x}, y)$  **do**
- 4:         Predict majority vote class (splitting ties randomly)

$$\hat{h}(x) = \text{sign} \left( \sum_{t=1}^T \alpha_t h_t(x) \right)$$

- 5:         **if** a mistake is made  $\hat{h}(x) \neq y$  **then**
  - 6:             **for** each classifier  $t \in \{1, \dots, T\}$  **do**
  - 7:                 **if**  $h_t(x) \neq y$ , then  $\alpha_t \leftarrow \beta \alpha_t$
-

# Weighted Majority Algorithm

## Theorems (Littlestone & Warmuth, 1994)

For the general case where  $WM$  is applied to a pool  $\mathcal{A}$  of algorithms we show the following upper bounds on the number of mistakes made in a given sequence of trials:

1.  $O(\log |\mathcal{A}| + m)$ , if one algorithm of  $\mathcal{A}$  makes at most  $m$  mistakes.
2.  $O(\log \frac{|\mathcal{A}|}{k} + m)$ , if each of a subpool of  $k$  algorithms of  $\mathcal{A}$  makes at most  $m$  mistakes.
3.  $O(\log \frac{|\mathcal{A}|}{k} + \frac{m}{k})$ , if the total number of mistakes of a subpool of  $k$  algorithms of  $\mathcal{A}$  is at most  $m$ .



These are  
“mistake  
bounds” of the  
variety we saw  
for the  
Perceptron  
algorithm

# ADABOOST

# Comparison

## **Weighted Majority Algorithm**

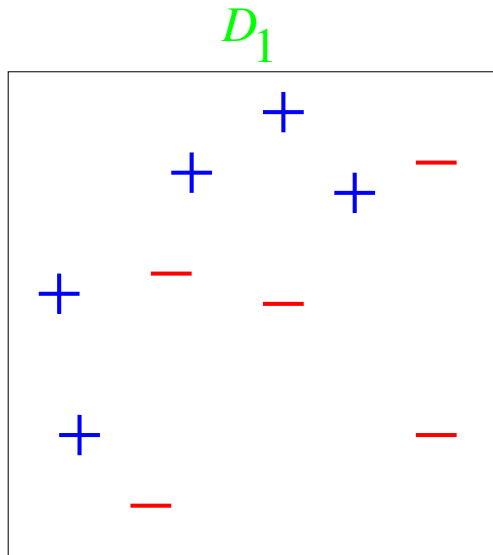
- an example of an ensemble method
- assumes the classifiers are learned ahead of time
- only learns (majority vote) weight for each classifiers

## **AdaBoost**

- an example of a boosting method
- simultaneously learns:
  - the classifiers themselves
  - (majority vote) weight for each classifiers

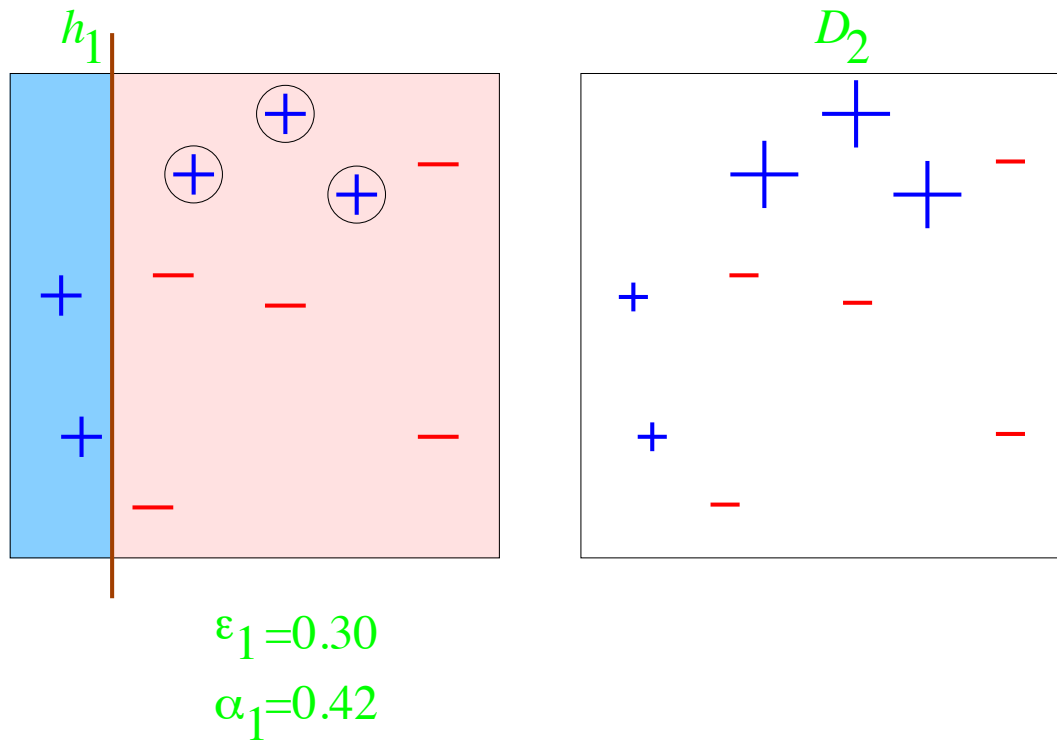


# AdaBoost: Toy Example

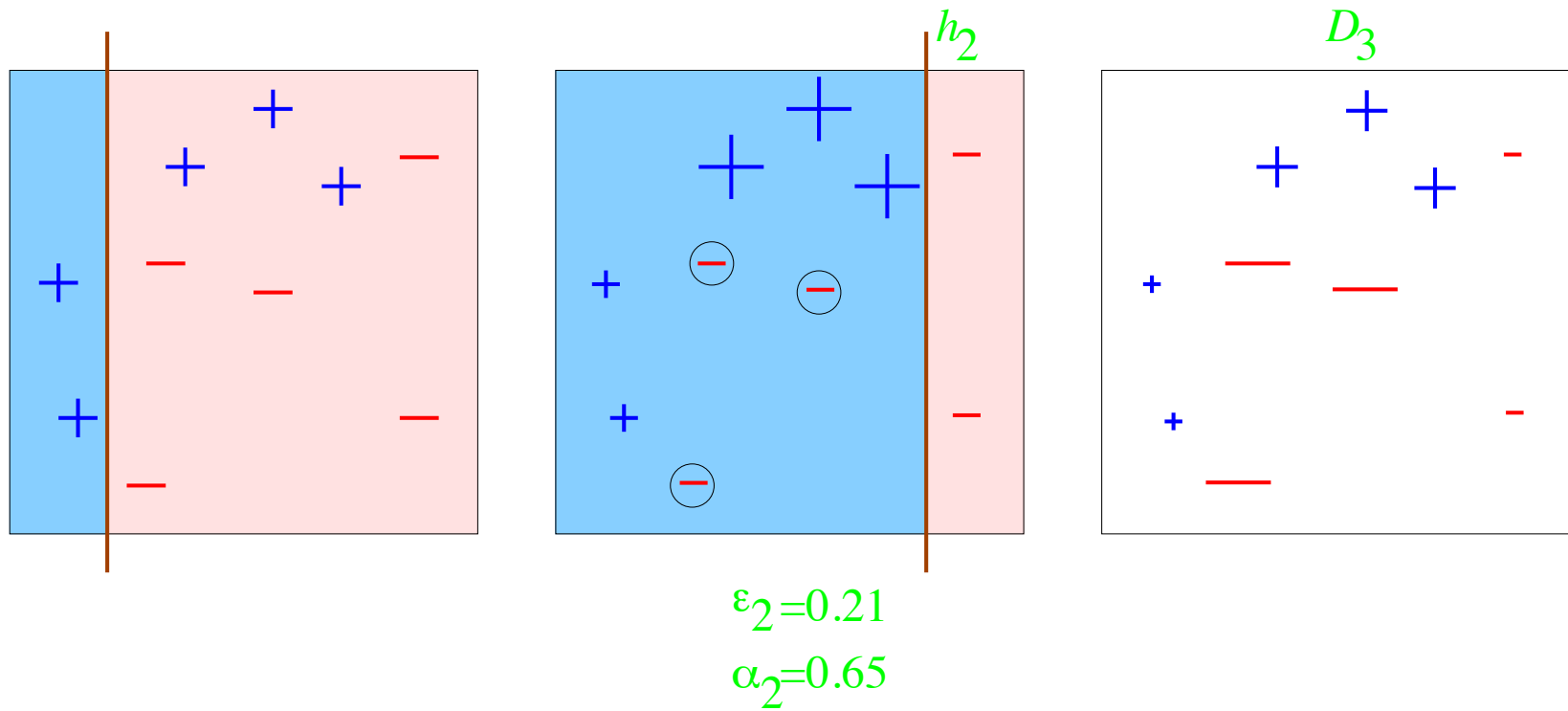


weak classifiers = vertical or horizontal half-planes

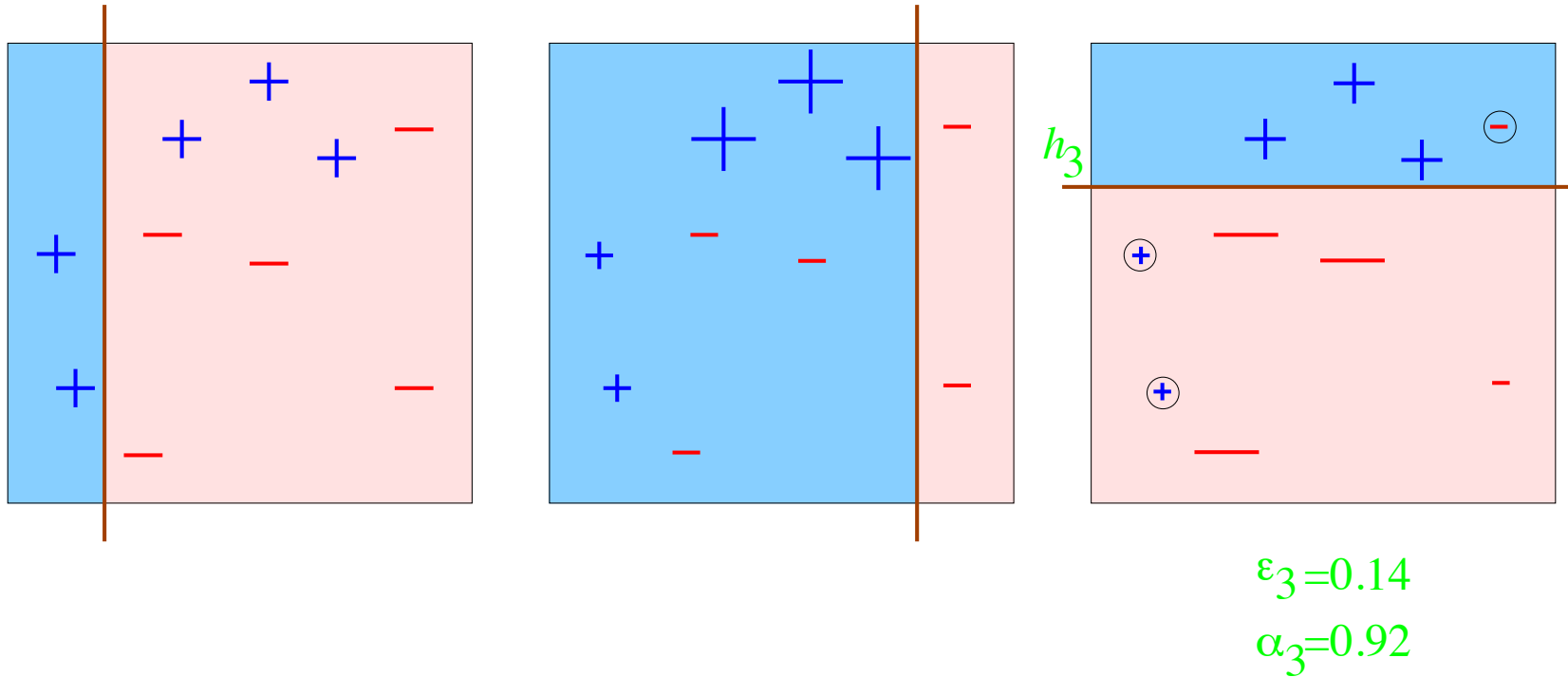
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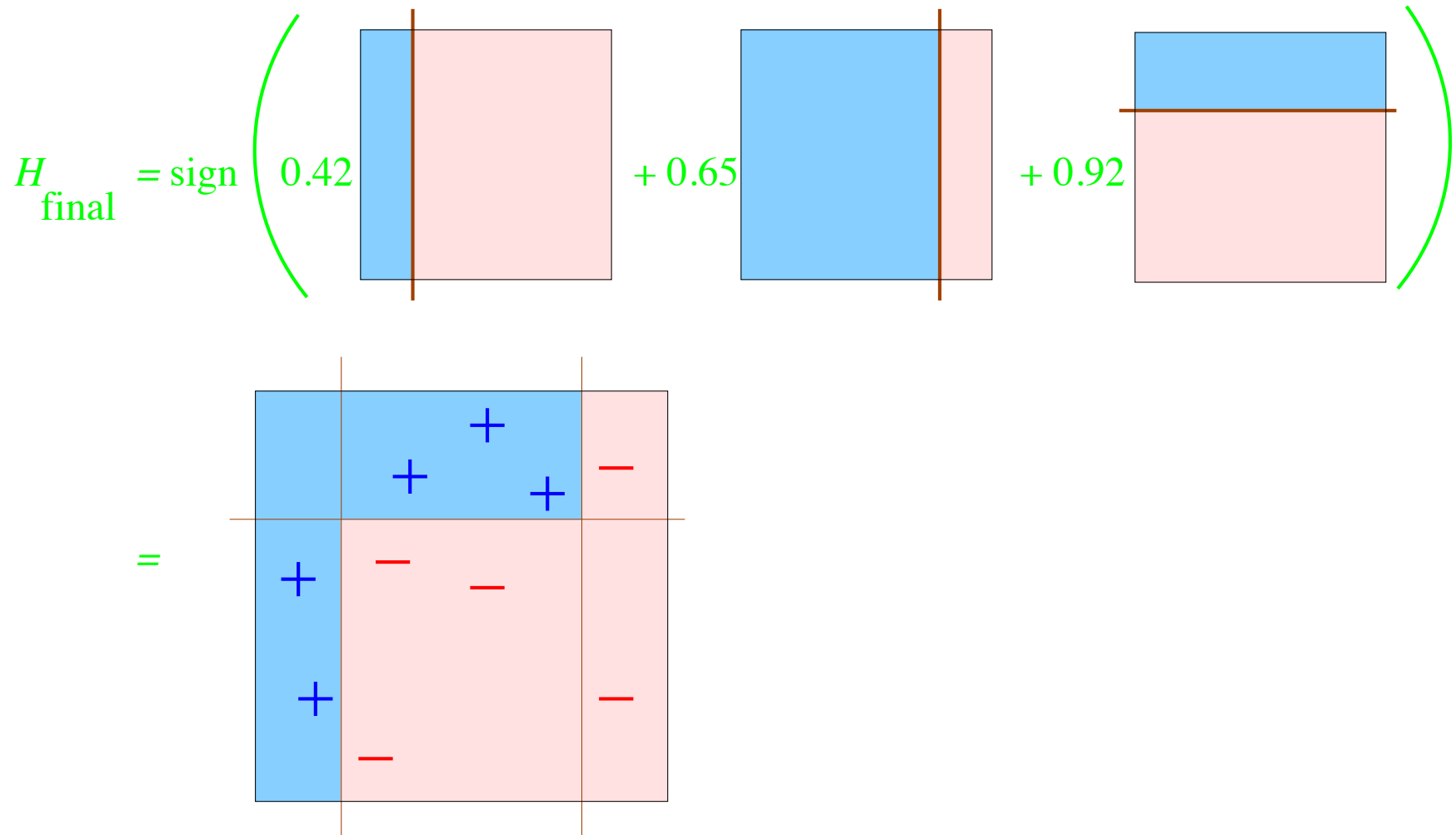
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# AdaBoost: Toy Example



# AdaBoost: Toy Example



# AdaBoost

Given:  $(x_1, y_1), \dots, (x_m, y_m)$  where  $x_i \in X, y_i \in Y = \{-1, +1\}$

Initialize  $D_1(i) = 1/m$ .

For  $t = 1, \dots, T$ :

- Train weak learner using distribution  $D_t$ .
- Get weak hypothesis  $h_t : X \rightarrow \{-1, +1\}$  with error

$$\epsilon_t = \Pr_{i \sim D_t} [h_t(x_i) \neq y_i].$$

- Choose  $\alpha_t = \frac{1}{2} \ln \left( \frac{1 - \epsilon_t}{\epsilon_t} \right)$ .
- Update:

$$\begin{aligned} D_{t+1}(i) &= \frac{D_t(i)}{Z_t} \times \begin{cases} e^{-\alpha_t} & \text{if } h_t(x_i) = y_i \\ e^{\alpha_t} & \text{if } h_t(x_i) \neq y_i \end{cases} \\ &= \frac{D_t(i) \exp(-\alpha_t y_i h_t(x_i))}{Z_t} \end{aligned}$$

where  $Z_t$  is a normalization factor (chosen so that  $D_{t+1}$  will be a distribution).

Output the final hypothesis:

$$H(x) = \text{sign} \left( \sum_{t=1}^T \alpha_t h_t(x) \right).$$

# AdaBoost

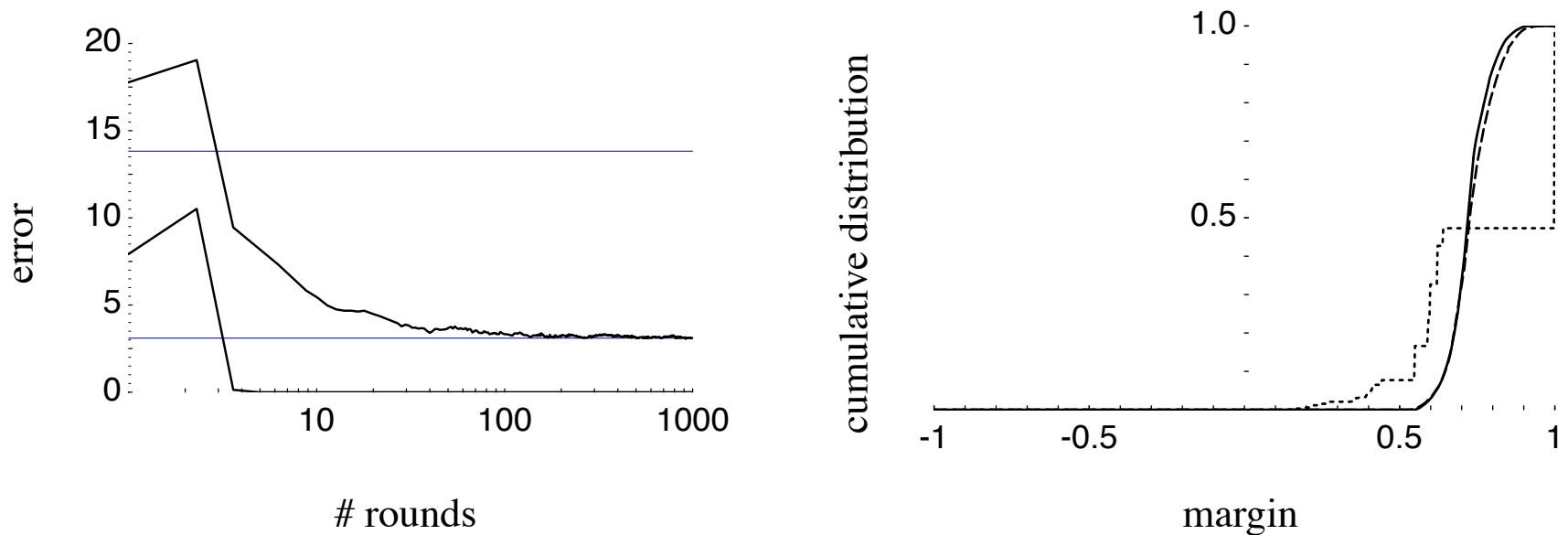


Figure 2: Error curves and the margin distribution graph for boosting C4.5 on the letter dataset as reported by Schapire et al. [41]. *Left*: the training and test error curves (lower and upper curves, respectively) of the combined classifier as a function of the number of rounds of boosting. The horizontal lines indicate the test error rate of the base classifier as well as the test error of the final combined classifier. *Right*: The cumulative distribution of margins of the training examples after 5, 100 and 1000 iterations, indicated by short-dashed, long-dashed (mostly hidden) and solid curves, respectively.

# Learning Objectives

## **Ensemble Methods / Boosting**

*You should be able to...*

1. Implement the Weighted Majority Algorithm
2. Implement AdaBoost
3. Distinguish what is learned in the Weighted Majority Algorithm vs. Adaboost
4. Contrast the theoretical result for the Weighted Majority Algorithm to that of Perceptron
5. Explain a surprisingly common empirical result regarding Adaboost train/test curves

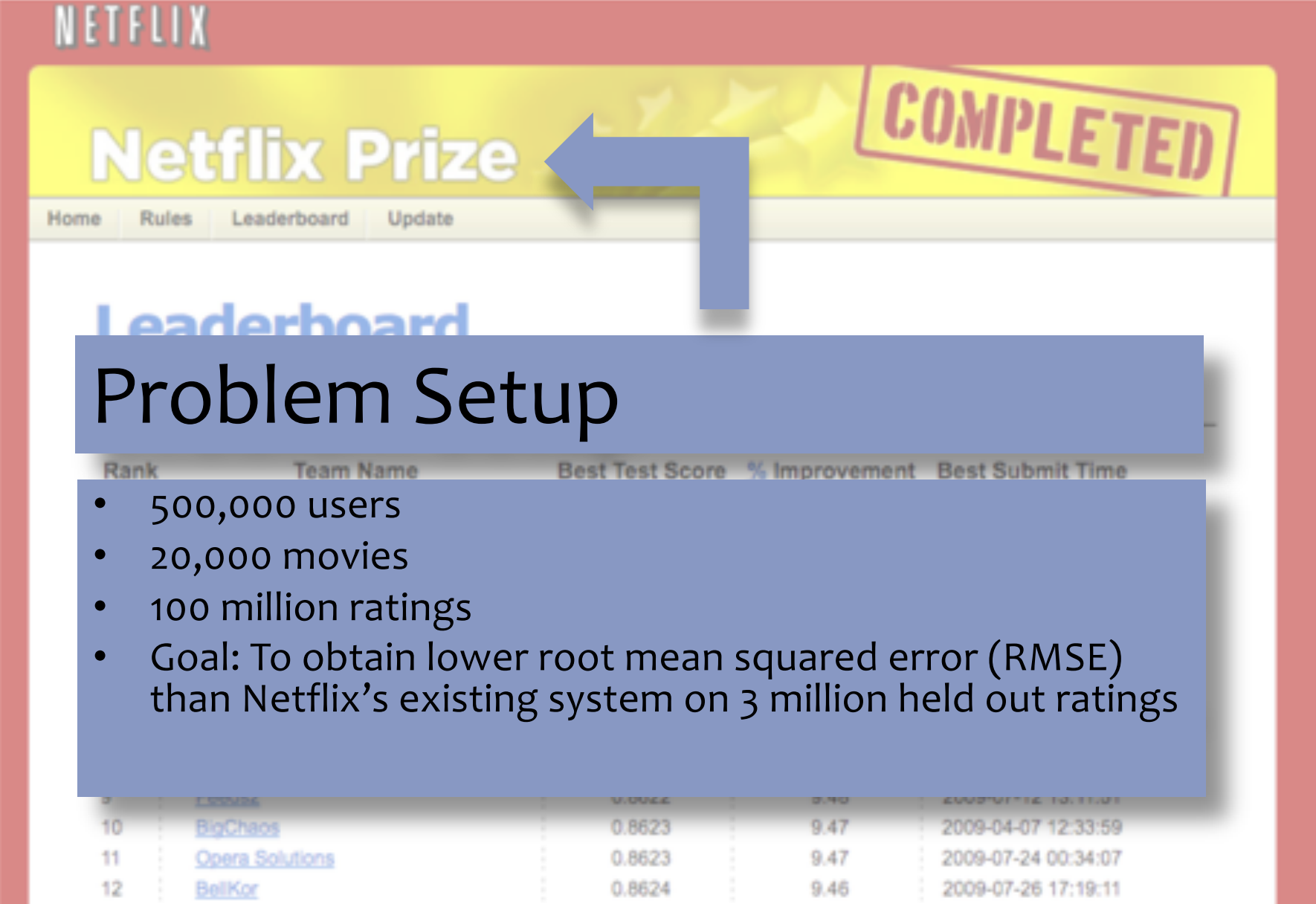


# Outline

- **Recommender Systems**
  - Content Filtering
  - Collaborative Filtering (CF)
  - CF: Neighborhood Methods
  - CF: Latent Factor Methods
- **Matrix Factorization**
  - Background: Low-rank Factorizations
  - Residual matrix
  - Unconstrained Matrix Factorization
    - Optimization problem
    - Gradient Descent, SGD, Alternating Least Squares
    - User/item bias terms (matrix trick)
  - Singular Value Decomposition (SVD)
  - Non-negative Matrix Factorization

# **RECOMMENDER SYSTEMS**

# Recommender Systems



The image shows a screenshot of the Netflix Prize website. At the top, the word 'NETFLIX' is visible in the upper left corner. Below it, a yellow banner contains the text 'Netflix Prize' and a 'COMPLETED' stamp. A blue arrow points from the 'COMPLETED' stamp towards the 'Netflix Prize' text. Below the banner is a navigation menu with links for 'Home', 'Rules', 'Leaderboard', and 'Update'. The main content area is titled 'Leaderboard' and features a table with the following columns: Rank, Team Name, Best Test Score, % Improvement, and Best Submit Time. A large blue box is overlaid on the page, containing the text 'Problem Setup' and a bulleted list of details about the competition.

## Problem Setup

- 500,000 users
- 20,000 movies
- 100 million ratings
- Goal: To obtain lower root mean squared error (RMSE) than Netflix's existing system on 3 million held out ratings

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
10	<a href="#">BigChaos</a>	0.8623	9.47	2009-04-07 12:33:59
11	<a href="#">Opera Solutions</a>	0.8623	9.47	2009-07-24 00:34:07
12	<a href="#">BellKor</a>	0.8624	9.46	2009-07-26 17:19:11

# Recommender Systems

**NETFLIX**

## Netflix Prize

**COMPLETED**

Home Rules Leaderboard Update

### Leaderboard

Showing Test Score. [Click here to show quiz score](#)

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	<a href="#">BellKor's Pragmatic Chaos</a>	0.8567	10.06	2009-07-26 18:18:28
2	<a href="#">The Ensemble</a>	0.8567	10.06	2009-07-26 18:38:22
3	<a href="#">Grand Prize Team</a>	0.8582	9.90	2009-07-10 21:24:40
4	<a href="#">Opera Solutions and Vandelay United</a>	0.8588	9.84	2009-07-10 01:12:31
5	<a href="#">Vandelay Industries I</a>	0.8591	9.81	2009-07-10 00:32:20
6	<a href="#">PragmaticTheory</a>	0.8594	9.77	2009-06-24 12:06:56
7	<a href="#">BellKor in BigChaos</a>	0.8601	9.70	2009-05-13 08:14:09
8	<a href="#">Dace_</a>	0.8612	9.59	2009-07-24 17:18:43
9	<a href="#">Feeds2</a>	0.8622	9.48	2009-07-12 13:11:51
10	<a href="#">BigChaos</a>	0.8623	9.47	2009-04-07 12:33:59
11	<a href="#">Opera Solutions</a>	0.8623	9.47	2009-07-24 00:34:07
12	<a href="#">BellKor</a>	0.8624	9.46	2009-07-26 17:19:11

# Recommender Systems

- **Setup:**

- **Items:**

- movies, songs, products, etc. (often many thousands)

- **Users:**

- watchers, listeners, purchasers, etc. (often many millions)

- **Feedback:**

- 5-star ratings, not-clicking 'next', purchases, etc.

- **Key Assumptions:**

- Can represent ratings numerically as a user/item matrix

- Users only rate a small number of items (the matrix is sparse)

	Doctor Strange	Star Trek: Beyond	Zootopia
Alice	1		5
Bob	3	4	
Charlie	3	5	2

# Two Types of Recommender Systems

## Content Filtering

- *Example:* **Pandora.com** music recommendations (Music Genome Project)
- **Con:** Assumes access to **side information** about items (e.g. properties of a song)
- **Pro:** Got a **new item** to add? No problem, just be sure to include the side information

## Collaborative Filtering

- *Example:* **Netflix** movie recommendations
- **Pro:** Does not assume access to **side information** about items (e.g. does not need to know about movie genres)
- **Con:** Does not work on **new items** that have no ratings

# **COLLABORATIVE FILTERING**

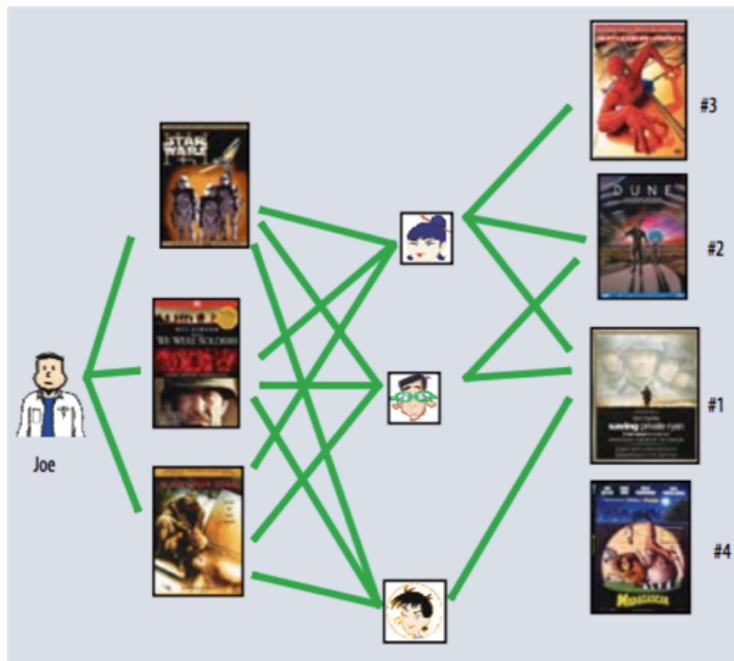
# Collaborative Filtering

- **Everyday Examples of Collaborative Filtering...**
  - Bestseller lists
  - Top 40 music lists
  - The “recent returns” shelf at the library
  - Unmarked but well-used paths thru the woods
  - The printer room at work
  - “Read any good books lately?”
  - ...
- **Common insight:** personal tastes are correlated
  - If Alice and Bob both like X and Alice likes Y then Bob is more likely to like Y
  - especially (perhaps) if Bob knows Alice

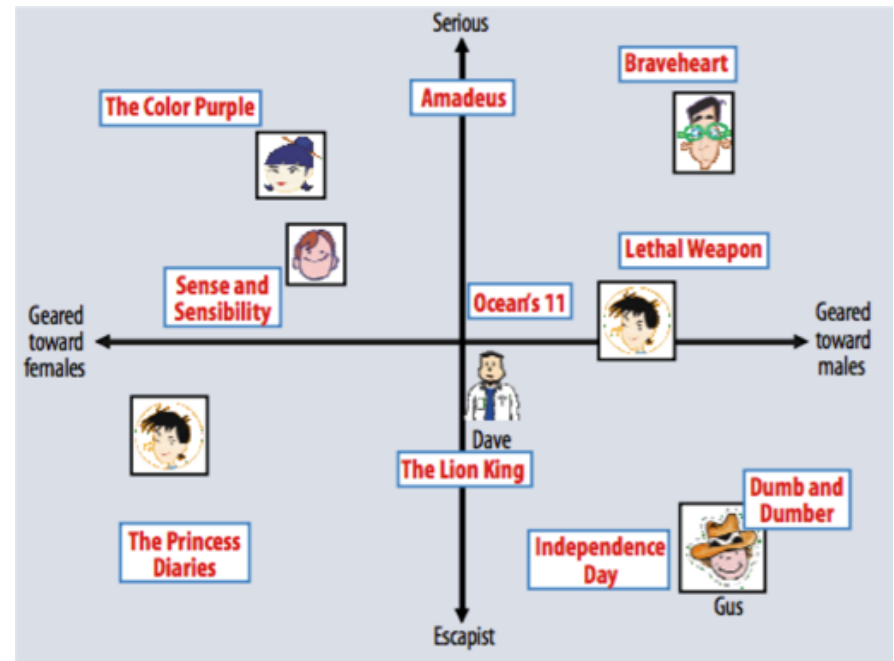


# Two Types of Collaborative Filtering

## 1. Neighborhood Methods

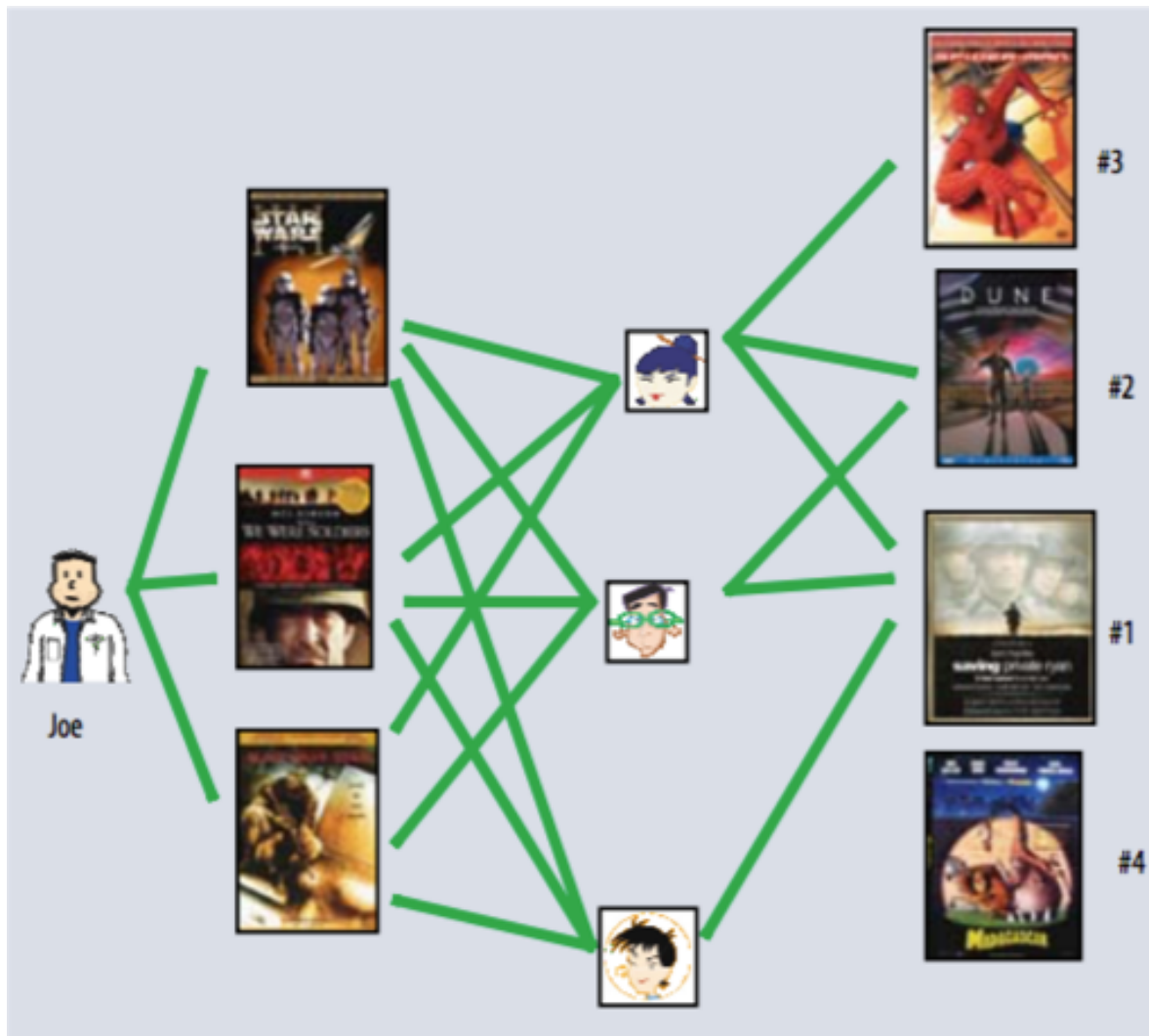


## 2. Latent Factor Methods



# Two Types of Collaborative Filtering

## 1. Neighborhood Methods



In the figure, assume that a green line indicates the movie was **watched**

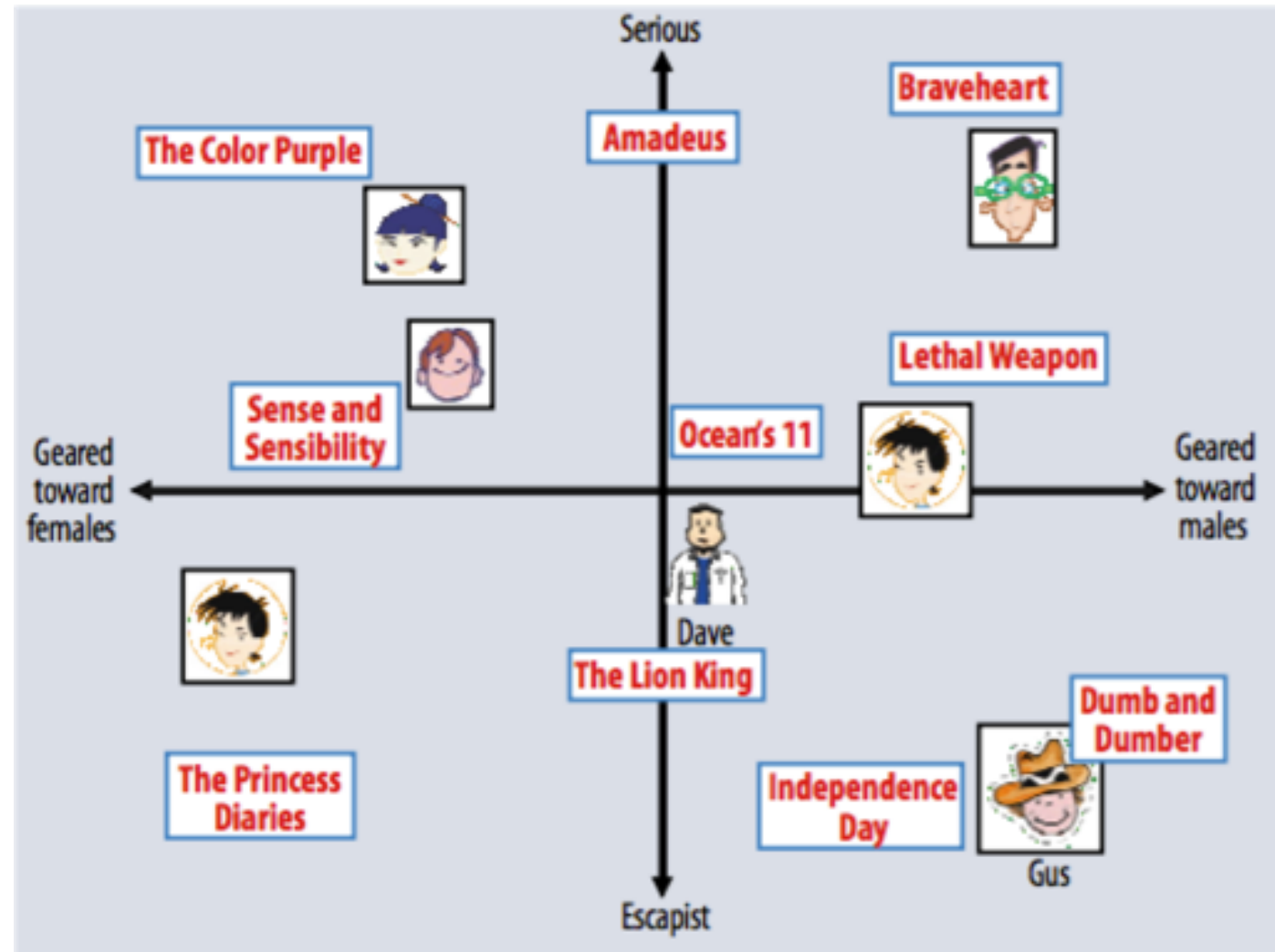
### Algorithm:

1. **Find neighbors** based on similarity of movie preferences
2. **Recommend** movies that those neighbors watched

# Two Types of Collaborative Filtering

## 2. Latent Factor Methods

- Assume that both movies and users live in some **low-dimensional space** describing their properties
- **Recommend** a movie based on its **proximity** to the user in the latent space
- **Example Algorithm:** Matrix Factorization



# Recommending Movies

## Question:

Applied to the Netflix Prize problem, which of the following methods *always* requires side information about the users and movies?

### Select all that apply

- A. collaborative filtering
- B. latent factor methods
- C. ensemble methods
- D. content filtering
- E. neighborhood methods
- F. recommender systems

## Answer:

# **MATRIX FACTORIZATION**

# Matrix Factorization

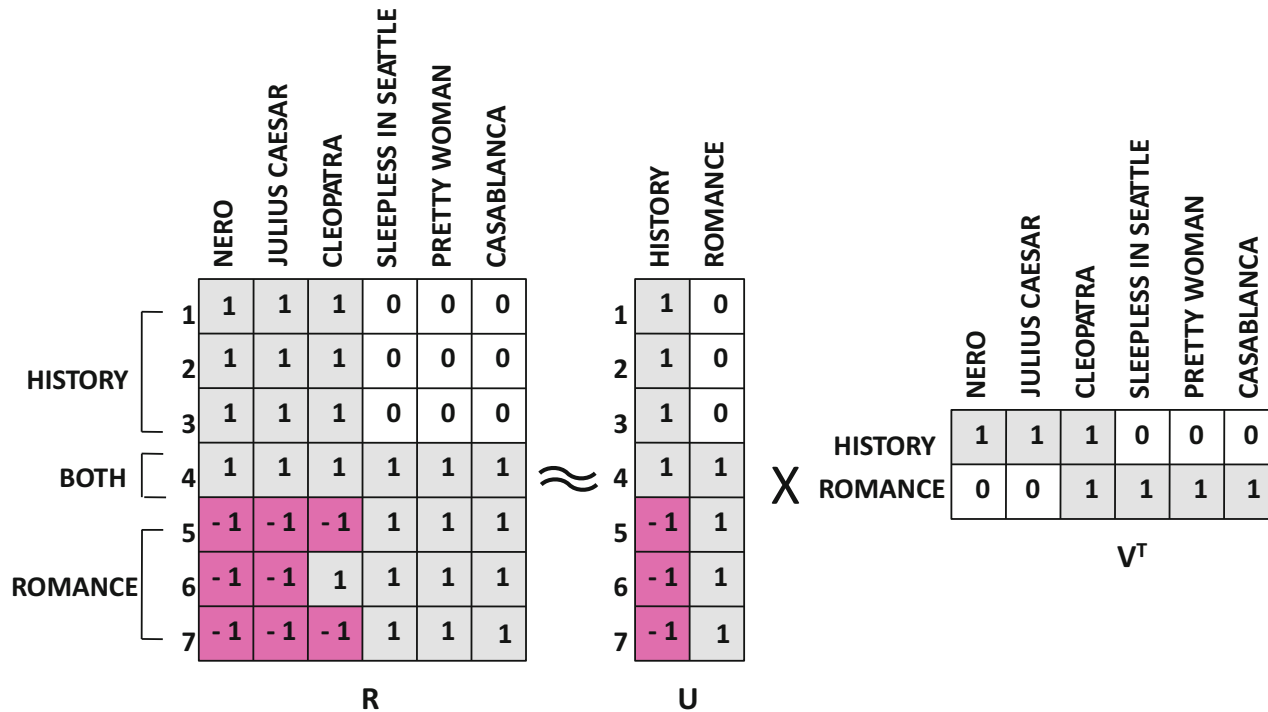
- Many different ways of factorizing a matrix
- We'll consider three:
  1. Unconstrained Matrix Factorization
  2. Singular Value Decomposition
  3. Non-negative Matrix Factorization
- MF is just another example of a **common recipe**:
  1. define a model
  2. define an objective function
  3. optimize with SGD

# Matrix Factorization

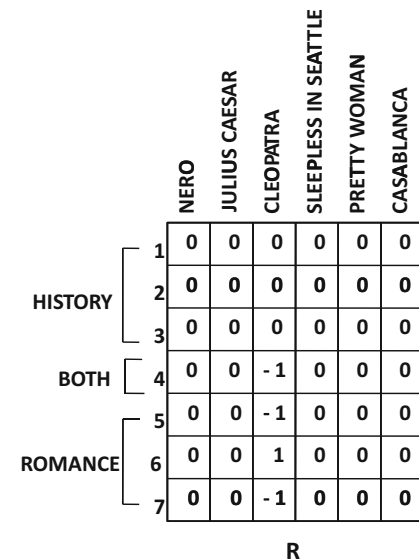
## *Whiteboard*

- Background: Low-rank Factorizations
- Residual matrix

# Example: MF for Netflix Problem



(a) Example of rank-2 matrix factorization

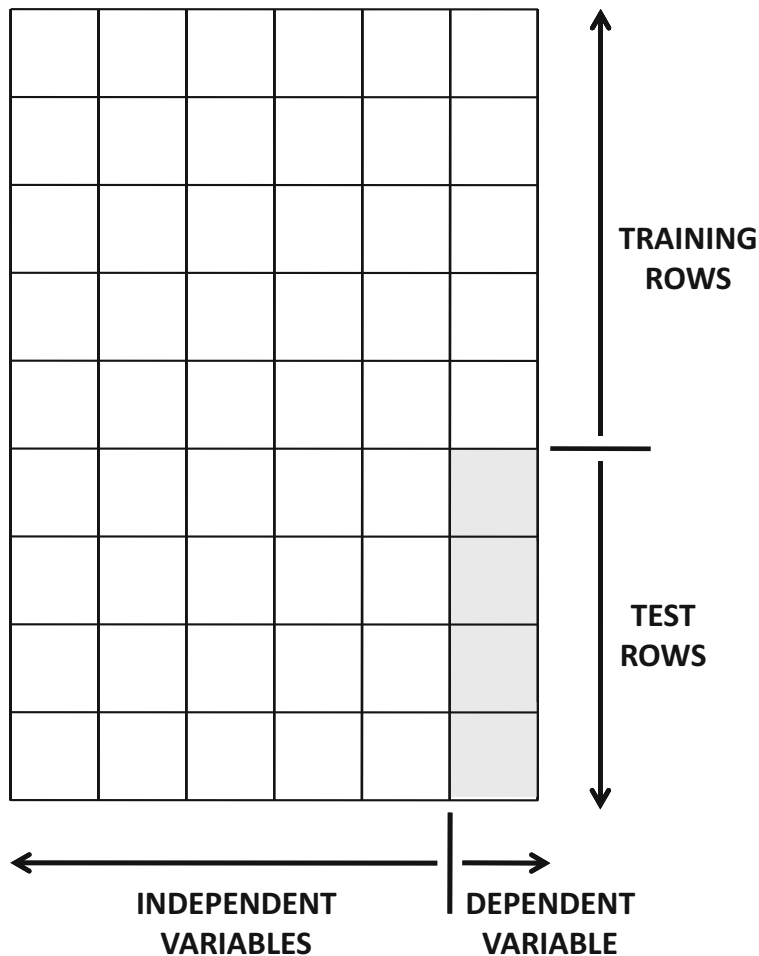


(b) Residual matrix

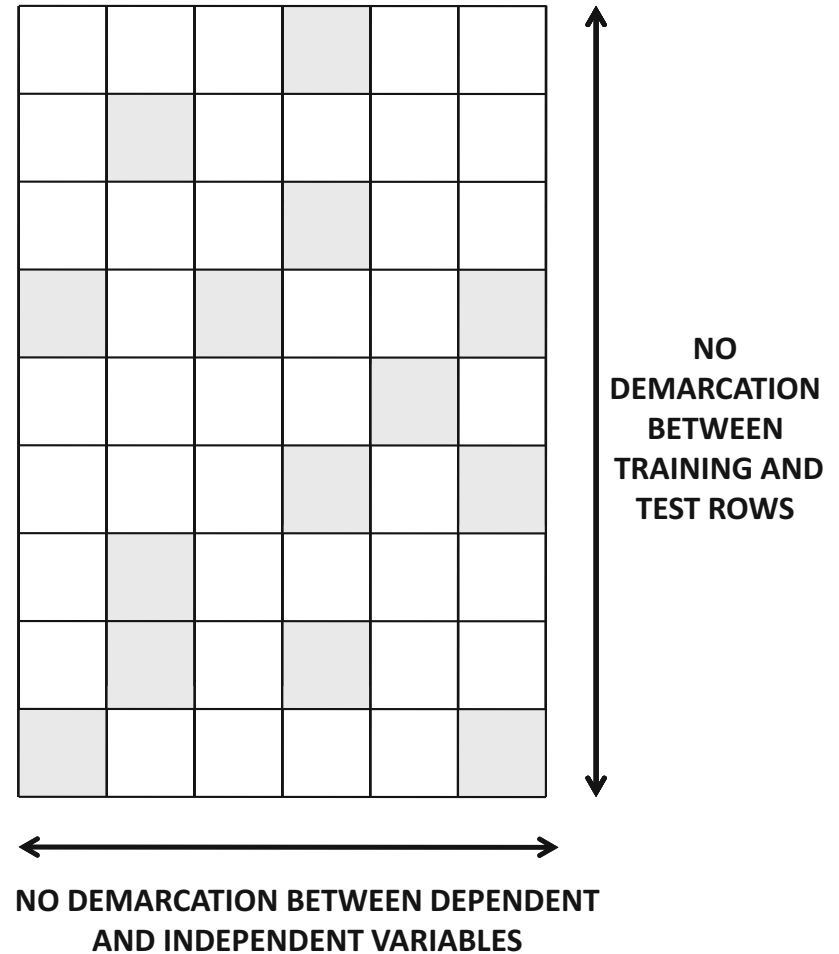


# Regression vs. Collaborative Filtering

## Regression



## Collaborative Filtering



# **UNCONSTRAINED MATRIX FACTORIZATION**

# Unconstrained Matrix Factorization

## *Whiteboard*

- Optimization problem
- SGD
- SGD with Regularization
- Alternating Least Squares
- User/item bias terms (matrix trick)

# Unconstrained Matrix Factorization

SGD for UMF:

While not converged:

① Sample  $(i, j)$  from  $Z$  uniformly at random

② Compute  $e_{ij} = r_{ij} - \vec{u}_i^T \vec{v}_j$

③ Update

$$\vec{u}_i \leftarrow \vec{u}_i - \gamma \nabla_{\vec{u}_i} J_{ij}(U, V)$$
$$\vec{v}_j \leftarrow \vec{v}_j - \gamma \nabla_{\vec{v}_j} J_{ij}(U, V)$$

W/Regularization

$$J_{ij}(U, V) = \frac{1}{2} (r_{ij} - \vec{u}_i^T \vec{v}_j)^2 + \lambda (\|\vec{u}_i\|_2^2 + \|\vec{v}_j\|_2^2)$$

$$\nabla_{\vec{u}_i} J_{ij}(U, V) = -e_{ij} \vec{v}_j + \lambda \vec{u}_i$$

$$\nabla_{\vec{v}_j} J_{ij}(U, V) = -e_{ij} \vec{u}_i + \lambda \vec{v}_j$$

where  $e_{ij} = r_{ij} - \vec{u}_i^T \vec{v}_j$

# Unconstrained Matrix Factorization

SGD for UMF:

User/Item Bias terms

$$\hat{r}_{ij} = o_i + p_j + \vec{U}_i^T \vec{V}_j$$

matrix trick:

$$U = \begin{bmatrix} | & | & | & | \\ o_1 & o_2 & \dots & o_n \\ | & | & | & | \end{bmatrix}$$

$$V = \begin{bmatrix} | & | & | & | \\ p_1 & p_2 & \dots & p_n \\ | & | & | & | \end{bmatrix}$$

# Unconstrained Matrix Factorization

## Alternating Least Squares (ALS) for UMF:

Block Coord. Descent:

While not converged:

$$\textcircled{1} U = \underset{U}{\operatorname{argmin}} J(U, V)$$

$$\textcircled{2} V = \underset{V}{\operatorname{argmin}} J(U, V)$$

convex and easy to solve in closed form

$$J(U, V) = \frac{1}{2} \sum_{(i,j) \in \mathcal{Z}} (r_{ij} - \vec{u}_i^T \vec{v}_j)^2$$

if  $U$  is fixed  $\rightarrow$  Least Squares in  $V$   
if  $V$  is fixed  $\rightarrow$  Least Squares in  $U$

Lin. Reg.

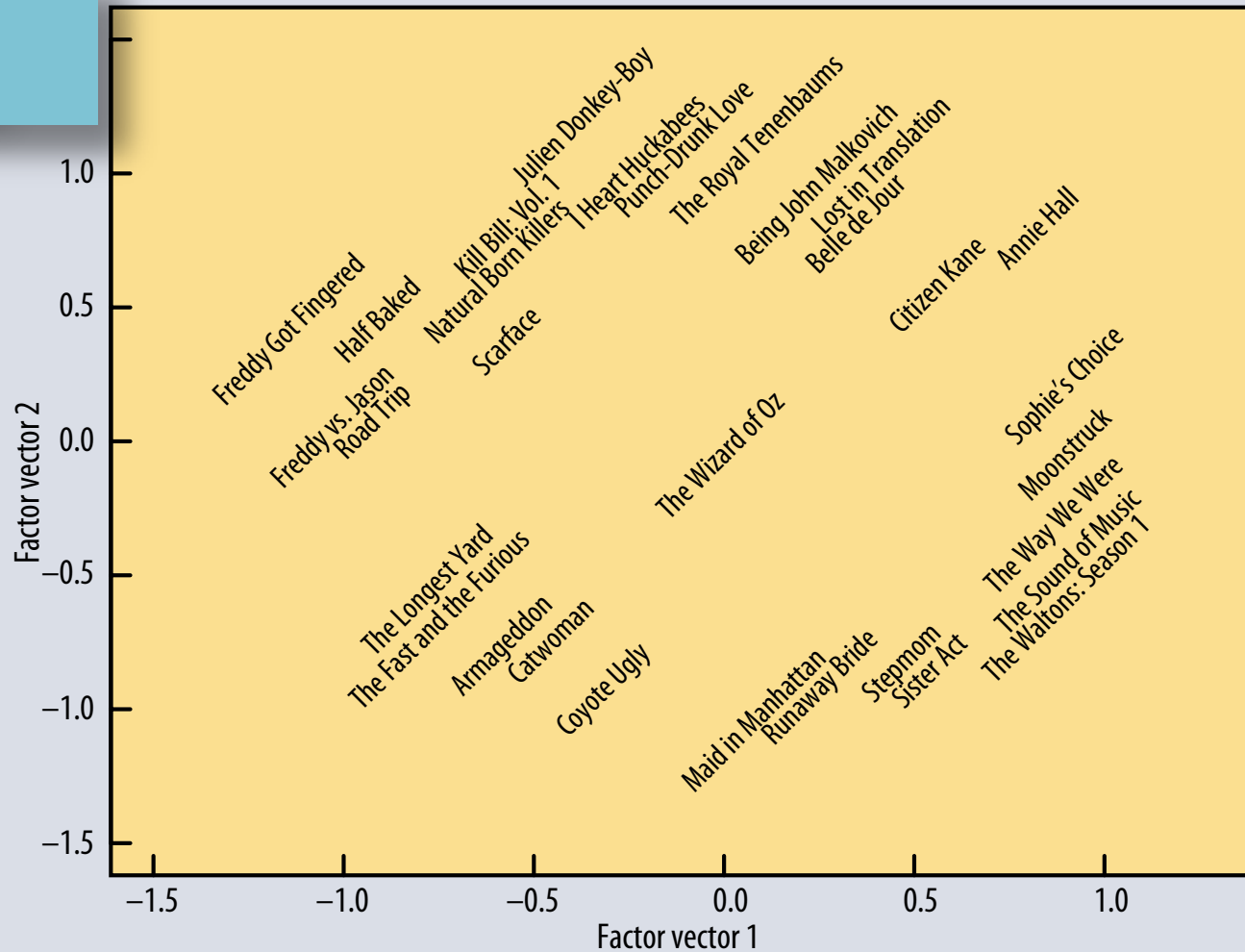
$$J(\Theta) = \frac{1}{2} \sum_{i=1}^N (y_i - \vec{\Theta}^T \vec{x}_i)^2$$

Option #1: take derivatives, set to zero and solve in closed form

★ solving  $J(U, V)$  in closed form directly isn't easy and  $J(U, V)$  is nonconvex

# Matrix Factorization

## Example Factors



**Figure 3.** The first two vectors from a matrix decomposition of the Netflix Prize data. Selected movies are placed at the appropriate spot based on their factor vectors in two dimensions. The plot reveals distinct genres, including clusters of movies with strong female leads, fraternity humor, and quirky independent films.

# Matrix Factorization

## Comparison of Optimization Algorithms

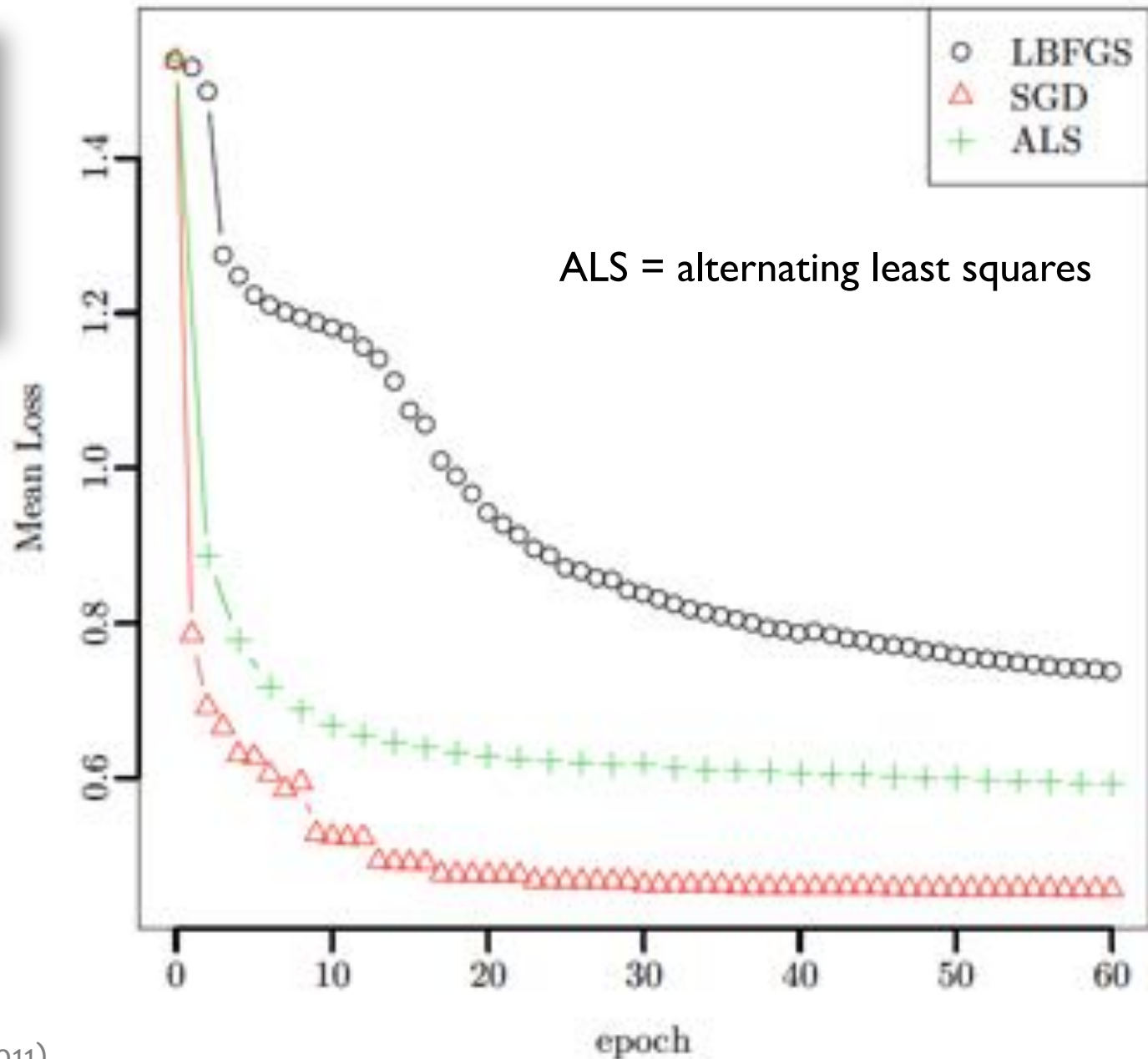


Figure from Gemulla et al. (2011)



# **SVD FOR COLLABORATIVE FILTERING**

# Singular Value Decomposition for Collaborative Filtering

For any arbitrary matrix  $\mathbf{A}$ , SVD gives a decomposition:

$$\mathbf{A} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^T$$

where  $\mathbf{\Lambda}$  is a diagonal matrix, and  $\mathbf{U}$  and  $\mathbf{V}$  are orthogonal matrices.

Suppose we have the SVD of our ratings matrix

$$R = Q\Sigma P^T,$$

but then we truncate each of  $Q$ ,  $\Sigma$ , and  $P$  s.t.  $Q$  and  $P$  have only  $k$  columns and  $\Sigma$  is  $k \times k$ :

$$R \approx Q_k \Sigma_k P_k^T$$

For collaborative filtering, let:

$$U \triangleq Q_k \Sigma_k$$

$$V \triangleq P_k$$

$$\Rightarrow U, V = \operatorname{argmin}_{U, V} \frac{1}{2} \|R - UV^T\|_2^2$$

s.t. columns of  $U$  are mutually orthogonal

s.t. columns of  $V$  are mutually orthogonal

**Theorem:** If  $R$  fully observed and no regularization, the optimal  $UV^T$  from SVD equals the optimal  $UV^T$  from Unconstrained MF

# **NON-NEGATIVE MATRIX FACTORIZATION**

# Implicit Feedback Datasets

- What information does a five-star rating contain?



- Implicit Feedback Datasets:
  - In many settings, users don't have a way of expressing *dislike* for an item (e.g. can't provide negative ratings)
  - The only mechanism for feedback is to “like” something
- Examples:
  - Facebook has a “Like” button, but no “Dislike” button
  - Google's “+1” button
  - Pinterest pins
  - Purchasing an item on Amazon indicates a preference for it, but there are many reasons you might *not* purchase an item (besides dislike)
  - Search engines collect click data but don't have a clear mechanism for observing dislike of a webpage

# Non-negative Matrix Factorization

**Constrained Optimization Problem:**

$$U, V = \operatorname{argmin}_{U, V} \frac{1}{2} \|R - UV^T\|_2^2$$

$$\text{s.t. } U_{ij} \geq 0$$

$$\text{s.t. } V_{ij} \geq 0$$

**Multiplicative Updates:** simple iterative algorithm for solving just involves multiplying a few entries together

# Fighting Fire with Fire: Using Antidote Data to Improve Polarization and Fairness of Recommender Systems

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where  $S_j = \sum_{i \in \Omega_j} u_i u_i^T + \hat{U}\hat{U}^T + \lambda I_d$ .

By using (9) instead of the general formula in (5) we can significantly reduce the number of computations required for finding the gradient of the utility function with respect to the antidote data. Furthermore, the term  $g_j^T U^T S_j^{-1}$  appears in all the partial derivatives that correspond to elements in column  $j$  of  $\hat{X}$  and can be precomputed in each iteration of the algorithm and reused for computing partial derivatives with respect to different antidote users.

## 5 SOCIAL OBJECTIVE FUNCTIONS

The previous section developed a general framework for improving various properties of recommender systems; in this section we show how to apply that framework specifically to issues of polarization and fairness.

As described in Section 2, polarization is the degree to which opinions, views, and sentiments diverge within a population. Recommender systems can capture this effect through the ratings that they present for items. To formalize this notion, we define polarization in terms of the variability of predicted ratings when compared across users. In fact, we note that both very high variability, and very low variability of ratings may be undesirable. In the case of high variability, users have strongly divergent opinions, leading to conflict. Recent analyses of the YouTube recommendation system have suggested that it can enhance this effect [29, 30]. On the other hand, the convergence of user preferences, i.e., very low variability of ratings given to each item across users, corresponds to increased homogeneity, an undesirable phenomenon that may occur as users interact with a recommender system [11]. As a result, in what follows we consider using antidote data in both ways: to either increase or decrease polarization.

As also described in Section 2, unfairness is a topic of growing interest in machine learning. Following the discussion in that section, we consider a recommender system fair if it provides equal quality of service (i.e., prediction accuracy) to all users or all groups of users [36].

Next we formally define the metrics that specify the objective functions associated with each of the above objectives. Since the gradient of each objective function is used in the optimization algorithm, for reproducibility we provide the details about derivation of the gradients in appendix A.2.

### 5.1 Polarization

To capture polarization, we seek to measure the extent to which the user ratings disagree. Thus, to measure user polarization we consider the estimated ratings  $\hat{X}$ , and we define the polarization metric as the normalized sum of pairwise euclidean distances between estimated user ratings, i.e., between rows of  $\hat{X}$ . In particular:

$$R_{\text{pol}}(\hat{X}) = \frac{1}{n^2 d} \sum_{k=1}^n \sum_{l>k}^n \|\hat{x}^k - \hat{x}^l\|^2 \quad (10)$$

The normalization term  $\frac{1}{n^2 d}$  in (10) makes the polarization metric identical to the following definition:<sup>4</sup>

$$R_{\text{pol}}(\hat{X}) = \frac{1}{d} \sum_{j=1}^d \sigma_j^2 \quad (11)$$

where  $\sigma_j^2$  is the variance of estimated user ratings for item  $j$ . Thus this polarization metric can be interpreted either as the average of the variances of estimated ratings in each item, or equivalently as the average user disagreement over all items.

### 5.2 Fairness

**Individual fairness.** For each user  $i$ , we define  $\ell_i$ , the loss of user  $i$ , as the mean squared estimation error over known ratings of user  $i$ :

$$\ell_i = \frac{\|P_{\Omega_i}(\hat{X} - \mathbf{x}^i)\|_2^2}{|\Omega_i|} \quad (12)$$

Then we define the individual unfairness as the variance of the user losses:<sup>5</sup>

$$R_{\text{indv}}(\mathbf{X}, \hat{X}) = \frac{1}{n^2} \sum_{k=1}^n \sum_{l>k}^n (\ell_k - \ell_l)^2 \quad (13)$$

To improve individual fairness, we seek to minimize  $R_{\text{indv}}$ .

**Group fairness.** Let  $I$  be the set of all users/items and  $G = \{G_1, \dots, G_g\}$  be a partition of users/items into  $g$  groups, i.e.,  $I = \bigcup_{i \in \{1, \dots, g\}} G_i$ . We define the loss of group  $i$  as the mean squared estimation error over all known ratings in group  $i$ :

$$L_i = \frac{\|P_{\Omega_i}(\hat{X} - \mathbf{X})\|_2^2}{|\Omega_i|} \quad (14)$$

For a given partition  $G$ , we define the group unfairness as the variance of all group losses:

$$R_{\text{grp}}(\mathbf{X}, \hat{X}, G) = \frac{1}{g^2} \sum_{k=1}^g \sum_{l>k}^g (L_k - L_l)^2 \quad (15)$$

Again, to improve group fairness, we seek to minimize  $R_{\text{grp}}$ .

### 5.3 Accuracy vs. Social Welfare

Adding antidote data to the system to improve a social utility will also have an effect on the overall prediction accuracy. Previous works have considered social objectives as regularizers or constraints added to the recommender model (eg. [8, 25, 37]), implying a trade-off between the prediction accuracy and a social objective.

However, in the case of the metrics we define here, the relationship is not as simple. Considering polarization, we find that in general, increasing or decreasing polarization will tend to decrease system accuracy. In either case we find that system accuracy only declines slightly in our experiments; we report on the specific values in Section 6. Considering either individual or group unfairness, the situation is more subtle. Note that our unfairness metrics will be exactly zero for a system with zero error (perfect accuracy). As a

<sup>4</sup>We can derive it by rewriting (10) as  $R_{\text{pol}}(\hat{X}) = \frac{1}{d} \sum_{j=1}^d \frac{1}{n^2} \sum_{k=1}^n \sum_{l>k}^n (x_{kj} - x_{lj})^2$ .

<sup>5</sup>Note that for a set of equally likely values  $x_1, \dots, x_n$  the variance can be expressed without referring to the mean as:  $\frac{1}{n} \sum_{i=1}^n \sum_{j=1}^n (x_i - x_j)^2$ .

# Summary

- Recommender systems solve many **real-world** (\*large-scale) **problems**
- Collaborative filtering by Matrix Factorization (MF) is an **efficient** and **effective** approach
- MF is just another example of a **common recipe**:
  1. define a model
  2. define an objective function
  3. optimize with your favorite black box optimizer (e.g. SGD, Gradient Descent, Block Coordinate Descent aka. Alternating Least Squares)

**EXTRA SLIDES ON UMF**



# Unconstrained Matrix Factorization

## In-Class Exercise

Derive a block coordinate descent algorithm for the Unconstrained Matrix Factorization problem.

- User vectors:

$$\mathbf{w}_u \in \mathbb{R}^r$$

- Item vectors:

$$\mathbf{h}_i \in \mathbb{R}^r$$

- Rating prediction:

$$v_{ui} = \mathbf{w}_u^T \mathbf{h}_i$$

- Set of non-missing entries

$$\mathcal{Z} = \{(u, i) : v_{ui} \text{ is observed}\}$$

- Objective:

$$\operatorname{argmin}_{\mathbf{w}, \mathbf{h}} \sum_{(u, i) \in \mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2$$

# Matrix Factorization (with matrices)

- User vectors:

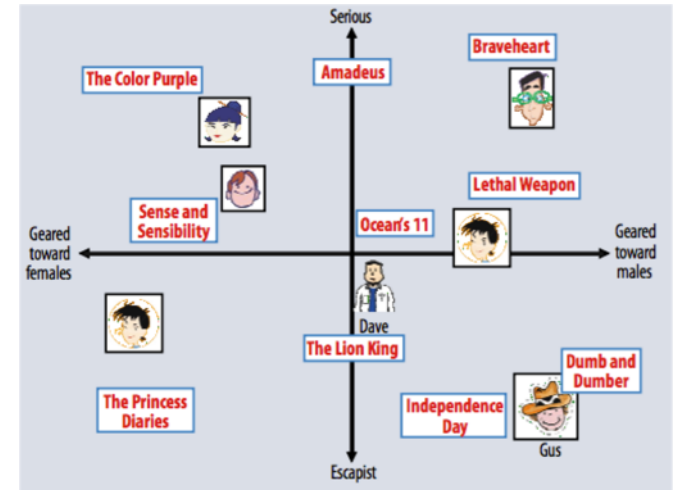
$$(W_{u*})^T \in \mathbb{R}^r$$

- Item vectors:

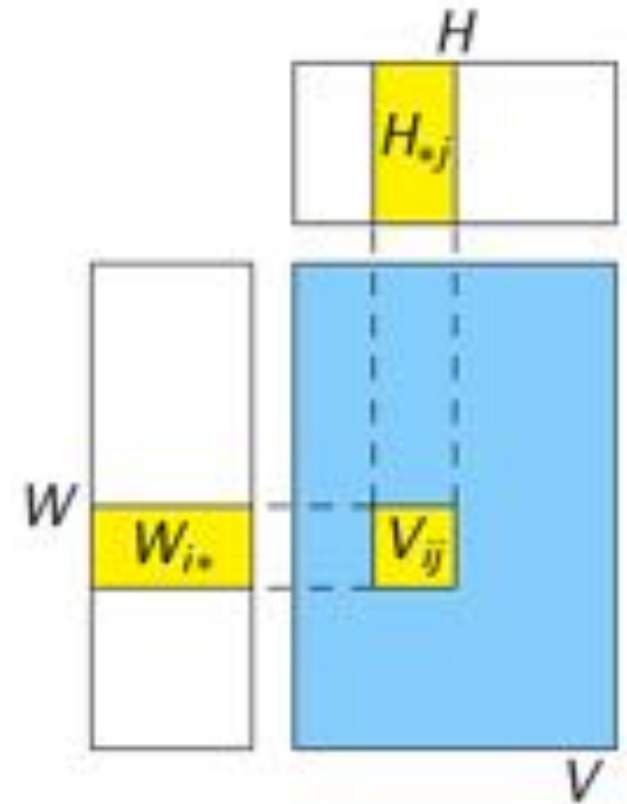
$$H_{*i} \in \mathbb{R}^r$$

- Rating prediction:

$$\begin{aligned} V_{ui} &= W_{u*} H_{*i} \\ &= [WH]_{ui} \end{aligned}$$



Figures from Koren et al. (2009)



Figures from Gemulla et al. (2011)<sub>90</sub>

# Matrix Factorization (with vectors)

- User vectors:

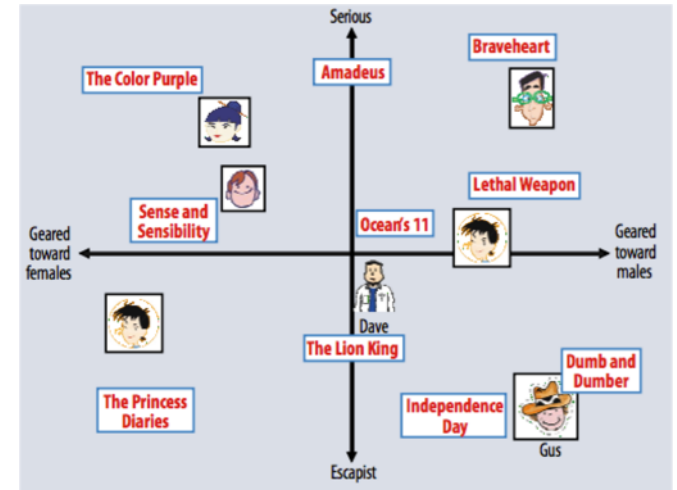
$$\mathbf{w}_u \in \mathbb{R}^r$$

- Item vectors:

$$\mathbf{h}_i \in \mathbb{R}^r$$

- Rating prediction:

$$v_{ui} = \mathbf{w}_u^T \mathbf{h}_i$$

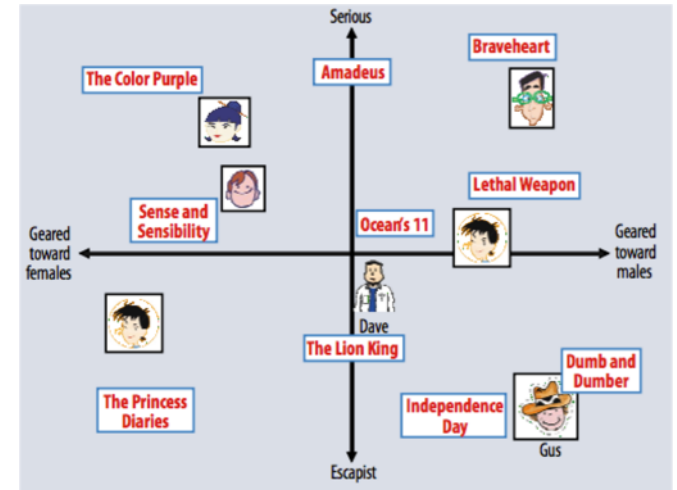


Figures from Koren et al. (2009)

# Matrix Factorization (with vectors)

- Set of non-missing entries:  
 $\mathcal{Z} = \{(u, i) : v_{ui} \text{ is observed}\}$
- Objective:

$$\operatorname{argmin}_{\mathbf{w}, \mathbf{h}} \sum_{(u, i) \in \mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2$$

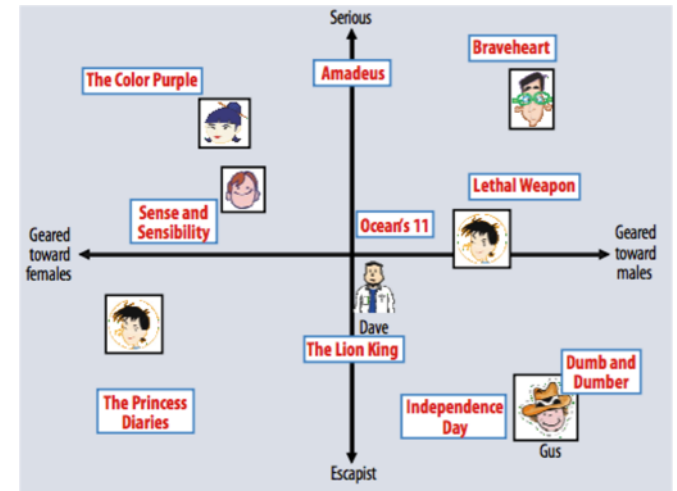


Figures from Koren et al. (2009)

# Matrix Factorization (with vectors)

- Regularized Objective:

$$\begin{aligned} \operatorname{argmin}_{\mathbf{w}, \mathbf{h}} \sum_{(u, i) \in \mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2 \\ + \lambda \left( \sum_i \|\mathbf{w}_i\|^2 + \sum_u \|\mathbf{h}_u\|^2 \right) \end{aligned}$$



Figures from Koren et al. (2009)

# Matrix Factorization (with vectors)

- Regularized Objective:

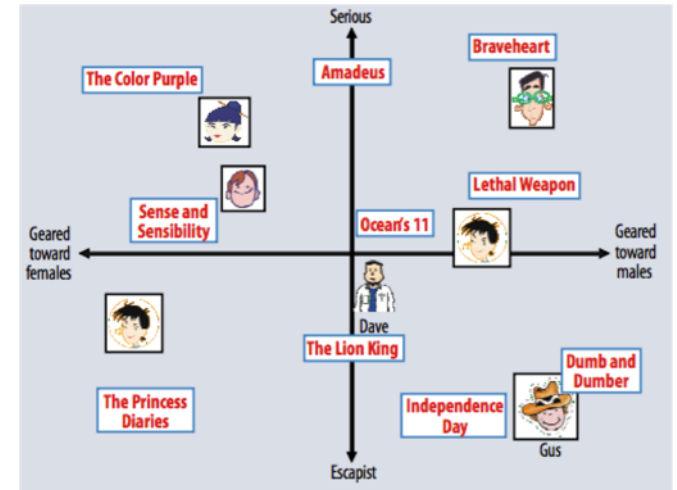
$$\begin{aligned} \operatorname{argmin}_{\mathbf{w}, \mathbf{h}} \sum_{(u,i) \in \mathcal{Z}} (v_{ui} - \mathbf{w}_u^T \mathbf{h}_i)^2 \\ + \lambda \left( \sum_i \|\mathbf{w}_i\|^2 + \sum_u \|\mathbf{h}_u\|^2 \right) \end{aligned}$$

- SGD update for random  $(u,i)$ :

$$e_{ui} \leftarrow v_{ui} - \mathbf{w}_u^T \mathbf{h}_i$$

$$\mathbf{w}_u \leftarrow \mathbf{w}_u + \gamma (e_{ui} \mathbf{h}_i - \lambda \mathbf{w}_u)$$

$$\mathbf{h}_i \leftarrow \mathbf{h}_i + \gamma (e_{ui} \mathbf{w}_u - \lambda \mathbf{h}_i)$$



Figures from Koren et al. (2009)

# Matrix Factorization (with matrices)

- User vectors:

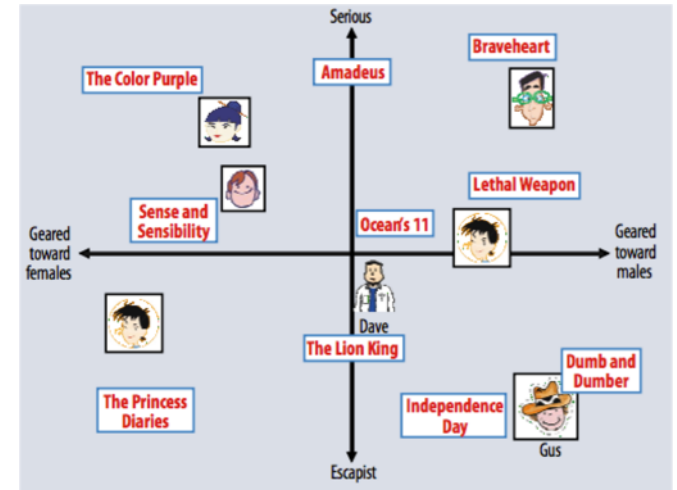
$$(W_{u*})^T \in \mathbb{R}^r$$

- Item vectors:

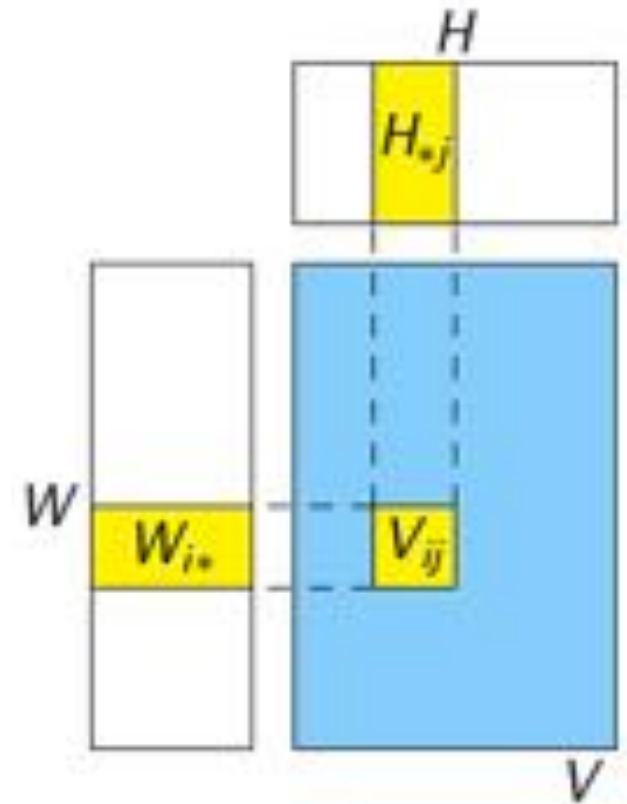
$$H_{*i} \in \mathbb{R}^r$$

- Rating prediction:

$$\begin{aligned} V_{ui} &= W_{u*} H_{*i} \\ &= [WH]_{ui} \end{aligned}$$



Figures from Koren et al. (2009)



Figures from Gemulla et al. (2011)<sub>95</sub>

# Matrix Factorization (with matrices)

- SGD

require that the loss can be written as

$$L = \sum_{(i,j) \in Z} l(V_{ij}, W_{i*}, H_{*j})$$

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## Algorithm 1 SGD for Matrix Factorization

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**Require:** A training set  $Z$ , initial values  $W_0$  and  $H_0$

**while** not converged **do** {step}

    Select a training point  $(i, j) \in Z$  uniformly at random.

$$W'_{i*} \leftarrow W_{i*} - \epsilon_n N \frac{\partial}{\partial W_{i*}} l(V_{ij}, W_{i*}, H_{*j})$$

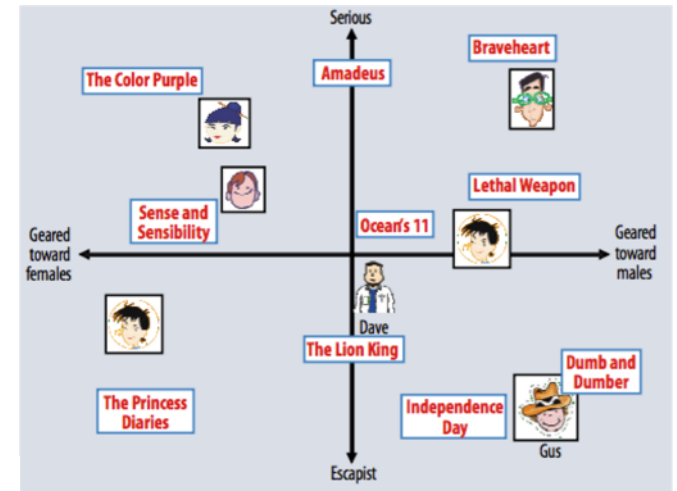
$$H_{*j} \leftarrow H_{*j} - \epsilon_n N \frac{\partial}{\partial H_{*j}} l(V_{ij}, W_{i*}, H_{*j})$$

$$W_{i*} \leftarrow W'_{i*}$$

**end while**

step size

Figure from Gemulla et al. (2011)



Figures from Koren et al. (2009)

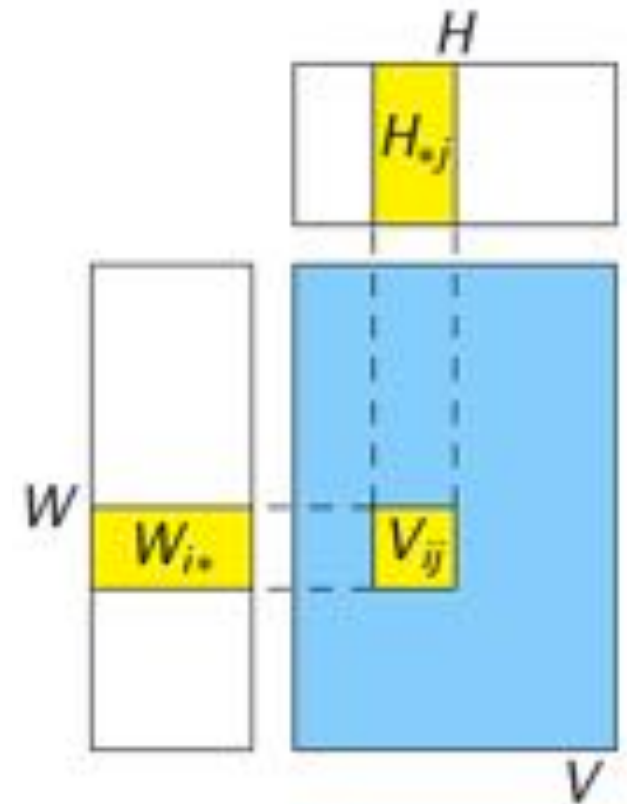


Figure from Gemulla et al. (2011)<sub>96</sub>